For Thought

1. False, the augmented matrix is a 2×3 matrix.

2. False, the required matrix is $\left[\begin{array}{rr|r} 1 & -1 & 4 \\ 3 & 1 & 5 \end{array}\right]$.

3. True **4.** True

5. True, since the row operation done is $R_1 + R_2 \to R_2$.

6. True, since the row operation done is $-R_1 + R_2 \to R_2$.

7. False, since it corresponds to

$$\begin{aligned} x &= 2 \\ y &= 7. \end{aligned}$$

8. True, since $0 \cdot x + 0 \cdot y = 7$ has no solution.

9. False, the system is dependent. **10.** True

9.1 Exercises

1. matrix

2. rows, columns

3. size

4. square

5. entry, element

6. augmented

7. diagonal

8. identity

9. 1×3 **10.** 3×1 **11.** 1×1

12. 2×2 **13.** 3×2 **14.** 3×3

15.
$$\left[\begin{array}{rr|r} 1 & -2 & 4 \\ 3 & 2 & -5 \end{array}\right]$$

16.
$$\left[\begin{array}{rr|r} 4 & -1 & 1 \\ 1 & 3 & 5 \end{array}\right]$$

17.
$$\left[\begin{array}{rrr|r} 1 & -1 & -1 & 4 \\ 1 & 3 & -1 & 1 \\ 0 & 2 & -5 & -6 \end{array}\right]$$

18.
$$\left[\begin{array}{rrr|r} 1 & 3 & 0 & 5 \\ 0 & 1 & -4 & 8 \\ -2 & 0 & 5 & 7 \end{array}\right]$$

19.
$$\left[\begin{array}{rrr|r} 1 & 3 & -1 & 5 \\ 1 & 0 & 1 & 0 \end{array}\right]$$

20.
$$\left[\begin{array}{rrr|r} 1 & -1 & 0 & 6 \\ 1 & 0 & 1 & 7 \end{array}\right]$$

21.
$$\begin{aligned} 3x + 4y &= -2 \\ 3x - 5y &= 0 \end{aligned}$$

22.
$$\begin{aligned} x &= -7 \\ y &= 5 \end{aligned}$$

23.
$$\begin{aligned} 5x &= 6 \\ -4x + 2z &= -1 \\ 4x + 4y &= 7 \end{aligned}$$

24.
$$\begin{aligned} x + z &= 2 \\ y - z &= -6 \\ x - y + z &= 5 \end{aligned}$$

25.
$$\begin{aligned} x - y + 2z &= 1 \\ y + 4z &= 3 \end{aligned}$$

26.
$$\begin{aligned} x + y + z &= 3 \\ y + 2z &= 7 \end{aligned}$$

27. Interchange R_1 and R_2

$$\left[\begin{array}{cc|c} 1 & 2 & 0 \\ -2 & 4 & 1 \end{array}\right]$$

28. Interchange R_1 and R_2

$$\left[\begin{array}{cc|c} 1 & 0 & 5 \\ 4 & 9 & 3 \end{array}\right]$$

29. Multiply $\frac{1}{2}$ to R_1

$$\left[\begin{array}{cc|c} 1 & 4 & 1 \\ 0 & 3 & 6 \end{array}\right]$$

30. Multiply $-\frac{1}{3}$ to R_1

$$\left[\begin{array}{cc|c} 1 & -2 & -4 \\ 0 & 9 & 3 \end{array}\right]$$

31. Multiply 3 to R_1 then add the product to R_2. This is the new R_2.

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ -3 & 5 & 0 \end{array}\right] =$$

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 3\cdot 1+(-3) & 3\cdot(-2)+5 & 3\cdot 1+0 \end{array}\right] =$$

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & -1 & 3 \end{array}\right]$$

32. Multiply -2 to R_2 then add the product to R_1. This is the new R_1.

$$\left[\begin{array}{cc|c} 1 & 2 & 7 \\ 0 & 1 & 4 \end{array}\right] =$$

$$\left[\begin{array}{cc|c} -2(0)+1 & -2(1)+2 & -2(4)+7 \\ 0 & 1 & 4 \end{array}\right] =$$

$$\left[\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & 4 \end{array}\right]$$

33. From the augmented matrix

$$\left[\begin{array}{cc|c} 2 & 4 & 14 \\ 5 & 4 & 5 \end{array}\right]$$

the system is

$$\begin{aligned} 2x+4y &= 14 \\ 5x+4y &= 5. \end{aligned}$$

From the final augmented matrix

$$\left[\begin{array}{cc|c} 1 & 0 & -3 \\ 0 & 1 & 5 \end{array}\right]$$

we obtain that the solution set is $\{(-3,5)\}$.

The row operations are $\frac{1}{2}R_1 \to R_1$, $-5R_1+R_2 \to R_2$, $-\frac{1}{6}R_2 \to R_2$, and $-2R_2+R_1 \to R_1$.

34. From the augmented matrix

$$\left[\begin{array}{cc|c} 3 & 5 & -2 \\ 1 & 2 & -1 \end{array}\right]$$

the system is

$$\begin{aligned} 3x+5y &= -2 \\ x+2y &= -1. \end{aligned}$$

From the final augmented matrix

$$\left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & -1 \end{array}\right]$$

we obtain that the solution set is $\{(1,-1)\}$.

The row operations are $R_1 \leftrightarrow R_2$, $-3R_1+R_2 \to R_2$, $-1R_2 \to R_2$, and $-2R_2+R_1 \to R_1$.

35. On $\left[\begin{array}{cc|c} 1 & 1 & 5 \\ -2 & 1 & -1 \end{array}\right]$ use $2R_1+R_2 \to R_2$ to get

$\left[\begin{array}{cc|c} 1 & 1 & 5 \\ 0 & 3 & 9 \end{array}\right]$, use $\frac{1}{3}R_2 \to R_2$ to get

$\left[\begin{array}{cc|c} 1 & 1 & 5 \\ 0 & 1 & 3 \end{array}\right]$, use $-R_2+R_1 \to R_1$ to get

$\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 3 \end{array}\right]$, solution set is $\{(2,3)\}$,

and the system is independent.

36. On $\left[\begin{array}{rr|r} 1 & -1 & 2 \\ 3 & -1 & 12 \end{array}\right]$ use $-3R_1 + R_2 \to R_2$ to get

$\left[\begin{array}{rr|r} 1 & -1 & 2 \\ 0 & 2 & 6 \end{array}\right]$, use $\frac{1}{2}R_2 \to R_2$ to get

$\left[\begin{array}{rr|r} 1 & -1 & 2 \\ 0 & 1 & 3 \end{array}\right]$, use $R_1 + R_2 \to R_1$ to get

$\left[\begin{array}{rr|r} 1 & 0 & 5 \\ 0 & 1 & 3 \end{array}\right]$, solution set is $\{(5,3)\}$,

and the system is independent.

37. On $\left[\begin{array}{rr|r} 2 & 2 & 8 \\ -3 & -1 & -6 \end{array}\right]$ use $\frac{1}{2}R_1 \to R_1$ to get

$\left[\begin{array}{rr|r} 1 & 1 & 4 \\ -3 & -1 & -6 \end{array}\right]$, use $3R_1 + R_2 \to R_2$ to get

$\left[\begin{array}{rr|r} 1 & 1 & 4 \\ 0 & 2 & 6 \end{array}\right]$, use $\frac{1}{2}R_2 \to R_2$ to get

$\left[\begin{array}{rr|r} 1 & 1 & 4 \\ 0 & 1 & 3 \end{array}\right]$, use $-1R_2 + R_1 \to R_1$ to get

$\left[\begin{array}{rr|r} 1 & 0 & 1 \\ 0 & 1 & 3 \end{array}\right]$, solution set is $\{(1,3)\}$,

and the system is independent.

38. On $\left[\begin{array}{rr|r} 3 & -6 & 9 \\ 2 & 1 & -4 \end{array}\right]$ use $\frac{1}{3}R_1 \to R_1$ to get

$\left[\begin{array}{rr|r} 1 & -2 & 3 \\ 2 & 1 & -4 \end{array}\right]$, use $-2R_1 + R_2 \to R_2$ to get

$\left[\begin{array}{rr|r} 1 & -2 & 3 \\ 0 & 5 & -10 \end{array}\right]$, use $\frac{1}{5}R_2 \to R_2$ to get

$\left[\begin{array}{rr|r} 1 & -2 & 3 \\ 0 & 1 & -2 \end{array}\right]$, use $2R_2 + R_1 \to R_1$ to get

$\left[\begin{array}{rr|r} 1 & 0 & -1 \\ 0 & 1 & -2 \end{array}\right]$, solution set is $\{(-1,-2)\}$,

independent.

39. On $\left[\begin{array}{rr|r} 2 & -1 & 3 \\ 3 & 2 & 15 \end{array}\right]$ use $-R_1 + R_2 \to R_1$ to get

$\left[\begin{array}{rr|r} 1 & 3 & 12 \\ 3 & 2 & 15 \end{array}\right]$, use $-3R_1 + R_2 \to R_2$ to get

$\left[\begin{array}{rr|r} 1 & 3 & 12 \\ 0 & -7 & -21 \end{array}\right]$, use $-\frac{1}{7}R_2 \to R_2$ to get

$\left[\begin{array}{rr|r} 1 & 3 & 12 \\ 0 & 1 & 3 \end{array}\right]$, use $-3R_2 + R_1 \to R_1$ to get

$\left[\begin{array}{rr|r} 1 & 0 & 3 \\ 0 & 1 & 3 \end{array}\right]$, solution set is $\{(3,3)\}$,

and the system is independent.

40. On $\left[\begin{array}{rr|r} 2 & -3 & -1 \\ 3 & -2 & 1 \end{array}\right]$ use $-R_1 + R_2 \to R_1$ to get

$\left[\begin{array}{rr|r} 1 & 1 & 2 \\ 3 & -2 & 1 \end{array}\right]$, use $-3R_1 + R_2 \to R_2$ to get

$\left[\begin{array}{rr|r} 1 & 1 & 2 \\ 0 & -5 & -5 \end{array}\right]$, use $-\frac{1}{5}R_2 \to R_2$ to get

$\left[\begin{array}{rr|r} 1 & 1 & 2 \\ 0 & 1 & 1 \end{array}\right]$, use $-R_2 + R_1 \to R_1$ to get

$\left[\begin{array}{rr|r} 1 & 0 & 1 \\ 0 & 1 & 1 \end{array}\right]$, solution set is $\{(1,1)\}$,

independent.

41. On $\left[\begin{array}{rr|r} 0.4 & -0.2 & 0 \\ 1 & 1.5 & 2 \end{array}\right]$ use $5R_1 \to R_1$ and $2R_2 \to R_2$ to get

$\left[\begin{array}{rr|r} 2 & -1 & 0 \\ 2 & 3 & 4 \end{array}\right]$, use $R_1 + (-R_2) \to R_2$ to get

$\left[\begin{array}{rr|r} 2 & -1 & 0 \\ 0 & -4 & -4 \end{array}\right]$, use $-\frac{1}{4}R_2 \to R_2$ to get

$\left[\begin{array}{rr|r} 2 & -1 & 0 \\ 0 & 1 & 1 \end{array}\right]$, use $R_1 + R_2 \to R_1$ to get

$\left[\begin{array}{rr|r} 2 & 0 & 1 \\ 0 & 1 & 1 \end{array}\right]$, use $\frac{1}{2}R_1 \to R_1$ to get

$\left[\begin{array}{rr|r} 1 & 0 & 0.5 \\ 0 & 1 & 1 \end{array}\right]$, solution set is $\{(0.5,1)\}$,

and the system is independent.

42. On $\left[\begin{array}{rr|r} 0.2 & 0.6 & 0.7 \\ 0.5 & -1 & 0.5 \end{array}\right]$ use $10R_1 \to R_1$ and $2R_2 \to R_2$ to get

$\left[\begin{array}{rr|r} 2 & 6 & 7 \\ 1 & -2 & 1 \end{array}\right]$, use $R_1 + (-2R_2) \to R_2$ to get

$\left[\begin{array}{cc|c} 2 & 6 & 7 \\ 0 & 10 & 5 \end{array}\right]$, use $\frac{1}{10}R_2 \to R_2$

and $\frac{1}{2}R_1 \to R_1$ to get

$\left[\begin{array}{cc|c} 1 & 3 & 3.5 \\ 0 & 1 & 0.5 \end{array}\right]$, use $-3R_2 + R_1 \to R_1$ to get

$\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 0.5 \end{array}\right]$, solution set is $\{(2, 0.5)\}$, independent.

43. On $\left[\begin{array}{cc|c} 3 & -5 & 7 \\ -3 & 5 & 4 \end{array}\right]$ use $R_1 + R_2 \to R_2$ to get

$\left[\begin{array}{cc|c} 3 & -5 & 7 \\ 0 & 0 & 11 \end{array}\right]$, inconsistent system, and the solution set is $\emptyset$.

44. On $\left[\begin{array}{cc|c} 2 & -3 & 9 \\ 4 & -6 & 1 \end{array}\right]$ use $-2R_1 + R_2 \to R_2$ to get

$\left[\begin{array}{cc|c} 2 & -3 & 9 \\ 0 & 0 & -17 \end{array}\right]$, inconsistent system, and the solution set is $\emptyset$.

45. On $\left[\begin{array}{cc|c} 0.5 & 1.5 & 2 \\ 3 & 9 & 12 \end{array}\right]$ use $2R_1 \to R_1$ to get

$\left[\begin{array}{cc|c} 1 & 3 & 4 \\ 3 & 9 & 12 \end{array}\right]$, use $-3R_1 + R_2 \to R_2$ to get

$\left[\begin{array}{cc|c} 1 & 3 & 4 \\ 0 & 0 & 0 \end{array}\right]$, dependent system, and solution set is $\{(u, v) \mid u + 3v = 4\}$.

46.

On $\left[\begin{array}{cc|c} 1 & -2.5 & 0.5 \\ -4 & 10 & -2 \end{array}\right]$ use $4R_1 + R_2 \to R_2$ to

get $\left[\begin{array}{cc|c} 1 & -2.5 & 0.5 \\ 0 & 0 & 0 \end{array}\right]$, use $2R_1 \to R_1$ to get

$\left[\begin{array}{cc|c} 2 & -5 & 1 \\ 0 & 0 & 0 \end{array}\right]$, dependent system, and solution set is $\{(m, n) \mid 2m - 5n = 1\}$.

47. Rewrite system as

$$\begin{aligned} 2x + y &= 4 \\ x - y &= 8. \end{aligned}$$

On $\left[\begin{array}{cc|c} 2 & 1 & 4 \\ 1 & -1 & 8 \end{array}\right]$ use $R_1 + (-2R_2) \to R_2$ to get

$\left[\begin{array}{cc|c} 2 & 1 & 4 \\ 0 & 3 & -12 \end{array}\right]$, use $\frac{1}{3}R_2 \to R_2$ to get

$\left[\begin{array}{cc|c} 2 & 1 & 4 \\ 0 & 1 & -4 \end{array}\right]$, use $-1R_2 + R_1 \to R_1$ to get

$\left[\begin{array}{cc|c} 2 & 0 & 8 \\ 0 & 1 & -4 \end{array}\right]$, use $\frac{1}{2}R_1 \to R_1$ to get

$\left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & -4 \end{array}\right]$, solution set is $\{(4, -4)\}$, and the system is independent.

48. Rewrite system as

$$\begin{aligned} 3x - 2y &= 1 \\ x + y &= 2. \end{aligned}$$

On $\left[\begin{array}{cc|c} 3 & -2 & 1 \\ 1 & 1 & 2 \end{array}\right]$ use $-3R_2 + R_1 \to R_2$ to get

$\left[\begin{array}{cc|c} 3 & -2 & 1 \\ 0 & -5 & -5 \end{array}\right]$, use $-\frac{1}{5}R_2 \to R_2$ to get

$\left[\begin{array}{cc|c} 3 & -2 & 1 \\ 0 & 1 & 1 \end{array}\right]$, use $2R_2 + R_1 \to R_1$ to get

$\left[\begin{array}{cc|c} 3 & 0 & 3 \\ 0 & 1 & 1 \end{array}\right]$, use $\frac{1}{3}R_1 \to R_1$ to get

$\left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 1 \end{array}\right]$, solution set is $\{(1, 1)\}$, independent.

49.

On $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & -1 & -1 & 0 \\ 0 & 2 & -1 & 3 \end{array}\right]$ use $-1R_2 + R_1 \to R_2$ to get

$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 2 & 2 & 6 \\ 0 & 2 & -1 & 3 \end{array}\right]$, use $\frac{1}{2}R_2 \to R_2$ to get

$\left[\begin{array}{rrr|r} 1 & 1 & 1 & 6 \\ 0 & 1 & 1 & 3 \\ 0 & 2 & -1 & 3 \end{array}\right]$, use $-1R_2 + R_1 \to R_1$ to get

$\left[\begin{array}{rrr|r} 1 & 0 & 0 & 3 \\ 0 & 1 & 1 & 3 \\ 0 & 2 & -1 & 3 \end{array}\right]$, use $-2R_2 + R_3 \to R_3$ to get

$\left[\begin{array}{rrr|r} 1 & 0 & 0 & 3 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & -3 & -3 \end{array}\right]$, use $-\frac{1}{3}R_3 \to R_3$ to get

$\left[\begin{array}{rrr|r} 1 & 0 & 0 & 3 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 1 & 1 \end{array}\right]$, use $-1R_3 + R_2 \to R_2$ to get

$\left[\begin{array}{rrr|r} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right]$, solution set is $\{(3, 2, 1)\}$,

and the system is independent.

50.

On $\left[\begin{array}{rrr|r} 1 & -1 & 1 & 2 \\ -1 & 1 & 1 & 4 \\ -1 & 0 & 1 & 2 \end{array}\right]$ use $-1R_3 + R_2 \to R_2$ to get

$\left[\begin{array}{rrr|r} 1 & -1 & 1 & 2 \\ 0 & 1 & 0 & 2 \\ -1 & 0 & 1 & 2 \end{array}\right]$, use $R_1 + R_2 \to R_1$ to get

$\left[\begin{array}{rrr|r} 1 & 0 & 1 & 4 \\ 0 & 1 & 0 & 2 \\ -1 & 0 & 1 & 2 \end{array}\right]$, use $R_1 + R_3 \to R_3$ to get

$\left[\begin{array}{rrr|r} 1 & 0 & 1 & 4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 2 & 6 \end{array}\right]$, use $\frac{1}{2}R_3 \to R_3$ to get

$\left[\begin{array}{rrr|r} 1 & 0 & 1 & 4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$, use $-1R_3 + R_1 \to R_1$ to get

$\left[\begin{array}{rrr|r} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$, solution set is $\{(1, 2, 3)\}$,

independent.

51.

Rewrite system as

$$\begin{aligned} 2x + y - z &= 2 \\ x + 2y - z &= 2 \\ x - y + 2z &= 2. \end{aligned}$$

On $\left[\begin{array}{rrr|r} 2 & 1 & -1 & 2 \\ 1 & 2 & -1 & 2 \\ 1 & -1 & 2 & 2 \end{array}\right]$, use $-1R_2 + R_1 \to R_1$ and $-1R_3 + R_2 \to R_3$ to get

$\left[\begin{array}{rrr|r} 1 & -1 & 0 & 0 \\ 1 & 2 & -1 & 2 \\ 0 & 3 & -3 & 0 \end{array}\right]$, use $-1R_2 + R_1 \to R_2$ and $\frac{1}{3}R_3 \to R_3$ to get

$\left[\begin{array}{rrr|r} 1 & -1 & 0 & 0 \\ 0 & -3 & 1 & -2 \\ 0 & 1 & -1 & 0 \end{array}\right]$, use $R_1 + R_3 \to R_1$ and $R_3 + R_2 \to R_2$ to get

$\left[\begin{array}{rrr|r} 1 & 0 & -1 & 0 \\ 0 & -2 & 0 & -2 \\ 0 & 1 & -1 & 0 \end{array}\right]$, use $-\frac{1}{2}R_2 \to R_2$ to get

$\left[\begin{array}{rrr|r} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & -1 & 0 \end{array}\right]$, use $-1R_3 + R_2 \to R_3$ to get

$\left[\begin{array}{rrr|r} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right]$, use $R_1 + R_3 \to R_1$ to get

$\left[\begin{array}{rrr|r} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right]$, solution set is $\{(1, 1, 1)\}$,

and the system is independent.

52.

Rewrite system as

$$\begin{aligned} 3x - y &= 4 \\ x + y - z &= -1 \\ x + 2z &= 3. \end{aligned}$$

On $\left[\begin{array}{rrr|r} 3 & -1 & 0 & 4 \\ 1 & 1 & -1 & -1 \\ 1 & 0 & 2 & 3 \end{array}\right]$, use $R_2 + R_1 \to R_1$ to get

$\left[\begin{array}{rrr|r} 4 & 0 & -1 & 3 \\ 1 & 1 & -1 & -1 \\ 1 & 0 & 2 & 3 \end{array}\right]$, use $-1R_3 + R_2 \to R_2$ to get

$\left[\begin{array}{rrr|r} 4 & 0 & -1 & 3 \\ 0 & 1 & -3 & -4 \\ 1 & 0 & 2 & 3 \end{array}\right]$, use $-1R_3 + R_1 \to R_1$ to get

$\left[\begin{array}{rrr|r} 3 & 0 & -3 & 0 \\ 0 & 1 & -3 & -4 \\ 1 & 0 & 2 & 3 \end{array}\right]$, use $\frac{1}{3}R_1 \to R_1$ to get

$\left[\begin{array}{rrr|r} 1 & 0 & -1 & 0 \\ 0 & 1 & -3 & -4 \\ 1 & 0 & 2 & 3 \end{array}\right]$, use $-1R_3 + R_1 \to R_3$ to get

$\left[\begin{array}{rrr|r} 1 & 0 & -1 & 0 \\ 0 & 1 & -3 & -4 \\ 0 & 0 & -3 & -3 \end{array}\right]$, use $-\frac{1}{3}R_3 \to R_3$ to get

$\left[\begin{array}{rrr|r} 1 & 0 & -1 & 0 \\ 0 & 1 & -3 & -4 \\ 0 & 0 & 1 & 1 \end{array}\right]$, use $R_1 + R_3 \to R_1$ to get

$\left[\begin{array}{rrr|r} 1 & 0 & 0 & 1 \\ 0 & 1 & -3 & -4 \\ 0 & 0 & 1 & 1 \end{array}\right]$, use $3R_3 + R_2 \to R_2$ to get

$\left[\begin{array}{rrr|r} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right]$, solution set is $\{(1, -1, 1)\}$, independent.

53.

Interchange rows of $\left[\begin{array}{rrr|r} 2 & -2 & 1 & -2 \\ 1 & 1 & -3 & 3 \\ 1 & -3 & 1 & -5 \end{array}\right]$ to get

$\left[\begin{array}{rrr|r} 1 & 1 & -3 & 3 \\ 1 & -3 & 1 & -5 \\ 2 & -2 & 1 & -2 \end{array}\right]$, use $-1R_2 + R_1 \to R_2$ and $-1R_3 + R_1 + R_2 \to R_3$ to get

$\left[\begin{array}{rrr|r} 1 & 1 & -3 & 3 \\ 0 & 4 & -4 & 8 \\ 0 & 0 & -3 & 0 \end{array}\right]$, use $\frac{1}{4}R_2 \to R_2$ and $-\frac{1}{3}R_3 \to R_3$ to get

$\left[\begin{array}{rrr|r} 1 & 1 & -3 & 3 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 1 & 0 \end{array}\right]$, use $R_2 + R_3 \to R_2$ to get

$\left[\begin{array}{rrr|r} 1 & 1 & -3 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{array}\right]$, use $-1R_2 + R_1 \to R_1$ to get

$\left[\begin{array}{rrr|r} 1 & 0 & -3 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{array}\right]$, use $3R_3 + R_1 \to R_1$ to get

$\left[\begin{array}{rrr|r} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{array}\right]$, solution set is $\{(1, 2, 0)\}$, and the system is independent.

54.

On $\left[\begin{array}{rrr|r} 1 & -3 & -1 & -3 \\ -1 & -1 & 2 & 1 \\ -1 & 2 & -1 & -2 \end{array}\right]$, use $R_2 + R_1 \to R_2$ and $R_1 + R_3 \to R_3$ to get

$\left[\begin{array}{rrr|r} 1 & -3 & -1 & -3 \\ 0 & -4 & 1 & -2 \\ 0 & -1 & -2 & -5 \end{array}\right]$, use $-3R_3 + R_1 \to R_1$

$\left[\begin{array}{rrr|r} 1 & 0 & 5 & 12 \\ 0 & -4 & 1 & -2 \\ 0 & -1 & -2 & -5 \end{array}\right]$, use $-4R_3 + R_2 \to R_3$

$\left[\begin{array}{rrr|r} 1 & 0 & 5 & 12 \\ 0 & -4 & 1 & -2 \\ 0 & 0 & 9 & 18 \end{array}\right]$, use $\frac{1}{9}R_3 \to R_3$ and $-1R_3 + R_2 \to R_2$ to get

$\left[\begin{array}{rrr|r} 1 & 0 & 5 & 12 \\ 0 & -4 & -8 & -20 \\ 0 & 0 & 1 & 2 \end{array}\right]$, use $-\frac{1}{4}R_2 \to R_2$ and $-5R_3 + R_1 \to R_1$ to get

$\left[\begin{array}{rrr|r} 1 & 0 & 0 & 2 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 1 & 2 \end{array}\right]$, use $-2R_3 + R_2 \to R_2$ to get

$\left[\begin{array}{rrr|r} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right]$, solution set is $\{(2, 1, 2)\}$, independent.

55.

Rewrite system as

$$\begin{aligned} x-3y+z &= 0\\ x-y-3z &= 4\\ x+y+2z &= -1. \end{aligned}$$

On $\left[\begin{array}{rrr|r} 1 & -3 & 1 & 0\\ 1 & -1 & -3 & 4\\ 1 & 1 & 2 & -1 \end{array}\right]$, use $-1R_2+R_1 \to R_2$ and $-1R_3+R_1 \to R_3$ to get

$\left[\begin{array}{rrr|r} 1 & -3 & 1 & 0\\ 0 & -2 & 4 & -4\\ 0 & -4 & -1 & 1 \end{array}\right]$, use $-\dfrac{1}{2}R_2 \to R_2$ and $-1R_3 \to R_3$ to get

$\left[\begin{array}{rrr|r} 1 & -3 & 1 & 0\\ 0 & 1 & -2 & 2\\ 0 & 4 & 1 & -1 \end{array}\right]$, use $-1R_3+R_1 \to R_1$ to get

$\left[\begin{array}{rrr|r} 1 & -7 & 0 & 1\\ 0 & 1 & -2 & 2\\ 0 & 4 & 1 & -1 \end{array}\right]$, use $-4R_2+R_3 \to R_3$ to get

$\left[\begin{array}{rrr|r} 1 & -7 & 0 & 1\\ 0 & 1 & -2 & 2\\ 0 & 0 & 9 & -9 \end{array}\right]$, use $\dfrac{1}{9}R_3 \to R_3$ to get

$\left[\begin{array}{rrr|r} 1 & -7 & 0 & 1\\ 0 & 1 & -2 & 2\\ 0 & 0 & 1 & -1 \end{array}\right]$, use $2R_3+R_2 \to R_2$ to get

$\left[\begin{array}{rrr|r} 1 & -7 & 0 & 1\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & -1 \end{array}\right]$, use $R_1+7R_2 \to R_1$ to get

$\left[\begin{array}{rrr|r} 1 & 0 & 0 & 1\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & -1 \end{array}\right]$, solution set is $\{(1,0,-1)\}$, and the system is independent.

56.

Rewrite system as

$$\begin{aligned} -x+z &= 2\\ 2x-y &= 1\\ y+3z &= 15. \end{aligned}$$

On $\left[\begin{array}{rrr|r} -1 & 0 & 1 & 2\\ 2 & -1 & 0 & 1\\ 0 & 1 & 3 & 15 \end{array}\right]$, use $R_1+R_2 \to R_1$ to get

$\left[\begin{array}{rrr|r} 1 & -1 & 1 & 3\\ 2 & -1 & 0 & 1\\ 0 & 1 & 3 & 15 \end{array}\right]$, use $-1R_1+R_2 \to R_1$ to get

$\left[\begin{array}{rrr|r} 1 & 0 & -1 & -2\\ 2 & -1 & 0 & 1\\ 0 & 1 & 3 & 15 \end{array}\right]$, use $-2R_1+R_2 \to R_2$ to get

$\left[\begin{array}{rrr|r} 1 & 0 & -1 & -2\\ 0 & -1 & 2 & 5\\ 0 & 1 & 3 & 15 \end{array}\right]$, use $R_2+R_3 \to R_3$ to get

$\left[\begin{array}{rrr|r} 1 & 0 & -1 & -2\\ 0 & -1 & 2 & 5\\ 0 & 0 & 5 & 20 \end{array}\right]$, use $\dfrac{1}{5}R_3 \to R_3$ to get

$\left[\begin{array}{rrr|r} 1 & 0 & -1 & -2\\ 0 & -1 & 2 & 5\\ 0 & 0 & 1 & 4 \end{array}\right]$, use $R_3+R_1 \to R_1$ and $-1R_2 \to R_2$ to get

$\left[\begin{array}{rrr|r} 1 & 0 & 0 & 2\\ 0 & 1 & -2 & -5\\ 0 & 0 & 1 & 4 \end{array}\right]$, use $2R_3+R_2 \to R_2$ to get

$\left[\begin{array}{rrr|r} 1 & 0 & 0 & 2\\ 0 & 1 & 0 & 3\\ 0 & 0 & 1 & 4 \end{array}\right]$, solution set is $\{(2,3,4)\}$, and the system is independent.

57.

On $\left[\begin{array}{rrr|r} 1 & -2 & 3 & 1\\ 2 & -4 & 6 & 2\\ -3 & 6 & -9 & -3 \end{array}\right]$, use $\dfrac{1}{2}R_2 \to R_2$ and $-\dfrac{1}{3}R_3 \to R_3$ to get

$\left[\begin{array}{rrr|r} 1 & -2 & 3 & 1\\ 1 & -2 & 3 & 1\\ 1 & -2 & 3 & 1 \end{array}\right]$, use $-1R_1+R_2 \to R_2$ and $-1R_1+R_3 \to R_3$ to get

$\left[\begin{array}{rrr|r} 1 & -2 & 3 & 1\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 \end{array}\right]$, dependent system, and solution set is $\{(x, y, z) \mid x - 2y + 3z = 1\}$.

58.
On $\left[\begin{array}{rrr|r} 4 & -2 & 6 & 4 \\ 2 & -1 & 3 & 2 \\ -2 & 1 & -3 & -2 \end{array}\right]$, use $\frac{1}{2}R_1 \to R_1$

and $-1R_3 \to R_3$ to get

$\left[\begin{array}{rrr|r} 2 & -1 & 3 & 2 \\ 2 & -1 & 3 & 2 \\ 2 & -1 & 3 & 2 \end{array}\right]$, use $-1R_1 + R_2 \to R_2$

and $-1R_1 + R_3 \to R_3$ to get

$\left[\begin{array}{rrr|r} 2 & -1 & 3 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$, dependent system,

and solution set is $\{(x, y, z) \mid 2x - y + 3z = 2\}$.

59.
On $\left[\begin{array}{rrr|r} 1 & -1 & 1 & 2 \\ 2 & 1 & -1 & 1 \\ 2 & -2 & 2 & 5 \end{array}\right]$, use $-\frac{1}{2}R_3 + R_1 \to R_3$

to get $\left[\begin{array}{rrr|r} 1 & -1 & 1 & 2 \\ 2 & 1 & -1 & 1 \\ 0 & 0 & 0 & -1/2 \end{array}\right]$, inconsistent,

and the solution set is $\emptyset$.

60.
On $\left[\begin{array}{rrr|r} 1 & -1 & 1 & 4 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & -1 & 3 \end{array}\right]$, use $-1R_3 + R_2 \to R_3$

to get $\left[\begin{array}{rrr|r} 1 & -1 & 1 & 4 \\ 1 & 1 & -1 & 1 \\ 0 & 0 & 0 & -2 \end{array}\right]$, inconsistent,

and the solution set is $\emptyset$.

61.
On $\left[\begin{array}{rrr|r} 1 & 1 & -1 & 3 \\ 3 & 1 & 1 & 7 \\ 1 & -1 & 3 & 1 \end{array}\right]$, use $-1R_3 + R_1 \to R_3$

and $3R_1 + (-1R_2) \to R_2$ to get

$\left[\begin{array}{rrr|r} 1 & 1 & -1 & 3 \\ 0 & 2 & -4 & 2 \\ 0 & 2 & -4 & 2 \end{array}\right]$, use $-1R_3 + R_2 \to R_3$ to get

$\left[\begin{array}{rrr|r} 1 & 1 & -1 & 3 \\ 0 & 2 & -4 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$, use $\frac{1}{2}R_2 \to R_2$ to get

$\left[\begin{array}{rrr|r} 1 & 1 & -1 & 3 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$, use $-1R_2 + R_1 \to R_1$ to get

$\left[\begin{array}{rrr|r} 1 & 0 & 1 & 2 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$. Substitute $z = 2 - x$

into $y = 1 + 2z$. Then $y = 1 + 2(2 - x) = 5 - 2x$. The solution set is

$$\{(x, 5 - 2x, 2 - x) \mid x \text{ is any real number}\}$$

and the system is dependent.

62.
On $\left[\begin{array}{rrr|r} 1 & 2 & 2 & 4 \\ 2 & 1 & 1 & 1 \\ -1 & 1 & 1 & 3 \end{array}\right]$, use

$2R_1 + (-1R_2) \to R_2$ to get

$\left[\begin{array}{rrr|r} 1 & 2 & 2 & 4 \\ 0 & 3 & 3 & 7 \\ -1 & 1 & 1 & 3 \end{array}\right]$, use $R_1 + R_3 \to R_3$ to get

$\left[\begin{array}{rrr|r} 1 & 2 & 2 & 4 \\ 0 & 3 & 3 & 7 \\ 0 & 3 & 3 & 7 \end{array}\right]$, use $-1R_2 + R_3 \to R_3$ to get

$\left[\begin{array}{rrr|r} 1 & 2 & 2 & 4 \\ 0 & 3 & 3 & 7 \\ 0 & 0 & 0 & 0 \end{array}\right]$, use $\frac{1}{3}R_2 \to R_2$ to get

$\left[\begin{array}{rrr|r} 1 & 2 & 2 & 4 \\ 0 & 1 & 1 & 7/3 \\ 0 & 0 & 0 & 0 \end{array}\right]$, use $-2R_2 + R_1 \to R_1$ to get

$\left[\begin{array}{rrr|r} 1 & 0 & 0 & -2/3 \\ 0 & 1 & 1 & 7/3 \\ 0 & 0 & 0 & 0 \end{array}\right]$. So $x = -\frac{2}{3}$ and

$y = \frac{7}{3} - z$. The solution set is

$$\left\{\left(-\frac{2}{3}, \frac{7 - 3z}{3}, z\right) \mid z \text{ is a real number}\right\}$$

and the system is dependent.

63.
On $\left[\begin{array}{rrr|r} 2 & -1 & 3 & 1 \\ 1 & 1 & -1 & 4 \end{array}\right]$, use $-2R_2 + R_1 \to R_2$

to get
$\left[\begin{array}{ccc|c} 2 & -1 & 3 & 1 \\ 0 & -3 & 5 & -7 \end{array}\right]$, use $-3R_1 + R_2 \to R_1$
to get
$\left[\begin{array}{ccc|c} -6 & 0 & -4 & -10 \\ 0 & -3 & 5 & -7 \end{array}\right]$, use $-\frac{1}{6}R_1 \to R_1$
and $-\frac{1}{3}R_2 \to R_2$ to get
$\left[\begin{array}{ccc|c} 1 & 0 & 2/3 & 5/3 \\ 0 & 1 & -5/3 & 7/3 \end{array}\right]$. Note $x = \frac{5-2z}{3}$ and $y = \frac{7+5z}{3}$. Solving for z, we get $z = \frac{5-3x}{2}$.
Then $y = \frac{7+5\cdot\frac{5-3x}{2}}{3} = \frac{7+5\cdot\frac{5-3x}{2}}{3}\cdot\frac{2}{2} = \frac{39-15x}{6} = \frac{13-5x}{2}$. The solution set is

$$\left\{\left(x, \frac{13-5x}{2}, \frac{5-3x}{2}\right) \mid x \text{ is any real number}\right\}$$

and the system is dependent.

64.
On $\left[\begin{array}{ccc|c} 1 & 3 & 1 & 6 \\ -1 & 1 & -1 & 2 \end{array}\right]$, use $R_2 + R_1 \to R_2$
to get
$\left[\begin{array}{ccc|c} 1 & 3 & 1 & 6 \\ 0 & 4 & 0 & 8 \end{array}\right]$, use $\frac{1}{4}R_2 \to R_2$ to get
$\left[\begin{array}{ccc|c} 1 & 3 & 1 & 6 \\ 0 & 1 & 0 & 2 \end{array}\right]$, use $-3R_2 + R_1 \to R_1$ to get
$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 2 \end{array}\right]$. Then $z = -x$ and $y = 2$.
The solution set is

$$\{(x, 2, -x) \mid x \text{ is any real number}\}$$

and the system is dependent.

65.
On $\left[\begin{array}{cccc|c} 1 & -1 & 1 & -1 & 2 \\ -1 & 2 & -1 & -1 & -1 \\ 2 & -1 & -1 & 1 & 4 \\ 1 & 3 & -2 & -3 & 6 \end{array}\right]$,
use $2R_2 + R_3 \to R_3$ and $R_2 + R_4 \to R_4$ to get
$\left[\begin{array}{cccc|c} 1 & -1 & 1 & -1 & 2 \\ 0 & 1 & 0 & -2 & 1 \\ 0 & 3 & -3 & -1 & 2 \\ 0 & 5 & -3 & -4 & 5 \end{array}\right]$, use
$-3R_2 + R_3 \to R_3$ and $-5R_2 + R_4 \to R_4$ to get
$\left[\begin{array}{cccc|c} 1 & -1 & 1 & -1 & 2 \\ 0 & 1 & 0 & -2 & 1 \\ 0 & 0 & -3 & 5 & -1 \\ 0 & 0 & -3 & 6 & 0 \end{array}\right]$, use
$R_1 + R_2 \to R_1$ and $-\frac{1}{3}R_4 \to R_4$ to get
$\left[\begin{array}{cccc|c} 1 & 0 & 1 & -3 & 3 \\ 0 & 1 & 0 & -2 & 1 \\ 0 & 0 & -3 & 5 & -1 \\ 0 & 0 & 1 & -2 & 0 \end{array}\right]$, use $R_3 \to R_4$
and $R_4 \to R_3$ to get
$\left[\begin{array}{cccc|c} 1 & 0 & 1 & -3 & 3 \\ 0 & 1 & 0 & -2 & 1 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & -3 & 5 & -1 \end{array}\right]$, use
$-3R_3 + (-1R_4) \to R_4$ to get
$\left[\begin{array}{cccc|c} 1 & 0 & 1 & -3 & 3 \\ 0 & 1 & 0 & -2 & 1 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$, use $-1R_3 + R_1 \to R_1$
to get
$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 3 \\ 0 & 1 & 0 & -2 & 1 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$, use $2R_4 + R_3 \to R_3$ to
get
$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 3 \\ 0 & 1 & 0 & -2 & 1 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$, use $2R_4 + R_2 \to R_2$ to
get
$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 3 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$, use $R_4 + R_1 \to R_1$ to
get
$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$, the solution set is

$\{(4,3,2,1)\}$, and the system is independent.

66. On $\left[\begin{array}{rrrr|r} 3 & -2 & 1 & 1 & 0 \\ 1 & -1 & 1 & -1 & -4 \\ -2 & 1 & 3 & -2 & 5 \\ 2 & 3 & -1 & -1 & -3 \end{array}\right]$, use

$R_1 \leftrightarrow R_2$ to get

$\left[\begin{array}{rrrr|r} 1 & -1 & 1 & -1 & -4 \\ 3 & -2 & 1 & 1 & 0 \\ -2 & 1 & 3 & -2 & 5 \\ 2 & 3 & -1 & -1 & -3 \end{array}\right]$, use

$R_3 + R_4 \to R_3$ and $-2R_1 + R_4 \to R_4$ to get

$\left[\begin{array}{rrrr|r} 1 & -1 & 1 & -1 & -4 \\ 3 & -2 & 1 & 1 & 0 \\ 0 & 4 & 2 & -3 & 2 \\ 0 & 5 & -3 & 1 & 5 \end{array}\right]$, use

$-3R_1 + R_2 \to R_2$ to get

$\left[\begin{array}{rrrr|r} 1 & -1 & 1 & -1 & -4 \\ 0 & 1 & -2 & 4 & 12 \\ 0 & 4 & 2 & -3 & 2 \\ 0 & 5 & -3 & 1 & 5 \end{array}\right]$, use

$-4R_2 + R_3 \to R_3$ and $-5R_2 + R_4 \to R_4$ to get

$\left[\begin{array}{rrrr|r} 1 & -1 & 1 & -1 & -4 \\ 0 & 1 & -2 & 4 & 12 \\ 0 & 0 & 10 & -19 & -46 \\ 0 & 0 & 7 & -19 & -55 \end{array}\right]$, use

$R_1 + R_2 \to R_1$ to get

$\left[\begin{array}{rrrr|r} 1 & 0 & -1 & 3 & 8 \\ 0 & 1 & -2 & 4 & 12 \\ 0 & 0 & 10 & -19 & -46 \\ 0 & 0 & 7 & -19 & -55 \end{array}\right]$, use

$-1R_4 + R_3 \to R_3$ to get

$\left[\begin{array}{rrrr|r} 1 & 0 & -1 & 3 & 8 \\ 0 & 1 & -2 & 4 & 12 \\ 0 & 0 & 3 & 0 & 9 \\ 0 & 0 & 7 & -19 & -55 \end{array}\right]$, use

$\frac{1}{3}R_3 \to R_3$ to get

$\left[\begin{array}{rrrr|r} 1 & 0 & -1 & 3 & 8 \\ 0 & 1 & -2 & 4 & 12 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 7 & -19 & -55 \end{array}\right]$, use

$2R_3 + R_2 \to R_2$ to get

$\left[\begin{array}{rrrr|r} 1 & 0 & -1 & 3 & 8 \\ 0 & 1 & 0 & 4 & 18 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 7 & -19 & -55 \end{array}\right]$, use

$R_1 + R_3 \to R_1$ and $\frac{7}{19}R_3 + \left(-\frac{1}{19}R_4\right) \to R_4$ to get

$\left[\begin{array}{rrrr|r} 1 & 0 & 0 & 3 & 11 \\ 0 & 1 & 0 & 4 & 18 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \end{array}\right]$, use

$-3R_4 + R_1 \to R_1$ and $-4R_4 + R_2 \to R_2$ to get

$\left[\begin{array}{rrrr|r} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \end{array}\right]$, solution set is

$\{(-1,2,3,4)\}$, and the system is independent.

67. Let x and y be the number of hours Mike worked at Burgers and the Soap Opera, respectively. The augmented matrix is

$A = \left[\begin{array}{rr|r} 1 & 1 & 60 \\ 8 & 9 & 502 \end{array}\right]$. On A use $-8R_1 + R_2 \to R_2$ to get

$\left[\begin{array}{rr|r} 1 & 1 & 60 \\ 0 & 1 & 22 \end{array}\right]$, use $-R_2 + R_1 \to R_1$ to get

$\left[\begin{array}{rr|r} 1 & 0 & 38 \\ 0 & 1 & 22 \end{array}\right]$. Mike worked $x = 38$ hours at Burgers and $y = 22$ hours at Soap Opera.

68. Let x and y be the number of women and men, respectively. The augmented matrix is

$A = \left[\begin{array}{rr|r} 1 & 1 & 60 \\ 28 & 44 & 2320 \end{array}\right]$.

On A use $-28R_1 + R_2 \to R_2$ to get

$\left[\begin{array}{rr|r} 1 & 1 & 60 \\ 0 & 16 & 640 \end{array}\right]$, use $\frac{1}{16}R_2 \to R_2$ to get

$\left[\begin{array}{rr|r} 1 & 1 & 60 \\ 0 & 1 & 40 \end{array}\right]$, use $-1R_2 + R_1 \to R_1$ to get

$\left[\begin{array}{rr|r} 1 & 0 & 20 \\ 0 & 1 & 40 \end{array}\right]$. There were $x = 20$ women and $y = 40$ men.

69. Let x, y, and z be the amounts invested in a mutual fund, in treasury bills, and in bonds, respectively. Augmented matrix is

$$A = \left[\begin{array}{rrr|r} 1 & 1 & 1 & 40,000 \\ 0.08 & 0.09 & 0.12 & 3,660 \\ 1 & -1 & -1 & 0 \end{array}\right].$$

On A use $100R_2 \to R_2$ to get

$$\left[\begin{array}{rrr|r} 1 & 1 & 1 & 40,000 \\ 8 & 9 & 12 & 366,000 \\ 1 & -1 & -1 & 0 \end{array}\right], \text{ use}$$

$-8R_1 + R_2 \to R_2$ to get

$$\left[\begin{array}{rrr|r} 1 & 1 & 1 & 40,000 \\ 0 & 1 & 4 & 46,000 \\ 1 & -1 & -1 & 0 \end{array}\right], \text{ use}$$

$-1R_3 + R_1 \to R_3$ and $-1R_2 + R_1 \to R_1$ to get

$$\left[\begin{array}{rrr|r} 1 & 0 & -3 & -6,000 \\ 0 & 1 & 4 & 46,000 \\ 0 & 2 & 2 & 40,000 \end{array}\right], \text{ use } \frac{1}{2}R_3 \to R_3 \text{ to}$$

get

$$\left[\begin{array}{rrr|r} 1 & 0 & -3 & -6,000 \\ 0 & 1 & 4 & 46,000 \\ 0 & 1 & 1 & 20,000 \end{array}\right], \text{ use}$$

$-1R_3 + R_2 \to R_3$ to get

$$\left[\begin{array}{rrr|r} 1 & 0 & -3 & -6,000 \\ 0 & 1 & 4 & 46,000 \\ 0 & 0 & 3 & 26,000 \end{array}\right], \text{ use}$$

$-1R_3 + R_2 \to R_2$ and $\frac{1}{3}R_3 \to R_3$ to get

$$\left[\begin{array}{rrr|r} 1 & 0 & -3 & -6,000 \\ 0 & 1 & 1 & 20,000 \\ 0 & 0 & 1 & 8,666.67 \end{array}\right], \text{ use } 3R_3 + R_1 \to R_1$$

and $-1R_3 + R_2 \to R_2$ to get

$$\left[\begin{array}{rrr|r} 1 & 0 & 0 & 20,000 \\ 0 & 1 & 0 & 11,333.33 \\ 0 & 0 & 1 & 8,666.67 \end{array}\right]. \text{ Investments were}$$

$x = \$20,000$ in a mutual fund, $y = \$11,333.33$ in treasury bills, and $z = \$8,666.67$ in bonds.

70. Let x, y, and z be the number of ounces of Kix, Quick Oats, and Muesli, respectively.

Using the RDA's we obtain the system

$$\begin{aligned} 0.04x + 0.1y + 0.08z &= 0.98 \\ 0.02x + 0.1y + 0.08z &= 0.84 \\ 0.04x + 0.02z &= 0.38. \end{aligned}$$

Then multiply each equation by 50. The augmented matrix of the resulting system is given below.

$$A = \left[\begin{array}{rrr|r} 2 & 5 & 4 & 49 \\ 1 & 5 & 4 & 42 \\ 2 & 0 & 1 & 19 \end{array}\right]$$

On A interchange rows R_1 and R_2 to get

$$\left[\begin{array}{rrr|r} 1 & 5 & 4 & 42 \\ 2 & 5 & 4 & 49 \\ 2 & 0 & 1 & 19 \end{array}\right], \text{ use } -1R_2 + R_1 \to R_2$$

to get

$$\left[\begin{array}{rrr|r} 1 & 5 & 4 & 42 \\ -1 & 0 & 0 & -7 \\ 2 & 0 & 1 & 19 \end{array}\right], \text{ use } 2R_2 + R_3 \to R_3$$

to get

$$\left[\begin{array}{rrr|r} 1 & 5 & 4 & 42 \\ -1 & 0 & 0 & -7 \\ 0 & 0 & 1 & 5 \end{array}\right], \text{ use } -1R_2 \to R_1$$

and $R_1 \to R_2$ to get

$$\left[\begin{array}{rrr|r} 1 & 0 & 0 & 7 \\ 1 & 5 & 4 & 42 \\ 0 & 0 & 1 & 5 \end{array}\right], \text{ use } -1R_1 + R_2 \to R_2 \text{ to get}$$

$$\left[\begin{array}{rrr|r} 1 & 0 & 0 & 7 \\ 0 & 5 & 4 & 35 \\ 0 & 0 & 1 & 5 \end{array}\right], \text{ use } \frac{1}{5}R_2 \to R_2 \text{ to get}$$

$$\left[\begin{array}{rrr|r} 1 & 0 & 0 & 7 \\ 0 & 1 & 4/5 & 7 \\ 0 & 0 & 1 & 5 \end{array}\right], \text{ use } -\frac{4}{5}R_3 + R_2 \to R_2$$

to get

$$\left[\begin{array}{rrr|r} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \end{array}\right]. \text{ Hulk Hogan ate}$$

$x = 7$ oz of Kix, $y = 3$ oz of Quick Oats, and $z = 5$ oz of Muesli.

71. The augmented matrix is

$$A = \left[\begin{array}{rrr|r} -1 & -1 & 1 & 4 \\ 1 & 1 & 1 & 2 \\ 8 & 2 & 1 & 7 \end{array}\right].$$

On A use $R_1 \to R_2$ and $R_2 \to R_1$ to get
$\left[\begin{array}{rrr|r} 1 & 1 & 1 & 2 \\ -1 & -1 & 1 & 4 \\ 8 & 2 & 1 & 7 \end{array}\right]$, use $8R_1 + (-1R_3) \to R_3$
and $-\frac{8}{3}R_2 + \left(-\frac{1}{3}R_3\right) \to R_2$ to get
$\left[\begin{array}{rrr|r} 1 & 1 & 1 & 2 \\ 0 & 2 & -3 & -13 \\ 0 & 6 & 7 & 9 \end{array}\right]$, use $-3R_2 + R_3 \to R_3$
to get
$\left[\begin{array}{rrr|r} 1 & 1 & 1 & 2 \\ 0 & 2 & -3 & -13 \\ 0 & 0 & 16 & 48 \end{array}\right]$, use $\frac{1}{2}R_2 \to R_2$
and $\frac{1}{16}R_3 \to R_3$ to get
$\left[\begin{array}{rrr|r} 1 & 1 & 1 & 2 \\ 0 & 1 & -3/2 & -13/2 \\ 0 & 0 & 1 & 3 \end{array}\right]$, use
$\frac{3}{2}R_3 + R_2 \to R_2$ to get
$\left[\begin{array}{rrr|r} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right]$, use $-1R_3 + R_1 \to R_1$
to get
$\left[\begin{array}{rrr|r} 1 & 1 & 0 & -1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right]$, use $-1R_2 + R_1 \to R_1$
to get
$\left[\begin{array}{rrr|r} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right]$.
Then $a = 1, b = -2$, and $c = 3$.

72. The augmented matrix is

$$A = \left[\begin{array}{rrr|r} 1 & -1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 9 & 3 & 1 & 0 \end{array}\right].$$

On A use $-1R_2 + R_1 \to R_2$ and
$-9R_1 + R_3 \to R_3$ to get
$\left[\begin{array}{rrr|r} 1 & -1 & 1 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 12 & -8 & 0 \end{array}\right]$, use $-\frac{1}{2}R_2 \to R_2$
and $-\frac{1}{4}R_3 \to R_3$ to get
$\left[\begin{array}{rrr|r} 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -3 & 2 & 0 \end{array}\right]$, use $R_1 + R_2 \to R_1$
and $3R_2 + R_3 \to R_3$ to get
$\left[\begin{array}{rrr|r} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \end{array}\right]$, use $\frac{1}{2}R_3 \to R_3$ to get
$\left[\begin{array}{rrr|r} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$, use $-1R_3 + R_1 \to R_1$ to get
$\left[\begin{array}{rrr|r} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$.
The solutions are $a = b = c = 0$.

73. Since the number of cars entering M.L. King Dr. and Washington St. is 750 and $x + y$ is the number of cars leaving the intersection of M.L. King Dr. and Washington St. then $x + y = 750$.

On the intersection of M.L. King Dr. and JFK Blvd., the number of cars entering this intersection is $450 + x$ and the number of cars leaving is $700 + z$. So $450 + x = 700 + z$.

Simplifying, one gets $y = 750 - x$ and $z = x - 250$; and since y and z are nonnegative, $250 \le x \le 750$. The values of x, y, and z that realizes this traffic flow must satisfy

$$\begin{aligned} y &= 750 - x \\ z &= x - 250 \\ 250 \le x &\le 750 \end{aligned}$$

If $z = 50$, then $50 = x - 250$ or $x = 300$, and $y = 750 - 300 = 450$.

74. As in Exercise 73, on each of the three intersections the number of cars entering must equal the number of cars leaving. So $x + y = 350 + 400$, $600 + z = 450 + x$, and $200 + 300 = z + y$. Simplifying, we obtain

$$\begin{aligned} x + y &= 750 \\ x - z &= 150 \\ y + z &= 500. \end{aligned}$$

By adding the last two equations, $x+y=650$. This contradicts the first equation and so the system has no solution. Thus, traffic will not flow, i.e., a gridlock.

Note, there are 1100 (i.e. 600+200+300) cars leaving the streets. One could reduce 400 to 300 so that the number of cars entering the streets is also 1100(=300+350+450); with this reduction traffic will flow.

75. No, since $(-2,1)$ does not satisfy $y > x+1$.

76. On $\left[\begin{array}{rr|r} 3 & -5 & -5 \\ 4 & 3 & 26 \end{array}\right]$ use $-4R_1 \to R_1$ and $3R_2 \to R_2$ to get $\left[\begin{array}{rr|r} -12 & 20 & 20 \\ 12 & 9 & -78 \end{array}\right]$.

Use $R_1+R_2 \to R_2$ to get $\left[\begin{array}{rr|r} -12 & 20 & 20 \\ 0 & 29 & -58 \end{array}\right]$,

Use $-\dfrac{1}{12}R_1 \to R_1$ and $\dfrac{1}{29}R_2 \to R_2$ to get $\left[\begin{array}{rr|r} 1 & -\frac{5}{3} & -\frac{5}{3} \\ 0 & 1 & -2 \end{array}\right]$.

Use $\dfrac{5}{3}R_2 + R_1 \to R_1$ to get $\left[\begin{array}{rr|r} 1 & 0 & -5 \\ 0 & 1 & -2 \end{array}\right]$.

The solution set is $\{(-5,-2)\}$.

77.

On $\left[\begin{array}{rrr|r} 1 & 1 & 0 & 4 \\ 0 & 1 & 1 & -3 \\ 1 & 0 & 1 & -17 \end{array}\right]$, use $-R_1 + R_3 \to R_3$

to get $\left[\begin{array}{rrr|r} 1 & 1 & 0 & 4 \\ 0 & 1 & 1 & -3 \\ 0 & -1 & 1 & -21 \end{array}\right]$.

Use $R_2 + R_3 \to R_3$ to get

$$\left[\begin{array}{rrr|r} 1 & 1 & 0 & 4 \\ 0 & 1 & 1 & -3 \\ 0 & 0 & 2 & -24 \end{array}\right]$$

Use $\dfrac{1}{2}R_3 \to R_3$ to get

$$\left[\begin{array}{rrr|r} 1 & 1 & 0 & 4 \\ 0 & 1 & 1 & -3 \\ 0 & 0 & 1 & -12 \end{array}\right]$$

Use $-R_3 + R_2 \to R_2$ to get

$$\left[\begin{array}{rrr|r} 1 & 1 & 0 & 4 \\ 0 & 1 & 0 & 9 \\ 0 & 0 & 1 & -12 \end{array}\right].$$

Use $-R_2 + R_1 \to R_1$ to get

$$\left[\begin{array}{rrr|r} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & 9 \\ 0 & 0 & 1 & -12 \end{array}\right]$$

The solution set is $\{(-5,9,-12)\}$.

78. **a)** $\log_5 1 - \log_5 25 = 0 - 2 = -2$

b) $\dfrac{1}{2} \cdot \log 10 = \dfrac{1}{2} \cdot 1 = \dfrac{1}{2}$

c) $3 \ln e = 3 \cdot 1 = 3$

79. Rewrite without absolute values:

$$\begin{array}{rclcrcl} x-3 & = & 2x+5 & \text{or} & x-3 & = & -2x-5 \\ -8 & = & x & \text{or} & 3x & = & -2 \end{array}$$

The solution set is $\{-2/3, -8\}$.

80. Let $w = 2x-1$. Then

$$\begin{aligned} 15w^2 - 2w - 8 &= 0 \\ (3w+2)(5w-4) &= 0 \\ w &= -\frac{2}{3}, \frac{4}{5} \end{aligned}$$

Substitute and solve for x:

$$\begin{array}{rclcrcl} 2x-1 & = & -\frac{2}{3} & \text{or} & 2x-1 & = & \frac{4}{5} \\ 2x & = & \frac{1}{3} & \text{or} & 2x & = & \frac{9}{5} \end{array}$$

The solution set is $\{1/6, 9/10\}$.

Thinking Outside the Box LXXIX

Let x be the amount of water that is already in, y the amount leaking in per hour, and z the amount pumped out per hour by one pump. Then we obtain a system of equations

$$\begin{aligned} 12(3)(z) &= x+3y \\ 5(10)(z) &= x+10y \\ n(2)(z) &= x+2y. \end{aligned}$$

From the first two equations, we find

$$14z = 7y \text{ or } 2z = y.$$

Using the first equation, we get

$$36z = x + 6z \text{ or } 30z = x.$$

Solving for n, we find

$$n = \frac{x+2y}{2z} = \frac{x+4z}{2z} = \frac{30z+4z}{2z} = 17.$$

Thus, 17 pumps are needed.

9.1 Pop Quiz

1. 2×3-matrix

2. $\left[\begin{array}{cc|c} 2 & -3 & -9 \\ 1 & 4 & 23 \end{array}\right]$

3. On $\left[\begin{array}{cc|c} 2 & -3 & -9 \\ 1 & 4 & 23 \end{array}\right]$ use $R_1 \leftrightarrow R_2$ to get

$\left[\begin{array}{cc|c} 1 & 4 & 23 \\ 2 & -3 & -9 \end{array}\right]$, use $-2R_1 + R_2 \to R_2$ to get

$\left[\begin{array}{cc|c} 1 & 4 & 23 \\ 0 & -11 & -55 \end{array}\right]$, use $-\frac{1}{11}R_2 \to R_2$ to get

$\left[\begin{array}{cc|c} 1 & 4 & 23 \\ 0 & 1 & 5 \end{array}\right]$, use $-4R_2 + R_1 \to R_1$ to get

$\left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 5 \end{array}\right]$. The solution set is $\{(3, 5)\}$.

4.

On $\left[\begin{array}{ccc|c} 1 & -1 & 1 & 4 \\ -1 & 2 & 1 & -1 \\ -1 & 1 & 4 & 6 \end{array}\right]$, use $R_1 + R_2 \to R_2$

and $R_1 + R_3 \to R_3$ to get

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 5 & 10 \end{array}\right]$$

Use $\frac{1}{5}R_3 \to R_3$ to get

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

Use $-R_3 + R_1 \to R_1$ and $-2R_3 + R_2 \to R_2$ to get

$$\left[\begin{array}{ccc|c} 1 & -1 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

Use $R_2 + R_1 \to R_1$ to get

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

The solution set is $\{(1, -1, 2)\}$.

9.1 Linking Concepts

a) At intersection C, one obtains $300 + 600 + a = 400 + b + f$ or equivalently $b + f - a = 500$.

At intersection B, one obtains $400 + b + e = 600 + 300 + a$ or equivalently $e + b - a = 500$.

At intersection D, one obtains $400 + c + f = 500 + d + 400$ or equivalently $c + f - d = 500$.

At intersection A, one obtains $500 + 500 + d = 500 + c + e$ or equivalently $e + c - d = 500$.

b) From part a), one obtains the augmented matrix (from 4 equations and 6 variables)

$$\left[\begin{array}{cccccc|c} -1 & 1 & 0 & 0 & 0 & 1 & 500 \\ 0 & 0 & 1 & -1 & 0 & 1 & 500 \\ -1 & 1 & 0 & 0 & 1 & 0 & 500 \\ 0 & 0 & 1 & -1 & 1 & 0 & 500 \end{array}\right].$$

Applying $-1R_1 + R_3 \to R_3$ and $-1R_2 + R_4 \to R_4$, one obtains

$$\left[\begin{array}{cccccc|c} -1 & 1 & 0 & 0 & 0 & 1 & 500 \\ 0 & 0 & 1 & -1 & 0 & 1 & 500 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \end{array}\right].$$

Applying $-1R_3 + R_4 \to R_4$, one obtains

$$\left[\begin{array}{cccccc|c} -1 & 1 & 0 & 0 & 0 & 1 & 500 \\ 0 & 0 & 1 & -1 & 0 & 1 & 500 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right].$$

It follows that the system of equations has infinitely many solutions and it does not have a unique solution.

c) From part b), the last augmented matrix, one finds $a = b + f - 500$, $c = d - f + 500$, and $e = f$. If one has $a = 300$, $c = 400$, and $f = 200$, then $300 = b + 200 - 500$ or $b = 600$. Similarly, one obtains $d = 100$ and $e = 200$.

d) At intersection A, instead of the one from part a), one would have $800 + 500 + d = 500 + c + e$ or equivalently $c - d + e = 800$. At the other intersections, the equations derived as in part a) still hold. Then

$$\left[\begin{array}{rrrrrr|r} -1 & 1 & 0 & 0 & 0 & 1 & 500 \\ 0 & 0 & 1 & -1 & 0 & 1 & 500 \\ -1 & 1 & 0 & 0 & 1 & 0 & 500 \\ 0 & 0 & 1 & -1 & 1 & 0 & 800 \end{array}\right].$$

Applying $-1R_1 + R_3 \to R_3$ and $-1R_2 + R_4 \to R_4$, one obtains

$$\left[\begin{array}{rrrrrr|r} -1 & 1 & 0 & 0 & 0 & 1 & 500 \\ 0 & 0 & 1 & -1 & 0 & 1 & 500 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 300 \end{array}\right].$$

The last two equations in the last augmented matrix, i.e., $e - f = 0$ and $e - f = 300$, show that the system is inconsistent and there is gridlock.

For Thought

1. True

2. False, since the orders of matrices A and C are different.

3. False, $A + B = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$.

4. True, $C + D = \begin{bmatrix} 1-3 & 1+5 \\ 3+1 & 3-2 \end{bmatrix} =$

$= \begin{bmatrix} -2 & 6 \\ 4 & 1 \end{bmatrix}$.

5. True, $A - B = \begin{bmatrix} 1-1 \\ 3-3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

6. False, $3B = 3\begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ 9 \end{bmatrix}$.

7. False, $-A = -\begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} -1 \\ -3 \end{bmatrix}$.

8. False, matrices of different orders cannot be added. **9.** False, matrices of different orders cannot be subtracted.

10. False, $C + 2D = \begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix} + \begin{bmatrix} -6 & 10 \\ 2 & -4 \end{bmatrix} =$

$= \begin{bmatrix} -5 & 11 \\ 5 & -1 \end{bmatrix}$.

9.2 Exercises

1. $x = 2,\ y = 5$ **2.** $x = 2/5,\ z = -1/2$

3. Since $2x = 6$ and $4y = 16$, $x = 3$ and $y = 4$. Also $3z = z + y$ and so $z = y/2 = 4/2 = 2$. Then $x = 3$, $y = 4$, and $z = 2$.

4. Since $-x = 3$ and $2y = -6$, $x = -3$ and $y = -3$. Also $x + y = 4z$ and so $4z = -6$. Then $x = -3$, $y = -3$, and $z = -3/2$.

5. $\begin{bmatrix} 3+2 \\ 5+1 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$ **6.** $\begin{bmatrix} -1+0 \\ 2+3 \end{bmatrix} = \begin{bmatrix} -1 \\ 5 \end{bmatrix}$

7. $\begin{bmatrix} -0.5+2 & -0.03+1 \\ 2-0.05 & -0.33+1 \end{bmatrix} = \begin{bmatrix} 1.5 & 0.97 \\ 1.95 & 0.67 \end{bmatrix}$

8. $\begin{bmatrix} -0.05+1 & -0.1-2 \\ 0.2-3 & -1+0.01 \end{bmatrix} = \begin{bmatrix} 0.95 & -2.1 \\ -2.8 & -0.99 \end{bmatrix}$

9. $\begin{bmatrix} 2+1 & -3-1 & 4+1 \\ 4+0 & -6+1 & 8-1 \\ 6+0 & -3+0 & 1+1 \end{bmatrix} = \begin{bmatrix} 3 & -4 & 5 \\ 4 & -5 & 7 \\ 6 & -3 & 2 \end{bmatrix}$

10. $\begin{bmatrix} -3-4 & 5-8 & 1-3 \\ -8+5 & 2+0 & 4-6 \\ 4-3 & 5+4 & -3-1 \end{bmatrix} =$

$\begin{bmatrix} -7 & -3 & -2 \\ -3 & 2 & -2 \\ 1 & 9 & -4 \end{bmatrix}$

11. $-A = -\begin{bmatrix} 1 & -4 \\ -5 & 6 \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 5 & -6 \end{bmatrix}$

and $A + (-A) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

12. $-A = -\begin{bmatrix} -3 & 5 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ 0 & -2 \end{bmatrix}$

and $A + (-A) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

13. $-A = -\begin{bmatrix} 3 & 0 & -1 \\ 8 & -2 & 1 \\ -3 & 6 & 3 \end{bmatrix} =$

$\begin{bmatrix} -3 & 0 & 1 \\ -8 & 2 & -1 \\ 3 & -6 & -3 \end{bmatrix}$ and

$A + (-A) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

14. $-A = -\begin{bmatrix} 4 & 8 & 3 \\ -5 & 0 & 6 \\ -1 & 1 & -9 \end{bmatrix} = \begin{bmatrix} -4 & -8 & -3 \\ 5 & 0 & -6 \\ 1 & -1 & 9 \end{bmatrix}$

and $A + (-A) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

15. $B - A = \begin{bmatrix} -1+4 & -2-1 \\ 7-3 & 4-0 \end{bmatrix} = \begin{bmatrix} 3 & -3 \\ 4 & 4 \end{bmatrix}$

16. $A - B = \begin{bmatrix} -4+1 & 1+2 \\ 3-7 & 0-4 \end{bmatrix} = \begin{bmatrix} -3 & 3 \\ -4 & -4 \end{bmatrix}$

17. $B - C = \begin{bmatrix} -1+3 & -2+4 \\ 7-2 & 4+5 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 5 & 9 \end{bmatrix}$

18. $C - B = \begin{bmatrix} -3+1 & -4+2 \\ 2-7 & 5+4 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ -5 & -9 \end{bmatrix}$

19. $B-E$ is undefined since B and E have different sizes

20. $A-D$ is undefined since A and D have different sizes

21. $3A = 3\begin{bmatrix} -4 & 1 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} -12 & 3 \\ 9 & 0 \end{bmatrix}$

22. $5B = 5\begin{bmatrix} -1 & -2 \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} -5 & -10 \\ 35 & 20 \end{bmatrix}$

23. $-1D = -1\begin{bmatrix} -4 \\ 5 \end{bmatrix} = \begin{bmatrix} 4 \\ -5 \end{bmatrix}$

24. $-3E = -3\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ -6 \end{bmatrix}$

25.

$3A + 3C = \begin{bmatrix} -12 & 3 \\ 9 & 0 \end{bmatrix} + \begin{bmatrix} -9 & -12 \\ 6 & -15 \end{bmatrix} =$

$\begin{bmatrix} -21 & -9 \\ 15 & -15 \end{bmatrix}$

26.

$3(A + C) = 3\begin{bmatrix} -7 & -3 \\ 5 & -5 \end{bmatrix} = \begin{bmatrix} -21 & -9 \\ 15 & -15 \end{bmatrix}$

27.

$2A - B = \begin{bmatrix} -8 & 2 \\ 6 & 0 \end{bmatrix} - \begin{bmatrix} -1 & -2 \\ 7 & 4 \end{bmatrix} =$

$\begin{bmatrix} -7 & 4 \\ -1 & -4 \end{bmatrix}$

28.

$-C - 3B = \begin{bmatrix} 3 & 4 \\ -2 & 5 \end{bmatrix} - \begin{bmatrix} -3 & -6 \\ 21 & 12 \end{bmatrix} =$

$\begin{bmatrix} 6 & 10 \\ -23 & -7 \end{bmatrix}$

29.

$2D - 3E = \begin{bmatrix} -8 \\ 10 \end{bmatrix} - \begin{bmatrix} -3 \\ 6 \end{bmatrix} = \begin{bmatrix} -5 \\ 4 \end{bmatrix}$

30.

$4D + 0E = \begin{bmatrix} -16 \\ 20 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -16 \\ 20 \end{bmatrix}$

31. $D+A$ is undefined since D and A have different sizes

32. $E+C$ is undefined since E and C have different sizes

33.

$(A + B) + C = \begin{bmatrix} -5 & -1 \\ 10 & 4 \end{bmatrix} + \begin{bmatrix} -3 & -4 \\ 2 & -5 \end{bmatrix} =$

$\begin{bmatrix} -8 & -5 \\ 12 & -1 \end{bmatrix}$

34.

$A + (B + C) = \begin{bmatrix} -4 & 1 \\ 3 & 0 \end{bmatrix} + \begin{bmatrix} -4 & -6 \\ 9 & -1 \end{bmatrix} =$

$\begin{bmatrix} -8 & -5 \\ 12 & -1 \end{bmatrix}$

35. $\begin{bmatrix} 0.2+0.2 & 0.1+0.05 \\ 0.4+0.3 & 0.3+0.8 \end{bmatrix} = \begin{bmatrix} 0.4 & 0.15 \\ 0.7 & 1.1 \end{bmatrix}$

36. $\begin{bmatrix} 1/2+1/2 & 1/3-1/6 \\ 1/4-1 & 1-1/3 \end{bmatrix} = \begin{bmatrix} 1 & 1/6 \\ -3/4 & 2/3 \end{bmatrix}$

37. $\begin{bmatrix} 1/2 & 3/2 \\ 3 & -12 \end{bmatrix}$ **38.** $\begin{bmatrix} 2 & -8 \\ -6 & -2 \end{bmatrix}$

39. $\begin{bmatrix} -2 & 4 \\ 6 & 8 \end{bmatrix} - \begin{bmatrix} -12 & 4 \\ 8 & -8 \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ -2 & 16 \end{bmatrix}$

40. $\begin{bmatrix} 1/2 \\ 2/3 \end{bmatrix} + \begin{bmatrix} 3/4 \\ 1/2 \end{bmatrix} = \begin{bmatrix} 5/4 \\ 7/6 \end{bmatrix}$

41. Undefined since we cannot add matrices with different sizes

42. Undefined since we cannot add matrices with different sizes

43. $\begin{bmatrix} -1 & 13 \\ -9 & 3 \\ 6 & -2 \end{bmatrix}$ **44.** $\begin{bmatrix} 3 & 3 & 4 \\ 4 & 7 & 8 \\ 6 & 3 & 2 \end{bmatrix}$

45. $\begin{bmatrix} 3\sqrt{2} & 2 & 3\sqrt{3} \end{bmatrix}$

46.

$$2\begin{bmatrix} -\sqrt{2} \\ \sqrt{5} \\ 3\sqrt{3} \end{bmatrix} - \begin{bmatrix} -2\sqrt{2} \\ 2\sqrt{5} \\ -\sqrt{3} \end{bmatrix} = \begin{bmatrix} -2\sqrt{2}+2\sqrt{2} \\ 2\sqrt{5}-2\sqrt{5} \\ 6\sqrt{3}+\sqrt{3} \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 7\sqrt{3} \end{bmatrix}$$

47.

$$\begin{bmatrix} 2a \\ 2b \end{bmatrix} + \begin{bmatrix} 6a \\ 12b \end{bmatrix} + \begin{bmatrix} 5a \\ -15b \end{bmatrix} = \begin{bmatrix} 13a \\ -b \end{bmatrix}$$

48.

$$\begin{bmatrix} -2a+3a-6a \\ 2b-4b+6b \\ 2c+c-12c \end{bmatrix} = \begin{bmatrix} -5a \\ 4b \\ -9c \end{bmatrix}$$

49.

$$\begin{bmatrix} -0.4x-0.6x & 0.4y-0.9y \\ 0.8x-1.5x & 3.2y+0.3y \end{bmatrix} =$$

$$\begin{bmatrix} -x & -0.5y \\ -0.7x & 3.5y \end{bmatrix}$$

50.

$$\begin{bmatrix} a^2 & ab \\ ac & 2ab \end{bmatrix} - \begin{bmatrix} b^2 & ab \\ 0 & 3ab \end{bmatrix} =$$

$$\begin{bmatrix} a^2-b^2 & 0 \\ ac & -ab \end{bmatrix}$$

51.

$$\begin{bmatrix} 2x & 2y & 2z \\ -2x & 4y & 6z \\ 2x & -2y & -6z \end{bmatrix} - \begin{bmatrix} -x & 0 & 3z \\ 4x & y & -z \\ 2x & 5y & z \end{bmatrix} =$$

$$\begin{bmatrix} 3x & 2y & -z \\ -6x & 3y & 7z \\ 0 & -7y & -7z \end{bmatrix}$$

52.

$$\begin{bmatrix} 1 & -4 & -2 \\ 7 & 8 & 5 \\ 2 & -3 & 1 \end{bmatrix} + \begin{bmatrix} 2 & -3 & 0 \\ 0 & 4 & -2 \\ 7 & -6 & -1 \end{bmatrix} =$$

$$\begin{bmatrix} 3 & -7 & -2 \\ 7 & 12 & 3 \\ 9 & -9 & 0 \end{bmatrix}$$

53. Equate the corresponding entries.

$$\begin{aligned} x+y &= 5 \\ x-y &= 1 \end{aligned}$$

Adding the two equations, we get $2x = 6$. Substitute $x = 3$ into $x+y = 5$. Then $3+y = 5$ and $y = 2$. Solution set is $\{(3, 2)\}$.

54. Equate the corresponding entries.

$$\begin{aligned} x-y &= -1 \\ 2x+y &= 4 \end{aligned}$$

Adding the two equations, we find $3x = 3$. Substitute $x = 1$ into $x-y = -1$. Then $1-y = -1$ and $y = 2$. Solution set is $\{(1, 2)\}$.

55. Equate the corresponding entries.

$$\begin{aligned} 2x+3y &= 7 \\ x-4y &= -13 \end{aligned}$$

Multiply second equation by -2 and add to the first one.

$$\begin{aligned} 2x+3y &= 7 \\ -2x+8y &= 26 \\ \hline 11y &= 33 \\ y &= 3 \end{aligned}$$

Substitute $y = 3$ into $x - 4y = -13$.
Then $x - 12 = -13$ and $x = -1$.
The solution set is $\{(-1, 3)\}$.

56. Equate the corresponding entries.

$$\begin{aligned} x - 3y &= 1 \\ 2x + y &= -5 \end{aligned}$$

Multiply first equation by -2 and add to the second one.

$$\begin{array}{rl} -2x + 6y &= -2 \\ 2x + y &= -5 \\ \hline 7y &= -7 \\ y &= -1 \end{array}$$

Substitute $y = -1$ into $x - 3y = 1$.
Then $x + 3 = 1$ and $x = -2$.
Solution set is $\{(-2, -1)\}$.

57. Equate the corresponding entries.

$$\begin{aligned} x + y + z &= 8 \\ x - y - z &= -7 \\ x - y + z &= 2 \end{aligned}$$

Adding the first and second equations, $2x = 1$ and $x = 0.5$. Multiply second equation by -1 and add to the third.

$$\begin{array}{rl} -x + y + z &= 7 \\ x - y + z &= 2 \\ \hline 2z &= 9 \\ z &= 4.5 \end{array}$$

Substitute $x = 0.5$ and $z = 4.5$ into $x + y + z = 8$. Then $y + 5 = 8$ and $y = 3$.
The solution set is $\{(0.5, 3, 4.5)\}$.

58. Equate the corresponding entries.

$$\begin{aligned} 2x + y + z &= 7 \\ x - 2y - z &= -6 \\ x - y + z &= 2 \end{aligned}$$

Multiply second equation by -2 and add to first equation. Also, multiply third equation by -1 and add to the second one.

$$\begin{array}{rl} 2x + y + z &= 7 \\ -2x + 4y + 2z &= 12 \\ \hline 5y + 3z &= 19 \end{array}$$

$$\begin{array}{rl} -x + y - z &= -2 \\ x - 2y - z &= -6 \\ \hline -y - 2z &= -8 \end{array}$$

Multiply $-y - 2z = -8$ by 5 and add to $5y + 3z = 19$.

$$\begin{array}{rl} 5y + 3z &= 19 \\ -5y - 10z &= -40 \\ \hline -7z &= -21 \end{array}$$

Substitute $z = 3$ into $5y + 3z = 19$. Then $5y + 9 = 19$ and $y = 2$. From $2x + y + z = 7$, we obtain $2x + 2 + 3 = 7$ and $x = 1$.
The solution set is $\{(1, 2, 3)\}$.

59. The matrices for January,February and March are, respectively,

$J = \begin{bmatrix} 120 \\ 30 \\ 40 \end{bmatrix}$, $F = \begin{bmatrix} 130 \\ 70 \\ 50 \end{bmatrix}$, and

$M = \begin{bmatrix} 140 \\ 60 \\ 45 \end{bmatrix}$. The sum

$J + F + M = \begin{bmatrix} \$390 \\ \$160 \\ \$135 \end{bmatrix}$ represents the

total expenses on food, clothing and utilities for the three months.

60.

The matrices are $K = [110 \quad 2 \quad 40]$, $Q = [100 \quad 4 \quad 100]$, and $M = [120 \quad 3 \quad 115]$. The sum $2K + 2Q + 3M = [780 \quad 21 \quad 625]$ represents the amounts of calories, protein, and potassium from 2 servings of Kix, 2 servings of Quick Oats, and 3 servings of Mueslix.

61. The supply matrix for the first week is

$S = \begin{bmatrix} 40 & 80 \\ 30 & 90 \\ 80 & 200 \end{bmatrix}$. Next week's supply matrix

is $S + 0.5S = \begin{bmatrix} 40 & 80 \\ 30 & 90 \\ 80 & 200 \end{bmatrix} + \begin{bmatrix} 20 & 40 \\ 15 & 45 \\ 40 & 100 \end{bmatrix}$

$= \begin{bmatrix} 60 & 120 \\ 45 & 135 \\ 120 & 300 \end{bmatrix}$.

62. The matrices are $K = [4 \quad 2 \quad 4]$ and $M = [11 \quad 4 \quad 16]$. The matrix $K + 2M = [26 \quad 10 \quad 36]$ represents the percentages of the U.S. Recommended Daily Allowances from a 1 oz serving of Kix and 1 cup of milk.

63. yes, yes

64. yes

65. yes, yes

66. yes

67. yes, yes

68. yes

69. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

70. yes

71. On $\left[\begin{array}{cc|c} 2 & 3 & 4 \\ 1 & -4 & -31 \end{array}\right]$, apply

$-2R_2 + R_1 \to R_2$ to get $\left[\begin{array}{cc|c} 2 & 3 & 4 \\ 0 & 11 & 66 \end{array}\right]$.

Use $\dfrac{1}{11}R_2 \to R_2$ to get $\left[\begin{array}{cc|c} 2 & 3 & 4 \\ 0 & 1 & 6 \end{array}\right]$.

Use $-3R_2 + R_1 \to R_2$ to get

$$\left[\begin{array}{cc|c} 2 & 0 & -14 \\ 0 & 1 & 6 \end{array}\right].$$

Use $\dfrac{1}{2}R_1 \to R_1$ to get $\left[\begin{array}{cc|c} 1 & 0 & -7 \\ 0 & 1 & 6 \end{array}\right]$.

The solution set is $\{(-7, 6)\}$.

72. Multiply the equation below by the LCD

$$\frac{8x^2 + 17x + 12}{x(x+1)(x+2)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x+2}$$

to obtain

$$8x^2 + 17x + 12 = A(x+1)(x+2) + Bx(x+2) + Cx(x+1).$$

If we substitute $x = 0$ in the above equation we find $12 = 2A$ or $A = 6$.

Similarly, if we substitute $x = -1$, we find $3 = -B$ or $B = -3$.

Likewise, if we substitute $x = -2$, we obtain $10 = 2C$ or $C = 5$.

Hence,

$$\frac{8x^2 + 17x + 12}{x(x+1)(x+2)} = \frac{6}{x} + \frac{-3}{x+1} + \frac{5}{x+2}$$

73. **a)** $bx(ax + y) + z(ax + y) = (ax + y)(bx + z)$

b) $x(6x^2 - 23xy + 20y^2) = x(3x - 4y)(2x - 5y)$

74. $\ln \sqrt{x} + \ln z - \ln(y^3) = \ln\left(\dfrac{z\sqrt{x}}{y^3}\right)$

75. $\dfrac{\ln x}{\ln 7}$

76. Let $f(x) = (x^2 - 9)(x^2 - 1) < 0$.
Note, the zeros of $f(x)$ are $x = \pm 1, \pm 3$.

If $x = 4$, then $f(4) > 0$.
If $x = 2$, then $f(2) < 0$.
If $x = 0$, then $f(0) > 0$.
If $x = -2$, then $f(-2) < 0$.
If $x = -4$, then $f(-4) > 0$.

+ 0 − 0 + 0 − 0 +

$-3 \quad -1 \quad 1 \quad 3$

The solution set is $(-3, -1) \cup (1, 3)$.

Thinking Outside the Box

LXXX Let the ordered triple (x, y, z) represent the amount of antifreeze in the 8-quart radiator, the 5-quart container, and the 3-quart container, respectively.

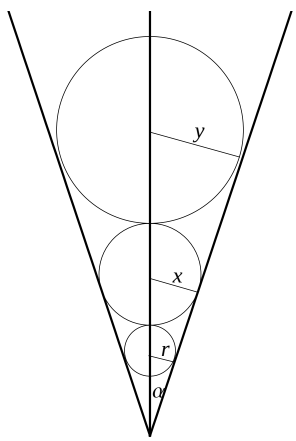
To leave four quarts of antifreeze in the radiator, perform the sequence of operations defined by the ordered triples:

$$(8, 0, 0), (5, 0, 3), (5, 3, 0), (2, 3, 3),$$

$$(2, 5, 1), (7, 0, 1), (7, 1, 0), (4, 1, 3).$$

Afterwards, fill the radiator with water.

LXXXI We study a general case of three tangent circles with radii $r < x < y$ that are tangent to the right most line, as shown below. The angle α is formed from the middle line to the right most line.



Using similar triangles, we find

$$\frac{x}{r} = \frac{x + r + r\csc\alpha}{r\csc\alpha} \quad (1)$$

$$\frac{x}{r} = \frac{x + r}{r\csc\alpha} + 1$$

$$x\csc\alpha = x + r + r\csc\alpha$$

$$x(\csc\alpha - 1) = r(1 + \csc\alpha)$$

$$\frac{\csc\alpha - 1}{1 + \csc\alpha} = \frac{r}{x} \quad (2)$$

Likewise, by similar triangles we have

$$\frac{y}{r} = \frac{y + 2x + r + r\csc\alpha}{r\csc\alpha}. \quad (3)$$

If we subtract (1) from (3), we obtain

$$\frac{y - x}{r} = \frac{y + x}{r\csc\alpha}$$

$$(y - x)\csc\alpha = y + x$$

$$y \cdot \frac{(\csc\alpha - 1)}{1 + \csc\alpha} = x$$

$$y \cdot \frac{r}{x} = x \quad \text{By (2)}$$

$$\sqrt{yr} = x \quad (4)$$

In particular, assume $r = 16$ in. and there is another circle of radius 54 in. that is tangent to the circle of radius y and tangent to the left and right most lines. Applying (4), we find

$$y = \sqrt{54x} \qquad \text{and} \qquad x = \sqrt{16y}$$

Solve for y as follows:

$$y = \sqrt{54\sqrt{16y}}$$

$$y^4 = 54^2(16y)$$

$$y^3 = 54^2(16)$$

$$y = 36.$$

Thus, $x = \sqrt{16y} = \sqrt{16(36)} = 24$. Hence, the radii are $x = 24$ in. and $y = 36$ in.

9.2 Pop Quiz

1. $A + B = \begin{bmatrix} 0 \\ 8 \end{bmatrix}$

2. $3A - B = \begin{bmatrix} 3 \\ 9 \end{bmatrix} - \begin{bmatrix} -1 \\ 5 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$

3. $-A + A = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

4. $A + C$ is undefined

5. $C + 2D = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 3 & 6 \end{bmatrix}$

For Thought

1. True **2.** True **3.** False, they cannot be multiplied since the number of columns of A is not the same as the number of rows of C.

4. False, the order of CA is 2×1. **5.** True

6. True, $BC = [7 \cdot 2 + 9 \cdot 4 \quad 7 \cdot 3 + 9 \cdot 5] = $
$= [14 + 36 \quad 21 + 45] = [50 \quad 66]$.

7. True, $AB = \begin{bmatrix} 1 \\ 6 \end{bmatrix} [7 \quad 9] = \begin{bmatrix} 1 \cdot 7 & 1 \cdot 9 \\ 6 \cdot 7 & 6 \cdot 9 \end{bmatrix} =$
$= \begin{bmatrix} 7 & 9 \\ 42 & 54 \end{bmatrix}$.

8. True, $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix} =$

$\begin{bmatrix} 4+0 & -2+9 \\ 8+0 & -4+15 \end{bmatrix} = \begin{bmatrix} 4 & 7 \\ 8 & 11 \end{bmatrix}$.

9. True, $BA = [7 \quad 9]\begin{bmatrix} 1 \\ 6 \end{bmatrix} = [7+54] = [61]$.

10. False, since $EC = \begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} =$

$= \begin{bmatrix} 4-4 & 6-5 \\ 0+12 & 0+15 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 12 & 15 \end{bmatrix}$ and from Exercise 8 one sees $EC \neq CE$.

9.3 Exercises

1. 3×5 **2.** 3×3 **3.** 1×1

4. 4×5 **5.** 5×5 **6.** 2×6

7. 3×3 **8.** 4×1

9. undefined **10.** undefined

11. $[-3(4) + 2(1)] = [-10]$

12. $[-1(4) + 2(2)] = [0]$

13. $\begin{bmatrix} 1(1)+3(3) \\ 2(1)+(-4)(3) \end{bmatrix} = \begin{bmatrix} 10 \\ -10 \end{bmatrix}$

14. $\begin{bmatrix} 0(2)+2(5) \\ -3(2)+1(5) \end{bmatrix} = \begin{bmatrix} 10 \\ -1 \end{bmatrix}$

15. $\begin{bmatrix} 5(1)+1(3) & 5(2)+1(1) \\ 2(1)+1(3) & 2(2)+1(1) \end{bmatrix} = \begin{bmatrix} 8 & 11 \\ 5 & 5 \end{bmatrix}$

16. $\begin{bmatrix} 3(0)+1(1) & 3(3)+1(6) \\ 4(0)+0(1) & 4(3)+0(6) \end{bmatrix} = \begin{bmatrix} 1 & 15 \\ 0 & 12 \end{bmatrix}$

17. $\begin{bmatrix} 3(5) & 3(6) \\ 1(5) & 1(6) \end{bmatrix} = \begin{bmatrix} 15 & 18 \\ 5 & 6 \end{bmatrix}$

18. $\begin{bmatrix} -2(4) & -2(3) \\ 2(4) & 2(3) \end{bmatrix} = \begin{bmatrix} -8 & -6 \\ 8 & 6 \end{bmatrix}$

19. $AB = \begin{bmatrix} 1(1)+3(-1) & 1(0)+3(1) & 1(1)+3(0) \\ 2(1)+4(-1) & 2(0)+4(1) & 2(1)+4(0) \\ 5(1)+6(-1) & 5(0)+6(1) & 5(1)+6(0) \end{bmatrix}$

$= \begin{bmatrix} -2 & 3 & 1 \\ -2 & 4 & 2 \\ -1 & 6 & 5 \end{bmatrix}$ and

$BA = \begin{bmatrix} 1(1)+0(2)+1(5) & 1(3)+0(4)+1(6) \\ -1(1)+1(2)+0(5) & -1(3)+1(4)+0(6) \end{bmatrix}$

$= \begin{bmatrix} 6 & 9 \\ 1 & 1 \end{bmatrix}$

20. $AB =$

$\begin{bmatrix} 1(2)+1(3) & 1(1)+1(2) & 1(0)+1(0) \\ -1(2)+0(3) & -1(1)+0(2) & -1(0)+0(0) \\ 1(2)+(-1)(3) & 1(1)+(-1)(2) & 1(0)+(-1)(0) \end{bmatrix}$

$= \begin{bmatrix} 5 & 3 & 0 \\ -2 & -1 & 0 \\ -1 & -1 & 0 \end{bmatrix}$ and

$BA =$

$\begin{bmatrix} 2(1)+1(-1)+0(1) & 2(1)+1(0)+0(-1) \\ 3(1)+2(-1)+0(1) & 3(1)+2(0)+0(-1) \end{bmatrix}$

$= \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$

21. $AB =$

$\begin{bmatrix} 1(1)+2(0)+3(0) & 1(1)+2(1)+3(0) & 1(1)+2(1)+3(1) \\ 2(1)+1(0)+3(0) & 2(1)+1(1)+3(0) & 2(1)+1(1)+3(1) \\ 3(1)+2(0)+1(0) & 3(1)+2(1)+1(0) & 3(1)+2(1)+1(1) \end{bmatrix}$

$= \begin{bmatrix} 1 & 3 & 6 \\ 2 & 3 & 6 \\ 3 & 5 & 6 \end{bmatrix}$ and

$BA =$

$\begin{bmatrix} 1(1)+1(2)+1(3) & 1(2)+1(1)+1(2) & 1(3)+1(3)+1(1) \\ 0(1)+1(2)+1(3) & 0(2)+1(1)+1(2) & 0(3)+1(3)+1(1) \\ 0(1)+0(2)+1(3) & 0(2)+0(1)+1(2) & 0(3)+0(3)+1(1) \end{bmatrix}$

$= \begin{bmatrix} 6 & 5 & 7 \\ 5 & 3 & 4 \\ 3 & 2 & 1 \end{bmatrix}$

22. $AB =$

$\begin{bmatrix} 2(0)+0(2)+0(0) & 2(0)+0(0)+0(3) & 2(1)+0(0)+0(0) \\ 2(0)+2(2)+0(0) & 2(0)+2(0)+0(3) & 2(1)+2(0)+0(0) \\ 2(0)+2(2)+2(0) & 2(0)+2(0)+2(3) & 2(1)+2(0)+2(0) \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 2 \\ 4 & 0 & 2 \\ 4 & 6 & 2 \end{bmatrix}$ and

$BA =$

$$\begin{bmatrix} 0(2)+0(2)+1(2) & 0(0)+0(2)+1(2) & 0(0)+0(0)+1(2) \\ 2(2)+0(2)+0(2) & 2(0)+0(2)+0(2) & 2(0)+0(0)+0(2) \\ 0(2)+3(2)+0(2) & 0(0)+3(2)+0(2) & 0(0)+3(0)+0(2) \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 & 2 \\ 4 & 0 & 0 \\ 6 & 6 & 0 \end{bmatrix}$$

23. $$AB = \begin{bmatrix} 2\cdot 2 & 2\cdot 3 & 2\cdot 4 \\ -3\cdot 2 & -3\cdot 3 & -3\cdot 4 \\ 1\cdot 2 & 1\cdot 3 & 1\cdot 4 \end{bmatrix} =$$

$$\begin{bmatrix} 4 & 6 & 8 \\ -6 & -9 & -12 \\ 2 & 3 & 4 \end{bmatrix}$$

24. $\begin{bmatrix} 4+12+4 & 6+15+0 \end{bmatrix} = \begin{bmatrix} 20 & 21 \end{bmatrix}$

25. $\begin{bmatrix} 2+0+0 & 2+3+0 & 2+3+4 \end{bmatrix} =$

$\begin{bmatrix} 2 & 5 & 9 \end{bmatrix}$

26. $[4-9+4] = [-1]$ **27.** undefined

28. $$DE = \begin{bmatrix} 2+0+0 & 2-1+0 & 2-1+1 \\ 0+0+0 & 0+3+0 & 0+3+2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 2 \\ 0 & 3 & 5 \end{bmatrix}$$

29. $$EC = \begin{bmatrix} 2+4+1 & 3+5+0 \\ 0+4+1 & 0+5+0 \\ 0+0+1 & 0+0+0 \end{bmatrix} = \begin{bmatrix} 7 & 8 \\ 5 & 5 \\ 1 & 0 \end{bmatrix}$$

30. undefined

31. $$DC = \begin{bmatrix} 4-4+1 & 6-5+0 \\ 0+12+2 & 0+15+0 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 1 \\ 14 & 15 \end{bmatrix}$$

32. $$CD = \begin{bmatrix} 4+0 & -2+9 & 2+6 \\ 8+0 & -4+15 & 4+10 \\ 2+0 & -1+0 & 1+0 \end{bmatrix} =$$

$$\begin{bmatrix} 4 & 7 & 8 \\ 8 & 11 & 14 \\ 2 & -1 & 1 \end{bmatrix}$$

33. undefined

34. $$EE = \begin{bmatrix} 1+0+0 & 1+1+0 & 1+1+1 \\ 0+0+0 & 0+1+0 & 0+1+1 \\ 0+0+0 & 0+0+0 & 0+0+1 \end{bmatrix} =$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

35. $$EA = \begin{bmatrix} 2-3+1 \\ 0-3+1 \\ 0+0+1 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$$

36. $$DA = \begin{bmatrix} 4+3+1 \\ 0-9+2 \end{bmatrix} = \begin{bmatrix} 8 \\ -7 \end{bmatrix}$$

37. undefined **38.** undefined

39. We will use the answer in Exercise 23.

$AB + 2E =$

$$\begin{bmatrix} 4 & 6 & 8 \\ -6 & -9 & -12 \\ 2 & 3 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 2 & 2 \\ 0 & 2 & 2 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 8 & 10 \\ -6 & -7 & -10 \\ 2 & 3 & 6 \end{bmatrix}$$

40. We will use the answer in Exercise 35.

$EA - 2A =$

$$\begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix} - \begin{bmatrix} 4 \\ -6 \\ 2 \end{bmatrix} = \begin{bmatrix} -4 \\ 4 \\ -1 \end{bmatrix}$$

41. $A^2 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} =$

$$\begin{bmatrix} 1(1)+0(1) & 1(0)+0(1) \\ 1(1)+1(1) & 1(0)+1(1) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

42. We will use the answer in Exercise 41.

$A^3 = A^2A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} =$

$\begin{bmatrix} 1(1)+0(1) & 1(0)+0(1) \\ 2(1)+1(1) & 2(0)+1(1) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$

43. We will use the answer in Exercise 42.

$A^4 = A^3A = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} =$

$\begin{bmatrix} 1(1)+0(1) & 1(0)+0(1) \\ 3(1)+1(1) & 3(0)+1(1) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}$

44. We will use the answer in Exercise 43.

$A^5 = A^4A = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} =$

$\begin{bmatrix} 1(1)+0(1) & 1(0)+0(1) \\ 4(1)+1(1) & 4(0)+1(1) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$

45.

$\begin{bmatrix} 2\cdot 1+0\cdot 0 & 2\cdot 1+0\cdot 1 \\ 3\cdot 1+1\cdot 0 & 3\cdot 1+1\cdot 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 3 & 4 \end{bmatrix}$

46.

$\begin{bmatrix} -1\cdot 2+2\cdot 5 & -1\cdot(-1)+2\cdot 2 \\ 3\cdot 2+4\cdot 5 & 3\cdot(-1)+4\cdot 2 \end{bmatrix} =$

$\begin{bmatrix} 8 & 5 \\ 26 & 5 \end{bmatrix}$

47.

$\begin{bmatrix} 7\cdot 3+4\cdot(-5) & 7\cdot(-4)+4\cdot 7 \\ 5\cdot 3+3\cdot(-5) & 5\cdot(-4)+3\cdot 7 \end{bmatrix} =$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

48.

$\begin{bmatrix} (-2)\cdot(-8)+(-3)\cdot 5 & 6-6 \\ 5\cdot(-8)+8\cdot 5 & 5\cdot(-3)+8\cdot 2 \end{bmatrix} =$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

49.

$\begin{bmatrix} -0.5\cdot 1+4\cdot 0 & -0.5\cdot 0+4\cdot 1 \\ 9\cdot 1+0.7\cdot 0 & 9\cdot 0+0.7\cdot 1 \end{bmatrix} =$

$\begin{bmatrix} -0.5 & 4 \\ 9 & 0.7 \end{bmatrix}$

50.

$\begin{bmatrix} 1\cdot(-0.7)+0\cdot 3 & 1\cdot 1.2+0\cdot 1.1 \\ 0\cdot(-0.7)+1\cdot 3 & 0\cdot 1.2+1\cdot 1.1 \end{bmatrix} =$

$\begin{bmatrix} -0.7 & 1.2 \\ 3 & 1.1 \end{bmatrix}$

51. $[-2a+6a \quad -6b+3b] = [4a \quad -3b]$

52.

$\begin{bmatrix} -2x+5y \\ 6x+4y \end{bmatrix}$

53.

$\begin{bmatrix} -2a+0\cdot 1 & 5a+0\cdot 4 & 3a+0\cdot 6 \\ 0\cdot(-2)+b & 0\cdot 5+4b & 0\cdot 3+6b \end{bmatrix} =$

$\begin{bmatrix} -2a & 5a & 3a \\ b & 4b & 6b \end{bmatrix}$

54. $\begin{bmatrix} x+y \\ x-y \end{bmatrix}$

55. $[1\cdot 1+2\cdot 0+3\cdot 1 \quad 1\cdot 0+2\cdot 1+3\cdot 0 \quad 1\cdot 1+2\cdot 1+3\cdot 1]$
$= [4 \quad 2 \quad 6]$

56.

$\begin{bmatrix} 1\cdot(-2)+1\cdot 3+0\cdot 5 \\ 1\cdot(-2)+0\cdot 3+1\cdot 5 \\ 0\cdot(-2)+1\cdot 3+1\cdot 5 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 8 \end{bmatrix}$

57. $[-1\cdot(-5)+0\cdot 1+3\cdot 4] = [17]$

58.

$\begin{bmatrix} -5\cdot(-1) & -5\cdot 0 & -5\cdot 3 \\ 1\cdot(-1) & 1\cdot 0 & 1\cdot 3 \\ 4\cdot(-1) & 4\cdot 0 & 4\cdot 3 \end{bmatrix} =$

$\begin{bmatrix} 5 & 0 & -15 \\ -1 & 0 & 3 \\ -4 & 0 & 12 \end{bmatrix}$

59.

$\begin{bmatrix} x^2 & xy \\ xy & y^2 \end{bmatrix}$

60. $[x^2+y^2]$

61.

$\begin{bmatrix} -1\cdot\sqrt{2}+2\cdot 0+3\sqrt{2} \\ 3\cdot\sqrt{2}+4\cdot 0+4\sqrt{2} \end{bmatrix} = \begin{bmatrix} 2\sqrt{2} \\ 7\sqrt{2} \end{bmatrix}$

62. $[2 \cdot 0 + 3 \cdot (-1) \quad 2\sqrt{2} + 3\sqrt{8} \quad 2 \cdot 5 + 3 \cdot 0] =$ $[-3 \quad 8\sqrt{2} \quad 10]$

63.
$$\begin{bmatrix} (1/2)\cdot(-8) + (1/3)\cdot(-5) & 6 + (1/3)\cdot 15 \\ (1/4)\cdot(-8) + (1/5)\cdot(-5) & 3 + (1/5)\cdot 15 \end{bmatrix}$$
$$= \begin{bmatrix} -4 - (5/3) & 6+5 \\ -2-1 & 3+3 \end{bmatrix} = \begin{bmatrix} -17/3 & 11 \\ -3 & 6 \end{bmatrix}$$

64.
$$\begin{bmatrix} (-1/8) + (1/4) & (1/16) + (1/8) \\ (-1/16) - (1/4) & (1/32) - (1/8) \end{bmatrix} =$$
$$\begin{bmatrix} 1/8 & 3/16 \\ -5/16 & -3/32 \end{bmatrix}$$

65. undefined **66.** undefined

67.
$$\begin{bmatrix} 9+0-7 & 8+0-8 & 10+0-4 \\ 0+3+0 & 0+5+0 & 0+2+0 \\ 9+3+7 & 8+5+8 & 10+2+4 \end{bmatrix} =$$
$$\begin{bmatrix} 2 & 0 & 6 \\ 3 & 5 & 2 \\ 19 & 21 & 16 \end{bmatrix}$$

68.
$$\begin{bmatrix} -2+2-3 & 3+5+0 & -4+7-6 \\ 0+2-3 & 0+5+0 & 0+7-6 \\ 0+0-3 & 0+0+0 & 0+0-6 \end{bmatrix} =$$
$$\begin{bmatrix} -3 & 8 & -3 \\ -1 & 5 & 1 \\ -3 & 0 & -6 \end{bmatrix}$$

69.
$$\begin{bmatrix} 1-0.6-0.6 & 1.5+1.2-1 \\ 0.8+0.4+1.8 & 1.2-0.8+3 \\ 0.4+0.6-2.4 & 0.6-1.2-4 \end{bmatrix} =$$
$$\begin{bmatrix} -0.2 & 1.7 \\ 3 & 3.4 \\ -1.4 & -4.6 \end{bmatrix}$$

70.
$$\begin{bmatrix} 4+1+42 & 6+2+35 & 8+5+28 \\ 6+3+24 & 9+6+20 & 12+15+16 \end{bmatrix} =$$
$$\begin{bmatrix} 47 & 43 & 41 \\ 33 & 35 & 43 \end{bmatrix}$$

71. System of equations is
$$\begin{aligned} 2x - 3y &= 0 \\ x + 2y &= 7. \end{aligned}$$
Multiply second equation by -2 and add to the first one.
$$\begin{array}{rcr} 2x - 3y &=& 0 \\ -2x - 4y &=& -14 \\ \hline -7y &=& -14 \end{array}$$
Substitute $y = 2$ into $x + 2y = 7$. Then $x + 4 = 7$ and $x = 3$. Solution set is $\{(3, 2)\}$.

72. System of equations is
$$\begin{aligned} x + 5y &= 2 \\ -2x + 4y &= 10. \end{aligned}$$
Multiply first equation by 2 and add to the second one.
$$\begin{array}{rcr} 2x + 10y &=& 4 \\ -2x + 4y &=& 10 \\ \hline 14y &=& 14 \end{array}$$
Substitute $y = 1$ into $x + 5y = 2$. Then $x+5 = 2$ and $x = -3$. Solution set is $\{(-3, 1)\}$.

73. System of equations is
$$\begin{aligned} 2x + 3y &= 5 \\ 4x + 6y &= 9 \end{aligned}$$
Multiply first equation by -2 and add to the second one.
$$\begin{array}{rcr} -4x - 6y &=& -10 \\ 4x + 6y &=& 9 \\ \hline 0 &=& -1 \end{array}$$
Inconsistent and the solution set is $\emptyset$.

74. System of equations is
$$\begin{aligned} x - 3y &= 1 \\ -2x + 6y &= -2 \end{aligned}$$
Multiply first equation by 2 and add to the second one.
$$\begin{array}{rcr} 2x - 6y &=& 2 \\ -2x + 6y &=& -2 \\ \hline 0 &=& 0 \end{array}$$
Dependent system and the solution set is $\{(x, y) \mid x - 3y = 1\}$.

75. System of equations is

$$\begin{aligned} x+y+z &= 4 \\ y+z &= 5 \\ z &= 6. \end{aligned}$$

Substitute $z=6$ into $y+z=5$ to get $y+6=5$ and $y=-1$. From $x+y+z=4$, we have $x-1+6=4$ and $x=-1$. Solution set is $\{(-1,-1,6)\}$.

76. System of equations is

$$\begin{aligned} 2x+3y+z &= 0 \\ y+4z &= 3 \\ 2z &= 6. \end{aligned}$$

Substitute $z=3$ into $y+4z=3$. Then $y+12=3$ and $y=-9$. From $2x+3y+z=0$, we have $2x-27+3=0$ and $x=12$. The solution set is $\{(12,-9,3)\}$.

77.

$$\begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \end{bmatrix}$$

78.

$$\begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -7 \\ 8 \end{bmatrix}$$

79.

$$\begin{bmatrix} 1 & 2 & -1 \\ 3 & -1 & 3 \\ 2 & 1 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$$

80.

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 1 & -1 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}$$

81.

$$A = \begin{bmatrix} \$24,000 & \$40,000 \\ \$38,000 & \$70,000 \end{bmatrix}, \; Q = \begin{bmatrix} 4 \\ 7 \end{bmatrix}, \text{ and}$$

matrix product $AQ = \begin{bmatrix} \$376,000 \\ \$642,000 \end{bmatrix}$

represents the costs for labor and material for building 4 economy houses and 7 deluxe models.

82.

$$A = \begin{bmatrix} 2 & 3 \\ 23 & 28 \\ 2 & 2 \end{bmatrix}, \; Q = \begin{bmatrix} 4 \\ 3 \end{bmatrix} \text{ and}$$

matrix product $AQ = \begin{bmatrix} 17 \\ 176 \\ 14 \end{bmatrix}$ represents

the amounts of protein, carbohydrates and fat consumed in 4 servings of Almond Delight and 3 servings of Basic 4.

83. False

84. True

85. True

86. True

87. True

88. True

89. $A+3B = \begin{bmatrix} 1 & 2 \\ -3 & 5 \end{bmatrix} + \begin{bmatrix} -3 & 9 \\ 6 & 12 \end{bmatrix} =$

$$\begin{bmatrix} -2 & 11 \\ 3 & 17 \end{bmatrix}$$

90. The augmented matrix is

$$A = \left[\begin{array}{ccc|c} 1 & 1 & -1 & 5 \\ 0 & 1 & 1 & 7 \\ 0 & 1 & -1 & 6 \end{array}\right].$$

On A use $-R_2+R_3 \to R_3$ to get

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 5 \\ 0 & 1 & 1 & 7 \\ 0 & 0 & -2 & -1 \end{array}\right]$$

Apply $-\frac{1}{2}R_3 \to R_3$ to get

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 5 \\ 0 & 1 & 1 & 7 \\ 0 & 0 & 1 & 1/2 \end{array}\right]$$

Use $-R_3+R_2 \to R_2$ to get

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 5 \\ 0 & 1 & 0 & 13/2 \\ 0 & 0 & 1 & 1/2 \end{array}\right]$$

Use $R_3 + R_1 \to R_1$ to get

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & \frac{11}{2} \\ 0 & 1 & 0 & 13/2 \\ 0 & 0 & 1 & 1/2 \end{array}\right]$$

Use $-R_2 + R_1 \to R_1$ to get

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 13/2 \\ 0 & 0 & 1 & 1/2 \end{array}\right]$$

The solution set is $\{-1, 13/2, 1/2\}$.

91. Independent since the lines have different slopes, namely, -9 and 500.

92. $y \to -\infty$ as $x \to \infty$, since the leading term is $-x^4$.

93. Let $f(x) = \dfrac{x-9}{x+99} \le 0$.
If $x = 10$, then $f(10) > 0$.
If $x = 0$, then $f(0) < 0$.
If $x = -100$, then $f(100) > 0$.

+	U	−	0	+
0	−99	10	9	100

The solution set is $(-99, 9]$.

94. $\left(\dfrac{1}{4x^8y^{18}}\right)^{-1/2} = 2x^4y^9$

Thinking Outside the Box LXXXII

There were 15 soccer teams in the north section, and 9 teams in the south section. In the south section, it is possible that Springville won three games and tied five games for a total of 5.5 points.

9.3 Pop Quiz

1. AB is undefined

2. AC is undefined

3. $\begin{bmatrix} 1 & 3 \\ 5 & 7 \end{bmatrix}\begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 1(2)+3(4) \\ 5(2)+7(4) \end{bmatrix} = \begin{bmatrix} 14 \\ 38 \end{bmatrix}$

4.

$$\begin{bmatrix} 1 & 3 \\ 5 & 7 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ 0 & 8 \end{bmatrix} = \begin{bmatrix} 1(2)+3(0) & 1(4)+3(8) \\ 5(2)+7(0) & 5(4)+7(8) \end{bmatrix} = \begin{bmatrix} 2 & 28 \\ 10 & 76 \end{bmatrix}$$

5.

$$\begin{bmatrix} 2 & 4 \\ 0 & 8 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 2(1)+4(5) & 2(3)+4(7) \\ 0(1)+8(5) & 0(3)+8(7) \end{bmatrix} = \begin{bmatrix} 22 & 34 \\ 40 & 56 \end{bmatrix}$$

9.3 Linking Concepts

a)

If $M = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ then

$$M^2 = M \cdot M = \begin{bmatrix} 0 & 0 & 1 & 2 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}.$$

b) The matrix M^2, in part a), gives the number of secondary points obtained. For instance, in the first row, A gets 1 secondary point from C and 2 secondary points from B.

c) Note, $M + M^2 = \begin{bmatrix} 0 & 1 & 2 & 2 \\ 1 & 0 & 1 & 2 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$

and $(M + M^2)\,T = \begin{bmatrix} 5 \\ 4 \\ 2 \\ 3 \end{bmatrix}$.

d) The entries in $(M + M^2)\,T$ represent the total number of points earned by each team. For instance, B won 4 points.

e) It is not possible for a team, say team A, to have a better win-loss record than another team, say team B, and to be ranked lower (in this point scheme) than the other team. To see this, one could analyze all the cases.

Case 1. If B has a 0-3 loss record, then in this point scheme B will get no points. Since A has at least one win, then A will get at least 1 point in the scheme. Then A will still be ranked higher than B.

Case 2. If B has a 1-2 loss record, then A either has a 3-0 or 2-1 win-loss record. Assume A has the 3-0 win-loss record. Since B has 1 win, A has at least 4 points in the scheme. If B ranks higher than A in the scheme, then B must have at least 5 points in the scheme. Then the team defeated by B must have won four times which is impossible since each team played exactly 3 games. A ranks higher than B in the scheme.

On the other hand, suppose A has a 2-1 win-loss record. If A defeated B, then A has at least 3 points in the scheme. In addition, if B has a higher rank than A in the scheme, then B must have at least 4 point in the scheme; consequently, the team defeated by B must have won 3 games, i.e., the team defeated by B also won against B, an impossibility. Now, let us suppose B defeated A. Then A (with a 2-1 win-loss record) defeated C and D; C and D both defeated B (with a 1-2 win-loss record). Then A would have at least 4 points and B would have 3 points in the scheme. Therefore, if B has a 1-2 loss record and A has a better win-loss record, then A will always rank higher in the point scheme.

Case 3. Suppose B has a 2-1 record and A has a 3-0 record. Then C and D lost to both A and B. Then A has 6 points (since C either won or lost against D) and B has 3 points.

f)

Choose $M = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \end{bmatrix}$.

Then $(M + M^2)T = \begin{bmatrix} 4 \\ 5 \\ 14 \\ 2 \\ 11 \\ 7 \end{bmatrix}$.

It is possible for a team, see Team B, with a lower percentage of wins but still be ranked higher in this point scheme than a team with a higher percentage of wins, see Team A. Tabulated below are the winning percentages of the 6 teams.

Team	Win%
A	40%
B	20%
C	80%
D	20%
E	80%
F	60%

g)

$(2M + M^2)T = \begin{bmatrix} 6 \\ 6 \\ 18 \\ 3 \\ 15 \\ 10 \end{bmatrix}$

Each team gets 2 points for each team they defeat, plus a point for each win their defeated teams register.

Team A earned 6 points. A breakdown is: Team A got 4 points for defeating teams B and D; since B and D each won 1 game, team A earned 2 secondary points.

h) Yes, it is possible for the ranking to change. From parts f) and g), one sees that the ranking of team A was better using $(M + M^2)T$.

For Thought

1. True, $AB = \begin{bmatrix} 10-9 & -6+6 \\ 15-15 & -9+10 \end{bmatrix} =$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 10-9 & 15-15 \\ -6+6 & -9+10 \end{bmatrix} = BA.$

2. True, $AB = BA = I$ by Exercise 1.

3. True, A is the inverse of B by Exercise 1.

4. False, AC is undefined, although CA is defined. **5.** True

6. False, a non-square matrix has no inverse.

7. False, the coefficient matrix is $\begin{bmatrix} 2 & 3 \\ 3 & 1 \end{bmatrix}$.

8. True, since $A^{-1} = B$ then $A^{-1}D =$

$= \begin{bmatrix} 5 & -3 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 11 \\ 19 \end{bmatrix} = \begin{bmatrix} -2 \\ 5 \end{bmatrix}.$

9. False, $(-2, 5)$ does not satisfy $3x + y = 19$.

10. False, the solution is $\begin{bmatrix} 2 & 3 \\ 3 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 3 \\ -7 \end{bmatrix}$.

9.4 Exercises

1.

$$\begin{aligned} AI &= \begin{bmatrix} 1 & 3 \\ 4 & 6 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1(1)+3(0) & 1(0)+3(1) \\ 4(1)+6(0) & 4(0)+6(1) \end{bmatrix} \\ &= \begin{bmatrix} 1 & 3 \\ 4 & 6 \end{bmatrix} \\ &= A \end{aligned}$$

and

$$\begin{aligned} IA &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 4 & 6 \end{bmatrix} \\ &= \begin{bmatrix} 1(1)+0(4) & 1(3)+0(6) \\ 0(1)+1(4) & 0(3)+1(6) \end{bmatrix} \\ &= \begin{bmatrix} 1 & 3 \\ 4 & 6 \end{bmatrix} \\ &= A \end{aligned}$$

2.

$$\begin{aligned} AI &= \begin{bmatrix} 3 & 2 \\ 5 & 9 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 3(1)+2(0) & 3(0)+2(1) \\ 5(1)+9(0) & 5(0)+9(1) \end{bmatrix} \\ &= \begin{bmatrix} 3 & 2 \\ 5 & 9 \end{bmatrix} \\ &= A \end{aligned}$$

and

$$\begin{aligned} IA &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 5 & 9 \end{bmatrix} \\ &= \begin{bmatrix} 1(3)+0(5) & 1(2)+0(9) \\ 0(3)+1(5) & 0(2)+1(9) \end{bmatrix} \\ &= \begin{bmatrix} 3 & 2 \\ 5 & 9 \end{bmatrix} \\ &= A \end{aligned}$$

3. $AI = \begin{bmatrix} 3 & 2 & 1 \\ 5 & 6 & 2 \\ 7 & 8 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} =$

$\begin{bmatrix} 3(1)+2(0)+1(0) & 3(0)+2(1)+1(0) & 3(0)+2(0)+1(1) \\ 5(1)+6(0)+2(0) & 5(0)+6(1)+2(0) & 5(0)+6(0)+2(1) \\ 7(1)+8(0)+3(0) & 7(0)+8(1)+3(0) & 7(0)+8(0)+3(1) \end{bmatrix}$

$= \begin{bmatrix} 3 & 2 & 1 \\ 5 & 6 & 2 \\ 7 & 8 & 3 \end{bmatrix} = A$

and

$IA = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 & 1 \\ 5 & 6 & 2 \\ 7 & 8 & 3 \end{bmatrix} =$

$\begin{bmatrix} 1(3)+0(5)+0(7) & 1(2)+0(6)+0(8) & 1(1)+0(2)+0(3) \\ 0(3)+1(5)+0(7) & 0(2)+1(6)+0(8) & 0(1)+1(2)+0(3) \\ 0(3)+0(5)+1(7) & 0(2)+0(6)+1(8) & 0(1)+0(2)+1(3) \end{bmatrix}$

$= \begin{bmatrix} 3 & 2 & 1 \\ 5 & 6 & 2 \\ 7 & 8 & 3 \end{bmatrix} = A$

4. $AI = \begin{bmatrix} 3 & 4 & 1 \\ 2 & 2 & 5 \\ 0 & 3 & 6 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} =$

$\begin{bmatrix} 3(1)+4(0)+1(0) & 3(0)+4(1)+1(0) & 3(0)+4(0)+1(1) \\ 2(1)+2(0)+5(0) & 2(0)+2(1)+5(0) & 2(0)+2(0)+5(1) \\ 0(1)+3(0)+6(0) & 0(0)+3(1)+6(0) & 0(0)+3(0)+6(1) \end{bmatrix}$

$$= \begin{bmatrix} 3 & 4 & 1 \\ 2 & 2 & 5 \\ 0 & 3 & 6 \end{bmatrix} = A$$

and

$$IA = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 4 & 1 \\ 2 & 2 & 5 \\ 0 & 3 & 6 \end{bmatrix} =$$

$$\begin{bmatrix} 1(3)+0(2)+0(0) & 1(4)+0(2)+0(3) & 1(1)+0(5)+0(6) \\ 0(3)+1(2)+0(0) & 0(4)+1(2)+0(3) & 0(1)+1(5)+0(6) \\ 0(3)+0(2)+1(0) & 0(4)+0(2)+1(3) & 0(1)+0(5)+1(6) \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 4 & 1 \\ 2 & 2 & 5 \\ 0 & 3 & 6 \end{bmatrix} = A$$

5.

$$I \begin{bmatrix} -3 & 5 \\ 12 & 6 \end{bmatrix} = \begin{bmatrix} -3 & 5 \\ 12 & 6 \end{bmatrix}$$

6.

$$\begin{bmatrix} 4 & 8 \\ 9 & -2 \end{bmatrix} I = \begin{bmatrix} 4 & 8 \\ 9 & -2 \end{bmatrix}$$

7.

$$\begin{bmatrix} -8+9 & -6+6 \\ 12-12 & 9-8 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

8.

$$\begin{bmatrix} 3-2 & -2+2 \\ 3-3 & -2+3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

9.

$$\begin{bmatrix} 5-4 & -4+4 \\ 5-5 & -4+5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

10.

$$\begin{bmatrix} -5/3+8/3 & 2/3-2/3 \\ -20/3+20/3 & 8/3-5/3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

11.

$$\begin{bmatrix} 3 & 5 & 1 \\ 4 & 5 & 7 \\ 4 & 9 & 2 \end{bmatrix} I = \begin{bmatrix} 3 & 5 & 1 \\ 4 & 5 & 7 \\ 4 & 9 & 2 \end{bmatrix}$$

12.

$$I \begin{bmatrix} 4 & 0 & 5 \\ 0 & 7 & 9 \\ 3 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 5 \\ 0 & 7 & 9 \\ 3 & 1 & 2 \end{bmatrix}$$

13.

$$\begin{bmatrix} 0+0+1 & 1+0-1 & -3+0+3 \\ 0+0+0 & 1+0+0 & -3+3+0 \\ 0+0+0 & 0+0+0 & 0+1+0 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

14.

$$\begin{bmatrix} 0.8+0.2+0 & 0.4-0.4+0 & -0.8+0.8+0 \\ 0.4+0-0.4 & 0.2+0+0.8 & -0.4+0+0.4 \\ 0+0.2-0.2 & 0-0.4+0.4 & 0+0.8+0.2 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

15.

$$\begin{bmatrix} 0.5+0.5+0 & -0.5+0.5+0 & 0.5-0.5+0 \\ 0+0.5-0.5 & 0+0.5+0.5 & 0-0.5+0.5 \\ 0.5+0-0.5 & -0.5+0+0.5 & 0.5+0+0.5 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

16.

$$\begin{bmatrix} 0.8+0.2+0 & 0.4+0-0.4 & 0+0.4-0.4 \\ 0.4-0.4+0 & 0.2+0+0.8 & 0-0.8+0.8 \\ -0.4+0.4+0 & -0.2+0+0.2 & 0+0.8+0.2 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

17.

Yes, since $\begin{bmatrix} 3 & 1 \\ 11 & 4 \end{bmatrix} \begin{bmatrix} 4 & -1 \\ -11 & 3 \end{bmatrix} =$

$\begin{bmatrix} 12-11 & -3+3 \\ 44-44 & -11+12 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and

similarly $\begin{bmatrix} 4 & -1 \\ -11 & 3 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 11 & 4 \end{bmatrix} = I.$

18.

Yes, since $\begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} =$

$\begin{bmatrix} (1/2)2 & 0 \\ 0 & (1/2)2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and

similarly $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix} = I.$

19.

No, since $\begin{bmatrix} 1/2 & -1 \\ 3 & -12 \end{bmatrix} \begin{bmatrix} 4 & 2 \\ 1 & 1 \end{bmatrix} =$

$\begin{bmatrix} 2-1 & 1-1 \\ 12-12 & 6-12 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -6 \end{bmatrix} \neq I.$

20. Yes, since $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} =$

$\begin{bmatrix} 1 & -2+2 & 1-4+3 \\ 0 & 1 & -2+2 \\ 0 & 0 & 1 \end{bmatrix} = I$ and

similarly $\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = I.$

21. No, since only square matrices may have inverses. **22.** No, since only square matrices may have inverses.

23. On $\left[\begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ 0 & 2 & 0 & 1 \end{array}\right]$, use $-2R_2 + R_1 \to R_1$ to get

$\left[\begin{array}{cc|cc} 1 & 0 & 1 & -2 \\ 0 & 2 & 0 & 1 \end{array}\right]$, use $\frac{1}{2}R_2 \to R_2$ to get

$\left[\begin{array}{cc|cc} 1 & 0 & 1 & -2 \\ 0 & 1 & 0 & 1/2 \end{array}\right]$. Then $A^{-1} = \begin{bmatrix} 1 & -2 \\ 0 & 1/2 \end{bmatrix}$.

24. On $\left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{array}\right]$, use $3R_2 + R_1 \to R_1$

to get $\left[\begin{array}{cc|cc} 1 & 0 & 1 & 3 \\ 0 & -1 & 0 & 1 \end{array}\right]$, use $-1R_2 \to R_2$ to get

$\left[\begin{array}{cc|cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 0 & -1 \end{array}\right]$. So $A^{-1} = \begin{bmatrix} 1 & 3 \\ 0 & -1 \end{bmatrix}$.

25. On $\left[\begin{array}{cc|cc} 1 & 6 & 1 & 0 \\ 1 & 9 & 0 & 1 \end{array}\right]$, use $-1R_1 + R_2 \to R_2$ to get

$\left[\begin{array}{cc|cc} 1 & 6 & 1 & 0 \\ 0 & 3 & -1 & 1 \end{array}\right]$, use $-2R_2 + R_1 \to R_1$ to get

$\left[\begin{array}{cc|cc} 1 & 0 & 3 & -2 \\ 0 & 3 & -1 & 1 \end{array}\right]$, use $\frac{1}{3}R_2 \to R_2$ to get

$\left[\begin{array}{cc|cc} 1 & 0 & 3 & -2 \\ 0 & 1 & -1/3 & 1/3 \end{array}\right]$.

Thus, $A^{-1} = \begin{bmatrix} 3 & -2 \\ -1/3 & 1/3 \end{bmatrix}$.

26. On $\left[\begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ 3 & 8 & 0 & 1 \end{array}\right]$, use $-3R_1 + R_2 \to R_2$ to

get $\left[\begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ 0 & -4 & -3 & 1 \end{array}\right]$, use $R_1 + R_2 \to R_1$ to

get $\left[\begin{array}{cc|cc} 1 & 0 & -2 & 1 \\ 0 & -4 & -3 & 1 \end{array}\right]$, use $-\frac{1}{4}R_2 \to R_2$

to get $\left[\begin{array}{cc|cc} 1 & 0 & -2 & 1 \\ 0 & 1 & 3/4 & -1/4 \end{array}\right]$.

Then $A^{-1} = \begin{bmatrix} -2 & 1 \\ 3/4 & -1/4 \end{bmatrix}$.

27. On $\left[\begin{array}{cc|cc} -2 & -3 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{array}\right]$, use $R_2 + R_1 \to R_1$ to get

$\left[\begin{array}{cc|cc} 1 & 1 & 1 & 1 \\ 3 & 4 & 0 & 1 \end{array}\right]$, use $-3R_1 + R_2 \to R_2$ to get

$\left[\begin{array}{cc|cc} 1 & 1 & 1 & 1 \\ 0 & 1 & -3 & -2 \end{array}\right]$, use $-1R_2 + R_1 \to R_1$ to get

$\left[\begin{array}{cc|cc} 1 & 0 & 4 & 3 \\ 0 & 1 & -3 & -2 \end{array}\right]$. So $A^{-1} = \begin{bmatrix} 4 & 3 \\ -3 & -2 \end{bmatrix}$.

28. On $\left[\begin{array}{cc|cc} 3 & 4 & 1 & 0 \\ 4 & 5 & 0 & 1 \end{array}\right]$, use $-1R_1 + R_2 \to R_1$ to get

$\left[\begin{array}{cc|cc} 1 & 1 & -1 & 1 \\ 4 & 5 & 0 & 1 \end{array}\right]$, use $-4R_1 + R_2 \to R_2$ to get

$\left[\begin{array}{cc|cc} 1 & 1 & -1 & 1 \\ 0 & 1 & 4 & -3 \end{array}\right]$, use $-1R_2 + R_1 \to R_1$ to get

$\left[\begin{array}{cc|cc} 1 & 0 & -5 & 4 \\ 0 & 1 & 4 & -3 \end{array}\right]$. So $A^{-1} = \begin{bmatrix} -5 & 4 \\ 4 & -3 \end{bmatrix}$.

29. On $\left[\begin{array}{cc|cc} 1 & -5 & 1 & 0 \\ -1 & 3 & 0 & 1 \end{array}\right]$, use $R_1 + R_2 \to R_2$ to get

$\left[\begin{array}{rr|rr} 1 & -5 & 1 & 0 \\ 0 & -2 & 1 & 1 \end{array}\right]$, use $-\frac{1}{2}R_2 \to R_2$ to get

$\left[\begin{array}{rr|rr} 1 & -5 & 1 & 0 \\ 0 & 1 & -1/2 & -1/2 \end{array}\right]$, use

$5R_2 + R_1 \to R_1$ to get

$\left[\begin{array}{rr|rr} 1 & 0 & -3/2 & -5/2 \\ 0 & 1 & -1/2 & -1/2 \end{array}\right]$.

Then $A^{-1} = \left[\begin{array}{rr} -3/2 & -5/2 \\ -1/2 & -1/2 \end{array}\right]$.

30.

On $\left[\begin{array}{rr|rr} 4 & 3 & 1 & 0 \\ -3 & -2 & 0 & 1 \end{array}\right]$, use $R_1 + R_2 \to R_1$ to get

$\left[\begin{array}{rr|rr} 1 & 1 & 1 & 1 \\ -3 & -2 & 0 & 1 \end{array}\right]$, use $3R_1 + R_2 \to R_2$ to get

$\left[\begin{array}{rr|rr} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 4 \end{array}\right]$, use $-1R_2 + R_1 \to R_1$ to get

$\left[\begin{array}{rr|rr} 1 & 0 & -2 & -3 \\ 0 & 1 & 3 & 4 \end{array}\right]$. So $A^{-1} = \left[\begin{array}{rr} -2 & -3 \\ 3 & 4 \end{array}\right]$.

31.

On $\left[\begin{array}{rr|rr} -1 & 5 & 1 & 0 \\ 2 & -10 & 0 & 1 \end{array}\right]$, use $R_1 + R_2 \to R_1$ and $\frac{1}{2}R_2 \to R_2$ to get

$\left[\begin{array}{rr|rr} 1 & -5 & 1 & 1 \\ 1 & -5 & 0 & 1/2 \end{array}\right]$, use $-1R_2 + R_1 \to R_1$ to get

$\left[\begin{array}{rr|rr} 0 & 0 & 1 & 1/2 \\ 1 & -5 & 0 & 1/2 \end{array}\right]$. So A has no inverse.

32.

On $\left[\begin{array}{rr|rr} 2 & 6 & 1 & 0 \\ 1 & 3 & 0 & 1 \end{array}\right]$, use $2R_2 + (-1R_1) \to R_1$ to get

$\left[\begin{array}{rr|rr} 0 & 0 & -1 & 2 \\ 1 & 3 & 0 & 1 \end{array}\right]$. Thus, A has no inverse.

33.

On $\left[\begin{array}{rrr|rrr} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & -1 & -1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 & 1 \end{array}\right]$, use

$R_2 + R_1 \to R_1$ and $-1R_3 + R_1 \to R_3$ to get

$\left[\begin{array}{rrr|rrr} 1 & 0 & -1 & 1 & 1 & 0 \\ 0 & -1 & -1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & -1 \end{array}\right]$, use

$R_2 + R_3 \to R_2$ to get

$\left[\begin{array}{rrr|rrr} 1 & 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & -1 \\ 0 & 1 & 1 & 1 & 0 & 1 \end{array}\right]$. So A^{-1} does not exist.

34.

On $\left[\begin{array}{rrr|rrr} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 2 & 1 & 5 & 0 & 0 & -1 \end{array}\right]$, use

$R_1 + R_2 + (-1R_3) \to R_3$ to get

$\left[\begin{array}{rrr|rrr} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & -1 \end{array}\right]$.

So A has no inverse.

35.

On $\left[\begin{array}{rrr|rrr} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & -1 & -1 & 0 & 1 & 0 \\ 1 & -1 & 1 & 0 & 0 & 1 \end{array}\right]$, use

$R_1 + R_2 \to R_2$ and $R_2 + (-1R_3) \to R_3$ to get

$\left[\begin{array}{rrr|rrr} 1 & 1 & 1 & 1 & 0 & 0 \\ 2 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & -2 & 0 & 1 & -1 \end{array}\right]$, use

$-2R_1 + R_2 \to R_2$ and $-\frac{1}{2}R_3 \to R_3$ to get

$\left[\begin{array}{rrr|rrr} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -2 & -2 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1/2 & 1/2 \end{array}\right]$, use

$-\frac{1}{2}R_2 \to R_2$ to get

$\left[\begin{array}{rrr|rrr} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1/2 & -1/2 & 0 \\ 0 & 0 & 1 & 0 & -1/2 & 1/2 \end{array}\right]$, use

$-1R_2 + R_1 \to R_1$ to get

$\left[\begin{array}{rrr|rrr} 1 & 0 & 0 & 1/2 & 1/2 & 0 \\ 0 & 1 & 1 & 1/2 & -1/2 & 0 \\ 0 & 0 & 1 & 0 & -1/2 & 1/2 \end{array}\right]$, use

$-1R_3 + R_2 \to R_2$ to get

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & 1/2 & 0 \\ 0 & 1 & 0 & 1/2 & 0 & -1/2 \\ 0 & 0 & 1 & 0 & -1/2 & 1/2 \end{array}\right].$$

So $A^{-1} = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & -1/2 \\ 0 & -1/2 & 1/2 \end{bmatrix}$.

36.

On $\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 1 & 3 & 0 & 0 & 0 & 1 \end{array}\right]$, use

$-1R_3 + R_1 \to R_3$ to get

$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 0 & -3 & 2 & 1 & 0 & -1 \end{array}\right]$, use

$-1R_2 + (-1R_3) \to R_2$ to get

$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & -2 & -1 & -1 & 1 \\ 0 & -3 & 2 & 1 & 0 & -1 \end{array}\right]$, use

$R_1 + R_2 \to R_2$ to get

$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & -1 & 1 \\ 0 & -3 & 2 & 1 & 0 & -1 \end{array}\right]$, use

$-1R_1 + R_2 \to R_2$ to get

$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & -2 & -1 & -1 & 1 \\ 0 & -3 & 2 & 1 & 0 & -1 \end{array}\right]$, use

$R_1 + R_2 \to R_1$ and $R_3 + R_2 \to R_2$ to get

$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & -1 & 1 \\ 0 & -2 & 0 & 0 & -1 & 0 \\ 0 & -3 & 2 & 1 & 0 & -1 \end{array}\right]$, use

$-\frac{1}{2}R_2 \to R_2$ and $\frac{1}{2}R_3 \to R_3$ to get

$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & 0 & 1/2 & 0 \\ 0 & -3/2 & 1 & 1/2 & 0 & -1/2 \end{array}\right]$, use

$-1R_2 + R_1 \to R_1$ to get

$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & -3/2 & 1 \\ 0 & 1 & 0 & 0 & 1/2 & 0 \\ 0 & -3/2 & 1 & 1/2 & 0 & -1/2 \end{array}\right]$, use

$\frac{3}{2}R_2 + R_3 \to R_3$ to get

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & -3/2 & 1 \\ 0 & 1 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & 1/2 & 3/4 & -1/2 \end{array}\right].$$

Thus, $A^{-1} = \begin{bmatrix} 0 & -3/2 & 1 \\ 0 & 1/2 & 0 \\ 1/2 & 3/4 & -1/2 \end{bmatrix}$.

37.

On $\left[\begin{array}{ccc|ccc} 0 & 2 & 0 & 1 & 0 & 0 \\ 3 & 3 & 2 & 0 & 1 & 0 \\ 2 & 5 & 1 & 0 & 0 & 1 \end{array}\right]$, use

$-1R_3 + R_2 \to R_3$ to get

$\left[\begin{array}{ccc|ccc} 0 & 2 & 0 & 1 & 0 & 0 \\ 3 & 3 & 2 & 0 & 1 & 0 \\ 1 & -2 & 1 & 0 & 1 & -1 \end{array}\right]$, use

$R_3 \to R_1$ and $R_1 \to R_3$ to get

$\left[\begin{array}{ccc|ccc} 1 & -2 & 1 & 0 & 1 & -1 \\ 3 & 3 & 2 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 & 0 & 0 \end{array}\right]$, use

$R_3 + R_1 \to R_1$ and $-1R_3 + R_2 \to R_2$ to get

$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 1 & -1 \\ 3 & 1 & 2 & -1 & 1 & 0 \\ 0 & 2 & 0 & 1 & 0 & 0 \end{array}\right]$, use

$-3R_1 + R_2 \to R_2$ to get

$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 1 & -1 \\ 0 & 1 & -1 & -4 & -2 & 3 \\ 0 & 2 & 0 & 1 & 0 & 0 \end{array}\right]$, use

$-2R_2 + R_3 \to R_3$ to get

$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 1 & -1 \\ 0 & 1 & -1 & -4 & -2 & 3 \\ 0 & 0 & 2 & 9 & 4 & -6 \end{array}\right]$, use

$\frac{1}{2}R_3 \to R_3$ to get

$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 1 & -1 \\ 0 & 1 & -1 & -4 & -2 & 3 \\ 0 & 0 & 1 & 9/2 & 2 & -3 \end{array}\right]$, use

$R_2 + R_3 \to R_2$ to get

$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 1 & -1 \\ 0 & 1 & 0 & 1/2 & 0 & 0 \\ 0 & 0 & 1 & 9/2 & 2 & -3 \end{array}\right]$, use

$-1R_3 + R_1 \to R_1$ to get

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -7/2 & -1 & 2 \\ 0 & 1 & 0 & 1/2 & 0 & 0 \\ 0 & 0 & 1 & 9/2 & 2 & -3 \end{array}\right].$$

So $A^{-1} = \begin{bmatrix} -7/2 & -1 & 2 \\ 1/2 & 0 & 0 \\ 9/2 & 2 & -3 \end{bmatrix}$.

38.

On $\left[\begin{array}{rrr|rrr} 4 & 1 & -3 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ -3 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$, use

$R_1 + R_3 \to R_1$ to get

$\left[\begin{array}{rrr|rrr} 1 & 2 & -1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ -3 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$, use

$3R_1 + R_3 \to R_3$ to get

$\left[\begin{array}{rrr|rrr} 1 & 2 & -1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 7 & -1 & 3 & 0 & 4 \end{array}\right]$, use

$-1R_3 + R_2 \to R_3$ to get

$\left[\begin{array}{rrr|rrr} 1 & 2 & -1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & -6 & 1 & -3 & 1 & -4 \end{array}\right]$, use

$-2R_2 + R_1 \to R_1$ to get

$\left[\begin{array}{rrr|rrr} 1 & 0 & -1 & 1 & -2 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & -6 & 1 & -3 & 1 & -4 \end{array}\right]$, use

$6R_2 + R_3 \to R_3$ to get

$\left[\begin{array}{rrr|rrr} 1 & 0 & -1 & 1 & -2 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -3 & 7 & -4 \end{array}\right]$, use

$R_1 + R_3 \to R_1$ to get

$\left[\begin{array}{rrr|rrr} 1 & 0 & 0 & -2 & 5 & -3 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -3 & 7 & -4 \end{array}\right]$,

So $A^{-1} = \begin{bmatrix} -2 & 5 & -3 \\ 0 & 1 & 0 \\ -3 & 7 & -4 \end{bmatrix}$.

39.

On $\left[\begin{array}{rrr|rrr} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 2 & 2 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$, use

$2R_1 + (-1R_3) \to R_3$ and $\frac{1}{2}R_2 \to R_2$ to get

$\left[\begin{array}{rrr|rrr} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1/2 & 0 \\ 0 & -1 & 2 & 2 & 0 & -1 \end{array}\right]$, use

$R_2 + R_3 \to R_3$ to get

$\left[\begin{array}{rrr|rrr} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1/2 & 0 \\ 0 & 0 & 3 & 2 & 1/2 & -1 \end{array}\right]$, use

$\frac{1}{3}R_3 \to R_3$ to get

$\left[\begin{array}{rrr|rrr} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & 2/3 & 1/6 & -1/3 \end{array}\right]$, use

$-1R_3 + R_2 \to R_2$ and $-1R_3 + R_1 \to R_1$ to get

$\left[\begin{array}{rrr|rrr} 1 & 0 & 0 & 1/3 & -1/6 & 1/3 \\ 0 & 1 & 0 & -2/3 & 1/3 & 1/3 \\ 0 & 0 & 1 & 2/3 & 1/6 & -1/3 \end{array}\right]$.

Then $A^{-1} = \begin{bmatrix} 1/3 & -1/6 & 1/3 \\ -2/3 & 1/3 & 1/3 \\ 2/3 & 1/6 & -1/3 \end{bmatrix}$.

40.

On $\left[\begin{array}{rrr|rrr} 1 & 3 & 0 & 1 & 0 & 0 \\ 0 & 3 & -2 & 0 & 1 & 0 \\ 0 & -5 & 3 & 0 & 0 & 1 \end{array}\right]$, use

$-1R_2 + R_1 \to R_1$ to get

$\left[\begin{array}{rrr|rrr} 1 & 0 & 2 & 1 & -1 & 0 \\ 0 & 3 & -2 & 0 & 1 & 0 \\ 0 & -5 & 3 & 0 & 0 & 1 \end{array}\right]$, use

$2R_2 + R_3 \to R_3$ and $\frac{1}{3}R_2 \to R_2$ to get

$\left[\begin{array}{rrr|rrr} 1 & 0 & 2 & 1 & -1 & 0 \\ 0 & 1 & -2/3 & 0 & 1/3 & 0 \\ 0 & 1 & -1 & 0 & 2 & 1 \end{array}\right]$, use

$-1R_3 + R_2 \to R_3$ to get

$\left[\begin{array}{rrr|rrr} 1 & 0 & 2 & 1 & -1 & 0 \\ 0 & 1 & -2/3 & 0 & 1/3 & 0 \\ 0 & 0 & 1/3 & 0 & -5/3 & -1 \end{array}\right]$, use

$2R_3 + R_2 \to R_2$ to get

$\left[\begin{array}{rrr|rrr} 1 & 0 & 2 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & -3 & -2 \\ 0 & 0 & 1/3 & 0 & -5/3 & -1 \end{array}\right]$, use

$3R_3 \to R_3$ to get

$\left[\begin{array}{rrr|rrr} 1 & 0 & 2 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & -3 & -2 \\ 0 & 0 & 1 & 0 & -5 & -3 \end{array}\right]$, use

$-2R_3 + R_1 \to R_1$ to get

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 9 & 6 \\ 0 & 1 & 0 & 0 & -3 & -2 \\ 0 & 0 & 1 & 0 & -5 & -3 \end{array}\right].$$

Then $A^{-1} = \begin{bmatrix} 1 & 9 & 6 \\ 0 & -3 & -2 \\ 0 & -5 & -3 \end{bmatrix}$.

41.

On $\left[\begin{array}{ccc|ccc} 0 & 4 & 2 & 1 & 0 & 0 \\ 0 & 3 & 2 & 0 & 1 & 0 \\ 1 & -1 & 1 & 0 & 0 & 1 \end{array}\right]$, use

$R_1 \to R_3$ and $R_3 \to R_1$ to get

$\left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 0 & 0 & 1 \\ 0 & 3 & 2 & 0 & 1 & 0 \\ 0 & 4 & 2 & 1 & 0 & 0 \end{array}\right]$, use

$-1R_2 + R_3 \to R_2$ to get

$\left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 4 & 2 & 1 & 0 & 0 \end{array}\right]$, use

$R_1 + R_2 \to R_1$ to get

$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & -1 & 1 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 4 & 2 & 1 & 0 & 0 \end{array}\right]$, use

$-4R_2 + R_3 \to R_3$ to get

$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & -1 & 1 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 2 & -3 & 4 & 0 \end{array}\right]$, use

$\frac{1}{2}R_3 \to R_3$ to get

$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & -1 & 1 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -3/2 & 2 & 0 \end{array}\right]$, use

$-1R_3 + R_1 \to R_1$ to get

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 5/2 & -3 & 1 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -3/2 & 2 & 0 \end{array}\right].$$

Then $A^{-1} = \begin{bmatrix} 5/2 & -3 & 1 \\ 1 & -1 & 0 \\ -3/2 & 2 & 0 \end{bmatrix}$.

42.

On $\left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 2 & 3 & -1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$, use

$2R_1 + (-1R_2) \to R_2$ to get

$\left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 2 & -1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$, use

$-1R_3 + R_2 \to R_3$ and
$-1R_2 + (-1R_3) + R_1 \to R_1$ to get

$\left[\begin{array}{ccc|ccc} 1 & 0 & -3 & -1 & 1 & -1 \\ 0 & 1 & 1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 & -1 \end{array}\right]$, use

$R_2 + R_3 \to R_2$ and $-1R_3 \to R_3$ to get

$\left[\begin{array}{ccc|ccc} 1 & 0 & -3 & -1 & 1 & -1 \\ 0 & 1 & 0 & 4 & -2 & -1 \\ 0 & 0 & 1 & -2 & 1 & 1 \end{array}\right]$, use

$4R_3 + R_1 \to R_1$ to get

$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & -9 & 5 & 3 \\ 0 & 1 & 0 & 4 & -2 & -1 \\ 0 & 0 & 1 & -2 & 1 & 1 \end{array}\right]$, use

$-1R_3 + R_1 \to R_1$ to get

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -7 & 4 & 2 \\ 0 & 1 & 0 & 4 & -2 & -1 \\ 0 & 0 & 1 & -2 & 1 & 1 \end{array}\right].$$

So $A^{-1} = \begin{bmatrix} -7 & 4 & 2 \\ 4 & -2 & -1 \\ -2 & 1 & 1 \end{bmatrix}$.

43.

On $\left[\begin{array}{cccc|cccc} 1 & 2 & 3 & 4 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$, use

$-2R_2 + R_1 \to R_1$ to get

$\left[\begin{array}{cccc|cccc} 1 & 0 & -1 & -2 & 1 & -2 & 0 & 0 \\ 0 & 1 & 2 & 3 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$, use

$-2R_4 + R_3 \to R_3$ to get

$\left[\begin{array}{cccc|cccc} 1 & 0 & -1 & -2 & 1 & -2 & 0 & 0 \\ 0 & 1 & 2 & 3 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$, use

$-2R_3 + R_2 \to R_2$ and $-2R_4 + R_3 \to R_3$ to get

$$\left[\begin{array}{cccc|cccc} 1 & 0 & -1 & -2 & 1 & -2 & 0 & 0 \\ 0 & 1 & 0 & 3 & 0 & 1 & -2 & 4 \\ 0 & 0 & 1 & -2 & 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right], \text{ use}$$

$R_1 + R_3 \to R_1$ to get

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & -4 & 1 & -2 & 1 & -4 \\ 0 & 1 & 0 & 3 & 0 & 1 & -2 & 4 \\ 0 & 0 & 1 & -2 & 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right], \text{ use}$$

$2R_4 + R_3 \to R_3$, $-3R_4 + R_2 \to R_2$ and $4R_4 + R_1 \to R_1$ to get

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & -2 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right].$$

Then $A^{-1} = \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

44.

On $\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 3 & -2 & 1 & 0 & 0 & 0 & 1 & 0 \\ 5 & 3 & -2 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$, use

$2R_1 + R_2 \to R_2$, $-3R_1 + R_3 \to R_3$, and $-5R_1 + R_4 \to R_4$ to get

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 & -3 & 0 & 1 & 0 \\ 0 & 3 & -2 & 1 & -5 & 0 & 0 & 1 \end{array}\right], \text{ use}$$

$2R_2 + R_3 \to R_3$ and $-3R_2 + R_4 \to R_4$ to get

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & -2 & 1 & -11 & -3 & 0 & 1 \end{array}\right], \text{ use}$$

$2R_3 + R_4 \to R_4$ to get

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 1 & -9 & 1 & 2 & 1 \end{array}\right].$$

Then $A^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ -9 & 1 & 2 & 1 \end{bmatrix}$.

45.

Since the coefficient matrix is $A = \begin{bmatrix} 1 & 6 \\ 1 & 9 \end{bmatrix}$,

$A^{-1}\begin{bmatrix} -3 \\ -6 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ -1/3 & 1/3 \end{bmatrix}\begin{bmatrix} -3 \\ -6 \end{bmatrix} =$

$= \begin{bmatrix} 3 \\ -1 \end{bmatrix}$. The solution set is $\{(3, -1)\}$.

46.

Since the coefficient matrix is $A = \begin{bmatrix} 1 & 4 \\ 3 & 8 \end{bmatrix}$,

$A^{-1}\begin{bmatrix} 5 \\ 7 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 3/4 & -1/4 \end{bmatrix}\begin{bmatrix} 5 \\ 7 \end{bmatrix} =$

$= \begin{bmatrix} -3 \\ 2 \end{bmatrix}$. The solution set is $\{(-3, 2)\}$.

47.

Since the coefficient matrix is $A = \begin{bmatrix} 1 & 6 \\ 1 & 9 \end{bmatrix}$,

$A^{-1}\begin{bmatrix} 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ -1/3 & 1/3 \end{bmatrix}\begin{bmatrix} 4 \\ 5 \end{bmatrix} =$

$= \begin{bmatrix} 2 \\ 1/3 \end{bmatrix}$. The solution set is $\{(2, 1/3)\}$.

48.

Since the coefficient matrix is $A = \begin{bmatrix} 1 & 4 \\ 3 & 8 \end{bmatrix}$,

$A^{-1}\begin{bmatrix} 1 \\ 5 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 3/4 & -1/4 \end{bmatrix}\begin{bmatrix} 1 \\ 5 \end{bmatrix} =$

$= \begin{bmatrix} 3 \\ -1/2 \end{bmatrix}$. The solution set is $\{(3, -1/2)\}$.

49.

Since the coefficient matrix is

$A = \begin{bmatrix} -2 & -3 \\ 3 & 4 \end{bmatrix}$, we get

$A^{-1}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} =$

$= \begin{bmatrix} 1 \\ -1 \end{bmatrix}$. The solution set is $\{(1, -1)\}$.

50.

Since the coefficient matrix is $A = \begin{bmatrix} 3 & 4 \\ 4 & 5 \end{bmatrix}$,

$$A^{-1}\begin{bmatrix}1\\2\end{bmatrix}=\begin{bmatrix}-5&4\\4&-3\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix}=$$

$=\begin{bmatrix}3\\-2\end{bmatrix}$. The solution set is $\{(3,-2)\}$.

51.
Since the coefficient matrix is
$A=\begin{bmatrix}1&-5\\-1&3\end{bmatrix}$, we obtain

$$A^{-1}\begin{bmatrix}-5\\1\end{bmatrix}=\begin{bmatrix}-3/2&-5/2\\-1/2&-1/2\end{bmatrix}\begin{bmatrix}-5\\1\end{bmatrix}=$$

$=\begin{bmatrix}5\\2\end{bmatrix}$. The solution set is $\{(5,2)\}$.

52.
Since the coefficient matrix is
$A=\begin{bmatrix}4&3\\-3&-2\end{bmatrix}$, we get

$$A^{-1}\begin{bmatrix}2\\-1\end{bmatrix}=\begin{bmatrix}-2&-3\\3&4\end{bmatrix}\begin{bmatrix}2\\-1\end{bmatrix}=$$

$=\begin{bmatrix}-1\\2\end{bmatrix}$. The solution set is $\{(-1,2)\}$.

53.
Since coefficient matrix is
$A=\begin{bmatrix}1&1&1\\1&-1&-1\\1&-1&1\end{bmatrix}$, we find $A^{-1}\begin{bmatrix}3\\-1\\5\end{bmatrix}=$

$$=\begin{bmatrix}1/2&1/2&0\\1/2&0&-1/2\\0&-1/2&1/2\end{bmatrix}\begin{bmatrix}3\\-1\\5\end{bmatrix}=\begin{bmatrix}1\\-1\\3\end{bmatrix}.$$

The solution set is $\{(1,-1,3)\}$.

54.
Since coefficient matrix is $A=\begin{bmatrix}1&0&2\\0&2&0\\1&3&0\end{bmatrix}$,

$$A^{-1}\begin{bmatrix}-4\\6\\7\end{bmatrix}=\begin{bmatrix}0&-3/2&1\\0&1/2&0\\1/2&3/4&-1/2\end{bmatrix}\begin{bmatrix}-4\\6\\7\end{bmatrix}$$

$=\begin{bmatrix}-2\\3\\-1\end{bmatrix}$. The solution set is $\{(-2,3,-1)\}$.

55.
Since the coefficient matrix is
$A=\begin{bmatrix}0&2&0\\3&3&2\\2&5&1\end{bmatrix}$, we get $A^{-1}\begin{bmatrix}6\\16\\19\end{bmatrix}=$

$$=\begin{bmatrix}-7/2&-1&2\\1/2&0&0\\9/2&2&-3\end{bmatrix}\begin{bmatrix}6\\16\\19\end{bmatrix}=\begin{bmatrix}1\\3\\2\end{bmatrix}.$$

The solution set is $\{(1,3,2)\}$.

56.
Since the coefficient matrix is
$A=\begin{bmatrix}4&1&-3\\0&1&0\\-3&1&2\end{bmatrix}$, we get $A^{-1}\begin{bmatrix}3\\-2\\-5\end{bmatrix}=$

$$=\begin{bmatrix}-2&5&-3\\0&1&0\\-3&7&-4\end{bmatrix}\begin{bmatrix}3\\-2\\-5\end{bmatrix}=\begin{bmatrix}-1\\-2\\-3\end{bmatrix}.$$

The solution set is $\{(-1,-2,-3)\}$.

57. The coefficient matrix of

$$\begin{aligned}0.3x+0.1y&=3\\2x+4y&=7\end{aligned}$$

is $A=\begin{bmatrix}0.3&0.1\\2&4\end{bmatrix}$. Note $A^{-1}\begin{bmatrix}3\\7\end{bmatrix}=$

$$\begin{bmatrix}4&-0.1\\-2&0.3\end{bmatrix}\begin{bmatrix}3\\7\end{bmatrix}=\begin{bmatrix}11.3\\-3.9\end{bmatrix}.$$

The solution set is $\{(11.3,-3.9)\}$.

58. The coefficient matrix of

$$\begin{aligned}2x-3y&=-7\\-x+y&=4\end{aligned}$$

is $A=\begin{bmatrix}2&-3\\-1&1\end{bmatrix}$. Note $A^{-1}\begin{bmatrix}-7\\4\end{bmatrix}=$

$$\begin{bmatrix}-1&-3\\-1&-2\end{bmatrix}\begin{bmatrix}-7\\4\end{bmatrix}=\begin{bmatrix}-5\\-1\end{bmatrix}.$$

The solution set is $\{(-5,-1)\}$.

59. Use the Gauss-Jordan method. On the augmented matrix $\left[\begin{array}{ccc|c}1&-1&1&5\\2&-1&3&1\\0&1&1&-9\end{array}\right]$,

use $-1R_1 + R_2 \to R_1$ to get

$$\left[\begin{array}{rrr|r} 1 & 0 & 2 & -4 \\ 2 & -1 & 3 & 1 \\ 0 & 1 & 1 & -9 \end{array}\right]\text{, use}$$

$-2R_1 + R_2 \to R_2$ to get

$$\left[\begin{array}{rrr|r} 1 & 0 & 2 & -4 \\ 0 & -1 & -1 & 9 \\ 0 & 1 & 1 & -9 \end{array}\right]\text{, use}$$

$R_2 + R_3 \to R_3$ and $-1R_2 \to R_2$ to get

$$\left[\begin{array}{rrr|r} 1 & 0 & 2 & -4 \\ 0 & 1 & 1 & -9 \\ 0 & 0 & 0 & 0 \end{array}\right]\text{. Since } y + z = -9$$

and $x + 2z = -4$, the solution set is

$\{(-2z - 4, -z - 9, z) \mid z \text{ is any real number }\}$.

60. Use the Gauss-Jordan method. On the augmented matrix $\left[\begin{array}{rrr|r} 1 & 1 & -1 & 4 \\ 2 & -3 & 1 & 2 \\ 4 & -1 & -1 & 6 \end{array}\right]$, use

$-1R_1 + R_2 \to R_1$ and $-2R_2 + R_3 \to R_2$ to get

$$\left[\begin{array}{rrr|r} 1 & -4 & 2 & -2 \\ 0 & 5 & -3 & 2 \\ 4 & -1 & -1 & 6 \end{array}\right]\text{, use}$$

$4R_1 + (-1R_3) \to R_3$ to get

$$\left[\begin{array}{rrr|r} 1 & -4 & 2 & -2 \\ 0 & 5 & -3 & 2 \\ 0 & -15 & 9 & -14 \end{array}\right]\text{, use}$$

$-\dfrac{1}{3}R_3 \to R_3$ to get

$$\left[\begin{array}{rrr|r} 1 & -4 & 2 & -2 \\ 0 & 5 & -3 & 2 \\ 0 & 5 & -3 & -14/3 \end{array}\right]\text{, use}$$

$-1R_3 + R_2 \to R_3$ to get

$$\left[\begin{array}{rrr|r} 1 & -4 & 2 & -2 \\ 0 & 5 & -3 & 2 \\ 0 & 0 & 0 & 20/3 \end{array}\right]\text{. Inconsistent and}$$

the solution set is $\emptyset$.

61.

Note coefficient matrix is $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 1 \\ 1 & 3 & 6 \end{bmatrix}$

and $A^{-1}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} =$

$$\begin{bmatrix} 7/4 & -1/4 & -1/4 \\ -11/12 & 5/12 & 1/12 \\ 1/6 & -1/6 & 1/6 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1/6 \\ 1/3 \end{bmatrix}.$$

The solution set is $\{(1/2, 1/6, 1/3)\}$.

62. Note the coefficient matrix is

$$A = \begin{bmatrix} .5 & -.25 & .1 \\ .2 & -.5 & .2 \\ .1 & .3 & -.5 \end{bmatrix}\text{. Then } A^{-1}\begin{bmatrix} 3 \\ -2 \\ -8 \end{bmatrix} =$$

$$= \begin{bmatrix} 5/2 & -5/4 & 0 \\ 30/19 & -65/19 & -20/19 \\ 55/38 & -175/76 & -50/19 \end{bmatrix}\begin{bmatrix} 3 \\ -2 \\ -8 \end{bmatrix} =$$

$$= \begin{bmatrix} 10 \\ 20 \\ 30 \end{bmatrix}\text{. The solution set is } \{(10, 20, 30)\}.$$

63.

Note $A^{-1} = \begin{bmatrix} -55/6 & 5/2 & 5/3 \\ 35/12 & -5/4 & 5/6 \\ 40/3 & 0 & -10/3 \end{bmatrix}$

and $A^{-1}\begin{bmatrix} 27 \\ 9 \\ 16 \end{bmatrix} = \begin{bmatrix} -165 \\ 97.5 \\ 240 \end{bmatrix}$.

The solution set is

$$\{(-165, 97.5, 240)\}.$$

64.

Note $A^{-1} = \begin{bmatrix} 7/30 & -1/30 & -1/3 \\ -1/60 & 13/60 & 1/6 \\ 1/10 & -3/10 & 0 \end{bmatrix}$ and

$A^{-1}\begin{bmatrix} 9 \\ -18 \\ 54 \end{bmatrix} = \begin{bmatrix} -15.3 \\ 4.95 \\ 6.3 \end{bmatrix}$.

The solution set is

$$\{(-15.3, 4.95, 6.3)\}.$$

65.

We get $A^{-1} = \begin{bmatrix} 68/133 & -36/133 & 8/19 \\ 10/19 & -12/19 & 6/19 \\ 127/133 & -122/133 & 6/19 \end{bmatrix}$

and $A^{-1}\begin{bmatrix} 16 \\ 24 \\ -8 \end{bmatrix} \approx \begin{bmatrix} -1.6842 \\ -9.2632 \\ -9.2632 \end{bmatrix}$.

The solution set is approximately

$$\{(-1.6842, -9.2632, -9.2632)\}.$$

66.

We get

$$A^{-1} = \frac{1}{17,229}\begin{bmatrix} 2,900 & -7,970 & 8,600 \\ -4,085 & -15,805 & 11,650 \\ -550 & -7,400 & 4,310 \end{bmatrix}$$

and $A^{-1}\begin{bmatrix} 100 \\ 250 \\ 300 \end{bmatrix} \approx \begin{bmatrix} 50.9316 \\ -50.1915 \\ -35.5215 \end{bmatrix}$.

Solution set is approximately

$$\{(50.9316, -50.1915, -35.5215)\}.$$

67.

Since $AA^{-1} = \begin{bmatrix} a & 7 \\ 3 & b \end{bmatrix}\begin{bmatrix} -b & 7 \\ 3 & -a \end{bmatrix} =$

$= \begin{bmatrix} 21-ab & 0 \\ 0 & 21-ab \end{bmatrix}$, $21 - ab = 1$ and

$ab = 20$. List of permissible pairs (a, b):

a	1	2	4	5	10	20
b	20	10	5	4	2	1

The matrices are $\begin{bmatrix} 1 & 7 \\ 3 & 20 \end{bmatrix}$, $\begin{bmatrix} 2 & 7 \\ 3 & 10 \end{bmatrix}$,

$\begin{bmatrix} 4 & 7 \\ 3 & 5 \end{bmatrix}$, $\begin{bmatrix} 5 & 7 \\ 3 & 4 \end{bmatrix}$,

$\begin{bmatrix} 10 & 7 \\ 3 & 2 \end{bmatrix}$, and $\begin{bmatrix} 20 & 7 \\ 3 & 1 \end{bmatrix}$.

68.

Since $AA^{-1} = \begin{bmatrix} a & a \\ 0 & c \end{bmatrix}\begin{bmatrix} a & a \\ 0 & c \end{bmatrix} =$

$= \begin{bmatrix} a^2 & a^2+ac \\ 0 & c^2 \end{bmatrix}$, then $a^2 = c^2 = 1$ and

$a^2 + ac = a(a+c) = 0$. Since $a = \pm 1$, $a + c = 0$ or $a = -c$. So $a = 1$, $c = -1$ or $a = -1$, $c = 1$. The matrices are

$\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$ and $\begin{bmatrix} -1 & -1 \\ 0 & 1 \end{bmatrix}$.

69. Let x and y be the costs of a dozen eggs and a magazine before taxes. Then

$$\begin{aligned} 0.08x + 0.05y &= 0.59 \\ x + y &= 8.79 - 0.59. \end{aligned}$$

If $A = \begin{bmatrix} .08 & .05 \\ 1 & 1 \end{bmatrix}$, then $A^{-1}\begin{bmatrix} .59 \\ 8.20 \end{bmatrix} =$

$= \begin{bmatrix} 100/3 & -5/3 \\ -100/3 & 8/3 \end{bmatrix}\begin{bmatrix} .59 \\ 8.20 \end{bmatrix} = \begin{bmatrix} 6 \\ 2.20 \end{bmatrix}$.

The eggs cost \$2.20 a dozen and the magazine costs \$6.

70. Let x and y be the prices of an iPhone and iPod, respectively. Then

$$\begin{aligned} x - y &= -80 \\ 120x + 48y &= 20640. \end{aligned}$$

If $A = \begin{bmatrix} 1 & -1 \\ 120 & 48 \end{bmatrix}$ then $A^{-1}\begin{bmatrix} -80 \\ 20640 \end{bmatrix} =$

$\begin{bmatrix} 48/168 & 1/168 \\ -120/168 & 1/168 \end{bmatrix}\begin{bmatrix} -80 \\ 20640 \end{bmatrix} = \begin{bmatrix} 100 \\ 180 \end{bmatrix}$.

A hot dog costs \$1.80 and a drink costs \$1.

71. Let x and y be the costs of one load of plywood and a load insulation, respectively. So

$$\begin{aligned} 4x + 6y &= 2500 \\ 3x + 5y &= 1950. \end{aligned}$$

If $A = \begin{bmatrix} 4 & 6 \\ 3 & 5 \end{bmatrix}$ then $A^{-1}\begin{bmatrix} 2500 \\ 1950 \end{bmatrix} =$

$= \begin{bmatrix} 5/2 & -3 \\ -3/2 & 2 \end{bmatrix}\begin{bmatrix} 2500 \\ 1950 \end{bmatrix} = \begin{bmatrix} 400 \\ 150 \end{bmatrix}$.

One load of plywood costs \$400 and a load of insulation costs \$150 .

72. Let x and y be the costs of an iPhone and iPod, respectively. Then

$$\begin{aligned} 12x + 6y &= 3300 \\ 4x + 8y &= 2000. \end{aligned}$$

If $A = \begin{bmatrix} 12 & 6 \\ 4 & 8 \end{bmatrix}$, then

$A^{-1} = \begin{bmatrix} 1/9 & -1/12 \\ -1/18 & 1/6 \end{bmatrix}$ and

$A^{-1}\begin{bmatrix} 3300 \\ 2000 \end{bmatrix} = \begin{bmatrix} 200 \\ 150 \end{bmatrix}$. Thus, Wednesday's shipment costs \$9950 $(= 25(200) + 33(150))$.

73.

One computes $A^{-1} = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$. To decode the message, we find

$$\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} 36 \\ 65 \end{bmatrix} = \begin{bmatrix} 7 \\ 15 \end{bmatrix} = \begin{bmatrix} g \\ o \end{bmatrix},$$

$$\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} 49 \\ 83 \end{bmatrix} = \begin{bmatrix} 15 \\ 4 \end{bmatrix} = \begin{bmatrix} o \\ d \end{bmatrix},$$

$$\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} 12 \\ 24 \end{bmatrix} = \begin{bmatrix} 0 \\ 12 \end{bmatrix} = \begin{bmatrix} space \\ l \end{bmatrix},$$

$$\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} 66 \\ 111 \end{bmatrix} = \begin{bmatrix} 21 \\ 3 \end{bmatrix} = \begin{bmatrix} u \\ c \end{bmatrix},$$

$$\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} 33 \\ 55 \end{bmatrix} = \begin{bmatrix} 11 \\ 0 \end{bmatrix} = \begin{bmatrix} k \\ space \end{bmatrix}.$$

The message is 'Good luck'.

74.

One computes $A^{-1} = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$. To decode the message, we find

$$\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} 15 \\ 29 \end{bmatrix} = \begin{bmatrix} 1 \\ 12 \end{bmatrix} = \begin{bmatrix} a \\ l \end{bmatrix},$$

$$\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} 26 \\ 45 \end{bmatrix} = \begin{bmatrix} 7 \\ 5 \end{bmatrix} = \begin{bmatrix} g \\ e \end{bmatrix},$$

$$\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} 24 \\ 46 \end{bmatrix} = \begin{bmatrix} 2 \\ 18 \end{bmatrix} = \begin{bmatrix} b \\ r \end{bmatrix},$$

$$\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} a \\ space \end{bmatrix},$$

$$\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} 46 \\ 83 \end{bmatrix} = \begin{bmatrix} 9 \\ 19 \end{bmatrix} = \begin{bmatrix} i \\ s \end{bmatrix},$$

$$\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} 6 \\ 12 \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \end{bmatrix} = \begin{bmatrix} space \\ f \end{bmatrix},$$

$$\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} 77 \\ 133 \end{bmatrix} = \begin{bmatrix} 21 \\ 14 \end{bmatrix} = \begin{bmatrix} u \\ n \end{bmatrix}.$$

The message is 'Algebra is fun'.

75. Let x and y be the amounts invested in the Asset Manager Fund and Magellan Fund, respectively. Since $0.86(60,000) = 51,600$, we get

$$\begin{aligned} x + y &= 60,000 \\ 0.76x + 0.90y &= 51,600. \end{aligned}$$

If $A = \begin{bmatrix} 1 & 1 \\ 0.76 & 0.90 \end{bmatrix}$, then

$A^{-1} = \begin{bmatrix} 45/7 & -50/7 \\ -38/7 & 50/7 \end{bmatrix}$ and

$A^{-1}\begin{bmatrix} 60,000 \\ 51,600 \end{bmatrix} = \begin{bmatrix} 17,142.86 \\ 42,857.14 \end{bmatrix}$. In the Asset Manager Fund the amount invested was \$17,142.86; in the Magellan Fund the amount invested was \$42,857.14.

76. Let x, y, and z be the amounts invested in Asset Manager Fund, Magellan Fund, and the Puritan Fund, respectively.
Since $0.74(50,000) = 37,000$ and $0.217(50,000) = 10,850$, we obtain

$$\begin{aligned} x + y + z &= 50,000 \\ 0.76x + 0.90y + 0.60z &= 37,000 \\ 0.20x + 0.09y + 0.33z &= 10,850. \end{aligned}$$

If $A = \begin{bmatrix} 1 & 1 & 1 \\ 0.76 & 0.90 & 0.60 \\ 0.20 & 0.09 & 0.33 \end{bmatrix}$, then

$A^{-1} = \begin{bmatrix} 405 & -400 & -500 \\ -218 & 650/3 & 800/3 \\ -186 & 550/3 & 700/3 \end{bmatrix}$ and

$$A^{-1}\begin{bmatrix} 50,000 \\ 37,000 \\ 10,850 \end{bmatrix} = \begin{bmatrix} 25,000 \\ 10,000 \\ 15,000 \end{bmatrix}.$$

The investor must invest \$25,000 in the Asset Manager Fund, \$10,000 in the Magellan Fund, and \$15,000 in the Puritan Fund.

77. Let x, y, and z be the prices of an animal totem, a trade-bead necklace, and a tribal mask, respectively. Then we obtain the system

$$\begin{aligned} 24x + 33y + 12z &= 202.23 \\ 19x + 40y + 22z &= 209.38 \\ 30x + 9y + 19z &= 167.66. \end{aligned}$$

The inverse of the coefficient matrix A is

$$A^{-1} = \frac{1}{11,007}\begin{bmatrix} 562 & -519 & 246 \\ 299 & 96 & -300 \\ 1029 & 774 & 333 \end{bmatrix}.$$

Since $A^{-1}\begin{bmatrix} 202.23 \\ 209.38 \\ 167.66 \end{bmatrix} \approx \begin{bmatrix} \$4.20 \\ \$2.75 \\ \$0.89 \end{bmatrix}$, we find animal totem costs \$4.20, a necklace costs \$2.75, and a tribal mask costs \$0.89.

78. Let x, y, z, w, and v be the prices of a jambalaya, crawfish, filé gumbo, iced tea, and a dessert, respectively. System is

$$\begin{aligned} 36x + 28y + 35z + 90w + 68v &= 344.35 \\ 37x + 19y + 56z + 84w + 75v &= 369.10 \\ 49x + 55y + 70z + 150w + 125v &= 588.90 \\ 58x + 34y + 52z + 122w + 132v &= 529.50 \\ 44x + 65y + 39z + 133w + 120v &= 521.65. \end{aligned}$$

Inverse of the coefficient matrix A is $A^{-1} \approx$

$$\begin{bmatrix} .052 & .132 & -.154 & -.036 & .088 \\ -.023 & .113 & -.095 & -.054 & .101 \\ -.032 & .072 & -.021 & -.027 & .024 \\ .044 & -.095 & .078 & .018 & -.066 \\ -.045 & -.028 & .029 & .031 & -.013 \end{bmatrix}$$

and $A^{-1}\begin{bmatrix} 344.35 \\ 369.10 \\ 588.90 \\ 529.50 \\ 521.65 \end{bmatrix} = \begin{bmatrix} 2.85 \\ 1.85 \\ 1.75 \\ 0.75 \\ 0.90 \end{bmatrix}$.

A serving of jambalaya costs \$2.85, a crawfish pie is \$1.85, a filé gumbo is \$1.75, iced tea is \$0.75, and a dessert is \$0.90 .

79.

On $\left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 1 & 2 & 3 & 12 \end{array}\right]$, apply $-R_1 + R_2 \to R_2$ to get

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 1 & 1 & 3 \end{array}\right]$$

Use $-R_2 + R_1 \to R_1$ to get

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 6 \\ 0 & 1 & 1 & 3 \end{array}\right]$$

Then the solution set is

$$\{(6 - z, 3 - z, z) : z \text{ is real}\}.$$

80.

$$\begin{bmatrix} 1 & 2 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} -1 & 3 \\ 2 & 4 \end{bmatrix} =$$

$$\begin{bmatrix} 1(-1) + 2(2) & 1(3) + 2(4) \\ -3(-1) + 5(2) & -3(3) + 5(4) \end{bmatrix} =$$

$$\begin{bmatrix} 3 & 11 \\ 13 & 11 \end{bmatrix}$$

81.

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} =$$

$$\begin{bmatrix} 1(3) & 1(4) \\ 1(1) + 1(5) & 1(2) + 1(6) \end{bmatrix} =$$

$$\begin{bmatrix} 3 & 4 \\ 6 & 8 \end{bmatrix}$$

82. Take the ln of both sides of $e^x = 2^{x-1}$. Then

$$\begin{aligned} x &= (x - 1)\ln 2 \\ x - x\ln 2 &= -\ln 2 \\ x(1 - \ln 2) &= -\ln 2 \\ x &= \frac{-\ln 2}{1 - \ln 2} \\ x &= \frac{\ln 2}{\ln 2 - 1} \end{aligned}$$

The solution set is $\left\{\frac{\ln 2}{\ln 2 - 1}\right\}$.

83. Use the method of completing the square.

$$\begin{aligned} x^2 - 4x &= -1 \\ x^2 - 4x + 4 &= 3 \\ (x-2)^2 &= 3 \\ x - 2 &= \pm\sqrt{3} \end{aligned}$$

The solution set is $\left\{2 \pm \sqrt{3}\right\}$.

84. Use the method of completing the square.

$$\begin{aligned} x^2 - 8x &= -20 \\ x^2 - 8x + 16 &= -4 \\ (x-4)^2 &= -4 \\ x - 4 &= \pm 2i \end{aligned}$$

The solution set is $\{4 \pm 2i\}$.

Thinking Outside the Box LXXXIII

a) Substituting $x = 0, 1, 2, ..., 10$ into $x(10-x)$, we find that the maximum product is 25. This maximum is achieved when $x = 5$. Thus, the two numbers are 5 and 5.

b) First, we enumerate all possible whole numbers satisfying

$$1 \le x \le y \le z \le w \le 10$$

and $x + y + z + w = 10$.

We find that the maximum product for $xyzw$ is 36, and this happens when $x = y = 2$ and $z = w = 3$.

c) The maximum product is 729, and the numbers are six 3's.

9.4 Pop Quiz

1.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix} =$$

$$\begin{bmatrix} 1(2)+0(6) & 1(4)+0(8) \\ 0(2)+1(6) & 0(4)+1(8) \end{bmatrix} =$$

$$\begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$$

2.

On $\left[\begin{array}{cc|cc} 2 & 5 & 1 & 0 \\ 3 & 8 & 0 & 1 \end{array}\right]$, use $\frac{1}{2}R_1 \to R_1$

to get

$\left[\begin{array}{cc|cc} 1 & 2.5 & 0.5 & 0 \\ 3 & 8 & 0 & 1 \end{array}\right]$, use $-3R_1 + R_2 \to R_2$

to get

$\left[\begin{array}{cc|cc} 1 & 2.5 & 0.5 & 0 \\ 0 & 0.5 & -1.5 & 1 \end{array}\right]$, use $2R_2 \to R_2$

to get

$\left[\begin{array}{cc|cc} 1 & 2.5 & 0.5 & 0 \\ 0 & 1 & -3 & 2 \end{array}\right]$, $-2.5R_2 + R_1 \to R_1$

to get

$\left[\begin{array}{cc|cc} 1 & 0 & 8 & -5 \\ 0 & 1 & -3 & 2 \end{array}\right]$.

Then $A^{-1} = \begin{bmatrix} 8 & -5 \\ -3 & 2 \end{bmatrix}$.

3.

The coefficient matrix is $A = \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$, and its inverse matrix is $A^{-1} = \begin{bmatrix} 8 & -5 \\ -3 & 2 \end{bmatrix}$.

Since $A^{-1}\begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 17 \\ -6 \end{bmatrix}$, the solution set is $\{(17, -6)\}$.

9.4 Linking Concepts

a) Let L_f, L_r, R_f, and R_r be the weights of the left front, left rear, right front, and right rear tires, respectively. Since $R_f = 288$ and $0.48(1250) = 600$, we get

$$\begin{aligned} L_f + L_r &= 625 \\ 288 + L_r &= 600. \end{aligned}$$

Then $L_r = 600 - 288 = 312$ lb and $L_f = 625 - 312 = 313$ lb. Finally, $R_r = 1250 - 313 - 312 - 288 = 337$ lb.

b) Since $0.62(1250) = 650$ lbs and no two tires have a combined weight of more than 650 lbs., yes the NASCAR rule is satisfied.

c) Note,

$$\begin{aligned} L_f + L_r &= 0.50(1300) = 650 \\ L_r + R_f &= 0.48(1300) = 624 \\ L_r + R_r &= 0.51(1300) = 663 \\ L_f + L_r + R_f + R_r &= 1300. \end{aligned}$$

Let $A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$.

Then $A^{-1} = \dfrac{1}{2} \begin{bmatrix} 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & -1 \\ -1 & 1 & -1 & 1 \\ -1 & -1 & 1 & 1 \end{bmatrix}$ and

$$\begin{bmatrix} L_f \\ L_r \\ R_f \\ R_r \end{bmatrix} = A^{-1} \begin{bmatrix} 650 \\ 624 \\ 663 \\ 1300 \end{bmatrix} = \begin{bmatrix} 331.5 \\ 318.5 \\ 305.5 \\ 344.5 \end{bmatrix}.$$

d) Note, 676 is 52% of 1300. Assume the weight each of the other three tires is x. Let y be the weight of the left front. Then $x = \dfrac{1300 - y}{3}$ and $y + x \le 676$. Then

$$\begin{aligned} y &\le 676 - x \\ y &\le 676 - \frac{1300 - y}{3} \\ 3y &\le 2028 - 1300 + y \\ 2y &\le 728 \\ y &\le 364. \end{aligned}$$

The maximum weight on the left front tire is 364 lb.

For Thought

1. False, $|A| = 12 - (-5) = 17$.

2. True, for $|A| \neq 0$.

3. True, $|B| = 4 \cdot 5 - (-2)(-10) = 0$.

4. False, since $|B| = 0$.

5. True, since the determinant of the coefficient matrix A is nonzero.

6. True, in general $|LM| = |L||M|$ for any square matrices L and M of the same size.

7. False, the system is not linear. **8.** True

9. False, $\begin{vmatrix} 2 & 0.1 \\ 100 & 5 \end{vmatrix} = 2 \cdot 5 - 100(0.1) = 0.$

10. False, because a 2×2 matrix is not equal to the number 27.

9.5 Exercises

1. $\begin{vmatrix} 1 & 3 \\ 0 & 2 \end{vmatrix} = 1 \cdot 2 - 0 \cdot 3 = 2$

2. $\begin{vmatrix} 0 & 4 \\ 2 & -1 \end{vmatrix} = 0(-1) - 2 \cdot 4 = -8$

3. $\begin{vmatrix} 3 & 4 \\ 2 & 9 \end{vmatrix} = 3(9) - 2(4) = 19$

4. $\begin{vmatrix} 7 & 2 \\ 3 & -2 \end{vmatrix} = 7(-2) - 3(2) = -20$

5. $\begin{vmatrix} -0.3 & -0.5 \\ -0.7 & 0.2 \end{vmatrix} = (-0.3)(0.2) - (-0.7)(-0.5)$
$= -0.41$

6. $\begin{vmatrix} -1/3 & 4/3 \\ -3 & 2/3 \end{vmatrix} = (-1/3)(2/3) - (-3)(4/3)$
$= -2/9 + 4 = 34/9$

7. $\begin{vmatrix} 1/8 & -3/8 \\ 2 & -1/4 \end{vmatrix} = (1/8)(-1/4) - (2)(-3/8)$
$= -1/32 + 3/4 = 23/32$

8. $\begin{vmatrix} -1 & -3 \\ -5 & -8 \end{vmatrix} = (-1)(-8) - (-5)(-3) = -7$

9. $\begin{vmatrix} 0.02 & 0.4 \\ 1 & 20 \end{vmatrix} = (.02)(20) - (.4)(1) = 0$

10.
$$\begin{vmatrix} -0.3 & 0.4 \\ 3 & -4 \end{vmatrix} = (-.3)(-4) - (3)(.4) = 0$$

11.
$$\begin{vmatrix} 3 & -5 \\ -9 & 15 \end{vmatrix} = (3)(15) - (-9)(-5) = 0$$

12.
$$\begin{vmatrix} -6 & 2 \\ 3 & -1 \end{vmatrix} = (-6)(-1) - (3)(2) = 0$$

13.
Since $\begin{vmatrix} a & 2 \\ 3 & 4 \end{vmatrix} = 4a - 6 = 10$, we find $4a = 16$ or $a = 4$.

14.
Since $\begin{vmatrix} 1 & 7 \\ 3 & a \end{vmatrix} = a - 21 = 5$, we find $a = 26$.

15.
Since $\begin{vmatrix} a & 8 \\ 2 & a \end{vmatrix} = a^2 - 16 = 0$, we find $a^2 = 16$ or $a = \pm 4$.

16.
Since $\begin{vmatrix} a & 1 \\ a & a \end{vmatrix} = a^2 - a = a(a-1) = 0$, we find $a = 0, 1$.

17.
Note $D = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 3$, $D_x = \begin{vmatrix} 5 & 1 \\ 7 & 2 \end{vmatrix} = 3$, and $D_y = \begin{vmatrix} 2 & 5 \\ 1 & 7 \end{vmatrix} = 9$.
Then $x = \dfrac{D_x}{D} = \dfrac{3}{3} = 1$ and $y = \dfrac{D_y}{D} = \dfrac{9}{3} = 3$.
Solution set is $\{(1,3)\}$.

18.
Note $D = \begin{vmatrix} 3 & 1 \\ 1 & 1 \end{vmatrix} = 2$, $D_x = \begin{vmatrix} 10 & 1 \\ 6 & 1 \end{vmatrix} = 4$, and $D_y = \begin{vmatrix} 3 & 10 \\ 1 & 6 \end{vmatrix} = 8$.
Then $x = \dfrac{D_x}{D} = \dfrac{4}{2} = 2$ and $y = \dfrac{D_y}{D} = \dfrac{8}{2} = 4$.
Solution set is $\{(2,4)\}$.

19.
Note $D = \begin{vmatrix} 1 & -2 \\ 1 & 2 \end{vmatrix} = 4$, $D_x = \begin{vmatrix} 7 & -2 \\ -5 & 2 \end{vmatrix} = 4$, and $D_y = \begin{vmatrix} 1 & 7 \\ 1 & -5 \end{vmatrix} = -12$.
So $x = \dfrac{D_x}{D} = \dfrac{4}{4} = 1$ and $y = \dfrac{D_y}{D} = \dfrac{-12}{4} = -3$. The solution set is $\{(1,-3)\}$.

20.
Note $D = \begin{vmatrix} 1 & -1 \\ 3 & 1 \end{vmatrix} = 4$, $D_x = \begin{vmatrix} 1 & -1 \\ 7 & 1 \end{vmatrix} = 8$, and $D_y = \begin{vmatrix} 1 & 1 \\ 3 & 7 \end{vmatrix} = 4$.
So $x = \dfrac{D_x}{D} = \dfrac{8}{4} = 2$ and $y = \dfrac{D_y}{D} = \dfrac{4}{4} = 1$.
Solution set is $\{(2,1)\}$.

21.
Note $D = \begin{vmatrix} 2 & -1 \\ 1 & 3 \end{vmatrix} = 7$, $D_x = \begin{vmatrix} -11 & -1 \\ 12 & 3 \end{vmatrix} = -21$, and $D_y = \begin{vmatrix} 2 & -11 \\ 1 & 12 \end{vmatrix} = 35$. Then
$x = \dfrac{D_x}{D} = -\dfrac{21}{7} = -3$ and $y = \dfrac{D_y}{D} = \dfrac{35}{7} = 5$.
Solution set is $\{(-3,5)\}$.

22.
Note $D = \begin{vmatrix} 3 & -2 \\ -5 & 4 \end{vmatrix} = 2$, $D_x = \begin{vmatrix} -4 & -2 \\ -1 & 4 \end{vmatrix} = -18$, and $D_y = \begin{vmatrix} 3 & -4 \\ -5 & -1 \end{vmatrix} = -23$. Then
$x = \dfrac{D_x}{D} = -\dfrac{18}{2} = -9$ and $y = \dfrac{D_y}{D} = -\dfrac{23}{2}$.
Solution set is $\{(-9,-23/2)\}$.

23. Rewrite system as
$$\begin{aligned} x - y &= 6 \\ x + y &= 5. \end{aligned}$$
Note $D = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = 2$, $D_x = \begin{vmatrix} 6 & -1 \\ 5 & 1 \end{vmatrix} = 11$, and $D_y = \begin{vmatrix} 1 & 6 \\ 1 & 5 \end{vmatrix} = -1$.

So $x = \frac{D_x}{D} = \frac{11}{2}$ and $y = \frac{D_y}{D} = -\frac{1}{2}$.
Solution set is $\{(11/2, -1/2)\}$.

24. Rewrite system as

$$\begin{aligned} 3x + y &= 7 \\ 4x - y &= -4 \end{aligned}$$

Note $D = \begin{vmatrix} 3 & 1 \\ 4 & -1 \end{vmatrix} = -7$, $D_x = \begin{vmatrix} 7 & 1 \\ -4 & -1 \end{vmatrix}$

$= -3$, and $D_y = \begin{vmatrix} 3 & 7 \\ 4 & -4 \end{vmatrix} = -40$.

Then $x = \frac{D_x}{D} = \frac{3}{7}$ and $y = \frac{D_y}{D} = \frac{40}{7}$.
Solution set is $\{(3/7, 40/7)\}$.

25.
Note $D = \begin{vmatrix} 1/2 & -1/3 \\ 1/4 & 1/2 \end{vmatrix} = 1/3$,

$D_x = \begin{vmatrix} 4 & -1/3 \\ 6 & 1/2 \end{vmatrix} = 4$, and

$D_y = \begin{vmatrix} 1/2 & 4 \\ 1/4 & 6 \end{vmatrix} = 2$. So $x = \frac{D_x}{D}$

$= \frac{4}{1/3} = 12$ and $y = \frac{D_y}{D} = \frac{2}{1/3} = 6$.
Solution set is $\{(12, 6)\}$.

26.
Note $D = \begin{vmatrix} 1/4 & 2/3 \\ 3/5 & -1/10 \end{vmatrix} = -17/40$,

$D_x = \begin{vmatrix} 25 & 2/3 \\ 12 & -1/10 \end{vmatrix} = -21/2$, and

$D_y = \begin{vmatrix} 1/4 & 25 \\ 3/5 & 12 \end{vmatrix} = -12$. Then

$x = \frac{D_x}{D} = \frac{-21/2}{-17/40} = \frac{420}{17}$ and

$y = \frac{D_y}{D} = \frac{-12}{-17/40} = \frac{480}{17}$.

Solution set is $\{(420/17, 480/17)\}$.

27.
Note $D = \begin{vmatrix} 0.2 & 0.12 \\ 1 & 1 \end{vmatrix} = 0.08$,

$D_x = \begin{vmatrix} 148 & 0.12 \\ 900 & 1 \end{vmatrix} = 40$, and

$D_y = \begin{vmatrix} 0.2 & 148 \\ 1 & 900 \end{vmatrix} = 32$. Then

$x = \frac{D_x}{D} = \frac{40}{0.08} = 500$ and

$y = \frac{D_y}{D} = \frac{32}{0.08} = 400$.

Solution set is $\{(500, 400)\}$.

28.
Note $D = \begin{vmatrix} 0.08 & 0.05 \\ 2 & -1 \end{vmatrix} = -0.18$,

$D_x = \begin{vmatrix} 72 & 0.05 \\ 0 & -1 \end{vmatrix} = -72$, and

$D_y = \begin{vmatrix} 0.08 & 72 \\ 2 & 0 \end{vmatrix} = -144$. So

$x = \frac{D_x}{D} = \frac{-72}{-0.18} = 400$ and

$y = \frac{D_y}{D} = \frac{-144}{-0.18} = 800$.

Solution set is $\{(400, 800)\}$.

29. Cramer's rule does not apply since
$D = \begin{vmatrix} 3 & 1 \\ -6 & -2 \end{vmatrix} = 0$. Dividing the second
equation by -2, one gets the first equation.
Solution set is $\{(x, y) \mid 3x + y = 6\}$.

30. Cramer's rule does not apply since
$D = \begin{vmatrix} 8 & -4 \\ 4 & -2 \end{vmatrix} = 0$. Dividing the first
equation by 2, one gets the second equation.
Solution set is $\{(x, y) \mid 4x - 2y = 1\}$.

31. Cramer's Rule does not apply since the determinant D is zero.

Adding the two equations, one gets $0 = 19$. Inconsistent and the solution set is $\emptyset$.

32. Cramer's Rule does not apply since the determinant D is zero.

Multiply second equation by -3 and add to the first one.

$$\begin{array}{rcr} 12x+3y &=& 9 \\ -12x-3y &=& -18 \\ \hline 0 &=& -9 \end{array}$$

Inconsistent and the solution set is $\emptyset$.

33. We use Cramer's Rule on the system

$$\begin{aligned} x-y &= 3 \\ 3x-y &= -9. \end{aligned}$$

Note, $D = \begin{vmatrix} 1 & -1 \\ 3 & -1 \end{vmatrix} = 2$,

$D_x = \begin{vmatrix} 3 & -1 \\ -9 & -1 \end{vmatrix} = -12$, and

$D_y = \begin{vmatrix} 1 & 3 \\ 3 & -9 \end{vmatrix} = -18$.

So $x = \dfrac{D_x}{D} = \dfrac{-12}{2} = -6$ and

$y = \dfrac{D_y}{D} = \dfrac{-18}{2} = -9$.

Solution set is $\{(-6,-9)\}$.

34. Use Cramer's Rule on the simplified system,

$$\begin{aligned} x-2y &= 3 \\ x+2y &= 15. \end{aligned}$$

Note, $D = \begin{vmatrix} 1 & -2 \\ 1 & 2 \end{vmatrix} = 4$,

$D_x = \begin{vmatrix} 3 & -2 \\ 15 & 2 \end{vmatrix} = 36$, and

$D_y = \begin{vmatrix} 1 & 3 \\ 1 & 15 \end{vmatrix} = 12$.

Then $x = \dfrac{D_x}{D} = \dfrac{36}{4} = 9$ and

$y = \dfrac{D_y}{D} = \dfrac{12}{4} = 3$.

Solution set is $\{(9,3)\}$.

35. Note, $D = \begin{vmatrix} \sqrt{2} & \sqrt{3} \\ 3\sqrt{2} & -2\sqrt{3} \end{vmatrix} = -5\sqrt{6}$,

$D_x = \begin{vmatrix} 4 & \sqrt{3} \\ -3 & -2\sqrt{3} \end{vmatrix} = -5\sqrt{3}$, and

$D_y = \begin{vmatrix} \sqrt{2} & 4 \\ 3\sqrt{2} & -3 \end{vmatrix} = -15\sqrt{2}$.

So $x = \dfrac{D_x}{D} = \dfrac{-5\sqrt{3}}{-5\sqrt{6}} = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

and $y = \dfrac{D_y}{D} = \dfrac{-15\sqrt{2}}{-5\sqrt{6}} = \dfrac{3}{\sqrt{3}} = \sqrt{3}$.

The solution set is $\{(\sqrt{2}/2, \sqrt{3})\}$.

36. Note, $D = \begin{vmatrix} \sqrt{3}/3 & 1 \\ 1 & -\sqrt{3} \end{vmatrix} = -2$,

$D_x = \begin{vmatrix} 1 & 1 \\ 0 & -\sqrt{3} \end{vmatrix} = -\sqrt{3}$, and

$D_y = \begin{vmatrix} \sqrt{3}/3 & 1 \\ 1 & 0 \end{vmatrix} = -1$.

Then $x = \dfrac{D_x}{D} = \dfrac{\sqrt{3}}{2}$ and $y = \dfrac{D_y}{D} = \dfrac{1}{2}$.

The solution set is $\{(\sqrt{3}/2, 1/2)\}$.

37. Multiply second equation by -1 and add to the first one.

$$\begin{array}{rcr} x^2+y^2 &=& 25 \\ -x^2+y &=& -5 \\ \hline y^2+y &=& 20 \\ y^2+y-20 &=& 0 \\ (y+5)(y-4) &=& 0 \end{array}$$

If $y=-5$, then $x^2=0$ and $x=0$.
If $y=4$, then $x^2=9$ and $x=\pm 3$.
The solution set is $\{(0,-5), (\pm 3, 4)\}$.

38. Multiply second equation by -1 and add to the first one.

$$\begin{array}{rcr} x^2+y &=& 8 \\ -x^2+y &=& -4 \\ \hline 2y &=& 4 \end{array}$$

Since $y=2$, $x^2=6$ and $x=\pm\sqrt{6}$.
Solution set is $\{(\pm\sqrt{6}, 2)\}$.

39. Solving for x in the second equation, we find $x = 4 + 2y$. Substituting into the first equation, we obtain

$$\begin{aligned} 4 + 2y - 2y &= y^2 \\ 4 &= y^2 \\ \pm 2 &= y. \end{aligned}$$

Using $y = 2$ in $x = 4 + 2y$, we get $x = 8$. Similarly, if $y = -2$ then $x = 0$. Solution set is $\{(8, 2), (0, -2)\}$.

40. Substitute $y = x^2$ into $x + y = 30$.

$$\begin{aligned} x + x^2 &= 30 \\ x^2 + x - 30 &= 0 \\ (x + 6)(x - 5) &= 0 \\ x &= -6, 5 \end{aligned}$$

If $x = -6$, then $y = (-6)^2 = 36$.
If $x = 5$, then $y = 5^2 = 25$.
The solution set is $\{(-6, 36), (5, 25)\}$.

41.

Invertible, since $\begin{vmatrix} 4 & 0.5 \\ 2 & 3 \end{vmatrix} = 12 - 1 = 11 \neq 0$

42.

Invertible, since $\begin{vmatrix} -5 & 2 \\ 4 & -1 \end{vmatrix} = 5 - 8 = -3 \neq 0$

43.

Not invertible, for $\begin{vmatrix} 3 & -4 \\ 9 & -12 \end{vmatrix} = -36 + 36 = 0$

44.

Not invertible, since $\begin{vmatrix} 1/2 & 12 \\ 1/3 & 8 \end{vmatrix} = 4 - 4 = 0$

45. Note, $D = \begin{vmatrix} 3.47 & 23.09 \\ 12.48 & 3.98 \end{vmatrix} = -274.3526$,

$$D_x = \begin{vmatrix} 5978.95 & 23.09 \\ 2765.34 & 3.98 \end{vmatrix} = -40,055.4796,$$

and

$$D_y = \begin{vmatrix} 3.47 & 5978.95 \\ 12.48 & 2765.34 \end{vmatrix} = -65,021.5662.$$

Then $x = \dfrac{D_x}{D} = 146$ and $y = \dfrac{D_y}{D} = 237$.
The solution set is $\{(146, 237)\}$.

46. Note, $D = \begin{vmatrix} 0.0875 & 0.1625 \\ 1 & 1 \end{vmatrix} = -0.075$,

$$D_x = \begin{vmatrix} 564.4 & 0.1625 \\ 4232 & 1 \end{vmatrix} = -123.3, \text{ and}$$

$$D_y = \begin{vmatrix} 0.0875 & 564.4 \\ 1 & 4232 \end{vmatrix} = -194.1\ .$$

Then $x = \dfrac{D_x}{D} = 1644$ and $y = \dfrac{D_y}{D} = 2588$.
The solution set is $\{(1644, 2588)\}$.

47. Let x and y be the number of boys and girls, respectively. Then

$$\begin{aligned} 0.44x + 0.35y &= 231 \\ x + y &= 615. \end{aligned}$$

Note, $D = \begin{vmatrix} 0.44 & 0.35 \\ 1 & 1 \end{vmatrix} = 0.09$,

$$D_x = \begin{vmatrix} 231 & 0.35 \\ 615 & 1 \end{vmatrix} = 15.75, \text{ and}$$

$$D_y = \begin{vmatrix} 0.44 & 231 \\ 1 & 615 \end{vmatrix} = 39.6.$$

There were $x = \dfrac{D_x}{D} = \dfrac{15.75}{0.09} = 175$ boys and $y = \dfrac{D_y}{D} = \dfrac{39.6}{0.09} = 440$ girls.

48. Let x and y be the number of women and men, respectively. Then

$$\begin{aligned} x + y &= 900 \\ 0.8x + 0.7y &= 680. \end{aligned}$$

Note, $D = \begin{vmatrix} 1 & 1 \\ 0.8 & 0.7 \end{vmatrix} = -0.1$,

$$D_x = \begin{vmatrix} 900 & 1 \\ 680 & 0.7 \end{vmatrix} = -50, \text{ and}$$

$$D_y = \begin{vmatrix} 1 & 900 \\ 0.8 & 680 \end{vmatrix} = -40.$$

There were $x = \dfrac{D_x}{D} = \dfrac{-50}{-0.1} = 500$ women and $y = \dfrac{D_y}{D} = \dfrac{-40}{-0.1} = 400$ men.

49. Let x and y be the measurements of the two acute angles. Then we obtain

$$\begin{aligned} x + y &= 90 \\ x - 2y &= 1. \end{aligned}$$

Note, $D = \begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} = -3$,

$D_x = \begin{vmatrix} 90 & 1 \\ 1 & -2 \end{vmatrix} = -181$, and

$D_y = \begin{vmatrix} 1 & 90 \\ 1 & 1 \end{vmatrix} = -89$.

The acute angles are $x = \dfrac{D_x}{D} = \dfrac{181}{3}$ degrees and $y = \dfrac{D_y}{D} = \dfrac{89}{3}$ degrees.

50. Let x be the number of degrees in each of the equal angles and let y be the measurement of the third angle. Then we have

$$\begin{aligned} 2x + y &= 180 \\ x - y &= 2. \end{aligned}$$

Note, $D = \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} = -3$,

$D_x = \begin{vmatrix} 180 & 1 \\ 2 & -1 \end{vmatrix} = -182$, and

$D_y = \begin{vmatrix} 2 & 180 \\ 1 & 2 \end{vmatrix} = -176$.

Thus, $x = \dfrac{D_x}{D} = \dfrac{182}{3}$ degrees and $y = \dfrac{D_y}{D} = \dfrac{176}{3}$ degrees.

The angles are $\dfrac{182}{3}$ degrees, $\dfrac{182}{3}$ degrees, and $\dfrac{176}{3}$ degrees.

51. Let x and y be the salaries of the president and vice-president, respectively. Then we have

$$\begin{aligned} x + y &= 400,000 \\ x - y &= 100,000. \end{aligned}$$

Note, $D = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = -2$,

$D_x = \begin{vmatrix} 400,000 & 1 \\ 100,000 & -1 \end{vmatrix} = -500,000$, and

$D_y = \begin{vmatrix} 1 & 400,000 \\ 1 & 100,000 \end{vmatrix} = -300,000$.

Thus, $x = \dfrac{D_x}{D} = \dfrac{-500,000}{-2} = 250,000$ and $y = \dfrac{D_y}{D} = \dfrac{-300,000}{-2} = 150,000.$

The president's salary is \$250,000 and the vice-president's salary is \$150,000.

52. Let x and y be the number of LCD TV's and Blue-ray players. Then we obtain

$$\begin{aligned} 8x + 2.5y &= 2,350 \\ 400x + 225y &= 147,500. \end{aligned}$$

Note, $D = \begin{vmatrix} 8 & 2.5 \\ 400 & 225 \end{vmatrix} = 800$,

$D_x = \begin{vmatrix} 2,350 & 2.5 \\ 147,500 & 225 \end{vmatrix} = 160,000$, and

$D_y = \begin{vmatrix} 8 & 2,350 \\ 400 & 147,500 \end{vmatrix} = 240,000.$

The number of LCD TV lost were $x = \dfrac{D_x}{D} = \dfrac{160,000}{800} = 200$ and the number of Blue-ray players lost were $y = \dfrac{D_y}{D} = \dfrac{240,000}{800} = 300.$

53. Yes, $|MN| = |M||N|$ since $|M| = 2$, $|N| = 3$, and $|MN| = \begin{vmatrix} 8 & 31 \\ 14 & 55 \end{vmatrix} = 6.$

54. Yes, $|M^{-1}| = 1/|M|$ since $|M^{-1}| = \begin{vmatrix} 2 & -1 \\ -5/2 & 3/2 \end{vmatrix} = 1/2$ and $|M| = 2.$

55.

$\left| \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} \right| =$

$\begin{vmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{vmatrix} =$

$(ae + bg)(cf + dh) - (ce + dg)(af + bh) =$
$aecf + bgcf + aedh + bgdh - ceaf - dgaf -$

$cebh - dgbh = bgcf + aedh - dgaf - cebh =$
$ad(eh-gf)-bc(eh-gf) = (ad-bc)(eh-gf) =$
$= \begin{vmatrix} a & b \\ c & d \end{vmatrix} \begin{vmatrix} e & f \\ g & h \end{vmatrix}$

56. Note $1 = |I| = |AA^{-1}|$ and by Exercise 51, we obtain $|AA^{-1}| = |A||A^{-1}|$.

Then $|A||A^{-1}| = 1$ and $|A^{-1}| = 1/|A|$.

57.

No, since $|-2M| = \begin{vmatrix} -6 & -4 \\ -10 & -8 \end{vmatrix} = 8$

and $-2|M| = -4$.

58. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Then $|kA| =$

$\begin{vmatrix} ka & kb \\ kc & kd \end{vmatrix} = k^2ad - k^2bc =$

$k^2(ad - bc) = k^2|A|$.

59.

On $\left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 1 & 3 & 0 & 1 \end{array}\right]$, use $-R_1 + R_2 \to R_2$

to get

$\left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & 2 & -1 & 1 \end{array}\right]$, use $\frac{1}{2}R_2 \to R_2$

to get

$\left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & 1 & -\frac{1}{2} & \frac{1}{2} \end{array}\right]$, use $-R_2 + R_1 \to R_1$

to get

$\left[\begin{array}{cc|cc} 1 & 0 & \frac{3}{2} & -\frac{1}{2} \\ 0 & 1 & -\frac{1}{2} & \frac{1}{2} \end{array}\right]$.

Then $A^{-1} = \begin{bmatrix} 3/2 & -1/2 \\ -1/2 & 1/2 \end{bmatrix}$.

60.

$$\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix} =$$
$$\begin{bmatrix} 1(3)+2(-1) & 1(-2)+2(1) \\ 1(3)+3(-1) & 1(-2)+3(1) \end{bmatrix} =$$
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

61. If we add the first two equations, the sum is $2x + 3y = 15$. Note, the third equation is similar to the sum but the right side is different, i.e., is $2x + 36 = 10$. Thus, there are no solutions. The solution set is the empty set

62. Since $y = -3x - 99$ and $y = 12x - 99$, then

$$\begin{aligned} -3x - 99 &= 12x - 99 \\ -3x &= 12x \\ -15x &= 0 \\ x &= 0 \end{aligned}$$

Then $y = -3(0) - 99 = -99$. The solution set is $\{0, -99\}$.

63. Since $0.91x = 72,800$, we find

$$x = \frac{72,800}{0.91} = 80,000.$$

The solution set is $\{80,000\}$.

64. A polynomial with zeros $-1, \pm 3$ is

$$\begin{aligned} f(x) &= (x-1)(x-3)(x+3) \\ &= (x-1)(x^2-9) \\ &= x^3 + x^2 - 9x - 9 \end{aligned}$$

Thinking Outside the Box LXXXIV

Take out one more of type A. Cut them in half one at a time and take half of each pill. Take the other halves the next day.

9.5 Pop Quiz

1.
$\begin{vmatrix} 4 & 2 \\ 3 & -1 \end{vmatrix} = -4 - 6 = -10$

2. We use Cramer's Rule on the system

$$\begin{aligned} 4x + 2y &= 3 \\ 3x - y &= 1. \end{aligned}$$

Note, $D = \begin{vmatrix} 4 & 2 \\ 3 & -1 \end{vmatrix} = -10,$

$D_x = \begin{vmatrix} 3 & 2 \\ 1 & -1 \end{vmatrix} = -5$, and

$D_y = \begin{vmatrix} 4 & 3 \\ 3 & 1 \end{vmatrix} = -5.$

So $x = \dfrac{D_x}{D} = \dfrac{-5}{-10} = \dfrac{1}{2}$ and

$y = \dfrac{D_y}{D} = \dfrac{-5}{-10} = \dfrac{1}{2}.$

The solution set is $\{(1/2, 1/2)\}$.

3. Yes, since the determinant is nonzero, i.e.,

$$\begin{vmatrix} 9 & 2 \\ 6 & 8 \end{vmatrix} = 72 - 12 = 60.$$

For Thought

1. False, the sign array of A is used in evaluating $|A|$.

2. False, the last term should be $1 \cdot \begin{vmatrix} 3 & 4 \\ 0 & 0 \end{vmatrix}$.

3. True

4. True, $|A|$ was expanded about the third row.

5. False, $|A|$ can be expanded only about a row or column. **6.** False, a minor is a 2×2 matrix only if it comes from a 3×3 matrix.

7. False, $x = \dfrac{D_x}{D}$. **8.** True

9. False, it can happen that $D = 0$ and there are infinitely many solutions.

10. False, Cramer's Rule applies only to a system of linear equations.

9.6 Exercises

1.
$$\begin{vmatrix} 5 & -6 \\ 9 & -8 \end{vmatrix} = -40 + 54 = 14$$

2.
$$\begin{vmatrix} 4 & -6 \\ 7 & -8 \end{vmatrix} = -32 + 42 = 10$$

3.
$$\begin{vmatrix} 4 & 5 \\ 7 & 9 \end{vmatrix} = 36 - 35 = 1$$

4.
$$\begin{vmatrix} -3 & 1 \\ 9 & -8 \end{vmatrix} = 24 - 9 = 15$$

5.
$$\begin{vmatrix} 2 & 1 \\ 7 & -8 \end{vmatrix} = -16 - 7 = -23$$

6.
$$\begin{vmatrix} 2 & -3 \\ 7 & 9 \end{vmatrix} = 18 + 21 = 39$$

7.
$$\begin{vmatrix} 2 & 1 \\ 4 & -6 \end{vmatrix} = -12 - 4 = -16$$

8.
$$\begin{vmatrix} 2 & -3 \\ 4 & 5 \end{vmatrix} = 10 + 12 = 22$$

9.
$$1\begin{vmatrix} 1 & -2 \\ -1 & 5 \end{vmatrix} - (-3)\begin{vmatrix} -4 & 0 \\ -1 & 5 \end{vmatrix} + 3\begin{vmatrix} -4 & 0 \\ 1 & -2 \end{vmatrix}$$
$$= 1(3) - (-3)(-20) + 3(8) = -33$$

10.
$$1\begin{vmatrix} 1 & -4 \\ 3 & 6 \end{vmatrix} - 3\begin{vmatrix} -3 & 2 \\ 3 & 6 \end{vmatrix} + 2\begin{vmatrix} -3 & 2 \\ 1 & -4 \end{vmatrix}$$
$$= 1(18) - 3(-24) + 2(10) = 110$$

11.
$$3\begin{vmatrix} 4 & -1 \\ 1 & -2 \end{vmatrix} - 0\begin{vmatrix} -1 & 2 \\ 1 & -2 \end{vmatrix} + 5\begin{vmatrix} -1 & 2 \\ 4 & -1 \end{vmatrix}$$
$$= 3(-7) - 0 + 5(-7) = -56$$

12.
$$-1\begin{vmatrix} 2 & -3 \\ 6 & -9 \end{vmatrix} - 0\begin{vmatrix} 3 & -1 \\ 6 & -9 \end{vmatrix} + 2\begin{vmatrix} 3 & -1 \\ 2 & -3 \end{vmatrix}$$
$$= -1(0) - 0 + 2(-7) = -14$$

13.
$$-2\begin{vmatrix} 0 & -1 \\ 2 & -7 \end{vmatrix} - (-3)\begin{vmatrix} 5 & 1 \\ 2 & -7 \end{vmatrix} + 0\begin{vmatrix} 5 & 1 \\ 0 & -1 \end{vmatrix}$$
$$= -2(2) - (-3)(-37) + 0 = -115$$

14.
$$0\begin{vmatrix} 4 & -2 \\ 3 & -1 \end{vmatrix} - (-1)\begin{vmatrix} -6 & 2 \\ 3 & -1 \end{vmatrix} + 5\begin{vmatrix} -6 & 2 \\ 4 & -2 \end{vmatrix}$$
$= 0 - (-1)(0) + 5(4) = 20$

15.
$$0.1\begin{vmatrix} 20 & 6 \\ 90 & 8 \end{vmatrix} - 0.4\begin{vmatrix} 30 & 1 \\ 90 & 8 \end{vmatrix} + 0.7\begin{vmatrix} 30 & 1 \\ 20 & 6 \end{vmatrix}$$
$= 0.1(-380) - 0.4(150) + 0.7(160) = 14$

16.
$$3\begin{vmatrix} 0.5 & 30 \\ 0.1 & 80 \end{vmatrix} - 5\begin{vmatrix} 0.3 & 10 \\ 0.1 & 80 \end{vmatrix} + 8\begin{vmatrix} 0.3 & 10 \\ 0.5 & 30 \end{vmatrix}$$
$= 3(37) - 5(23) + 8(4) = 28$

17. Expanding about the second row,
$$D = -(-2)\begin{vmatrix} 3 & 5 \\ 3 & -4 \end{vmatrix} = -(-2)(-27) = -54.$$

18. Expanding about the third column,
$$D = 1\begin{vmatrix} 3 & 4 \\ -2 & 1 \end{vmatrix} = 1(11) = 11.$$

19. Expanding about the first row,
$$D = 1\begin{vmatrix} 2 & 2 \\ 4 & 4 \end{vmatrix} - 1\begin{vmatrix} 2 & 2 \\ 4 & 4 \end{vmatrix} + 1\begin{vmatrix} 2 & 2 \\ 4 & 4 \end{vmatrix}$$
$= 1(0) - 1(0) + 1(0) = 0.$

20. Expanding about the first column,
$$D = 4\begin{vmatrix} -1 & 3 \\ -1 & 3 \end{vmatrix} - 4\begin{vmatrix} -1 & 3 \\ -1 & 3 \end{vmatrix} + 4\begin{vmatrix} -1 & 3 \\ -1 & 3 \end{vmatrix}$$
$= 4(0) - 4(0) + 4(0) = 0.$

21. Expanding about the first row,
$$D = -(-1)\begin{vmatrix} 3 & 6 \\ -2 & -5 \end{vmatrix} = -(-1)(-3) = -3.$$

22. Expanding about the first row,
$$D = 2\begin{vmatrix} 3 & -4 \\ 5 & -2 \end{vmatrix} = 2(14) = 28.$$

23. Expanding about the second column,
$$D = -9\begin{vmatrix} 2 & 1 \\ 4 & 6 \end{vmatrix} = -9(8) = -72.$$

24. By expanding about the third row, the determinant is 0.

25. Expanding about the first row,
$$3\begin{vmatrix} -3 & 2 & 0 \\ 3 & 1 & 2 \\ -4 & 1 & 3 \end{vmatrix} + 0 + 1\begin{vmatrix} 2 & -3 & 0 \\ -2 & 3 & 2 \\ 2 & -4 & 3 \end{vmatrix} -$$
$$5\begin{vmatrix} 2 & -3 & 2 \\ -2 & 3 & 1 \\ 2 & -4 & 1 \end{vmatrix} =$$
$= 3(-37) + 1(4) - 5(6) = -137.$

26. Expand about the fourth column.
$$-0 + (-3)\begin{vmatrix} 1 & -4 & 2 \\ 2 & 2 & 4 \\ 3 & 0 & -3 \end{vmatrix} - 1\begin{vmatrix} 1 & -4 & 2 \\ -2 & -1 & 0 \\ 3 & 0 & -3 \end{vmatrix} +$$
$$1\begin{vmatrix} 1 & -4 & 2 \\ -2 & -1 & 0 \\ 2 & 2 & 4 \end{vmatrix} =$$
$= -0 + (-3)(-90) - 1(33) + 1(-40) = 197.$

27. Expand the determinant about the second row.
$$-(1)\begin{vmatrix} -3 & 4 & 6 \\ 3 & 1 & -3 \\ 0 & 2 & 1 \end{vmatrix} + (-5)\begin{vmatrix} 2 & 4 & 6 \\ 1 & 1 & -3 \\ -2 & 2 & 1 \end{vmatrix} =$$
$= -(1)(3) + (-5)(58) = -293.$

28. Expand the determinant about the third column.
$$-(-3)\begin{vmatrix} -2 & 4 & 5 \\ 2 & -1 & 7 \\ 3 & 2 & -1 \end{vmatrix} = -(-3)(153) = 459.$$

29.

One finds $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 1 & 1 \end{vmatrix} = 2,$

$D_x = \begin{vmatrix} 6 & 1 & 1 \\ 2 & -1 & 1 \\ 7 & 1 & 1 \end{vmatrix} = 2,\ D_y = \begin{vmatrix} 1 & 6 & 1 \\ 1 & 2 & 1 \\ 2 & 7 & 1 \end{vmatrix} =$

4, and $D_z = \begin{vmatrix} 1 & 1 & 6 \\ 1 & -1 & 2 \\ 2 & 1 & 7 \end{vmatrix} = 6.$

Since $x = \dfrac{D_x}{D} = 2/2 = 1,\ y = \dfrac{D_y}{D} = 4/2 = 2,$

and $z = \dfrac{D_z}{D} = 6/2 = 3,$

the solution set is $\{(1,2,3)\}$.

30.

One finds $D = \begin{vmatrix} 2 & -2 & 3 \\ 1 & 1 & -1 \\ 3 & 1 & -2 \end{vmatrix} = -6$,

$D_x = \begin{vmatrix} 7 & -2 & 3 \\ -2 & 1 & -1 \\ 5 & 1 & -2 \end{vmatrix} = -10$,

$D_y = \begin{vmatrix} 2 & 7 & 3 \\ 1 & -2 & -1 \\ 3 & 5 & -2 \end{vmatrix} = 44$,

and $D_z = \begin{vmatrix} 2 & -2 & 7 \\ 1 & 1 & -2 \\ 3 & 1 & 5 \end{vmatrix} = 22$.

Since $x = \dfrac{D_x}{D} = 10/6 = 5/3$,

$y = \dfrac{D_y}{D} = -44/6 = -22/3$,

and $z = \dfrac{D_z}{D} = -22/6 = -11/3$,

the solution set is $\{(5/3, -22/3, -11/3)\}$.

31.

One finds $D = \begin{vmatrix} 1 & 2 & 0 \\ 1 & -3 & 1 \\ 2 & -1 & 0 \end{vmatrix} = 5$,

$D_x = \begin{vmatrix} 8 & 2 & 0 \\ -2 & -3 & 1 \\ 1 & -1 & 0 \end{vmatrix} = 10$,

$D_y = \begin{vmatrix} 1 & 8 & 0 \\ 1 & -2 & 1 \\ 2 & 1 & 0 \end{vmatrix} = 15$,

and $D_z = \begin{vmatrix} 1 & 2 & 8 \\ 1 & -3 & -2 \\ 2 & -1 & 1 \end{vmatrix} = 25$.

Since $x = \dfrac{D_x}{D} = 10/5 = 2$,

$y = \dfrac{D_y}{D} = 15/5 = 3$,

and $z = \dfrac{D_z}{D} = 25/5 = 5$,

the solution set is $\{(2,3,5)\}$.

32.

One finds $D = \begin{vmatrix} 2 & 1 & 0 \\ 0 & 3 & -1 \\ 1 & 0 & 3 \end{vmatrix} = 17$,

$D_x = \begin{vmatrix} -4 & 1 & 0 \\ -1 & 3 & -1 \\ -16 & 0 & 3 \end{vmatrix} = -17$,

$D_y = \begin{vmatrix} 2 & -4 & 0 \\ 0 & -1 & -1 \\ 1 & -16 & 3 \end{vmatrix} = -34$,

and $D_z = \begin{vmatrix} 2 & 1 & -4 \\ 0 & 3 & -1 \\ 1 & 0 & -16 \end{vmatrix} = -85$.

Since $x = \dfrac{D_x}{D} = -17/17 = -1$,

$y = \dfrac{D_y}{D} = -34/17 = -2$,

and $z = \dfrac{D_z}{D} = -85/17 = -5$,

the solution set is $\{(-1,-2,-5)\}$.

33.

One finds $D = \begin{vmatrix} 2 & -3 & 1 \\ 1 & 4 & -1 \\ 3 & -1 & 2 \end{vmatrix} = 16$,

$D_x = \begin{vmatrix} 1 & -3 & 1 \\ 0 & 4 & -1 \\ 0 & -1 & 2 \end{vmatrix} = 7$,

$D_y = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 0 & -1 \\ 3 & 0 & 2 \end{vmatrix} = -5$,

and $D_z = \begin{vmatrix} 2 & -3 & 1 \\ 1 & 4 & 0 \\ 3 & -1 & 0 \end{vmatrix} = -13$.

Then $x = \dfrac{D_x}{D} = 7/16$,

$y = \dfrac{D_y}{D} = -5/16$, and $z = \dfrac{D_z}{D} = -13/16$.

The solution set is $\{(7/16, -5/16, -13/16)\}$.

34.

One finds $D = \begin{vmatrix} -2 & 1 & -1 \\ 1 & -1 & 3 \\ 3 & 3 & 2 \end{vmatrix} = 23$,

$$D_x = \begin{vmatrix} 0 & 1 & -1 \\ 1 & -1 & 3 \\ 0 & 3 & 2 \end{vmatrix} = -5,$$

$$D_y = \begin{vmatrix} -2 & 0 & -1 \\ 1 & 1 & 3 \\ 3 & 0 & 2 \end{vmatrix} = -1,$$

and $D_z = \begin{vmatrix} -2 & 1 & 0 \\ 1 & -1 & 1 \\ 3 & 3 & 0 \end{vmatrix} = 9.$

Then $x = \dfrac{D_x}{D} = -5/23,$

$y = \dfrac{D_y}{D} = -1/23$, and $z = \dfrac{D_z}{D} = 9/23.$

The solution set is $\{(-5/23, -1/23, 9/23)\}$.

35.

One finds $D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & -1 & 3 \\ 3 & 1 & -1 \end{vmatrix} = 14,$

$$D_x = \begin{vmatrix} 2 & 1 & 1 \\ 0 & -1 & 3 \\ 0 & 1 & -1 \end{vmatrix} = -4,$$

$$D_y = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 0 & 3 \\ 3 & 0 & -1 \end{vmatrix} = 22,$$

and $D_z = \begin{vmatrix} 1 & 1 & 2 \\ 2 & -1 & 0 \\ 3 & 1 & 0 \end{vmatrix} = 10.$

Then $x = \dfrac{D_x}{D} = -4/14 = -2/7,$

$y = \dfrac{D_y}{D} = 22/14 = 11/7$, and

$z = \dfrac{D_z}{D} = 10/14 = 5/7.$

The solution set is $\{(-2/7, 11/7, 5/7)\}$.

36.

One finds $D = \begin{vmatrix} 1 & -2 & -1 \\ -1 & 1 & 3 \\ 1 & 3 & 1 \end{vmatrix} = -12,$

$$D_x = \begin{vmatrix} 0 & -2 & -1 \\ 0 & 1 & 3 \\ 3 & 3 & 1 \end{vmatrix} = -15,$$

$$D_y = \begin{vmatrix} 1 & 0 & -1 \\ -1 & 0 & 3 \\ 1 & 3 & 1 \end{vmatrix} = -6,$$

and $D_z = \begin{vmatrix} 1 & -2 & 0 \\ -1 & 1 & 0 \\ 1 & 3 & 3 \end{vmatrix} = -3.$

Then $x = \dfrac{D_x}{D} = 15/12 = 5/4,$

$y = \dfrac{D_y}{D} = 6/12 = 1/2$, and $z = \dfrac{D_z}{D} = 3/12 = 1/4$. The solution set is $\{(5/4, 1/2, 1/4)$.

37. This system is dependent since the sum of the first two equations is the third equation. Adding the first and third equations, we get $3x - 3z = 4$ or $z = x - 4/3$. Substituting into the first equation, we find

$$\begin{aligned} x + y - 2\left(x - \frac{4}{3}\right) &= 1 \\ y &= 1 - \frac{8}{3} + x \\ y &= x - \frac{5}{3}. \end{aligned}$$

Solution set is

$$\left\{\left(x, x - \frac{5}{3}, x - \frac{4}{3}\right) \mid x \text{ is any real number}\right\}.$$

38. This system is dependent since if one multiplies the first equation by 2 and adds to the second equation, one gets the third equation:

$$\begin{array}{rcl} 2x + 2y + 2z &=& 8 \\ -2x - y + 3z &=& 1 \\ \hline y + 5z &=& 9 \end{array}$$

So $y = 9 - 5z$. Substituting into the first equation, we get $x + (9 - 5z) + z = 4$ or $x = 4z - 5$. Solution set is

$$\{(4z - 5, 9 - 5z, z) \mid z \text{ is any real number}\}.$$

39. Multiply the first equation by -2 and add to the third equation.

$$\begin{array}{rcr} -2x + 2y - 2z &=& -10 \\ 2x - 2y + 2z &=& 16 \\ \hline 0 &=& 6 \end{array}$$

Inconsistent and the solution set is $\emptyset$.

40. Multiply the second equation by -3 and add it to the first equation.

$$\begin{array}{rcr} 3x+6y+9z &=& 12 \\ -3x-6y-9z &=& 0 \\ \hline 0 &=& 12 \end{array}$$

Inconsistent and the solution set is $\emptyset$.

41. Let x, y, and z be the ages of Jackie, Rochelle, and Alisha, respectively. Since $\dfrac{x+y}{2} = 33$, $\dfrac{y+z}{2} = 25$, and $\dfrac{x+z}{2} = 19$, we obtain

$$\begin{array}{rcl} x+y &=& 66 \\ y+z &=& 50 \\ x+z &=& 38. \end{array}$$

One finds $D = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = 2$,

$D_x = \begin{vmatrix} 66 & 1 & 0 \\ 50 & 1 & 1 \\ 38 & 0 & 1 \end{vmatrix} = 54$, $D_y = \begin{vmatrix} 1 & 66 & 0 \\ 0 & 50 & 1 \\ 1 & 38 & 1 \end{vmatrix} =$ 78, and $D_z = \begin{vmatrix} 1 & 1 & 66 \\ 0 & 1 & 50 \\ 1 & 0 & 38 \end{vmatrix} = 22$.

So Jackie is $x = \dfrac{D_x}{D} = 54/2 = 27$ years old,

Rochelle is $y = \dfrac{D_y}{D} = 78/2 = 39$ years old,

and Alisha is $z = \dfrac{D_z}{D} = 22/2 = 11$ years old.

42. Let x, y, and z be the number of nickels, dimes, and quarters, respectively.

$$\begin{array}{rcl} x+y+z &=& 49 \\ -x+y+z &=& 1 \\ 5x+10y+25z &=& 550 \end{array}$$

One finds $D = \begin{vmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \\ 5 & 10 & 25 \end{vmatrix} = 30$,

$D_x = \begin{vmatrix} 49 & 1 & 1 \\ 1 & 1 & 1 \\ 550 & 10 & 25 \end{vmatrix} = 720$,

$D_y = \begin{vmatrix} 1 & 49 & 1 \\ -1 & 1 & 1 \\ 5 & 550 & 25 \end{vmatrix} = 390$, and

$D_z = \begin{vmatrix} 1 & 1 & 49 \\ -1 & 1 & 1 \\ 5 & 10 & 550 \end{vmatrix} = 360$.

There were $x = \dfrac{D_x}{D} = 720/30 = 24$ nickels,

$y = \dfrac{D_y}{D} = 390/30 = 13$ dimes,

and $z = \dfrac{D_z}{D} = 360/30 = 12$ quarters.

43. Let x, y, and z be the scores in the first test, second test, and final exam, respectively. Then

$$\begin{array}{rcl} x+y+z &=& 180 \\ 0.2x+0.2y+0.6z &=& 76 \\ 0.1x+0.2y+0.7z &=& 83. \end{array}$$

One finds $D = \begin{vmatrix} 1 & 1 & 1 \\ 0.2 & 0.2 & 0.6 \\ 0.1 & 0.2 & 0.7 \end{vmatrix} = -0.04$,

$D_x = \begin{vmatrix} 180 & 1 & 1 \\ 76 & 0.2 & 0.6 \\ 83 & 0.2 & 0.7 \end{vmatrix} = -1.2$,

$D_y = \begin{vmatrix} 1 & 180 & 1 \\ 0.2 & 76 & 0.6 \\ 0.1 & 83 & 0.7 \end{vmatrix} = -2$,

and $D_z = \begin{vmatrix} 1 & 1 & 180 \\ 0.2 & 0.2 & 76 \\ 0.1 & 0.2 & 83 \end{vmatrix} = -4$.

Test scores are $x = \dfrac{D_x}{D} = 1.2/(0.04) = 30$,

$y = \dfrac{D_y}{D} = 2/(0.04) = 50$, and

the final exam is $z = \dfrac{D_z}{D} = 4/(0.04) = 100$.

44. Let x, y, and z be the number of batches of chocolate chips, oatmeal, and peanut butter cookies, respectively. Then we have

$$\begin{array}{rcl} 2x+y+4z &=& 18 \\ 2x+y+2z &=& 14 \\ x+2y+z &=& 13. \end{array}$$

One finds $D = \begin{vmatrix} 2 & 1 & 4 \\ 2 & 1 & 2 \\ 1 & 2 & 1 \end{vmatrix} = 6$,

$D_x = \begin{vmatrix} 18 & 1 & 4 \\ 14 & 1 & 2 \\ 13 & 2 & 1 \end{vmatrix} = 18$,

$D_y = \begin{vmatrix} 2 & 18 & 4 \\ 2 & 14 & 2 \\ 1 & 13 & 1 \end{vmatrix} = 24$,

and $D_z = \begin{vmatrix} 2 & 1 & 18 \\ 2 & 1 & 14 \\ 1 & 2 & 13 \end{vmatrix} = 12$.

Since $x = \dfrac{D_x}{D} = 18/6 = 3$,

$y = \dfrac{D_y}{D} = 24/6 = 4$, and $z = \dfrac{D_z}{D} = 12/6 = 2$,

the number of batches of chocolate chips, oatmeal cookies, and peanut butter cookies are 3, 4, and 2, respectively.

45. One finds $D = \begin{vmatrix} 0.2 & -0.3 & 1.2 \\ 0.25 & 0.35 & -0.9 \\ 2.4 & -1 & 1.25 \end{vmatrix} = -\dfrac{527}{800}$,

$D_x = \begin{vmatrix} 13.11 & -0.3 & 1.2 \\ -1.575 & 0.35 & -0.9 \\ 42.02 & -1 & 1.25 \end{vmatrix} = -\dfrac{11,067}{1000}$,

$D_y = \begin{vmatrix} 0.2 & 13.11 & 1.2 \\ 0.25 & -1.575 & -0.9 \\ 2.4 & 42.02 & 1.25 \end{vmatrix} = -\dfrac{64,821}{8000}$,

and $D_z = \begin{vmatrix} 0.2 & -0.3 & 13.11 \\ 0.25 & 0.35 & -1.575 \\ 2.4 & -1 & 42.02 \end{vmatrix} = -\dfrac{3689}{500}$.

Note, $\dfrac{D_x}{D} = 16.8$, $\dfrac{D_y}{D} = 12.3$, and $\dfrac{D_z}{D} = 11.2$.

The solution set is $\{(16.8, 12.3, 11.2)\}$.

46. One finds $D = \begin{vmatrix} 3.6 & 4.5 & 6.8 \\ 0.09 & 0.05 & 0.04 \\ 1 & 1 & -1 \end{vmatrix} = \dfrac{533}{1000}$,

$D_x = \begin{vmatrix} 45,300 & 4.5 & 6.8 \\ 474 & 0.05 & 0.04 \\ 0 & 1 & -1 \end{vmatrix} = \dfrac{6396}{5}$,

$D_y = \begin{vmatrix} 3.6 & 45,300 & 6.8 \\ 0.09 & 474 & 0.04 \\ 1 & 0 & -1 \end{vmatrix} = \dfrac{4797}{5}$,

and $D_z = \begin{vmatrix} 3.6 & 4.5 & 45,300 \\ 0.09 & 0.05 & 474 \\ 1 & 1 & 0 \end{vmatrix} = \dfrac{11,193}{5}$.

Note $\dfrac{D_x}{D} = 2400$, $\dfrac{D_y}{D} = 1800$, and $\dfrac{D_z}{D} = 4200$.
The solution set is $\{(2400, 1800, 4200)\}$.

47. Let x, y, and z be the prices per gallon of regular, plus, and supreme gasoline, respectively.
One finds

$D = \begin{vmatrix} 1270 & 980 & 890 \\ 1450 & 1280 & 1050 \\ 1340 & 1190 & 1060 \end{vmatrix} = 18,038,000$,

$D_x = \begin{vmatrix} 12,204.86 & 980 & 890 \\ 14,698.22 & 1280 & 1050 \\ 13,969.41 & 1190 & 1060 \end{vmatrix} = 68,526,400$,

$D_y = \begin{vmatrix} 1270 & 12,204.86 & 890 \\ 1450 & 14,698.22 & 1050 \\ 1340 & 13,969.41 & 1060 \end{vmatrix} = 70,330,200$,

$D_z = \begin{vmatrix} 1270 & 980 & 12,204.86 \\ 1450 & 1280 & 14,698.22 \\ 1340 & 1190 & 13,969.41 \end{vmatrix} = 72,134,000$.

Regular gas costs $\dfrac{D_x}{D} \approx \$3.799$,

plus costs $\dfrac{D_y}{D} \approx \$3.899$, and

supreme costs $\dfrac{D_z}{D} \approx \$3.999$ per gallon.

48. Let x, y, and z be the number of gallons of regular, plus, and supreme gasoline he sells every week, respectively. One finds

$D = \begin{vmatrix} 3.799 & 3.899 & 3.999 \\ 3.749 & 3.849 & 3.949 \\ 3.759 & 3.899 & 3.949 \end{vmatrix} = -0.00045$,

$D_x = \begin{vmatrix} 17,996.37 & 3.899 & 3.999 \\ 17,764.87 & 3.849 & 3.949 \\ 17,856.07 & 3.899 & 3.949 \end{vmatrix} = -0.8415$

$$D_y = \begin{vmatrix} 3.799 & 17,996.37 & 3.999 \\ 3.749 & 17,764.87 & 3.949 \\ 3.759 & 17,856.07 & 3.949 \end{vmatrix} = -0.6525,$$

and

$$D_z = \begin{vmatrix} 3.799 & 3.899 & 17,996.37 \\ 3.749 & 3.849 & 17,764.87 \\ 3.759 & 3.899 & 17,856.07 \end{vmatrix} = -0.5895.$$

Thus, the amounts sold are

$\frac{D_x}{D} = 1870$ gallons of regular,

$\frac{D_y}{D} = 1450$ gallons of plus, and

$\frac{D_z}{D} = 1310$ gallons of supreme.

49.

One finds $D = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 2 & -1 & 1 & 3 \\ 1 & 2 & -1 & 2 \\ 1 & -1 & -1 & 4 \end{vmatrix} = 11,$

$$D_w = \begin{vmatrix} 4 & 1 & 1 & 1 \\ 13 & -1 & 1 & 3 \\ -2 & 2 & -1 & 2 \\ 8 & -1 & -1 & 4 \end{vmatrix} = 11,$$

$$D_x = \begin{vmatrix} 1 & 4 & 1 & 1 \\ 2 & 13 & 1 & 3 \\ 1 & -2 & -1 & 2 \\ 1 & 8 & -1 & 4 \end{vmatrix} = -22,$$

$$D_y = \begin{vmatrix} 1 & 1 & 4 & 1 \\ 2 & -1 & 13 & 3 \\ 1 & 2 & -2 & 2 \\ 1 & -1 & 8 & 4 \end{vmatrix} = 33,$$

and $D_z = \begin{vmatrix} 1 & 1 & 1 & 4 \\ 2 & -1 & 1 & 13 \\ 1 & 2 & -1 & -2 \\ 1 & -1 & -1 & 8 \end{vmatrix} = 22.$

Note, $\frac{D_w}{D} = 1$, $\frac{D_x}{D} = -2$, $\frac{D_y}{D} = 3$, and $\frac{D_z}{D} = 2$. The solution set is $\{(1, -2, 3, 2)\}$.

50.

One finds $D = \begin{vmatrix} 2 & 2 & -2 & 1 \\ 1 & 1 & 1 & 1 \\ 4 & -3 & 2 & -5 \\ 1 & 3 & -1 & 9 \end{vmatrix} = 170,$

$$D_w = \begin{vmatrix} 11 & 2 & -2 & 1 \\ 10 & 1 & 1 & 1 \\ 6 & -3 & 2 & -5 \\ 20 & 3 & -1 & 9 \end{vmatrix} = 680,$$

$$D_x = \begin{vmatrix} 2 & 11 & -2 & 1 \\ 1 & 10 & 1 & 1 \\ 4 & 6 & 2 & -5 \\ 1 & 20 & -1 & 9 \end{vmatrix} = 510,$$

$$D_y = \begin{vmatrix} 2 & 2 & 11 & 1 \\ 1 & 1 & 10 & 1 \\ 4 & -3 & 6 & -5 \\ 1 & 3 & 20 & 9 \end{vmatrix} = 340,$$

and $D_z = \begin{vmatrix} 2 & 2 & -2 & 11 \\ 1 & 1 & 1 & 10 \\ 4 & -3 & 2 & 6 \\ 1 & 3 & -1 & 20 \end{vmatrix} = 170,$

Note, $\frac{D_w}{D} = 4$, $\frac{D_x}{D} = 3$, $\frac{D_y}{D} = 2$, and $\frac{D_z}{D} = 1$.

The solution set is $\{(4, 3, 2, 1)\}$.

51.

Note, $\begin{vmatrix} x & y & 1 \\ 3 & -5 & 1 \\ -2 & 6 & 1 \end{vmatrix} = 8 - 11x - 5y = 0.$

Since both points $(3, -5)$ and $(-2, 6)$ satisfies $8 - 11x - 5y = 0$, this is an equation of the line through the two points.

52.

One finds $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} =$

$xy_1 + yx_2 + x_1y_2 - x_2y_1 - xy_2 - x_1y = 0.$

If the points (x_1, y_1) and (x_2, y_2) satisfy this linear equation, then we will be done. Upon substitution of the first point, we have

$x_1y_1 + y_1x_2 + x_1y_2 - x_2y_1 - x_1y_2 - x_1y_1 = 0$

which is an identity. Substituting the second point we get

$x_2y_1 + y_2x_2 + x_1y_2 - x_2y_1 - x_2y_2 - x_1y_2 = 0$

which is also an identity.

53. We will prove the determinant is zero for one particular case and the other cases can proved similarly.

Let us suppose $A = \begin{bmatrix} a & b & c \\ 0 & 0 & 0 \\ g & h & i \end{bmatrix}$.

Then $|A| = a \cdot 0 \cdot i + b \cdot 0 \cdot g + c \cdot 0 \cdot h - g \cdot 0 \cdot c - h \cdot 0 \cdot a - i \cdot 0 \cdot b = 0$

54. We will prove it for one particular case and the other cases can be shown similarly.

Let $A = \begin{bmatrix} a & b & b \\ c & d & d \\ e & f & f \end{bmatrix}$.

Then $|A| = adf + bde + bcf - edb - fda - fcb$. Since all the terms cancel, we find $|A| = 0$.

55. We will prove it for one particular case and the other cases can be shown similarly.

Let us suppose $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ and

$B = \begin{bmatrix} a & b & c \\ d & e & f \\ kg & kh & ki \end{bmatrix}$. Then $|B| =$

$aeki + bfkg + cdkh - kgec - khfa - kidb = k(aei + bfg + cdh - gec - hfa - idb) = k|A|$.

56. We will prove this only for the case when the first and second rows are interchanged, and the other cases can be proved similarly.

Then we obtain

$\left|\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\right| =$

$(aei + bfg + cdh - gec - hfa - idb)$ and

$\left|\begin{bmatrix} d & e & f \\ a & b & c \\ g & h & i \end{bmatrix}\right| =$

$(dbi + ecg + fah - gbf - hcd - iae)$.

It follows that if we interchange the first and the second rows, then the determinant of the matrices differ only by a sign.

57.

$$\begin{vmatrix} 3 & -2 \\ -1 & 4 \end{vmatrix} = 3(4) - (-1)(-2) = 10$$

58. Expand about the 3rd row.

$$-5\begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix} + 7\begin{vmatrix} 1 & 2 \\ 2 & 0 \end{vmatrix} = -5(-2) + 7(-4) = -18$$

59. For each value of z, the system below

$$\begin{aligned} 5x - 9y &= 44 - 11z \\ 13x - 12y &= -9 - 31z \end{aligned}$$

has unique solution since the coefficient matrix $\begin{bmatrix} 5 & -9 \\ 13 & -12 \end{bmatrix}$ has a nonzero determinant. Then the given system of three equations is not independent since the solution is not unique. Hence, in addition since the system is not inconsistent, it follows by elimination that the system is dependent.

60. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} =$

$\begin{bmatrix} 1(0)+2(1) & 1(1)+2(0) & 1(0)+2(1) \\ 3(0)+4(1) & 3(1)+4(0) & 3(0)+4(1) \\ 5(0)+6(1) & 5(1)+6(0) & 5(0)+6(1) \end{bmatrix} =$

$\begin{bmatrix} 2 & 1 & 2 \\ 4 & 3 & 4 \\ 6 & 5 & 6 \end{bmatrix}$

61. If $(x-4)^2 = x^2$, then $x - 4 = \pm x$. Since $x-4 = x$ is inconsistent, we have $x-4 = -x$ or $2x = 4$. Then $x = 2$. Thus, $y = x^2 = 2^2 = 4$. The solution set is $\{2, 4\}$.

62. Let x be the number of minutes and let y be the population of bacteria. Since the population doubles every 15 minutes, we obtain

$$y = 1000 \cdot 2^{x/15}.$$

If $y = 2500$, then

$$\begin{aligned} 1000 \cdot 2^{x/15} &= 2500 \\ 2^{x/15} &= 2.5 \\ \frac{x}{15}\ln 2 &= \ln 2.5 \\ x &= \frac{15 \ln 2.5}{\ln 2} \\ x &\approx 19.8 \text{ min.} \end{aligned}$$

Thinking Outside the Box LXXXV

It makes no sense to do this computation. No one is holding \$29. Rather, three times \$9 (i.e., total paid by the three boys) is the same as \$25 (rent) plus \$2 (bellboy).

9.6 Pop Quiz

1. Expanding about the first row, we find

$$D = 1\begin{vmatrix} 1 & 1 \\ 3 & 4 \end{vmatrix} - 2\begin{vmatrix} 2 & 1 \\ 3 & 4 \end{vmatrix} + 3\begin{vmatrix} 2 & 1 \\ 3 & 3 \end{vmatrix}$$
$$= 1(1) - 2(5) + 3(3) = 0.$$

2. Apply Cramer's Rule. We find

$$D = \begin{vmatrix} 1 & 1 & -1 \\ 1 & -1 & 2 \\ 2 & 1 & 1 \end{vmatrix} = -3,$$

$$D_x = \begin{vmatrix} 1 & 1 & -1 \\ 9 & -1 & 2 \\ 12 & 1 & 1 \end{vmatrix} = -9,$$

$$D_y = \begin{vmatrix} 1 & 1 & -1 \\ 1 & 9 & 2 \\ 2 & 12 & 1 \end{vmatrix} = -6,$$

and $D_z = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 9 \\ 2 & 1 & 12 \end{vmatrix} = -12.$

Then $x = \dfrac{D_x}{D} = (-9)/(-3) = 3,$

$y = \dfrac{D_y}{D} = (-6)/(-3) = 2,$ and

$z = \dfrac{D_z}{D} = (-12)/(-3) = 4.$

The solution set is $\{(3, 2, 4)\}$.

3. Expanding about the first column, the determinant is

$$D = 1\begin{vmatrix} 2 & 6 \\ 5 & 15 \end{vmatrix} = 0.$$

Since the determinant is zero, the matrix is not invertible.

Review Exercises

1.
$$\begin{bmatrix} 2+3 & -3+7 \\ -2+1 & 4+2 \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ -1 & 6 \end{bmatrix}$$

2.
$$\begin{bmatrix} 2-3 & -3-7 \\ -2-1 & 4-2 \end{bmatrix} = \begin{bmatrix} -1 & -10 \\ -3 & 2 \end{bmatrix}$$

3.
$$\begin{bmatrix} 4 & -6 \\ -4 & 8 \end{bmatrix} - \begin{bmatrix} 3 & 7 \\ 1 & 2 \end{bmatrix} =$$
$$\begin{bmatrix} 1 & -13 \\ -5 & 6 \end{bmatrix}$$

4.
$$2A + 3B = \begin{bmatrix} 4 & -6 \\ -4 & 8 \end{bmatrix} + \begin{bmatrix} 9 & 21 \\ 3 & 6 \end{bmatrix} =$$
$$\begin{bmatrix} 13 & 15 \\ -1 & 14 \end{bmatrix}$$

5.
$$AB = \begin{bmatrix} 6-3 & 14-6 \\ -6+4 & -14+8 \end{bmatrix} =$$
$$\begin{bmatrix} 3 & 8 \\ -2 & -6 \end{bmatrix}$$

6.
$$BA = \begin{bmatrix} 6-14 & -9+28 \\ 2-4 & -3+8 \end{bmatrix} =$$
$$\begin{bmatrix} -8 & 19 \\ -2 & 5 \end{bmatrix}$$

7. $D + E$ is undefined

8. $F + G$ is undefined

9.
$$AC = \begin{bmatrix} -2-9 \\ 2+12 \end{bmatrix} = \begin{bmatrix} -11 \\ 14 \end{bmatrix}$$

10.
$$BD = \begin{bmatrix} 15-21 \\ 5-6 \end{bmatrix} = \begin{bmatrix} -6 \\ -1 \end{bmatrix}$$

11.
$$EF = \begin{bmatrix} 3 & 2 & -1 \\ -12 & -8 & 4 \\ 9 & 6 & -3 \end{bmatrix}$$

12. $FE = [3 - 8 - 3] = [-8]$

13. $FG = [-3+2+2 \quad 2-3 \quad -1] =$
$[1 \quad -1 \quad -1]$

14.
$$GE = \begin{bmatrix} -1 \\ 1-4 \\ -2-12+3 \end{bmatrix} = \begin{bmatrix} -1 \\ -3 \\ -11 \end{bmatrix}$$

15. GF is undefined **16.** EG is undefined

17.
On $\left[\begin{array}{rr|rr} 2 & -3 & 1 & 0 \\ -2 & 4 & 0 & 1 \end{array}\right]$, use $R_1 + R_2 \to R_2$ to get

$\left[\begin{array}{rr|rr} 2 & -3 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{array}\right]$, use $3R_2 + R_1 \to R_1$ to get

$\left[\begin{array}{rr|rr} 2 & 0 & 4 & 3 \\ 0 & 1 & 1 & 1 \end{array}\right]$, use $\frac{1}{2}R_1 \to R_1$ to get

$\left[\begin{array}{rr|rr} 1 & 0 & 2 & 1.5 \\ 0 & 1 & 1 & 1 \end{array}\right]$. So $A^{-1} = \begin{bmatrix} 2 & 1.5 \\ 1 & 1 \end{bmatrix}$.

18.
On $\left[\begin{array}{rr|rr} 3 & 7 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{array}\right]$, use $-3R_2 + R_1 \to R_2$ to get

$\left[\begin{array}{rr|rr} 3 & 7 & 1 & 0 \\ 0 & 1 & 1 & -3 \end{array}\right]$, use $-7R_2 + R_1 \to R_1$ to get

$\left[\begin{array}{rr|rr} 3 & 0 & -6 & 21 \\ 0 & 1 & 1 & -3 \end{array}\right]$, use $\frac{1}{3}R_1 \to R_1$ to get

$\left[\begin{array}{rr|rr} 1 & 0 & -2 & 7 \\ 0 & 1 & 1 & -3 \end{array}\right]$. So $B^{-1} = \begin{bmatrix} -2 & 7 \\ 1 & -3 \end{bmatrix}$.

19.
On $\left[\begin{array}{rrr|rrr} -1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ -2 & 3 & 1 & 0 & 0 & 1 \end{array}\right]$,

use $R_1 + R_2 \to R_2$, $2R_2 + R_3 \to R_3$, and $-1R_1 \to R_1$ to get

$\left[\begin{array}{rrr|rrr} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 5 & 1 & 0 & 2 & 1 \end{array}\right]$,

use $-5R_2 + R_3 \to R_3$ to get

$\left[\begin{array}{rrr|rrr} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & -5 & -3 & 1 \end{array}\right]$.

So $G^{-1} = \begin{bmatrix} -1 & 0 & 0 \\ 1 & 1 & 0 \\ -5 & -3 & 1 \end{bmatrix}$.

20.
$$A^{-1}C = \begin{bmatrix} 2 & 1.5 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix} =$$
$$\begin{bmatrix} -2+4.5 \\ -1+3 \end{bmatrix} = \begin{bmatrix} 2.5 \\ 2 \end{bmatrix}$$

21.
From Exercise 5, $(AB)^{-1} = \begin{bmatrix} 3 & 8 \\ -2 & -6 \end{bmatrix}^{-1}$.

On $\left[\begin{array}{rr|rr} 3 & 8 & 1 & 0 \\ -2 & -6 & 0 & 1 \end{array}\right]$, use $-\frac{1}{2}R_2 \to R_2$ to get

$\left[\begin{array}{rr|rr} 3 & 8 & 1 & 0 \\ 1 & 3 & 0 & -0.5 \end{array}\right]$, use $-3R_2 + R_1 \to R_1$ to get

$\left[\begin{array}{rr|rr} 0 & -1 & 1 & 1.5 \\ 1 & 3 & 0 & -0.5 \end{array}\right]$, use $3R_1 + R_2 \to R_2$ to get

$\left[\begin{array}{rr|rr} 0 & -1 & 1 & 1.5 \\ 1 & 0 & 3 & 4 \end{array}\right]$, use $R_1 \leftrightarrow R_2$ to get

$\left[\begin{array}{rr|rr} 1 & 0 & 3 & 4 \\ 0 & -1 & 1 & 1.5 \end{array}\right]$, use $-1R_2 \to R_2$ to get

$\left[\begin{array}{rr|rr} 1 & 0 & 3 & 4 \\ 0 & 1 & -1 & -1.5 \end{array}\right]$.

So $(AB)^{-1} = \begin{bmatrix} 3 & 4 \\ -1 & -1.5 \end{bmatrix}$.

22.
From Exercises 17 and 18,
$$A^{-1}B^{-1} = \begin{bmatrix} 2 & 1.5 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} -2 & 7 \\ 1 & -3 \end{bmatrix} =$$
$$= \begin{bmatrix} -4+1.5 & 14-4.5 \\ -2+1 & 7-3 \end{bmatrix} = \begin{bmatrix} -2.5 & 9.5 \\ -1 & 4 \end{bmatrix}.$$

23.
$$AA^{-1} = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

24.
$$GG^{-1} = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

25. $|A| = 8 - 6 = 2$

26. $|B| = 6 - 7 = -1$

27. Expanding about the third column,

$$|G| = 1 \cdot \begin{vmatrix} -1 & 0 \\ 1 & 1 \end{vmatrix} = 1(-1) = -1.$$

28. $|C|$ is undefined

29. The solution set is $\{(10/3, 17/3)\}$.

First solution is by Gaussian elimination.

On $\left[\begin{array}{cc|c} 1 & 1 & 9 \\ 2 & -1 & 1 \end{array}\right]$, use $-2R_1 + R_2 \to R_2$ to get

$\left[\begin{array}{cc|c} 1 & 1 & 9 \\ 0 & -3 & -17 \end{array}\right]$, use $-\dfrac{1}{3}R_2 \to R_2$ to get

$\left[\begin{array}{cc|c} 1 & 1 & 9 \\ 0 & 1 & 17/3 \end{array}\right]$, use $-1R_2 + R_1 \to R_1$ to get

$\left[\begin{array}{cc|c} 1 & 0 & 10/3 \\ 0 & 1 & 17/3 \end{array}\right]$.

Secondly, by matrix inversion note $A^{-1} = \begin{bmatrix} 1/3 & 1/3 \\ 2/3 & -1/3 \end{bmatrix}$ and $A^{-1}\begin{bmatrix} 9 \\ 1 \end{bmatrix} = \begin{bmatrix} 10/3 \\ 17/3 \end{bmatrix}$.

Thirdly, by Cramer's Rule note

$D = \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} = -3$, $D_x = \begin{vmatrix} 9 & 1 \\ 1 & -1 \end{vmatrix} = -10$,

and $D_y = \begin{vmatrix} 1 & 9 \\ 2 & 1 \end{vmatrix} = -17$.

So $\dfrac{D_x}{D} = 10/3$, $\dfrac{D_y}{D} = 17/3$.

30. The solution set is $\{(5/2, -1/4)\}$.

First solution is by Gaussian elimination.

On $\left[\begin{array}{cc|c} 1 & -2 & 3 \\ 1 & 2 & 2 \end{array}\right]$, use $R_1 + R_2 \to R_2$ to get

$\left[\begin{array}{cc|c} 1 & -2 & 3 \\ 2 & 0 & 5 \end{array}\right]$, use $-2R_1 + R_2 \to R_1$ to get

$\left[\begin{array}{cc|c} 0 & 4 & -1 \\ 2 & 0 & 5 \end{array}\right]$, use $\dfrac{1}{4}R_1 \to R_1$ and $\dfrac{1}{2}R_2 \to R_2$ to get

$\left[\begin{array}{cc|c} 0 & 1 & -1/4 \\ 1 & 0 & 5/2 \end{array}\right]$, use $R_1 \leftrightarrow R_2$ to get

$\left[\begin{array}{cc|c} 1 & 0 & 5/2 \\ 0 & 1 & -1/4 \end{array}\right]$.

Secondly, by matrix inversion note $A^{-1} = \begin{bmatrix} 1/2 & 1/2 \\ -1/4 & 1/4 \end{bmatrix}$ and $A^{-1}\begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 5/2 \\ -1/4 \end{bmatrix}$.

Thirdly, by Cramer's Rule note

$D = \begin{vmatrix} 1 & -2 \\ 1 & 2 \end{vmatrix} = 4$, $D_x = \begin{vmatrix} 3 & -2 \\ 2 & 2 \end{vmatrix} = 10$,

and $D_y = \begin{vmatrix} 1 & 3 \\ 1 & 2 \end{vmatrix} = -1$.

So $\dfrac{D_x}{D} = 5/2$, $\dfrac{D_y}{D} = -1/4$.

31. The solution set is $\{(-2, 3)\}$.

First solution is by Gaussian elimination. On $\left[\begin{array}{cc|c} 2 & 1 & -1 \\ 3 & 2 & 0 \end{array}\right]$, use $-\dfrac{3}{2}R_1 + R_2 \to R_2$ to get

$\left[\begin{array}{cc|c} 2 & 1 & -1 \\ 0 & 1/2 & 3/2 \end{array}\right]$, use $-2R_2 + R_1 \to R_1$ and $2R_2 \to R_2$ to get

$\left[\begin{array}{cc|c} 2 & 0 & -4 \\ 0 & 1 & 3 \end{array}\right]$, use $\dfrac{1}{2}R_2 \to R_2$ to get

$\left[\begin{array}{cc|c} 1 & 0 & -2 \\ 0 & 1 & 3 \end{array}\right]$.

Secondly, by matrix inversion note $A^{-1} = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$ and $A^{-1}\begin{bmatrix} -1 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$.

Thirdly, by Cramer's Rule note

$D = \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} = 1$, $D_x = \begin{vmatrix} -1 & 1 \\ 0 & 2 \end{vmatrix} = -2$, and $D_y = \begin{vmatrix} 2 & -1 \\ 3 & 0 \end{vmatrix} = 3$.

So $\frac{D_x}{D} = -2$, $\frac{D_y}{D} = 3$.

32. Solution set is $\{(2,5)\}$.

First solution is by Gaussian elimination.

On $\left[\begin{array}{cc|c} 3 & -1 & 1 \\ -2 & 1 & 1 \end{array}\right]$, use $R_1+R_2 \to R_1$ to get

$\left[\begin{array}{cc|c} 1 & 0 & 2 \\ -2 & 1 & 1 \end{array}\right]$, use $2R_1 + R_2 \to R_2$ to get

$\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 5 \end{array}\right]$.

Secondly, by matrix inversion note

$A^{-1} = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$ and $A^{-1}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$.

Thirdly, by Cramer's Rule note

$D = \begin{vmatrix} 3 & -1 \\ -2 & 1 \end{vmatrix} = 1$, $D_x = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = 2$,

and $D_y = \begin{vmatrix} 3 & 1 \\ -2 & 1 \end{vmatrix} = 5$.

So $\frac{D_x}{D} = 2$, $\frac{D_y}{D} = 5$.

33. Solution set is $\{(x,y) | x - 5y = 9\}$.

Solution is by Gaussian elimination.

On $\left[\begin{array}{cc|c} 1 & -5 & 9 \\ -2 & 10 & -18 \end{array}\right]$, use $2R_1 + R_2 \to R_2$

to get $\left[\begin{array}{cc|c} 1 & -5 & 9 \\ 0 & 0 & 0 \end{array}\right]$. Dependent system.

This cannot be solved by matrix inversion since $A^{-1} = \begin{bmatrix} 1 & -5 \\ -2 & 10 \end{bmatrix}^{-1}$ does not exist.

Nor can it be solved by Cramer's rule

since $|D| = \begin{vmatrix} 1 & -5 \\ -2 & 10 \end{vmatrix} = 0$.

34. The solution set is $\emptyset$.

First, apply the Gaussian elimination method.

On $\left[\begin{array}{cc|c} 3 & -1 & 4 \\ 6 & -2 & 6 \end{array}\right]$, use $-2R_1 + R_2 \to R_2$ to

get $\left[\begin{array}{cc|c} 3 & -1 & 4 \\ 0 & 0 & -2 \end{array}\right]$. Inconsistent system.

This cannot be solved by matrix inversion

since $A^{-1} = \begin{bmatrix} 3 & -1 \\ 6 & -2 \end{bmatrix}^{-1}$ does not exist.

Nor can it be solved by Cramer's rule

since $|D| = \begin{vmatrix} 3 & -1 \\ 6 & -2 \end{vmatrix} = 0$.

35. The solution set is $\emptyset$.

First, apply the Gaussian elimination method.

On $\left[\begin{array}{cc|c} 0.05 & 0.1 & 1 \\ 10 & 20 & 20 \end{array}\right]$, use

$-200R_1 + R_2 \to R_2$ to get

$\left[\begin{array}{cc|c} 0.05 & 0.1 & 1 \\ 0 & 0 & -180 \end{array}\right]$, which is inconsistent.

This cannot be solved by matrix inversion since

$A^{-1} = \begin{bmatrix} 0.05 & 0.1 \\ 10 & 20 \end{bmatrix}^{-1}$ does not exist.

Nor can it be solved by Cramer's rule

since $|D| = \begin{vmatrix} 0.05 & 0.1 \\ 10 & 20 \end{vmatrix} = 0$.

36. Solution set is $\{(x,y) | x - 5y = 75\}$.

Solution is by Gaussian elimination. On

$\left[\begin{array}{cc|c} 0.04 & -0.2 & 3 \\ 2 & -10 & 150 \end{array}\right]$, use $-50R_1 + R_2 \to R_2$

to get $\left[\begin{array}{cc|c} 0.04 & -0.2 & 3 \\ 0 & 0 & 0 \end{array}\right]$. Thus, we have a dependent system.

Divide $2x - 10y = 150$ by 2 to get $x - 5y = 75$. This descibes the infinite number of solutions.

This cannot be solved by matrix inversion since

$A^{-1} = \begin{bmatrix} 0.04 & -0.2 \\ 2 & -10 \end{bmatrix}^{-1}$ does not exist.

Nor can it be solved by Cramer's rule

since $|D| = \begin{vmatrix} 0.04 & -0.2 \\ 2 & -10 \end{vmatrix} = 0$.

37. Solution set is $\{(1,2,3)\}$.

First solution is by Gaussian elimination.

On $\left[\begin{array}{rrr|r} 1 & 1 & -2 & -3 \\ -1 & 2 & -1 & 0 \\ -1 & -1 & 3 & 6 \end{array}\right]$, use

$R_1 + R_2 \to R_2$ to get

$\left[\begin{array}{rrr|r} 1 & 1 & -2 & -3 \\ 0 & 3 & -3 & -3 \\ 0 & 0 & 1 & 3 \end{array}\right]$, use

$\frac{1}{3}R_2 \to R_2$ to get

$\left[\begin{array}{rrr|r} 1 & 1 & -2 & -3 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right]$, use

$R_3 + R_2 \to R_2$ and $2R_3 + R_1 \to R_1$ to get

$\left[\begin{array}{rrr|r} 1 & 1 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$, use $-1R_2 + R_1 \to R_1$

to get $\left[\begin{array}{rrr|r} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$.

Secondly, by matrix inversion note $A^{-1} =$

$\begin{bmatrix} 5/3 & -1/3 & 1 \\ 4/3 & 1/3 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ and $A^{-1}\begin{bmatrix} -3 \\ 0 \\ 6 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

Thirdly, by Cramer's Rule note

$$D = \begin{vmatrix} 1 & 1 & -2 \\ -1 & 2 & -1 \\ -1 & -1 & 3 \end{vmatrix} = 3,$$

$$D_x = \begin{vmatrix} -3 & 1 & -2 \\ 0 & 2 & -1 \\ 6 & -1 & 3 \end{vmatrix} = 3,$$

$$D_y = \begin{vmatrix} 1 & -3 & -2 \\ -1 & 0 & -1 \\ -1 & 6 & 3 \end{vmatrix} = 6,$$

and $D_z = \begin{vmatrix} 1 & 1 & -3 \\ -1 & 2 & 0 \\ -1 & -1 & 6 \end{vmatrix} = 9.$

Then $\frac{D_x}{D} = 1$, $\frac{D_y}{D} = 2$, and $\frac{D_z}{D} = 3$.

38. Solution set is $\{(2,0,3)\}$.

First solution is by Gaussian elimination.

On $\left[\begin{array}{rrr|r} 1 & -1 & 1 & 5 \\ 1 & 1 & 3 & 11 \\ -1 & 2 & -1 & -5 \end{array}\right]$, use

$-1R_1 + R_2 \to R_2$ and $R_1 + R_3 \to R_3$ to get

$\left[\begin{array}{rrr|r} 1 & -1 & 1 & 5 \\ 0 & 2 & 2 & 6 \\ 0 & 1 & 0 & 0 \end{array}\right]$, use

$R_3 + R_1 \to R_1$ and $\frac{1}{2}R_2 \to R_2$ to get

$\left[\begin{array}{rrr|r} 1 & 0 & 1 & 5 \\ 0 & 1 & 1 & 3 \\ 0 & 1 & 0 & 0 \end{array}\right]$, use

$-1R_3 + R_2 \to R_2$ to get

$\left[\begin{array}{rrr|r} 1 & 0 & 1 & 5 \\ 0 & 0 & 1 & 3 \\ 0 & 1 & 0 & 0 \end{array}\right]$, use

$-1R_2 + R_1 \to R_1$ to get

$\left[\begin{array}{rrr|r} 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 1 & 0 & 0 \end{array}\right]$, use $R_2 \leftrightarrow R_3$ to get

$\left[\begin{array}{rrr|r} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 \end{array}\right]$.

Secondly, by matrix inversion

$$A^{-1} = \begin{bmatrix} 7/2 & -1/2 & 2 \\ 1 & 0 & 1 \\ -3/2 & 1/2 & -1 \end{bmatrix}$$

and $A^{-1}\begin{bmatrix} 5 \\ 11 \\ -5 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}$.

Thirdly, by Cramer's Rule note

$$D = \begin{vmatrix} 1 & -1 & 1 \\ 1 & 1 & 3 \\ -1 & 2 & -1 \end{vmatrix} = -2,$$

$$D_x = \begin{vmatrix} 5 & -1 & 1 \\ 11 & 1 & 3 \\ -5 & 2 & -1 \end{vmatrix} = -4,$$

$$D_y = \begin{vmatrix} 1 & 5 & 1 \\ 1 & 11 & 3 \\ -1 & -5 & -1 \end{vmatrix} = 0,$$

and $D_z = \begin{vmatrix} 1 & -1 & 5 \\ 1 & 1 & 11 \\ -1 & 2 & -5 \end{vmatrix} = -6.$

So $\dfrac{D_x}{D} = 2$, $\dfrac{D_y}{D} = 0$, and $\dfrac{D_z}{D} = 3$.

39. The solution set is $\{(-3, 4, 1)\}$.

First solution is by Gaussian elimination.

On $\left[\begin{array}{ccc|c} 0 & 1 & -3 & 1 \\ 1 & 2 & 0 & 5 \\ 1 & 0 & 4 & 1 \end{array}\right]$, use

$-1R_3 + R_2 \to R_2$ and $R_1 \leftrightarrow R_3$ to get

$\left[\begin{array}{ccc|c} 1 & 0 & 4 & 1 \\ 0 & 2 & -4 & 4 \\ 0 & 1 & -3 & 1 \end{array}\right]$, use $\dfrac{1}{2}R_2 \to R_2$ to get

$\left[\begin{array}{ccc|c} 1 & 0 & 4 & 1 \\ 0 & 1 & -2 & 2 \\ 0 & 1 & -3 & 1 \end{array}\right]$, use $-1R_2 + R_3 \to R_3$

to get

$\left[\begin{array}{ccc|c} 1 & 0 & 4 & 1 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & -1 & -1 \end{array}\right]$, use

$-2R_3 + R_2 \to R_2$, $4R_3 + R_1 \to R_1$, and

$-1R_3 \to R_3$ to get

$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 1 \end{array}\right]$.

Secondly, by matrix inversion

$A^{-1} = \begin{bmatrix} 4 & -2 & 3 \\ -2 & 3/2 & -3/2 \\ -1 & 1/2 & -1/2 \end{bmatrix}$

and $A^{-1}\begin{bmatrix} 1 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 4 \\ 1 \end{bmatrix}$.

Thirdly, by Cramer's Rule note

$D = \begin{vmatrix} 0 & 1 & -3 \\ 1 & 2 & 0 \\ 1 & 0 & 4 \end{vmatrix} = 2,$

$D_x = \begin{vmatrix} 1 & 1 & -3 \\ 5 & 2 & 0 \\ 1 & 0 & 4 \end{vmatrix} = -6,$

$D_y = \begin{vmatrix} 0 & 1 & -3 \\ 1 & 5 & 0 \\ 1 & 1 & 4 \end{vmatrix} = 8,$

and $D_z = \begin{vmatrix} 0 & 1 & 1 \\ 1 & 2 & 5 \\ 1 & 0 & 1 \end{vmatrix} = 2.$

Then $\dfrac{D_x}{D} = -3$, $\dfrac{D_y}{D} = 4$, and $\dfrac{D_z}{D} = 1$.

40. The solution set is $\{(2, 2, 4)\}$.

First solution is by Gaussian elimination.

On $\left[\begin{array}{ccc|c} 3 & 1 & -2 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 1 & 3 & 14 \end{array}\right]$, use

$2R_3 + R_2 \to R_2$ and $-1R_3 + R_1 \to R_1$ to get

$\left[\begin{array}{ccc|c} 3 & 0 & -5 & -14 \\ 0 & 0 & 7 & 28 \\ 0 & 1 & 3 & 14 \end{array}\right]$, use

$\dfrac{1}{3}R_1 \to R_1$ and $R_3 \leftrightarrow R_2$ to get

$\left[\begin{array}{ccc|c} 1 & 0 & -5/3 & -14/3 \\ 0 & 1 & 3 & 14 \\ 0 & 0 & 7 & 28 \end{array}\right]$, use $\dfrac{1}{7}R_3 \to R_3$

to get

$\left[\begin{array}{ccc|c} 1 & 0 & -5/3 & -14/3 \\ 0 & 1 & 3 & 14 \\ 0 & 0 & 1 & 4 \end{array}\right]$, use

$-3R_3 + R_2 \to R_2$ and $\dfrac{5}{3}R_3 + R_1 \to R_1$ to get

$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 4 \end{array}\right]$.

Secondly, by matrix inversion

$A^{-1} = \begin{bmatrix} 1/3 & 5/21 & 1/7 \\ 0 & -3/7 & 1/7 \\ 0 & 1/7 & 2/7 \end{bmatrix}$

and $A^{-1}\begin{bmatrix} 0 \\ 0 \\ 14 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$.

Thirdly, by Cramer's Rule note

$D = \begin{vmatrix} 3 & 1 & -2 \\ 0 & -2 & 1 \\ 0 & 1 & 3 \end{vmatrix} = -21,$

$D_x = \begin{vmatrix} 0 & 1 & -2 \\ 0 & -2 & 1 \\ 14 & 1 & 3 \end{vmatrix} = -42,$

$D_y = \begin{vmatrix} 3 & 0 & -2 \\ 0 & 0 & 1 \\ 0 & 14 & 3 \end{vmatrix} = -42,$

and $D_z = \begin{vmatrix} 3 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 1 & 14 \end{vmatrix} = -84.$

So $\dfrac{D_x}{D} = 2$, $\dfrac{D_y}{D} = 2$, and $\dfrac{D_z}{D} = 4$.

41. The solution set is

$$\left\{ \left(\frac{3y+3}{2}, y, \frac{1-y}{2} \right) \mid y \text{ is any real number} \right\}.$$

First, apply the Gaussian elimination method.

On $\left[\begin{array}{rrr|r} 1 & -1 & 1 & 2 \\ 1 & -2 & -1 & 1 \\ 2 & -3 & 0 & 3 \end{array} \right]$, use

$-1R_1 + R_2 \to R_2$ and $-2R_1 + R_3 \to R_3$ to get

$\left[\begin{array}{rrr|r} 1 & -1 & 1 & 2 \\ 0 & -1 & -2 & -1 \\ 0 & -1 & -2 & -1 \end{array} \right]$, use

$-1R_2 + R_3 \to R_3$, $-1R_2 + R_1 \to R_1$,

and $-1R_2 \to R_2$ to get

$\left[\begin{array}{rrr|r} 1 & 0 & 3 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right]$. Since $z = \dfrac{1-y}{2}$, we get

$x = 3 - 3z = 3 - 3\left(\dfrac{1-y}{2}\right) = \dfrac{3y+3}{2}.$

This cannot be solved by matrix inversion

since $A^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -1 \\ 2 & -3 & 0 \end{bmatrix}^{-1}$ does not exist.

Nor can it be solved by Cramer's rule

since $|D| = \begin{vmatrix} 1 & -1 & 1 \\ 1 & -2 & -1 \\ 2 & -3 & 0 \end{vmatrix} = 0.$

42. Solution set is $\emptyset$ as seen by an application of the Gaussian elimination method.

On $\left[\begin{array}{rrr|r} 1 & 2 & 1 & 1 \\ 2 & 4 & 2 & 0 \\ -1 & -2 & -1 & 2 \end{array} \right]$, use

$R_1 + R_3 \to R_3$ and $-2R_1 + R_2 \to R_2$ to get

$\left[\begin{array}{rrr|r} 1 & 2 & 1 & 1 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 3 \end{array} \right]$. Inconsistent.

This cannot be solved by matrix inversion

since $A^{-1} = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ -1 & -2 & -1 \end{bmatrix}^{-1}$ does not

exist. Nor can it be solved by Cramer's rule

since $|D| = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ -1 & -2 & -1 \end{vmatrix} = 0.$

43. Solution set is $\emptyset$ as seen by an application of the Gaussian elimination method.

On $\left[\begin{array}{rrr|r} 1 & -3 & -1 & 2 \\ 1 & -3 & -1 & 1 \\ 1 & -3 & -1 & 0 \end{array} \right]$, use

$-1R_1 + R_2 \to R_2$ and $-1R_1 + R_3 \to R_3$ to get

$\left[\begin{array}{rrr|r} 1 & -3 & -1 & 2 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & -2 \end{array} \right]$. Inconsistent.

This cannot be solved by matrix inversion

since $A^{-1} = \begin{bmatrix} 1 & -3 & -1 \\ 1 & -3 & -1 \\ 1 & -3 & -1 \end{bmatrix}^{-1}$ does not exist.

Nor can it be solved by Cramer's rule

since $|D| = \begin{vmatrix} 1 & -3 & -1 \\ 1 & -3 & -1 \\ 1 & -3 & -1 \end{vmatrix} = 0.$

44. The solution set is

$$\{(1, y, 2-y) \mid y \text{ is any real number}\}.$$

First, apply the Gaussian elimination method.

On $\left[\begin{array}{rrr|r} 2 & -1 & -1 & 0 \\ 1 & 1 & 1 & 3 \\ 3 & 0 & 0 & 3 \end{array} \right]$, use

$R_1 + R_2 \to R_2$ and $\dfrac{1}{3}R_3 \to R_3$ to get

$$\left[\begin{array}{rrr|r} 2 & -1 & -1 & 0 \\ 3 & 0 & 0 & 3 \\ 1 & 0 & 0 & 1 \end{array}\right]$$, use

$-\frac{1}{3}R_2 + R_3 \to R_3$ and $\frac{1}{3}R_2 \to R_2$ to get

$$\left[\begin{array}{rrr|r} 2 & -1 & -1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$, use $-2R_2 + R_1 \to R_1$

to get

$$\left[\begin{array}{rrr|r} 0 & -1 & -1 & -2 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$, use $R_2 \to R_1$

and $-1R_1 \to R_2$ to get

$$\left[\begin{array}{rrr|r} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$. So $x = 1$ and $z = 2 - y$.

This cannot be solved by matrix inversion since

$$A^{-1} = \begin{bmatrix} 2 & -1 & -1 \\ 1 & 1 & 1 \\ 3 & 0 & 0 \end{bmatrix}^{-1} \text{ does not exist.}$$

Nor can it be solved by Cramer's rule

$$\text{since } |D| = \begin{vmatrix} 2 & -1 & -1 \\ 1 & 1 & 1 \\ 3 & 0 & 0 \end{vmatrix} = 0.$$

45. Using $x = 9$ in $x + y = -3$, we find $9 + y = -3$ and $y = -12$. The solution set is $\{(9, -12)\}$.

46. Since $x^2 = 4$, $x = \pm 2$. Using $x = 2$ in $x - y = 1$, we get $y = 1$. Similarly, if $x = -2$ then $-2 - y = 1$ and $y = -3$. The solution set is $\{(2, 1), (-2, -3)\}$.

47.

Note $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 6 \\ 8 \end{bmatrix} =$

$$\begin{bmatrix} -1 & 1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 6 \\ 8 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}.$$

The solution set is $\{(2, 4)\}$.

48. Since $0.5y = 7$, we get $y = 14$. Since $x + y = 9$, we find $x + 14 = 9$ and $x = -5$.
The solution set is $\{(-5, 14)\}$

49. System of equations can be written as

$$\begin{aligned} x + y &= -3 \\ -x &= 0. \end{aligned}$$

Using $x = 0$ in $x + y = -3$, we get $y = -3$. Solution set is $\{(0, -3)\}$.

50. System of equations can be written as

$$\begin{aligned} -x + y &= 4 \\ x - y &= 5. \end{aligned}$$

Adding the equations, we get $0 = 9$. Inconsistent and the solution set is $\emptyset$.

51. System of equations can be written as

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

By using matrix inversion, we obtain

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1/2 & -1/2 & 1/2 \\ 1/2 & 1/2 & -1/2 \\ -1/2 & 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} =$$

$\begin{bmatrix} 1/2 \\ 1/2 \\ 1/2 \end{bmatrix}$. The solution set is $\{(1/2, 1/2, 1/2)\}$.

52. System of equations can be written as

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 3 \end{bmatrix}.$$

By using matrix inversion, we find

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 1/2 & 1 \\ 0 & 1/2 & 1 \\ 0 & 1/2 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \\ 3 \end{bmatrix} = \begin{bmatrix} 7.5 \\ 5.5 \\ 2.5 \end{bmatrix}.$$

Solution set is $\{(7.5, 5.5, 2.5)\}$.

53. By using the inverse of the coefficient matrix, we get

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3/5 & -3/5 & 2/5 \\ 2/5 & 3/5 & -2/5 \\ -1/5 & 1/5 & 1/5 \end{bmatrix} \begin{bmatrix} -1 \\ 7 \\ 17 \end{bmatrix} =$$

$\begin{bmatrix} 2 \\ -3 \\ 5 \end{bmatrix}$. The solution set is $\{(2, -3, 5)\}$.

54. Since the coefficient matrix A has an inverse, we get $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1}\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.
The solution set is $\{(0,0,0)\}$.

55. Let x and y be the number of gallons of pollutant A and pollutant B, respectively. Then

$$\begin{aligned} 10x + 6y &= 4060 \\ \frac{3}{4} &= \frac{x}{y}. \end{aligned}$$

Substitute $x = \frac{3}{4}y$ in $10x + 6y = 4060$. Solving for x, one finds the quantities discharged are $x \approx 225.56$ gallons of pollutant A and $y \approx 300.74$ gallons of pollutant B.

56. Let x, y, and z be the amounts spent by Ross, Joey, and Chandler, respectively. Then

$$\begin{aligned} x + y + z &= 216 \\ x + y &= \frac{z}{2} \\ x + 12 &= y. \end{aligned}$$

Solving the system, one finds Ross spent $x = \$30$, Joey spent $y = \$42$, and Chandler spent $z = \$144$.

57. Let x, y, and z be the expenses including tax for water, gas, and electricity, respectively.

$$\begin{aligned} x + y + z &= 189.83 \\ \frac{x}{1.04} + \frac{y}{1.05} + \frac{z}{1.06} &= 180 \\ z &= 2y \end{aligned}$$

Solving the system, one finds the expenses including taxes are $x = \$22.88$ for water, $y = \$55.65$ for gas, and $z = \$111.30$ for electricity.

58. Substitute $(1, 38)$, $(2, 42)$, and $(3, 49)$ into $y = ax^2 + bx + c$. Then we obtain

$$\begin{aligned} a + b + c &= 38 \\ 4a + 2b + c &= 42 \\ 9a + 3b + c &= 49. \end{aligned}$$

Solving the system, one finds $a = 1.5$, $b = -0.5$, and $c = 37$. An equation of the parabola is

$$y = 1.5x^2 - 0.5x + 37.$$

The predicted sales for the 4th month is

$$y = 1.5(4)^2 - 0.5(4) + 37 = 59 \text{ cars}.$$

Thinking Outside the Box LXXXVI

Let $1 \le x \le 199$ be the number of dogs remaining. The only number less than 200 that is divisible by four of the five denominators below

$$3, 4, 5, 7, \text{ and } 9$$

is $x = 180$. Then there are $180/9 = 20$ dachshunds remaining, $3(20) = 60$ original beagles, $180/5 = 36$ beagles remaining, and $60 - 36 = 24$ beagles escaped.

Chapter 9 Test

1.

On $\left[\begin{array}{cc|c} 2 & -3 & 1 \\ 1 & 9 & 4 \end{array}\right]$, use $-2R_2 + R_1 \to R_2$ to get

$\left[\begin{array}{cc|c} 2 & -3 & 1 \\ 0 & -21 & -7 \end{array}\right]$, use $\frac{1}{2}R_1 \to R_1$ and $-\frac{1}{21}R_2 \to R_2$ to get

$\left[\begin{array}{cc|c} 1 & -3/2 & 1/2 \\ 0 & 1 & 1/3 \end{array}\right]$, use $\frac{3}{2}R_2 + R_1 \to R_1$ to get

$\left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 1/3 \end{array}\right]$. Solution set is $\{(1, 1/3)\}$.

2.

On $\left[\begin{array}{ccc|c} 2 & -1 & 1 & 5 \\ 1 & -2 & -1 & -2 \\ 3 & -1 & -1 & 6 \end{array}\right]$, use

$-\frac{1}{2}R_1 + R_2 \to R_2$ and

$-\frac{3}{2}R_1 + R_3 \to R_3$ to get

$\left[\begin{array}{rrr|r} 2 & -1 & 1 & 5 \\ 0 & -3/2 & -3/2 & -9/2 \\ 0 & 1/2 & -5/2 & -3/2 \end{array}\right]$, use

$-\frac{2}{3}R_2 \to R_2$ and $3R_3 + R_2 \to R_3$ to get

$\left[\begin{array}{rrr|r} 2 & -1 & 1 & 5 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & -9 & -9 \end{array}\right]$,

use $-\frac{1}{9}R_3 \to R_3$ and $R_2 + R_1 \to R_1$ to get

$\left[\begin{array}{rrr|r} 2 & 0 & 2 & 8 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 1 & 1 \end{array}\right]$, use

$-1R_3 + R_2 \to R_2$ and $\frac{1}{2}R_1 \to R_1$ to get

$\left[\begin{array}{rrr|r} 1 & 0 & 1 & 4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right]$, use $-1R_3 + R_1 \to R_1$

to get $\left[\begin{array}{rrr|r} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right]$.

The solution set is $\{(3, 2, 1)\}$.

3.

On $\left[\begin{array}{rrr|r} 1 & -1 & -1 & 1 \\ 2 & 1 & -1 & 0 \\ 5 & -2 & -4 & 3 \end{array}\right]$, use

$-2R_1 + R_2 \to R_2$ and

$-5R_1 + R_3 \to R_3$ to get

$\left[\begin{array}{rrr|r} 1 & -1 & -1 & 1 \\ 0 & 3 & 1 & -2 \\ 0 & 3 & 1 & -2 \end{array}\right]$, use

$-1R_3 + R_2 \to R_3$ and $\frac{1}{3}R_2 \to R_2$ to get

$\left[\begin{array}{rrr|r} 1 & -1 & -1 & 1 \\ 0 & 1 & 1/3 & -2/3 \\ 0 & 0 & 0 & 0 \end{array}\right]$, use

$R_2 + R_1 \to R_1$ to get

$\left[\begin{array}{rrr|r} 1 & 0 & -2/3 & 1/3 \\ 0 & 1 & 1/3 & -2/3 \\ 0 & 0 & 0 & 0 \end{array}\right]$.

Since $x = \frac{2}{3}z + \frac{1}{3}$, we find $3x = 2z + 1$ and $z = \frac{3x - 1}{2}$. Since $y = -\frac{1}{3}z - \frac{2}{3}$, we get

$$\begin{aligned} y &= -\frac{1}{3}\left(\frac{3x-1}{2}\right) - \frac{2}{3} \\ y &= \frac{-3x-3}{6} \\ y &= \frac{-x-1}{2} \end{aligned}$$

The solution set is

$\left\{\left(x, \frac{-x-1}{2}, \frac{3x-1}{2}\right) \mid x \text{ is any real number}\right\}$.

4.

$A + B = \begin{bmatrix} 3 & -4 \\ -6 & 10 \end{bmatrix}$

5.

$2A - B = \begin{bmatrix} 2 & -2 \\ -4 & 8 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ -4 & 6 \end{bmatrix} =$

$\begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix}$

6.

$AB = \begin{bmatrix} 2+4 & -3-6 \\ -4-16 & 6+24 \end{bmatrix} = \begin{bmatrix} 6 & -9 \\ -20 & 30 \end{bmatrix}$

7.

$AC = \begin{bmatrix} -2-1 \\ 4+4 \end{bmatrix} = \begin{bmatrix} -3 \\ 8 \end{bmatrix}$

8. CB is undefined

9. $FG = [-2 \quad 3-2 \quad 1+1] = [-2 \quad 1 \quad 2]$

10.

$EF = \begin{bmatrix} 2(1) & 2(0) & 2(-1) \\ 3(1) & 3(0) & 3(-1) \\ -1(1) & -1(0) & -1(-1) \end{bmatrix}$

$= \begin{bmatrix} 2 & 0 & -2 \\ 3 & 0 & -3 \\ -1 & 0 & 1 \end{bmatrix}$

11.

On $\left[\begin{array}{rr|rr} 1 & -1 & 1 & 0 \\ -2 & 4 & 0 & 1 \end{array}\right]$, use

$2R_1 + R_2 \to R_2$ to get

$\left[\begin{array}{rr|rr} 1 & -1 & 1 & 0 \\ 0 & 2 & 2 & 1 \end{array}\right]$, use

$\frac{1}{2}R_2 + R_1 \to R_1$ and $\frac{1}{2}R_2 \to R_2$

to get $\left[\begin{array}{cc|cc} 1 & 0 & 2 & 1/2 \\ 0 & 1 & 1 & 1/2 \end{array}\right]$.

Then $A^{-1} = \left[\begin{array}{cc} 2 & 1/2 \\ 1 & 1/2 \end{array}\right]$.

12.

On $\left[\begin{array}{ccc|ccc} -2 & 3 & 1 & 1 & 0 & 0 \\ -3 & 1 & 3 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$,

use $-\frac{1}{2}R_1 \to R_1$ and $-\frac{3}{2}R_1 + R_2 \to R_2$ to get

$\left[\begin{array}{ccc|ccc} 1 & -3/2 & -1/2 & -1/2 & 0 & 0 \\ 0 & -7/2 & 3/2 & -3/2 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$, use

$\frac{4}{7}R_2 + R_3 \to R_3$, $-\frac{3}{7}R_2 + R_1 \to R_1$,

and $-\frac{2}{7}R_2 \to R_2$ to get

$\left[\begin{array}{ccc|ccc} 1 & 0 & -8/7 & 1/7 & -3/7 & 0 \\ 0 & 1 & -3/7 & 3/7 & -2/7 & 0 \\ 0 & 0 & -1/7 & -6/7 & 4/7 & 1 \end{array}\right]$, use

$-3R_3 + R_2 \to R_2$, $-8R_3 + R_1 \to R_1$,

and $-7R_3 \to R_3$ to get

$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 7 & -5 & -8 \\ 0 & 1 & 0 & 3 & -2 & -3 \\ 0 & 0 & 1 & 6 & -4 & -7 \end{array}\right]$.

Then $G^{-1} = \left[\begin{array}{ccc} 7 & -5 & -8 \\ 3 & -2 & -3 \\ 6 & -4 & -7 \end{array}\right]$.

13. $|A| = 4 - 2 = 2$

14. $|B| = 12 - 12 = 0$

15. Expanding about the third row, we find

$|G| = -2\begin{vmatrix} -2 & 1 \\ -3 & 3 \end{vmatrix} + (-1)\begin{vmatrix} -2 & 3 \\ -3 & 1 \end{vmatrix} =$

$-2(-3) + (-1)(7) = -1$

16.

One finds $D = \begin{vmatrix} 1 & -1 \\ -2 & 4 \end{vmatrix} = 2$,

$D_x = \begin{vmatrix} 2 & -1 \\ 2 & 4 \end{vmatrix} = 10$, $D_y = \begin{vmatrix} 1 & 2 \\ -2 & 2 \end{vmatrix} = 6$.

Then $\frac{D_x}{D} = 5$ and $\frac{D_y}{D} = 3$.

The solution set is $\{(5, 3)\}$.

17. Cramer's Rule is not applicable since

$D = \begin{vmatrix} 2 & -3 \\ -4 & 6 \end{vmatrix} = 0$. Rather, multiply first

equation by 2 and add to the second one.

$$\begin{array}{rcr} 4x - 6y & = & 12 \\ -4x + 6y & = & 1 \\ \hline 0 & = & 13 \end{array}$$

Inconsistent and the solution set is $\emptyset$.

18.

One finds $D = \begin{vmatrix} -2 & 3 & 1 \\ -3 & 1 & 3 \\ 0 & 2 & -1 \end{vmatrix} = -1$,

$D_x = \begin{vmatrix} -2 & 3 & 1 \\ -4 & 1 & 3 \\ 0 & 2 & -1 \end{vmatrix} = -6$,

$D_y = \begin{vmatrix} -2 & -2 & 1 \\ -3 & -4 & 3 \\ 0 & 0 & -1 \end{vmatrix} = -2$, and

$D_z = \begin{vmatrix} -2 & 3 & -2 \\ -3 & 1 & -4 \\ 0 & 2 & 0 \end{vmatrix} = -4$.

Then $\frac{D_x}{D} = 6$, $\frac{D_y}{D} = 2$, and $\frac{D_z}{D} = 4$.

The solution set is $\{(6, 2, 4)\}$.

19. The inverse of coefficient matrix is given

by $A^{-1} = \left[\begin{array}{cc} 2 & 1/2 \\ 1 & 1/2 \end{array}\right]$. Since

$A^{-1}\left[\begin{array}{c} 1 \\ -8 \end{array}\right] = \left[\begin{array}{c} -2 \\ -3 \end{array}\right]$, the solution

set is $\{(-2, -3)\}$.

20. The inverse of coefficient matrix was found

in Exercise 12: $A^{-1} = \left[\begin{array}{ccc} 7 & -5 & -8 \\ 3 & -2 & -3 \\ 6 & -4 & -7 \end{array}\right]$.

Since $A^{-1}\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 15 \\ 6 \\ 13 \end{bmatrix}$, the solution set is $\{(15, 6, 13)\}$.

21. Corresponding system of equations is

$$\begin{aligned} x - y &= 12 \\ 35y - 10x &= 730. \end{aligned}$$

Solving this system, one finds $x = 46$ copies were bought and $y = 34$ copies were sold.

22. Substitute $(0, 3)$, $(1, -1/2)$, and $(4, 3)$ into $y = ax^2 + b\sqrt{x} + c$. Then we obtain

$$\begin{aligned} c &= 3 \\ a + b + c &= -\frac{1}{2} \\ 16a + 2b + c &= 3. \end{aligned}$$

Solving this system, one finds $a = 0.5$, $b = -4$, $c = 3$, and the graph is given by

$$y = 0.5x^2 - 4\sqrt{x} + 3.$$

Tying It All Together

1. Simplify the left-hand side.

$$\begin{aligned} 2x + 6 - 5x &= 7 \\ -3x &= 1 \end{aligned}$$

Solution set is $\{-1/3\}$.

2. Multiply each side of the equation by 30.

$$\begin{aligned} \frac{1}{2}x - \frac{1}{6} &= \frac{4}{5} \\ 15x - 5 &= 24 \\ 15x &= 29 \end{aligned}$$

Solution set is $\{29/15\}$.

3. Multiply $\frac{1}{2}$ to $(2x - 2)$.

$$\begin{aligned} (x-1)(6x-8) &= 4 \\ 6x^2 - 14x + 8 &= 4 \\ 6x^2 - 14x + 4 &= 0 \\ 2(3x^2 - 7x + 2) &= 0 \\ 2(3x-1)(x-2) &= 0 \end{aligned}$$

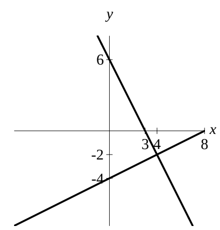
Solution set is $\{1/3, 2\}$.

4. Simplify left-hand side.

$$\begin{aligned} 1 - (4x - 2) &= 9 \\ -4x + 3 &= 9 \\ -4x &= 6 \end{aligned}$$

Solution set is $\{-3/2\}$.

5. The solution set is $\{(4, -2)\}$ as can be seen from the point of intersection.



6. Using $y = 1 - 2x$ in $2x + 6y = 2$, we get

$$\begin{aligned} 2x + 6(1 - 2x) &= 2 \\ 2x + 6 - 12x &= 2 \\ -10x &= -4. \end{aligned}$$

Using $x = \frac{2}{5}$ in $y = 1 - 2x$, we have $y = 1 - \frac{4}{5} = \frac{1}{5}$. Solution set is $\left\{\left(\frac{2}{5}, \frac{1}{5}\right)\right\}$.

7. Multiply second equation by 6 and add to the first one.

$$\begin{array}{rcr} 2x - 0.06y &=& 20 \\ 18x + 0.06y &=& 120 \\ \hline 20x &=& 140 \end{array}$$

Using $x = 7$ in $2x - 0.06y = 20$, we find

$$\begin{aligned} 14 - 0.06y &= 20 \\ -0.06y &= 6 \\ y &= -100. \end{aligned}$$

The solution set is $\{(7, -100)\}$.

8.

On $\left[\begin{array}{rr|r} 2 & -1 & -1 \\ 1 & 3 & -11 \end{array}\right]$, use $-2R_2 + R_1 \to R_2$ to get

$\left[\begin{array}{rr|r} 2 & -1 & -1 \\ 0 & -7 & 21 \end{array}\right]$, use $-\frac{1}{7}R_2 \to R_2$

and $\frac{1}{2}R_1 \to R_1$ to get

$\left[\begin{array}{cc|c} 1 & -1/2 & -1/2 \\ 0 & 1 & -3 \end{array}\right]$, use $\frac{1}{2}R_2 + R_1 \to R_1$

to get $\left[\begin{array}{cc|c} 1 & 0 & -2 \\ 0 & 1 & -3 \end{array}\right]$.

The solution set is $\{(-2,-3)\}$.

9. Inverse of the coefficient matrix A is

$A^{-1} = \begin{bmatrix} -1/2 & -5/2 \\ -1/2 & -3/2 \end{bmatrix}$.

Since $A^{-1}\begin{bmatrix} -7 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$,

the solution set is $\{(1,2)\}$.

10.

One finds $D = \begin{vmatrix} 4 & -3 \\ 3 & -5 \end{vmatrix} = -11$,

$D_x = \begin{vmatrix} 5 & -3 \\ 1 & -5 \end{vmatrix} = -22$, and

$D_y = \begin{vmatrix} 4 & 5 \\ 3 & 1 \end{vmatrix} = -11$.

Then $\frac{D_x}{D} = 2$ and $\frac{D_y}{D} = 1$.

The solution set is $\{(2,1)\}$.

11. Since $6 = 3x - 5y$ and $1 = 3x - 5y$, we get $6 = 1$. A contradiction. The solution set is $\emptyset$.

12. Multiply first equation by 2 and add the result to second equation.

$$\begin{array}{rcr} 2x - 4y & = & 6 \\ -2x + 4y & = & -6 \\ \hline 0 & = & 0 \end{array}$$

A dependent system and the solution set is

$$\{(x,y) \mid x - 2y = 3\}.$$

13. Substitute $x = y - 1$ in $x^2 + y^2 = 25$. Then

$$\begin{aligned} (y^2 - 2y + 1) + y^2 &= 25 \\ 2y^2 - 2y - 24 &= 0 \\ 2(y^2 - y - 12) &= 0 \\ 2(y-4)(y+3) &= 0. \end{aligned}$$

If $y = 4$, then $x = y - 1 = 4 - 1 = 3$.

If $y = -3$, then $x = (-3) - 1 = -4$.

The solution set is $\{(3,4), (-4,-3)\}$.

14. Substituting $y = x^2 - 1$ in $x + y = 1$, we get

$$\begin{aligned} x + (x^2 - 1) &= 1 \\ x^2 + x - 2 &= 0 \\ (x+2)(x-1) &= 0. \end{aligned}$$

If $x = -2$, then $y = x^2 - 1 = (-2)^2 - 1 = 3$.

If $x = 1$, then $y = 1^2 - 1 = 0$.

Solution set is $\{(-2,3), (1,0)\}$.

15. Adding the given equations, we obtain

$$\begin{array}{rcr} x^2 - y^2 & = & 1 \\ x^2 + y^2 & = & 3 \\ \hline 2x^2 & = & 4 \\ x^2 & = & 2 \\ x & = & \pm\sqrt{2}. \end{array}$$

Substitute $x = \pm\sqrt{2}$ into $x^2 + y^2 = 3$. Then $2 + y^2 = 3$ and so $y = \pm 1$. The solution set is $\{(\sqrt{2}, \pm 1), (-\sqrt{2}, \pm 1)\}$.

16. Multiply the first equation by -2 and add it to the second equation.

$$\begin{array}{rcr} -2x^2 + 2y^2 & = & -2 \\ 2x^2 + 3y^2 & = & 2 \\ \hline 5y^2 & = & 0 \\ y & = & 0 \end{array}$$

Substitute $y = 0$ into $x^2 - y^2 = 1$. Then $x^2 = 1$ and so $x = \pm 1$. The solution set is $\{(\pm 1, 0)\}$.

17. plane

18. nonlinear, linear

19. partial fraction decomposition

20. linear

21. constraints

22. objective

23. matrix

24. row

25. column

26. augmented

27. equivalent

28. Gaussian elimination