For Thought

1. True, since if $x = 2$ and $y = 3$ we get $2 + 3 = 5$.

2. False, since $(2, 3)$ does not satisfy $x - y = 1$.

3. False, it is independent since the two lines are perpendicular and intersect at only one point.

4. True. Multiply $x - 2y = 4$ by -3 and add to the second equation.

$$\begin{array}{rcr} -3x + 6y &=& -12 \\ 3x - 6y &=& 8 \\ \hline 0 &=& -4 \end{array}$$

Since $0 = -4$ is false, there is no solution.

5. True, adding gives $2x = 6$. **6.** True

7. False, it is dependent because substituting $x = 5 + 3y$ results in an identity.

$$\begin{aligned} 9y - 3(5 + 3y) &= -15 \\ 9y - 15 - 9y &= -15 \\ -15 &= -15 \end{aligned}$$

8. False, it is dependent and the solution set is $\{(5 + 3y, y)|y \text{ is any real number}\}$.

9. False, it is dependent, there are an infinite number of solutions.

10. True, since both lines have slope $1/2$.

8.1 Exercises

1. system

2. consistent

3. inconsistent

4. independent

5. dependent

6. equivalent

7. Note, $x = 1$ and $y = 3$ satisfies both $x + y = 4$ and $x - y = -2$. Yes, $(1, 2)$ is a solution.

8. Note, $x = -1$ and $y = 2$ satisfies both $x + y = 1$ and $2x - 3y = -8$. Yes, $(-1, 2)$ is a solution.

9. Note, $x = -1$ and $y = 5$ does not satisfy $x - 2y = -9$. Thus, $(-1, 5)$ is not a solution.

10. Note, $x = 3$ and $y = 2$ does not satisfy $2x + 4y = 16$. Thus, $(3, 2)$ is not a solution.

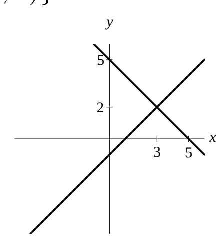
11. $\{(1, 2)\}$

12. $\{(-3, 1)\}$

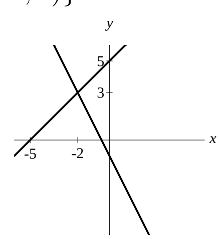
13. No solution since the lines are parallel. The solution set is $\emptyset$.

14. Since the lines are identical, the solution set is $\left\{(x, y) \mid y = \frac{1}{2}x + \frac{3}{2}\right\}$.

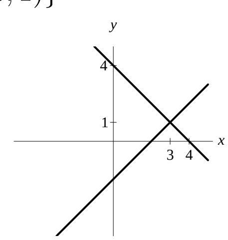
15. $\{(3, 2)\}$



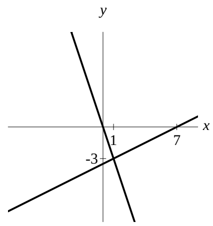
16. $\{(-2, 3)\}$



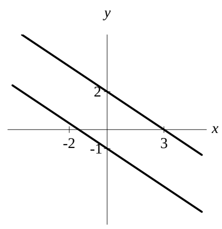
17. $\{(3, 1)\}$



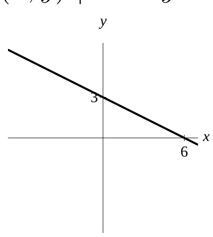
18. $\{(1, -3)\}$



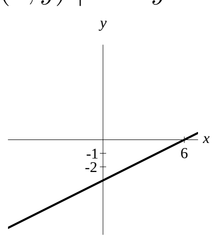
19. No solution since the lines are parallel. The solution set is $\emptyset$.



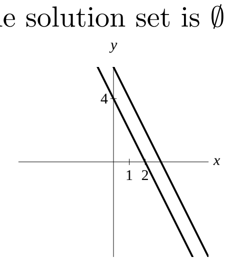
20. Since the lines are identical, the solution set is $\{(x, y) \mid x + 2y = 6\}$.



21. Since the lines are identical, the solution set is $\{(x, y) \mid x - 2y = 6\}$.



22. No solution since the lines are parallel. The solution set is $\emptyset$.



23. Substitute $y = 2x + 1$ into $3x - 4y = 1$.

$$\begin{aligned} 3x - 4(2x + 1) &= 1 \\ 3x - 8x - 4 &= 1 \\ -5x &= 5 \\ x &= -1 \end{aligned}$$

From $y = 2x + 1$, $y = 2(-1) + 1 = -1$. Independent and solution set is $\{(-1, -1)\}$.

24. Substitute $x = 6 - 3y$ into $5x - 6y = 23$.

$$\begin{aligned} 5(6 - 3y) - 6y &= 23 \\ 30 - 15y - 6y &= 23 \\ -21y &= -7 \\ y &= 1/3 \end{aligned}$$

From $x = 6 - 3y$, $x = 6 - 3(1/3) = 5$. Independent and solution set is $\{(5, 1/3)\}$.

25. Substitute $y = 1 - x$ into $2x - 3y = 8$.

$$\begin{aligned} 2x - 3(1 - x) &= 8 \\ 2x - 3 + 3x &= 8 \\ 5x &= 11 \\ x &= 11/5 \end{aligned}$$

From $y = 1 - x$, $y = 1 - 11/5 = -6/5$. Independent and solution set is $\{(11/5, -6/5)\}$.

26. Substitute $y = 5 - 2x$ into $x + 2y = 3$.

$$\begin{aligned} x + 2(5 - 2x) &= 3 \\ x + 10 - 4x &= 3 \\ -3x &= -7 \\ x &= 7/3 \end{aligned}$$

From $y = 5 - 2x$, $y = 5 - 2(7/3) = 1/3$. Independent and solution set is $\{(7/3, 1/3)\}$.

27. Substitute $y = 3x + 5$ into $3(x + 1) = y - 2$.

$$\begin{aligned} 3x + 3 &= (3x + 5) - 2 \\ 3x + 3 &= 3x + 3 \\ 3 &= 3 \end{aligned}$$

Dependent and solution set is $\{(x, y) \mid y = 3x + 5\}$.

28. Substitute $y = -2x$ into $2y = 1 - 4x$.

$$\begin{aligned} 2(-2x) &= 1 - 4x \\ -4x &= 1 - 4x \\ 0 &= 1 \end{aligned}$$

Inconsistent and the solution set is $\emptyset$.

29. Multiplying $\frac{1}{2}x + \frac{1}{3}y = 3$ by 6, we get $3x + 2y = 18$. Then substitute $2y = 6 - 3x$ into $3x + 2y = 18$. So

$$\begin{aligned} 3x + (6 - 3x) &= 18 \\ 6 &= 18 \end{aligned}$$

Inconsistent and the solution set is $\emptyset$.

30. Multiplying $\frac{1}{5}x + \frac{1}{2}y = 1$ by 10, we get $2x + 5y = 10$. Then substitute $2x = 10 - 5y$ into $2x + 5y = 10$.

$$\begin{aligned} (10 - 5y) + 5y &= 10 \\ 10 &= 10 \end{aligned}$$

Dependent and solution set is $\{(x, y) \mid 2x + 5y = 10\}$.

31. Multiplying $0.05x + 0.06y = 10.50$ by 100, we obtain $5x + 6y = 1050$. Then substitute $y = 200 - x$ into $5x + 6y = 1050$.

$$\begin{aligned} 5x + 6(200 - x) &= 1050 \\ 5x + 1200 - 6x &= 1050 \\ -x &= -150 \\ x &= 150 \end{aligned}$$

From $y = 200 - x$, $y = 200 - 150 = 50$. Independent and solution set is $\{(150, 50)\}$.

32. Multiplying $\frac{1}{2}x + \frac{1}{3}y = 80$ by 6, we obtain $3x + 2y = 480$. Then substitute $y = 300 - 2x$ into $3x + 2y = 480$.

$$\begin{aligned} 3x + 2(300 - 2x) &= 480 \\ 3x + 600 - 4x &= 480 \\ -x &= -120 \\ x &= 120 \end{aligned}$$

From $y = 300 - 2x$, $y = 300 - 2(120) = 60$. Independent and solution set is $\{(120, 60)\}$.

33. Since $3x + 1 = 3x - 7$ leads to $1 = -7$, the system is inconsistent and the solution set is $\emptyset$.

34. Substitute $y = 9 - 2x$ into $4x + 2y = 10$.

$$\begin{aligned} 4x + 2(9 - 2x) &= 10 \\ 4x + 18 - 4x &= 10 \\ 18 &= 10 \end{aligned}$$

Inconsistent and the solution set is $\emptyset$.

35. Multiplying the first and second equations by 6 and 4, respectively, we have $3x - 2y = 72$ and $x - 2y = 4$. Substitute $2y = x - 4$ into $3x - 2y = 72$.

$$\begin{aligned} 3x - (x - 4) &= 72 \\ 2x + 4 &= 72 \\ 2x &= 68 \\ x &= 34 \end{aligned}$$

From $2y = x - 4$, $y = \dfrac{34 - 4}{2} = 15$. Independent and solution set is $\{(34, 15)\}$.

36. Multiplying the first and second equations each by 100, we have $5x + 10y = 1000$ and $6x + 20y = 1600$, respectively. Substitute $10y = 1000 - 5x$ into $6x + 2 \cdot 10y = 1600$.

$$\begin{aligned} 6x + 2(1000 - 5x) &= 1600 \\ 6x + 2000 - 10x &= 1600 \\ -4x + 2000 &= 1600 \\ -4x &= -400 \\ x &= 100 \end{aligned}$$

From $5x + 10y = 1000$, we find

$$\begin{aligned} 5(100) + 10y &= 1000 \\ 500 + 10y &= 1000 \\ 10y &= 500 \\ y &= 50. \end{aligned}$$

Independent and solution set is $\{(100, 50)\}$.

37. Adding the two equations, we get $2x = 26$. So $x = 13$ and from $x + y = 20$, we obtain $13 + y = 20$ or $y = 7$. Independent and solution set is $\{(13, 7)\}$.

38. Adding the two equations, we get $-y = 12$. So $y = -12$ and from $3x - 2y = 7$, we obtain $3x + 24 = 7$ or $x = -17/3$. Independent & solution set is $\{(-17/3, -12)\}$.

39. Multiplying $x - y = 5$ by 2 and by adding to the second equation, we obtain

$$\begin{array}{rcr} 2x - 2y & = & 10 \\ 3x + 2y & = & 10 \\ \hline 5x & = & 20 \\ x & = & 4 \end{array}$$

From $x - y = 5$, $4 - y = 5$ or $y = -1$. Independent and solution set is $\{(4, -1)\}$.

40. Multiply $x - 4y = -3$ by 3 and add to the second equation.

$$\begin{array}{rcr} 3x - 12y & = & -9 \\ -3x + 5y & = & 2 \\ \hline -7y & = & -7 \\ y & = & 1 \end{array}$$

Since $x - 4y = -3$, $x - 4 = -3$ or $x = 1$. Independent and solution set is $\{(1, 1)\}$.

41. Adding the two equations leads to $0 = 12$. Inconsistent and the solution set is $\emptyset$.

42. Multiplying $2x - y = 6$ by 2 and adding to the second equation.

$$\begin{array}{rcr} 4x - 2y & = & 12 \\ -4x + 2y & = & 9 \\ \hline 0 & = & 21 \end{array}$$

Inconsistent and the solution set is $\emptyset$.

43. Multiply $2x + 3y = 1$ by -3 and $3x - 5y = -8$ by 2. Then add the equations.

$$\begin{array}{rcr} -6x - 9y & = & -3 \\ 6x - 10y & = & -16 \\ \hline -19y & = & -19 \\ y & = & 1 \end{array}$$

Since $2x + 3y = 1$, $2x + 3 = 1$ or $x = -1$. Independent and solution set is $\{(-1, 1)\}$.

44. Multiply $-2x+5y = 14$ by 7 and $7x+6y = -2$ by 2. Then add the equations.

$$\begin{array}{rcr} -14x + 35y & = & 98 \\ 14x + 12y & = & -4 \\ \hline 47y & = & 94 \\ y & = & 2 \end{array}$$

Since $-2x+5y = 14$, $-2x+10 = 14$ or $x = -2$. Independent and solution set is $\{(-2, 2)\}$.

45. Multiply $0.05x + 0.1y = 0.6$ by -100 and $x + 2y = 12$ by 5. Then add the equations.

$$\begin{array}{rcr} -5x - 10y & = & -60 \\ 5x + 10y & = & 60 \\ \hline 0 & = & 0 \end{array}$$

Dependent and the solution set is $\{(x, y) \mid x + 2y = 12\}$.

46. Multiplying $0.02x - 0.04y = 0.08$ by -50, $-x + 2y = -4$. Then add this to the second equation.

$$\begin{array}{rcr} -x + 2y & = & -4 \\ x - 2y & = & 4 \\ \hline 0 & = & 0 \end{array}$$

Dependent and the solution set is $\{(x, y)|x - 2y = 4\}$.

47. Multiplying $\dfrac{x}{2} + \dfrac{y}{2} = 5$ by 2 and $\dfrac{3x}{2} - \dfrac{2y}{3} = 2$ by 6, we have $x + y = 10$ and $9x - 4y = 12$, respectively. Then multiply $x+y = 10$ by 4 and add to the second equation.

$$\begin{array}{rcr} 4x + 4y & = & 40 \\ 9x - 4y & = & 12 \\ \hline 13x & = & 52 \\ x & = & 4 \end{array}$$

Since $x + y = 10$, $4 + y = 10$ and $y = 6$. Independent and the solution set is $\{(4, 6)\}$.

48. Multiplying $\frac{x}{4} + \frac{y}{3} = 0$ by $-\frac{1}{2}$, we obtain $-\frac{x}{8} - \frac{y}{6} = 0$. Add this to the second equation.

$$\begin{array}{rcr} -\frac{x}{8} - \frac{y}{6} & = & 0 \\ \frac{x}{8} - \frac{y}{6} & = & 2 \\ \hline -\frac{y}{3} & = & 2 \\ y & = & -6 \end{array}$$

Since $\frac{x}{4} + \frac{y}{3} = 0$, $\frac{x}{4} - 2 = 0$ and $x = 8$.
Independent and the solution set is $\{(8, -6)\}$.

49. Multiply $3x - 2.5y = -4.2$ by -4 and $0.12x + 0.09y = 0.4932$ by 100.
Then add the equations.

$$\begin{array}{rcr} -12x + 10y & = & 16.8 \\ 12x + 9y & = & 49.32 \\ \hline 19y & = & 66.12 \\ y & = & 3.48 \end{array}$$

From $3x - 2.5y = -4.2$, we find

$$\begin{array}{rcl} 3x - 2.5(3.48) & = & -4.2 \\ 3x - 8.7 & = & -4.2 \\ 3x & = & 4.5 \\ x & = & 1.5. \end{array}$$

Independent and the solution set is $\{(1.5, 3.48)\}$.

50. Multiply $1.5x - 2y = 8.5$ by -2 and then add to the second equation.

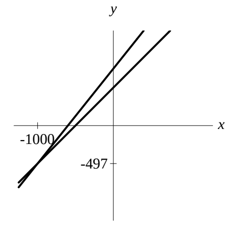
$$\begin{array}{rcr} -3x + 4y & = & -17 \\ 3x + 1.5y & = & 6 \\ \hline 5.5y & = & -11 \\ y & = & -2 \end{array}$$

Since $3x + 1.5y = 6$, $3x - 3 = 6$ and $x = 3$.
Independent and the solution set is $\{(3, -2)\}$.

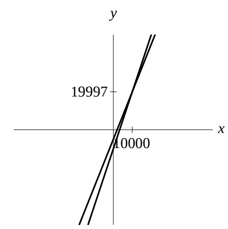
51. Independent **52.** Inconsistent

53. Dependent **54.** Independent

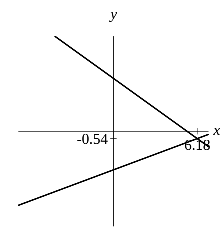
55. The point of intersection is $(-1000, -497)$.



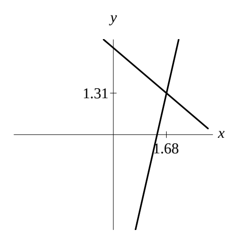
56. The point of intersection is $(10000, 19997)$.



57. The point of intersection is approximately $(6.18, -0.54)$



58. The point of intersection is approximately $(1.68, 1.31)$



59. Let x and y be Althea's and Vaughn's incomes, respectively. Then $x + y = 82,000$ and $x - y = 16,000$. By adding these two equations, we get $2x = 98,000$ or $x = 49,000$. Thus, Althea's income is \$49,000 and Vaughn's income is \$33,000.

60. Let x and y be number of males and females, respectively. Then $x + y = 76$ and $y - x = 2$. By adding these two equations, we get $2y = 78$ or $y = 39$. Thus, there were 39 females and 37 males.

61. Let x and y be the amounts invested at 10% and 8%, respectively.

$$\begin{aligned} x + y &= 25,000 \\ 0.1x + 0.08y &= 2200 \end{aligned}$$

Multiply the second equation by -10 and add to the first.

$$\begin{array}{rcr} x + y &=& 25,000 \\ -x - 0.8y &=& -22,000 \\ \hline 0.2y &=& 3000 \\ y &=& 15,000 \end{array}$$

Carmen invested \$15,000 at 8% and \$10,000 at 10%.

62. Let x and y be the amounts lost in the futures market and stock market, respectively.

$$\begin{aligned} 2y &= x \\ x + y &= 18,630 \end{aligned}$$

Substitute $x = 2y$ into the second equation.

$$\begin{aligned} 2y + y &= 18,630 \\ 3y &= 18,630 \\ y &= 6210 \end{aligned}$$

Gerhart lost \$6210 in the stock market and \$12,420 in the futures market.

63. Let x and y be the prices of an adult ticket and child ticket, respectively.

$$\begin{aligned} 2x + 5y &= 33 \\ x + 3y &= 18.50 \end{aligned}$$

Multiply the second equation by -2 and add to the first equation.

$$\begin{array}{rcr} 2x + 5y &=& 33 \\ -2x - 6y &=& -37 \\ \hline -y &=& -4 \\ y &=& 4 \end{array}$$

A child's ticket costs \$4. Since $x + 3y = 18.50$, $x + 12 = 18.50$ and so an adult's ticket costs $x = \$6.50$.

64. Let x and y be the prices of a paperback and hardback, respectively.

$$\begin{aligned} 6x + 3y &= 8.25 \\ 4x + 5y &= 9.25 \end{aligned}$$

Multiply the first equation by -2 and the second by 3. Then add resulting equations.

$$\begin{array}{rcr} -12x - 6y &=& -16.50 \\ 12x + 15y &=& 27.75 \\ \hline 9y &=& 11.25 \\ y &=& 1.25 \end{array}$$

Since $6x + 3y = 8.25$, $6x + 3.75 = 8.25$ and so $x = 0.75$. Todd's bill is $7(0.75) + 9(1.25) = \$16.50$.

65. If m and f are the number of male and female memberships, respectively, then a system of equations is

$$\begin{aligned} m + f &= 12 \\ 500m + 500f &= 6,000. \end{aligned}$$

Dividing the second equation by 500,one finds $m + f = 12$; the system of equations is dependent. Therefore, one cannot conclude the number of female memberships and male memberships.

All we know is that m and $12 - m$ are the number of male and female memberships where $0 \le m \le 12$.

66. If a and c are the prices of an adult's and a child's ticket, respectively, then a system of equations is

$$\begin{aligned} 3c + a &= 65.75 \\ 6c + 2a &= 131.50. \end{aligned}$$

Note, the second equation is two times the first equation, so the system of equations is dependent. Therefore, one cannot determine the price of an adult's ticket and the price of a child's ticket. There are many possibilities.

67. Let x and y be the number of cows and ostriches, respectively.

$$\begin{aligned} 2x + 2y &= 84 \\ 4x + 2y &= 122 \end{aligned}$$

Subtract the first equation from the second equation.

$$\begin{aligned} 2x &= 38 \\ x &= 19 \end{aligned}$$

Consequently, we have

$$\begin{aligned} 2x+2y &= 84 \\ 38+2y &= 84 \\ 2y &= 46 \\ y &= 23 \end{aligned}$$

Hence, there are $x = 19$ cows and $y = 23$ ostriches.

68. Let x and y be the number of snakes and iguanas, respectively.

$$\begin{aligned} 2x+2y &= 60 \\ 4y &= 68 \end{aligned}$$

Substitute $y = 17$ into the first equation.

$$\begin{aligned} 2x+34 &= 60 \\ 2x &= 26 \\ x &= 13 \end{aligned}$$

Hence, there are $x = 13$ snakes and $y = 17$ iguanas.

69. Let x and y be the number of cows and horses, respectively.

$$\begin{aligned} 4x+4y &= 96 \\ x+y &= 24 \end{aligned}$$

Divide the first equation by 4.

$$\begin{aligned} x+y &= 24 \\ x+y &= 24 \end{aligned}$$

Since the two equations are identical, the system of equations is dependent and there are infinitely many possible solutions.

70. Let x and y be the number of snakes and mice, respectively.

$$\begin{aligned} 2x+2y &= 78 \\ x+y &= 38 \end{aligned}$$

If we divide the first equation by two, we find

$$x+y = 39$$

This contradicts the second equation above. The system is inconsistent, and there are no solutions.

71. If c and m are the prices of a coffee and a muffin, respectively, then a system of equations is

$$\begin{aligned} 3c+7m &= 7.77 \\ 6c+14m &= 14.80. \end{aligned}$$

But if one multiplies the second equation by two, then one obtains $6c + 14m = 15.54$; this last equation contradicts the second equation in the system. Therefore, the system of equations is inconsistent and has no solution.

72. Let f and m be the number of females and males, respectively. A system of equations is

$$\begin{aligned} f+55 &= m \\ \frac{2}{3}m &= \frac{2}{3}f+30. \end{aligned}$$

But if one multiplies the second equation by $\frac{3}{2}$, then one obtains $m = f + 45$; this last equation contradicts the first equation in the system. Therefore, the system of equations is inconsistent and has no solution.

73. Let x and y be the number of male and female students, respectively.

$$\begin{aligned} 0.5x+0.3y &= 230 \\ 0.2x+0.6y &= 260 \end{aligned}$$

Multiply the first equation by -2 and the second by 5. Then add the resulting equations.

$$\begin{aligned} -x-0.6y &= -460 \\ x+3y &= 1300 \\ \hline 2.4y &= 840 \\ y &= 350 \end{aligned}$$

From $0.2x + 0.6y = 260$, $0.2x + 210 = 260$ and so $x = 250$. There are $250 + 350 = 600$ students at CHS.

74. Let x and y be the number of servings of Rice Krispies and Grape-nuts, respectively. Then we get

$$\begin{aligned} 2x + 3y &= 23 \\ 25x + 23y &= 215. \end{aligned}$$

Multiply the first equation by -25 and the second by 2. Then add the resulting equations.

$$\begin{array}{rcr} -50x - 75y &=& -575 \\ 50x + 46y &=& 430 \\ \hline -29y &=& -145 \\ y &=& 5 \end{array}$$

From $2x + 3y = 23$, $2x + 15 = 23$ and so $x = 4$. There must be 4 servings of Rice Krispies and 5 servings of Grape-nuts.

75. Let x and y be the number of nickels and pennies, respectively. We obtain

$$\begin{aligned} x + y &= 87 \\ 0.05x + 0.01y &= 1.75. \end{aligned}$$

Multiply the second equation by -100 and then add it to the first equation.

$$\begin{array}{rcr} x + y &=& 87 \\ -5x - y &=& -175 \\ \hline -4x &=& -88 \\ x &=& 22 \end{array}$$

From $x + y = 87$, $22 + y = 87$ and $y = 65$. Isabelle has 22 nickels and 65 pennies.

76. Let x and y be the number of quarters and dimes, respectively. Then we obtain

$$\begin{aligned} x + y &= 166 \\ 2.5(0.25x) + 2.5(0.1y) &= 61.75. \end{aligned}$$

Simplify the second equation.

$$\begin{aligned} x + y &= 166 \\ 0.625x + 0.25y &= 61.75 \end{aligned}$$

Multiply the second equation by -100 and the first by 25.

$$\begin{array}{rcr} 25x + 25y &=& 4150 \\ -62.5x - 25y &=& -6175 \\ \hline -37.5x &=& -2025 \\ x &=& 54 \end{array}$$

Since $x + y = 166$, we find $54 + y = 166$ and $y = 112$. Theodore has 54 quarters and 112 dimes.

77. The weights x and y must satisfy

$$\begin{aligned} 5x &= 3y \\ 3(4 + x + y) &= 6y. \end{aligned}$$

The second equation can be written as $12 + 3x = 3y$. Substituting into the first equation one finds

$$\begin{aligned} 12 + 3x &= 5x \\ 12 &= 2x \\ 6 &= x. \end{aligned}$$

Then $x = 6$ oz and $y = 10$ oz since $y = \dfrac{5x}{3} = \dfrac{5(6)}{3}$.

78. Let x and y be the rent of a single and double room, respectively.

$$\begin{aligned} x + 20 &= y \\ 26x + 15y &= 3949 \end{aligned}$$

Substitute $y = x+20$ into the second equation.

$$\begin{aligned} 26x + 15(x + 20) &= 3949 \\ 41x + 300 &= 3949 \\ 41x &= 3649 \\ x &= 89 \end{aligned}$$

A single room is \$89 and a double is \$109.

79. Let x be the number of months. Plan A costs $150x + 800$ and Plan B costs $200x + 200$. Thus, Plan A is cheaper in the long run. The number of months for which the costs are the same is given by

$$\begin{aligned} 150x + 800 &= 200x + 200 \\ 600 &= 50x \\ x &= 12 \text{ months}. \end{aligned}$$

80. Let x be the taxable income. We consider five cases. Suppose $0 < x \leq 8,025$. Under the simplified plan the federal tax is $100 + 0.25x$.

According to the 2008 Tax Schedule, the tax is $0.10x$. If these taxes are equal, then

$$\begin{aligned} 100 + 0.25x &= 0.10x \\ 0.15x &= -100 \\ x &\approx -\$667. \end{aligned}$$

So, the taxes are not equal if $0 < x \leq 8,025$.

Suppose $8,025 < x \leq 32,550$. Then

$$\begin{aligned} 100 + 0.25x &= 802.50 + 0.15(x - 802.50) \\ 100 + 0.25x &= 0.15x + 682.125 \\ x &= \$6,821.25 \end{aligned}$$

This contradicts $8,025 < x \leq 32,550$.

Suppose $32,550 < x \leq 78,850$. Then

$$\begin{aligned} 100 + 0.25x &= 4481.25 + 0.25(x - 32,550) \\ 100 &= -3656.25 \end{aligned}$$

We have an inconsistent equation.

Suppose $78,850 < x \leq 164,550$. Then

$$\begin{aligned} 100 + 0.25x &= 16,056.25 + 0.28(x - 78,850) \\ x &= \$204,058.33. \end{aligned}$$

This contradicts $78,850 < x \leq 164,550$.

Suppose $164,550 < x \leq 357,700$. Then

$$\begin{aligned} 100 + 0.25x &= 40,052.25 + 0.33(x - 164,550) \\ x &= \$179,365.63. \end{aligned}$$

Thus, the taxes will be the same under both plans if the income is \$179,365.63.

Finally, suppose $x > 357,000$. Then

$$\begin{aligned} 100 + 0.25x &= 103,791.75 + 0.35(x - 357,700) \\ x &= \$215,032.50. \end{aligned}$$

This contradicts $x > 319,000$.

Thus, the taxes will be the same under both plans if the income is \$179,365.63.

81. If we set the formulas equal to each other, then we find

$$\begin{aligned} 0.08aD &= \frac{D(a+1)}{24} \\ 0.08a &= \frac{(a+1)}{24} \\ 1.92a &= a + 1 \\ a &= \frac{1}{0.92} \\ a &\approx 1.09. \end{aligned}$$

The dosage is the same if the age is 1.1 yr.

82. Let x be the taxable income. In one plan, the tax is $0.17(x - 15000)$. In the second plan, the tax is $(0.15)(0.75)x$. The income when the taxes are equal is given by

$$\begin{aligned} 0.17(x - 15000) &= (0.15)(0.75)x \\ 0.17x - 2550 &= 0.1125x \\ x &\approx \$44,348. \end{aligned}$$

83. Suppose the fronts of the trucks are at the same points. Let t be the number of hours before the trucks pass each other. They will be passing each other when the ends of the trucks are at the same position, i.e., when the total distance driven by the trucks is 100 feet.

$$\begin{aligned} 40(5280)t + 50(5280)t &= 100 \\ 40t + 50t &= \frac{100}{5280} \\ 90t &= \frac{100}{5280} \\ t &= \frac{100}{(90)5280} \text{ hour} \\ t &= \frac{100}{(90)5280(3600)} \text{ sec} \\ t &= \frac{25}{33} \text{ sec} \\ t &\approx 0.76 \text{ sec} \end{aligned}$$

Hence, the trucks pass each other in 0.76 sec, approximately.

84. Suppose the front end of the faster truck is at the rear end of the slower truck. Let t be the number of hours before the faster truck passes the slower truck. The faster truck passes the slower truck exactly when the rear end of the faster truck is at the same position of the front end of the slower truck.

$$\begin{aligned} 40(5280)t &= 50(5280)t - 100 \\ 100 &= 10(5280)t \\ 1 &= 528t \\ t &= \frac{1}{528} \text{ hour} \\ t &= \frac{3600}{528} \text{ sec} \\ t &= \frac{75}{11} \text{ sec} \\ t &\approx 6.8 \text{ sec} \end{aligned}$$

Thus, the faster truck passes the slower truck in about 6.8 sec.

85. Solve for a and b.

$$\begin{aligned} -3a + b &= 9 \\ 2a + b &= -1. \end{aligned}$$

Multiply the second equation by -1 and add to the first equation.

$$\begin{array}{rcr} -3a + b &=& 9 \\ -2a - b &=& 1 \\ \hline -5a &=& 10 \\ a &=& -2 \end{array}$$

Substituting $a = -2$ into $2a + b = -1$, one finds $b = 3$. An equation of the line is $y = -2x + 3$.

86. Solve for a and b.

$$\begin{aligned} a + b &= -1 \\ 3a + b &= 7 \end{aligned}$$

Multiply the first equation by -1 and add to the second equation.

$$\begin{array}{rcr} -a - b &=& 1 \\ 3a + b &=& 7 \\ \hline 2a &=& 8 \\ a &=& 4 \end{array}$$

Substituting $a = 4$ into $a+b = -1$, one obtains $b = -5$. An equation of the line is $y = 4x - 5$.

87. Solve for a and b.

$$\begin{aligned} -2a + b &= 3 \\ 4a + b &= -7 \end{aligned}$$

Multiply the first equation by -1 and add to the second equation.

$$\begin{array}{rcr} 2a - b &=& -3 \\ 4a + b &=& -7 \\ \hline 6a &=& -10 \\ a &=& -\frac{5}{3} \end{array}$$

Substituting $a = -\frac{5}{3}$ into $-2a + b = 3$, one gets $b = -\frac{1}{3}$. An equation of the line is $y = -\frac{5}{3}x - \frac{1}{3}$.

88. Solve for a and b.

$$\begin{aligned} -3a + b &= -1 \\ 4a + b &= 9 \end{aligned}$$

Multiply the first equation by -1 and add to the second equation.

$$\begin{array}{rcr} 3a - b &=& 1 \\ 4a + b &=& 9 \\ \hline 7a &=& 10 \\ a &=& \frac{10}{7} \end{array}$$

Substituting $a = \dfrac{10}{7}$ into $-3a + b = -1$, we obtain $b = \dfrac{23}{7}$. An equation of the line is $y = \dfrac{10}{7}x + \dfrac{23}{7}$.

91. Independent system with solution $(2, -3)$:

$$\begin{aligned} x + y &= -1 \\ x - y &= 5 \end{aligned}$$

92. Dependent system with solution $(t, t+5)$:

$$\begin{aligned} y &= x + 5 \\ 2x - 2y &= -10 \end{aligned}$$

93. **a)** $f(\frac{2}{3}) = \left(8^{1/3}\right)^2 = 2^2 = 4$

b) $g(3) = 4^{2-3} = 4^{-1} = \dfrac{1}{4}$

c) $(f \circ g)(2) = f(g(2)) = f(1) = 8$

94. $y = 5$

95. Since $2^3 = 8$ and $2^2 = 4$, we find

$$\begin{aligned} \left(2^3\right)^{x-3} &= \left(2^2\right)^{x+5} \\ 2^{3x-9} &= 2^{2x+10} \\ 3x - 9 &= 2x + 10 \\ x &= 19 \end{aligned}$$

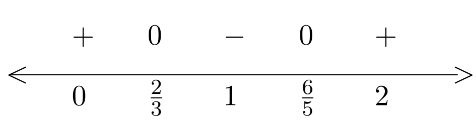
The solution set is $\{19\}$.

96. The axis of symmetry is $x = -\dfrac{b}{2a} = -\dfrac{-5}{2(-3)}$ or $x = -\dfrac{5}{6}$

97. The zeros of

$$f(x) = (3x - 2)(5x - 6).$$

are $x = 2/3, 6/5$.
If $x = 0$, then $f(0) > 0$.
If $x = 1$, then $f(1) < 0$.
If $x = 2$, then $f(2) > 0$.



The solution set of $f(x) \le 0$ is $\left[\frac{2}{3}, \frac{6}{5}\right]$.

98. The remainder is $f(2) = 2^8 - 2(2) + 1 = 253$.

Thinking Outside the Box

LXIX. Let p and f be the number of students who passed and failed, respectively. Since the mean score for the passing students is 65, the mean score for the failing students is 53, and the mean score for all students is 53, we obtain

$$\frac{65p + 35f}{p + f} = 53.$$

Solving for f, we find

$$f = \frac{2p}{3}.$$

Then the percentage of passing students in the class is

$$\frac{p}{p+f} \cdot 100 = \frac{p}{p + 2p/3} \cdot 100 = \frac{3}{5} \cdot 100 = 60\%.$$

LXX. Factoring the exponent, we find

$$x^3 - 9x^2 + 20x = x(x^2 - 9x + 20) = x(x-4)(x-5).$$

The zeros of the exponent are $x = 0, 4, 5$.
Factoring the base, we obtain

$$x^2 + 2x - 24 = (x+6)(x-4).$$

Since 0^0 is undefined, we find that $x = 4$ is not a solution of

$$(x^2 + 2x - 24)^{x^3 - 9x^2 + 20x} = 0.$$

If we set the base equal to 1, we find

$$\begin{aligned} x^2 + 2x - 24 &= 1 \\ (x+1)^2 &= 26 \\ x &= -1 \pm \sqrt{26}. \end{aligned}$$

Thus, the solution set is

$$\{0, 5, -1 \pm \sqrt{26}\}.$$

8.1 Pop Quiz

1. Substitute $y = 2x$ into $7x - 3y = 4$.

$$\begin{aligned} 7x - 3(2x) &= 4 \\ 7x - 6x &= 4 \\ x &= 4 \end{aligned}$$

Since $x = 4$, we find $y = 2x = 2(4) = 8$. Independent and the solution set is $\{(4, 8)\}$.

2. Adding the equations, we find

$$\begin{aligned} 3x - 5y &= 11 \\ 7x + 5y &= 19 \\ \hline 10x &= 30 \\ x &= 3 \end{aligned}$$

Substituting $x = 3$ into $3x - 5y = 11$, we obtain $9 - 5y = 11$ or $-2 = 5y$ or $y = -2/5$. Independent and the solution set is $\{(3, -2/5)\}$.

3. Multiply the first equation by 2 and multiply the second equation by 3. Then add the resulting products as follows

$$\begin{aligned} 15x - 6y &= -3 \\ 8x + 6y &= 26 \\ \hline 23x &= 23 \\ x &= 1 \end{aligned}$$

Substituting $x = 1$ into $5x - 2y = -1$, we obtain $5 - 2y = -1$ or $6 = 2y$ or $y = 3$. Independent and the solution set is $\{(1, 3)\}$.

4. Since $3x = 1 - 9y$, we get $3x + 9y = 1$. Multiply the equation $x + 3y = 8$ by -3 and add the result to the $3x + 9y = 1$.

$$\begin{aligned} 3x + 9y &= 1 \\ -3x - 9y &= -24 \\ \hline 0 &= -23 \end{aligned}$$

Thus, the system of equations is inconsistent and the solution set is the empty set $\emptyset$.

5. Since $y = x + 1$, we find $-x + y = 1$. Dividing the other equation $5x - 5y + 5 = 0$ by 1, we obtain $x - y + 1 = 0$ or $x - y = -1$. Then add the equations $-x + y = 1$ and $x - y = -1$.

$$\begin{aligned} -x + y &= 1 \\ x - y &= -1 \\ \hline 0 &= 0 \end{aligned}$$

Thus, the system is dependent and the solution set $\{(x, y) : y = x + 1\}$.

8.1 Linking Concepts

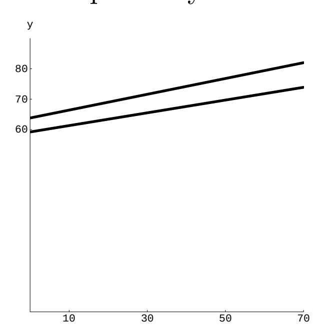
a) Using a calculator, we find the regression line is $y = 0.213452381x + 59.24166667$ in year $1930 + x$.

Approximately, $y = 0.21x + 59.2$ in year $1930 + x$.

b) With a calculator, we find the regression line is $y = 0.255x + 63.75$ in year $1930 + x$.

Approximately, $y = 0.26x + 63.8$ in year $1930 + x$.

c) The higher line represents the life expectancy of women. The other line represents the life expectancy of men.



d) Since the slope for women is larger, the life expectancy of men will not catch up to the life expectancy of women.

e) Solving for x in the system of equations

$$\begin{aligned} y &= 0.213452381x + 59.24166667 \\ y &= 0.255x + 63.75 \end{aligned}$$

one finds $x \approx -108.51$. Thus, life expectancy for men and women were equal in the year 1821 ($\approx 1930 - 108.51$).

f) Rate of change of life expectancy is 0.21 year/calendar year for men, and 0.26 year/calendar year for women.

g) A possible answer is there are more advances in health care for women than that for men.

For Thought

1. True

2. False, since $(1, 1, 0)$ does not satisfy $-x - y + z = 4$.

3. True, adding the first two equations gives $0 = 6$ which is false.

4. False, since $(2, 3, -1)$ does not satisfy $x - y - z = 8$.

5. True

6. True, adding the first two equations gives an identity. Also, multiplying the first by -2 and then adding to the third equation gives an identity.

$$\begin{array}{rcr} x - y + z & = & 1 \\ -x + y - z & = & -1 \\ \hline 0 & = & 0 \end{array}$$

$$\begin{array}{rcr} -2x + 2y - 2z & = & -2 \\ 2x - 2y + 2z & = & 2 \\ \hline 0 & = & 0 \end{array}$$

7. True. The calculations in Exercise 6 above show system (c) is dependent.

8. True

9. True, if $x = 1$ then $(x + 2, x, x - 1) = (3, 1, 0)$.

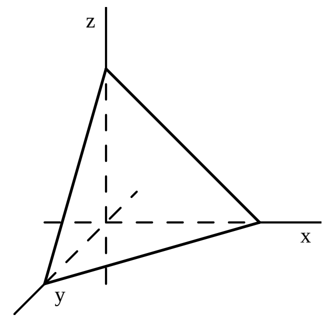
10. False, the value is $0.05x + 0.10y + 0.25z$ dollars.

8.2 Exercises

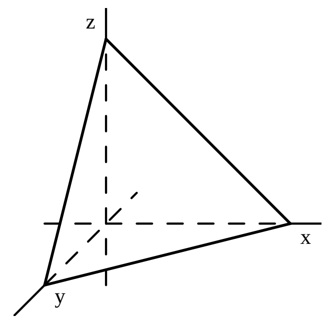
1. linear

2. independent

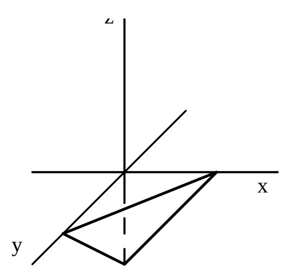
3. Points on the plane are $(5, 0, 0)$, $(0, 5, 0)$, and $(0, 0, 5)$.



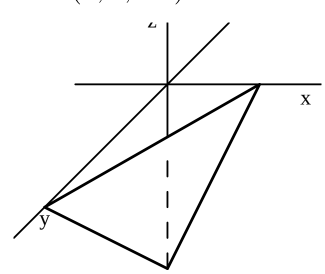
4. Points on the plane are $(6, 0, 0)$, $(0, 3, 0)$, and $(0, 0, 6)$.



5. Points on the plane are $(3, 0, 0)$, $(0, 3, 0)$, and $(0, 0, -3)$.



6. Points on the plane are $(3, 0, 0)$, $(0, 6, 0)$, and $(0, 0, -6)$.



7. Note, $x = 1$, $y = 3$, and $z = 2$ satisfy all the three equations in the system . Yes, $(1, 3, 2)$ is a solution.

8. Note, $x = -1$, $y = 2$, and $z = 4$ satisfy all the three equations in the system . Yes, $(1, 3, 2)$ is a solution.

9. Note, $x = -1$, $y = 5$, and $z = 2$ does not satisfy $x-y-2z = -8$. No, $(-1, 5, 2)$ is not a solution.

10. Note, $x = 3$, $y = 2$, and $z = 1$ does not satisfy $2x+y-z = -9$. No, $(3, 2, 1)$ is not a solution.

11. Add the first and second equations and add the first and third equations.

$$\begin{array}{rcr} x + y + z &=& 6 \\ 2x - 2y - z &=& -5 \\ \hline 3x - y &=& 1 \end{array}$$

$$\begin{array}{rcr} x + y + z &=& 6 \\ 3x + y - z &=& 2 \\ \hline 4x + 2y &=& 8 \end{array}$$

Divide $4x+2y = 8$ by 2 and add to $3x-y = 1$.

$$\begin{array}{rcr} 2x + y &=& 4 \\ 3x - y &=& 1 \\ \hline 5x &=& 5 \end{array}$$

So $x = 1$. From $3x - y = 1$, $3 - y = 1$ and $y = 2$. From $x + y + z = 6$, $1 + 2 + z = 6$ and $z = 3$. The solution set is $\{(1, 2, 3)\}$.

12. Add first and second equations. Also, multiply the third equation by -1 and add to the first.

$$\begin{array}{rcr} 3x - y + 2z &=& 14 \\ x + y - z &=& 0 \\ \hline 4x + z &=& 14 \end{array}$$

$$\begin{array}{rcr} 3x - y + 2z &=& 14 \\ -2x + y - 3z &=& -18 \\ \hline x - z &=& -4 \end{array}$$

Add $4x + z = 14$ to $x - z = -4$.

$$\begin{array}{rcr} 4x + z &=& 14 \\ x - z &=& -4 \\ \hline 5x &=& 10 \end{array}$$

So $x = 2$. From $x - z = -4$, $2 - z = -4$ and $z = 6$. From $x + y - z = 0$, $2 + y - 6 = 0$ and $y = 4$. The solution set is $\{(2, 4, 6)\}$.

13. Multiply the first equation by 2 and add to the second. Then multiply the first equation by -3 and add to the third equation.

$$\begin{array}{rcr} 6x + 4y + 2z &=& 2 \\ x + y - 2z &=& -4 \\ \hline 7x + 5y &=& -2 \end{array}$$

$$\begin{array}{rcr} -9x - 6y - 3z &=& -3 \\ 2x - 3y + 3z &=& 1 \\ \hline -7x - 9y &=& -2 \end{array}$$

Add $7x + 5y = -2$ to $-7x - 9y = -2$.

$$\begin{array}{rcr} 7x + 5y &=& -2 \\ -7x - 9y &=& -2 \\ \hline -4y &=& -4 \end{array}$$

So $y = 1$. From $7x + 5y = -2$, $7x + 5 = -2$ and $x = -1$. From $3x + 2y + z = 1$, $-3 + 2 + z = 1$ and $z = 2$.
The solution set is $\{(-1, 1, 2)\}$.

14. Multiply first equation by -2 and add to the second. Then multiply first equation by 3 and add to the third.

$$\begin{array}{rcr} -8x + 4y - 2z &=& -26 \\ 3x - y + 2z &=& 13 \\ \hline -5x + 3y &=& -13 \end{array}$$

$$\begin{array}{rcr} 12x - 6y + 3z &=& 39 \\ x + 3y - 3z &=& -10 \\ \hline 13x - 3y &=& 29 \end{array}$$

Add $-5x + 3y = -13$ to $13x - 3y = 29$.

$$\begin{array}{rcr} -5x + 3y &=& -13 \\ 13x - 3y &=& 29 \\ \hline 8x &=& 16 \end{array}$$

So $x = 2$. From $-5x + 3y = -13$, we find $-10 + 3y = -13$ and $y = -1$.
From $4x - 2y + z = 13$, we get $8 + 2 + z = 13$ and $z = 3$. Solution set is $\{(2, -1, 3)\}$.

15. Add first equation to third equation. Multiply second equation by 2 and add to the first.

$$\begin{array}{rcr} 2x + y - 2z &=& -15 \\ x + 3y + 2z &=& -5 \\ \hline 3x + 4y &=& -20 \end{array}$$

$$\begin{array}{rcr} 8x - 4y + 2z &=& 30 \\ 2x + y - 2z &=& -15 \\ \hline 10x - 3y &=& 15 \end{array}$$

Multiply $3x+4y = -20$ by 3 and $10x-3y = 15$ by 4 and then add the equations.

$$\begin{array}{rcr} 9x + 12y &=& -60 \\ 40x - 12y &=& 60 \\ \hline 49x &=& 0 \end{array}$$

So $x = 0$. From $3x + 4y = -20$, $4y = -20$ and $y = -5$. From $2x + y - 2z = -15$, we get $-5 - 2z = -15$ and $z = 5$.
The solution set is $\{(0, -5, 5)\}$.

16. Multiply first equation by -2 and add to second equation. Then multiply first equation by -3 and add to the third equation.

$$\begin{array}{rcr} -2x + 4y + 6z &=& -8 \\ 2x - 4y + 5z &=& -3 \\ \hline 11z &=& -11 \end{array}$$

$$\begin{array}{rcr} -3x + 6y + 9z &=& -12 \\ 5x - 6y + 4z &=& -7 \\ \hline 2x + 13z &=& -19 \end{array}$$

So $z = -1$. From $2x + 13z = -19$, we get $2x - 13 = -19$ and $x = -3$.
Since $x - 2y - 3z = 4$, $-3 - 2y + 3 = 4$ and $y = -2$. Solution set is $\{(-3, -2, -1)\}$.

17. If we substitute $x = 1, 2, 3$ in $(x, x + 3, x - 5)$, we obtain

$$(1, 4, -4),\ (2, 5, -3),\ \text{and } (3, 6, -2),$$

respectively.

18. If we substitute $x = 0, 1, 2$ in $(x, 2x-4, x-9)$, we obtain

$$(0, -4, -9),\ (1, -2, -8),\ \text{ and } (2, 0, -7),$$

respectively.

19. If we substitute $y = 1, 2, 3$ in $(2y, y, y - 7)$, we obtain

$$(2, 1, -6),\ (4, 2, -5),\ \text{ and } (6, 3, -4),$$

respectively.

20. If we substitute $z = -1, 0, 1$ in $(3-z, 2-z, z)$, we obtain

$$(4, 3, -1),\ (3, 2, 0),\ \text{ and } (2, 1, 1),$$

respectively.

21. Let $y = x + 3$. Then $x = y - 3$, $x - 5 = y - 8$, and

$$(x, x + 3, x - 5) = (y - 3, y, y - 8).$$

22. Let $y = 2x$. Then $x = y/2$, $3x = 3y/2$, and

$$(x, 2x, 3x) = \left(\frac{1}{2}y, y, \frac{3}{2}y\right).$$

23. Let $z = x - 1$. Then $x = z + 1$, $x + 1 = z + 2$, and

$$(x, x + 1, x - 1) = (z + 1, z + 2, z)\,.$$

24. Let $z = x + 5$. Then $x = z - 5$, $x - 1 = z - 6$, and

$$(x, x - 1, x + 5) = (z - 5, z - 6, z)\,.$$

25. Let $y = 2x + 1$. Then $x = (y - 1)/2$, $3x - 1 = (3y - 5)/2$, and

$$(x, 2x + 1, 3x - 1) = \left(\frac{y-1}{2}, y, \frac{3y-5}{2}\right).$$

26. Let $z = 2x - 4$. Then $x = (z + 4)/2$, $3x = (3z + 12)/2$, and

$$(x, 3x, 2x - 4) = \left(\frac{z+4}{2}, \frac{3z+12}{2}, z\right).$$

27. Adding the two equations, we have $4x - 4z = -20$ or $z = x + 5$.
From $x + 2y - 3z = -17$, we obtain

$$\begin{aligned} x + 2y - 3(x + 5) &= -17 \\ 2y - 2x - 15 &= -17 \\ 2y &= 2x - 2 \\ y &= x - 1. \end{aligned}$$

The solution set is
$\{(x, x - 1, x + 5) | x \text{ is any real number}\}$.

28. Adding the two equations, we have $3x + y = 7$ or $y = 7 - 3x$. From $x + 2y + z = 4$, one has

$$\begin{aligned} x + 2(7 - 3x) + z &= 4 \\ 14 - 5x + z &= 4 \\ z &= 5x - 10. \end{aligned}$$

Then solution set is $\{(x, 7 - 3x, 5x - 10) | x \text{ is any real number}\}$.

29. Adding the two equations, we obtain $2x - y = 7$ or $y = 2x - 7$. From $x + y - z = 2$, we obtain

$$\begin{aligned} z &= x + y - 2 \\ z &= x + (2x - 7) - 2 \\ z &= 3x - 9. \end{aligned}$$

The solution set is $\{(x, 2x - 7, 3x - 9) | x \text{ is any real number}\}$.

30. If we subtract the second equation from the first equation, we find $x - 2y = -12$ or $x = 2y - 12$. From $x - y - z = 3$, we obtain

$$\begin{aligned} z &= x - y - 3 \\ z &= (2y - 12) - y - 3 \\ z &= y - 15. \end{aligned}$$

The solution set is $\{(2y - 12, y, y - 15) | y \text{ is any real number}\}$.

31. Adding the two equations, we find $2x + 2z = 12$ or $x = 6 - z$. From $y + z = 5$, we obtain

$$y = 5 - z.$$

The solution set is $\{(6 - z, 5 - z, z) | z \text{ is any real number}\}$.

32. If we subtract the second equation from the first equation, we obtain $2x - 2y = -10$ or $x = y - 5$. From $x - z = -10$, we obtain

$$\begin{aligned} z &= x + 10 \\ z &= (y - 5) + 10 \\ z &= y + 5. \end{aligned}$$

The solution set is $\{(y - 5, y, y + 5) | y \text{ is any real number}\}$.

33. Adding the first two equations, we have $0 = 0$, which is an identity. Multiply the second equation by 2 and add to the third equation.

$$\begin{array}{rcr} -2x - 4y + 6z &=& -10 \\ 2x + 4y - 6z &=& 10 \\ \hline 0 &=& 0 \end{array}$$

The system is dependent and the solution set is $\{(x, y, z) | x + 2y - 3z = 5\}$.

34. Multiply first and second equations by -3 and 2, respectively, then add the resulting equations. Also, multiply first and third by -5 and 2, respectively, then add the resulting equations.

$$\begin{array}{rcr} -6x + 18y - 12z &=& -24 \\ 6x - 18y + 12z &=& 24 \\ \hline 0 &=& 0 \end{array}$$

$$\begin{array}{rcr} -10x + 30y - 20z &=& -40 \\ 10x - 30y + 20z &=& 40 \\ \hline 0 &=& 0 \end{array}$$

The system is dependent and the solution set is $\{(x, y, z) | x - 3y + 2z = 4\}$.

35. Multiply first equation by -2 and add to the second equation.

$$\begin{array}{rcr} -2x + 4y - 6z &=& -10 \\ 2x - 4y + 6z &=& 3 \\ \hline 0 &=& -7 \end{array}$$

This is false, so the solution set is $\emptyset$.

36. Add the first and third equations.

$$\begin{array}{rcr} -2x + y - 3z &=& 6 \\ 2x - y + 3z &=& 1 \\ \hline 0 &=& 7 \end{array}$$

This is false, so the solution set is $\emptyset$.

37. Adding the two equations, we get $3x = 6$ or $x = 2$. Substitute into the two equations. Then

$$\begin{aligned} 2 + y - z &= 2 \\ y - z &= 0 \end{aligned}$$

and

$$\begin{aligned} 4 - y + z &= 4 \\ -y + z &= 0. \end{aligned}$$

In any case $y = z$. The solution set is $\{(2, y, y) | y \text{ is any real number}\}$.

38. Adding the two equations, we get $y = 5$. Substitute into the two equations. We obtain

$$\begin{aligned} -2x + 10 - z &= 4 \\ -2x - z &= -6 \\ 2x + z &= 6 \end{aligned}$$

and

$$\begin{aligned} 2x - 5 + z &= 1 \\ 2x + z &= 6. \end{aligned}$$

In any case, $z = 6 - 2x$. The solution set is $\{(x, 5, 6 - 2x) | x \text{ is any real number}\}$.

39. If the second equation is multiplied by -1 and added to the first equation, the result is the third equation.

$$\begin{array}{rcr} x + y &=& 5 \\ -y + z &=& -2 \\ \hline x + z &=& 3 \end{array}$$

There ae infinitely number many solutions. Since $y = 5 - x$ and $z = 3 - x$, the solution set $\{(x, 5 - x, 3 - x) | x \text{ is any real number}\}$.

40. Adding the first two equations, we have $z - y = 0$ which is equivalent to the third equation. From the first equation, $y = 2x + 1$ and since $y = z$, the solution set is $\{(x, 2x + 1, 2x + 1) | x \text{ is any real number}\}$.

41. Multiply the first equation by 2 and add to the second equation.

$$\begin{array}{rcr} 2x - 2y + 2z &=& 14 \\ 2y - 3z &=& -13 \\ \hline 2x - z &=& 1 \end{array}$$

Multiply $2x - z = 1$ by -2 and add to the third equation.

$$\begin{array}{rcr} -4x + 2z &=& -2 \\ 3x - 2z &=& -3 \\ \hline -x &=& -5 \end{array}$$

So $x = 5$. From $2x - z = 1$, we get $10 - z = 1$ and $z = 9$. Since $2y - 3z = -13$, $2y - 27 = -13$ and $y = 7$. The solution set is $\{(5, 7, 9)\}$.

42. Multiply the second equation by 3 and add to twice the third equation.

$$\begin{array}{rcr} 6y + 9z &=& -42 \\ -6y - 4z &=& 22 \\ \hline 5z &=& -20 \end{array}$$

So $z = -4$. Since $2y + 3z = -14$, $2y - 12 = -14$, and $y = -1$. From $2x + y - z = 5$, we get $2x - 1 + 4 = 5$ and $x = 1$. The solution set is $\{(1, -1, -4)\}$.

43. Multiply first equation by -2 and add to third equation. Then multiply first equation by -4 and add to the second.

$$\begin{array}{rcr} -2x - 2y - 4z &=& -15 \\ 5x + 2y + 5z &=& 21 \\ \hline 3x + z &=& 6 \end{array}$$

$$\begin{array}{rcr} -4x - 4y - 8z &=& -30 \\ 3x + 4y + z &=& 12 \\ \hline -x - 7z &=& -18 \end{array}$$

Multiply $-x - 7z = -18$ by 3 and add to $3x + z = 6$.

$$\begin{array}{rcr} -3x - 21z &=& -54 \\ 3x + z &=& 6 \\ \hline -20z &=& -48 \end{array}$$

So $z = 48/20 = 2.4$. From $3x + z = 6$, we get $3x + 2.4 = 6$ and $x = 1.2$. From $x + y + 2z = 7.5$, $1.2 + y + 4.8 = 7.5$ and $y = 1.5$.
The solution set is $\{(1.2, 1.5, 2.4)\}$.

44. Divide all three equations by 100.

$$\begin{aligned} x + 2y + 5z &= 0.47 \\ 3.5x + 0.05y + 2.5z &= 0.339 \\ 2x + 0.80y + z &= 0.234 \end{aligned}$$

Multiply the first equation by −3.5 and add to the second one. Also, multiply first equation by −2 and add to the third.

$$\begin{array}{rcr} -3.5x - 7y - 17.5z &=& -1.645 \\ 3.5x + 0.05y + 2.5z &=& 0.339 \\ \hline -6.95y - 15z &=& -1.306 \end{array}$$

$$\begin{array}{rcr} -2x - 4y - 10z &=& -0.94 \\ 2x + 0.80y + z &=& 0.234 \\ \hline -3.20y - 9z &=& -0.706 \end{array}$$

Multiply $-3.20y - 9z = -0.706$ by -5 and add to three times $-6.95y - 15z = -1.306$.

$$\begin{array}{rcr} 16y + 45z &=& 3.53 \\ -20.85y - 45z &=& -3.918 \\ \hline -4.85y &=& -0.388 \end{array}$$

So $y = 0.08$. From $16y + 45z = 3.53$, $1.28 + 45z = 3.53$ and $z = 0.05$.
From $x + 2y + 5z = 0.47$, we have $x + 0.16 + 0.25 = 0.47$ and $x = 0.06$.
Solution set is $\{(0.06, 0.08, 0.05)\}$.

45. Multiply the first equation by −5 and add to 100 times the second one.

$$\begin{array}{rcr} -5x - 5y - 5z &=& -45,000 \\ 5x + 6y + 9z &=& 71,000 \\ \hline y + 4z &=& 26,000 \end{array}$$

Substitute $z = 3y$ into $y + 4z = 26,000$.

$$\begin{array}{rcl} y + 12y &=& 26,000 \\ 13y &=& 26,000 \\ y &=& 2,000 \end{array}$$

From $z = 3y$, we obtain $z = 6000$.
From $x + y + z = 9,000$, we have $x + 2000 + 6000 = 9000$ and $x = 1000$.
The solution set is $\{(1000, 2000, 6000)\}$.

46. Multiply first equation by −8 and add to 100 times the second one.

$$\begin{array}{rcr} -8x - 8y - 8z &=& -1,600,000 \\ 9x + 8y + 12z &=& 2,020,000 \\ \hline x + 4z &=& 420,000 \end{array}$$

Combine first and third equations.

$$\begin{array}{rcr} x + y + z &=& 200,000 \\ -x - y + z &=& 0 \\ \hline 2z &=& 200,000 \\ z &=& 100,000 \end{array}$$

From $x + 4z = 420,000$, we have $x + 400,000 = 420,000$ and $x = 20,000$.
From $z = x + y$, we get $100,000 = 20,000 + y$ and $y = 80,000$. The solution set is $\{(20,000,\ 80,000,\ 100,000)\}$.

47. Substitute $x = 2y - 1$ into $z = 2x - 3$ to get $z = 4y - 5$. Then substitute $z = 4y - 5$ into $y = 3z + 2$.

$$\begin{array}{rcl} y &=& 3(4y - 5) + 2 \\ y &=& 12y - 13 \\ -11y &=& -13 \\ y &=& 13/11 \end{array}$$

From $z = 4y - 5$, we obtain $z = 52/11 - 5 = -3/11$. Since $x = 2y - 1$, $x = 26/11 - 1 = 15/11$. The solution set is $\{(15/11, 13/11, -3/11)\}$.

48. Multiply first equation by −2 and add to second equation. Then multiply first equation by −3 and add to the third.

$$\begin{array}{rcl} -2x - 4y + 6z &=& 0 \\ 2x - y + z &=& 0 \\ \hline -5y + 7z &=& 0 \end{array}$$

$$\begin{array}{rcl} -3x - 6y + 9z &=& 0 \\ 3x + y - 4z &=& 0 \\ \hline -5y + 5z &=& 0 \end{array}$$

Multiply $-5y + 7z = 0$ by -1 and add to $-5y + 5z = 0$.

$$\begin{array}{rcl} 5y - 7z &=& 0 \\ -5y + 5z &=& 0 \\ \hline -2z &=& 0 \end{array}$$

So $z = 0$. Since $-5y + 5z = 0$, $-5y = 0$ and $y = 0$. From $x + 2y - 3z = 0$, we have $x + 0 - 0 = 0$ or $x = 0$.
The solution set is $\{(0, 0, 0)\}$.

49. Substitute $(-1,-2)$, $(2,1)$, $(-2,1)$ into $y = ax^2 + bx + c$. So

$$\begin{aligned} a - b + c &= -2 \\ 4a + 2b + c &= 1 \\ 4a - 2b + c &= 1. \end{aligned}$$

Multiply first equation by -1 and add to the second and third equations.

$$\begin{aligned} -a + b - c &= 2 \\ 4a + 2b + c &= 1 \\ \hline 3a + 3b &= 3 \end{aligned}$$

$$\begin{aligned} -a + b - c &= 2 \\ 4a - 2b + c &= 1 \\ \hline 3a - b &= 3 \end{aligned}$$

Multiply $3a + 3b = 3$ by -1 and add to $3a - b = 3$.

$$\begin{aligned} -3a - 3b &= -3 \\ 3a - b &= 3 \\ \hline -4b &= 0 \end{aligned}$$

So $b = 0$. From $3a - b = 3$, we get $3a = 3$ and $a = 1$. From $a - b + c = -2$, $1 + c = -2$ and $c = -3$. Since the solution is $(a,b,c) = (1,0,-3)$, the parabola is $y = x^2 - 3$.

50. Substitute $(1,2)$, $(2,3)$, $(3,6)$ into $y = ax^2 + bx + c$. Then

$$\begin{aligned} a + b + c &= 2 \\ 4a + 2b + c &= 3 \\ 9a + 3b + c &= 6. \end{aligned}$$

Multiply the first equation by -1 and add to the second and third equations.

$$\begin{aligned} -a - b - c &= -2 \\ 4a + 2b + c &= 3 \\ \hline 3a + b &= 1 \end{aligned}$$

$$\begin{aligned} -a - b - c &= -2 \\ 9a + 3b + c &= 6 \\ \hline 8a + 2b &= 4 \end{aligned}$$

Multiply $3a + b = 1$ by -2 and add to $8a + 2b = 4$.

$$\begin{aligned} -6a - 2b &= -2 \\ 8a + 2b &= 4 \\ \hline 2a &= 2 \end{aligned}$$

So $a = 1$. From $3a + b = 1$, $3 + b = 1$ and $b = -2$. From $a+b+c = 2$, $1-2+c = 2$ and $c = 3$. Since the solution is $(a,b,c) = (1,-2,3)$, the parabola is $y = x^2 - 2x + 3$.

51. Substitute $(0,0)$, $(1,3)$, $(2,2)$ into $y = ax^2 + bx + c$. Then

$$\begin{aligned} c &= 0 \\ a + b + c &= 3 \\ 4a + 2b + c &= 2. \end{aligned}$$

Multiply second one by -2 and add to third equation.

$$\begin{aligned} -2a - 2b - 2c &= -6 \\ 4a + 2b + c &= 2 \\ \hline 2a - c &= -4 \end{aligned}$$

Substituting $c = 0$ into $2a - c = -4$, we get $2a = -4$ and $a = -2$. From $a + b + c = 3$, $-2 + b = 3$ and $b = 5$. Since $(a,b,c) = (-2,5,0)$, the parabola is $y = -2x^2 + 5x$.

52. Substitute $(0,-6)$, $(1,-3)$, $(2,6)$ into $y = ax^2 + bx + c$.

$$\begin{aligned} c &= -6 \\ a + b + c &= -3 \\ 4a + 2b + c &= 6 \end{aligned}$$

Multiply second one by -2 and add to third equation.

$$\begin{aligned} -2a - 2b - 2c &= 6 \\ 4a + 2b + c &= 6 \\ \hline 2a - c &= 12 \end{aligned}$$

Substituting $c = -6$ into $2a - c = 12$, $2a + 6 = 12$ and $a = 3$. From $a + b + c = -3$, $3 + b - 6 = -3$ and $b = 0$. Since solution is $(a,b,c) = (3,0,-6)$, the parabola is $y = 3x^2 - 6$.

53. Substitute $(0,4)$, $(-2,0)$, $(-3,1)$ into $y = ax^2 + bx + c$. Then

$$\begin{aligned} c &= 4 \\ 4a - 2b + c &= 0 \\ 9a - 3b + c &= 1. \end{aligned}$$

Multiply second and third equations by 3 and -2, respectively, then add the equations.

$$\begin{array}{rcr} 12a - 6b + 3c & = & 0 \\ -18a + 6b - 2c & = & -2 \\ \hline -6a + c & = & -2 \end{array}$$

Substituting $c = 4$ into $-6a + c = -2$, $-6a+4 = -2$ and $a = 1$. From $4a-2b+c = 0$, $4 - 2b + 4 = 0$ and $b = 4$. Since $(a, b, c) = (1, 4, 4)$, the parabola is $y = x^2 + 4x + 4$.

54. Substitute $(0, 6)$, $(3, 0)$, $(-1, 12)$ into $y = ax^2 + bx + c$. Thus,

$$\begin{array}{rcl} c & = & 6 \\ 9a + 3b + c & = & 0 \\ a - b + c & = & 12. \end{array}$$

Multiply third equation by 3 and add to the second.

$$\begin{array}{rcr} 9a + 3b + c & = & 0 \\ 3a - 3b + 3c & = & 36 \\ \hline 12a + 4c & = & 36 \end{array}$$

Substituting $c = 6$ into $12a + 4c = 36$, we get $12a + 24 = 36$ and $a = 1$. From $a - b + c = 12$, $1 - b + 6 = 12$ and $b = -5$. Since $(a, b, c) = (1, -5, 6)$, the parabola is $y = x^2 - 5x + 6$.

55. By substituting the coordinates of the points $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$ into $ax + by + cz = 1$, we get $a = 1$, $b = 1$, and $c = 1$. A linear equation satisfied by the ordered triples is $x + y + z = 1$.

56. By substituting the coordinates of the points $(0, 0, 2)$, $(0, 1, 0)$, and $(1, 0, 0)$ into $ax + by + cz = 1$, we get $2c = 1$, $b = 1$, and $a = 1$. A linear equation satisfied by the ordered triples is $x + y + \frac{1}{2}z = 1$

57. By substituting the coordinates of the points $(1, 1, 1)$, $(0, 2, 0)$, and $(1, 0, 0)$ into $ax + by + cz = 1$, we get $a+b+c = 1$, $2b = 1$, and $a = 1$. Then $b- = \frac{1}{2}$ and $c = -\frac{1}{2}$. A linear equation satisfied by the ordered triples is

$x + \frac{1}{2}y - \frac{1}{2}z = 1$ or $2x + y - z = 2$

58. If $ax + by + cz = 1$ is satisfied by $(1, 0, 1)$, $(2, 1, 0)$, and $(0, 2, 1)$, then

$$\begin{array}{rcl} a + c & = & 1 \\ 2a + b & = & 1 \\ 2b + c & = & 1. \end{array}$$

The solution of this system is $a = \frac{2}{5}$, $b = \frac{1}{5}$, $c = \frac{3}{5}$. A linear equation satisfied by the ordered triples is $\frac{2}{5}x + \frac{1}{5}y + \frac{3}{5}z = 1$ or $2x + y + 3z = 5$.

59. Let x, y, z be the three numbers listed in increasing order.

$$\begin{array}{rcl} x + y + z & = & 40 \\ -x \quad\; + z & = & 12 \\ x + y - z & = & 0. \end{array}$$

If we add the first two equations and add the last two equations, we obtain (by subtracting the 2nd sum from the 1st sum)

$$\begin{array}{rcr} y + 2z & = & 52 \\ y \quad\;\; & = & 12 \\ \hline 2z & = & 40 \end{array}$$

Then $z = 20$. Since $-x + z = 12$, we find $-x+20 = 12$ or $x = 8$. Thus, the numbers are $8, 12$, and 20.

60. Let x, y, z be the lengths of the sides of a triangle listed from smallest to largest.

$$\begin{array}{rcl} x + y + z & = & 40 \\ x + y - z & = & 2 \\ -x \quad\; + z & = & 11. \end{array}$$

If we add the first two equations and add the last two equations, we obtain

$$\begin{array}{rcl} 2x + 2y & = & 42 \\ y & = & 13 \end{array}$$

Then $2x + 2(13) = 42$ or $2x = 16$ or $x = 8$. Since $-x + z = 11$, we find $-8 + z = 11$ or $z = 19$. Thus, the three sides are 8 m, 13 m, and 19 m.

61. Let x, y, z be the scores in the 1st, 2nd, and 3rd quizzes, respectively.

$$\begin{aligned} x+y+z &= 21 \\ x-y &= -1 \\ y-z &= -4. \end{aligned}$$

If we add the first two equations and add the last two equations, we obtain

$$\begin{aligned} 2x+z &= 20 \\ x-z &= -5 \\ \hline 3x &= 15 \end{aligned}$$

Since $x = 5$, the scores on the quizzes are 5, 6, and 10.

62. Let x, y, z be the scores in the 1st, 2nd, and 3rd quizzes, respectively.

$$\begin{aligned} x+y &= 16 \\ x+y+z &= 36 \\ x \quad -z &= -13. \end{aligned}$$

Multiply the first equation by -1, and add the last two equations. Then we obtain

$$\begin{aligned} -x-y &= -16 \\ 2x+y &= 23 \\ \hline x &= 7 \end{aligned}$$

Since $x = 7$, the scores on the quizzes are 7, 9, and 20.

63. Let x, y, and z be the amounts invested in stocks, bonds, and a mutual fund. Then

$$\begin{aligned} x+y+z &= 25,000 \\ 0.08x+0.10y+0.06z &= 1,860 \\ 2y &= z. \end{aligned}$$

Multiply the first equation by -8 and add to 100 times the second.

$$\begin{aligned} -8x-8y-8z &= -200,000 \\ 8x+10y+6z &= 186,000 \\ \hline 2y-2z &= -14,000 \end{aligned}$$

Substitute $z = 2y$ into $2y - 2z = -14,000$.

$$\begin{aligned} 2y-4y &= -14,000 \\ -2y &= -14,000 \\ y &= 7,000 \end{aligned}$$

Since $z = 2y$, we obtain $z = 14,000$.
Since $x + y + z = 25,000$, $x = 4000$.
Marita invested \$4,000 in stocks,
\$7000 in bonds, and
\$14,000 in a mutual fund.

64. Let x, y, and z be the number of people in the year 1980 under 20 years, the number of people in the 20-60 category, and the number of people over 60 years, respectively. Then

$$\begin{aligned} x+y+z &= 1911 \\ 1.1x+0.92y+\frac{4}{3}z &= 2136 \\ \frac{4}{3}z &= 1.1x+0.92y. \end{aligned}$$

Substitute the third equation into the second.

$$\begin{aligned} \frac{4}{3}z+\frac{4}{3}z &= 2136 \\ \frac{8}{3}z &= 2136 \\ z &= 801 \end{aligned}$$

The first equation simplifies to

$$\begin{aligned} x+y+801 &= 1911 \\ x &= 1110-y. \end{aligned}$$

Substitute $x = 1110 - y$ into
$1.1x + 0.92y + \frac{4}{3}z = 2136.$

$$\begin{aligned} 1.1(1110-y)+0.92y+\frac{4}{3}801 &= 2136 \\ -0.18y+1221+1068 &= 2136 \\ -0.18y &= -153 \\ y &= 850 \end{aligned}$$

Since $x = 1110 - y$, we find $x = 1110 - 850 = 260$. The number of people in each age group in 1980 are $x = 260$, $y = 850$, and $z = 801$.

65. Let x, y, and z be the prices last year of a hamburger, fries, and a Coke, respectively. So

$$\begin{aligned} x+y+z &= 3.80 \\ 1.1x+1.2y+1.25z &= 4.49 \\ 1.25z &= 1.1x-0.07. \end{aligned}$$

Multiply first equation by -12 and add to 10 times the second equation.

$$\begin{array}{rcr} -12x - 12y - 12z & = & -45.6 \\ 11x + 12y + 12.5z & = & 44.9 \\ \hline -x + 0.5z & = & -0.7 \end{array}$$

Substitute $x = 0.5z + 0.7$ into $1.25z = 1.1x - 0.07$.

$$\begin{aligned} 1.25z &= 1.1(0.5z + 0.7) - 0.07 \\ 0.7z &= 0.7 \\ z &= 1 \end{aligned}$$

Since $x = 0.5z + 0.7$, we find $x = 0.5(1) + 0.7 = 1.20$. From $x + y + z = 3.80$, we have $y = 1.60$. The prices last year of a hamburger, fries and Coke are \$1.20, \$1.60, and \$1, respectively.

66. Let x, y, and z be the hundred's digit, ten's digit, and unit's digit of Angelo's correct house number, respectively. Then

$$\begin{aligned} x + y + z &= 9 \\ 3x &= z \\ (100x + 10y + z) + 396 &= 100z + 10y + x. \end{aligned}$$

Simplify the third equation and substitute $z = 3x$.

$$\begin{aligned} 99x - 99z &= -396 \\ 99x - 99(3x) &= -396 \\ -198x &= -396 \\ x &= 2 \end{aligned}$$

Since $z = 3x$, we get $z = 3(2) = 6$. From $x + y + z = 9$, we obtain $y = 1$. The correct address is 216 Elm Street.

67. Let L_f and L_r be the weights on the left front tire and left rear tire, respectively. Let R_f and R_r be the weights on the right front tire and right rear tire, respectively. Since $1200(0.51) = 612$ and $1200(0.48) = 576$, we obtain

$$\begin{aligned} L_f + L_r &= 612 \\ R_r + L_r &= 576 \\ L_f, L_r, R_f, R_r &\geq 280. \end{aligned}$$

Three possible weight distributions are $(L_r, L_f, R_r, R_f) = (280, 332, 296, 292)$, $(L_r, L_f, R_r, R_f) = (285, 327, 291, 297)$, and $(L_r, L_f, R_r, R_f) = (290, 322, 286, 302)$.

68. Let b, f, and c be the prices of a burger, french fries, and a Coke, respectively. Then

$$\begin{aligned} 5b + 7f + 6c &= 11.25 \\ 6b + 8f + 7c &= 13.20. \end{aligned}$$

After rewriting the equations and adding them together, we obtain

$$\begin{array}{rcr} b + \frac{7}{5}f + \frac{6}{5}c & = & 2.25 \\ -b - \frac{4}{3}f - \frac{7}{6}c & = & -2.20 \\ \hline \frac{1}{15}f + \frac{1}{30}c & = & 0.05 \\ c & = & 1.5 - 2f. \end{array}$$

Since $c \geq 1.5 - 2f \geq 0$, then $f \leq 0.75$ and the cost of french fries is at most 75 cents. Marilyn did not pay more than 80 cents for the french fries.

Three possible prices are

- burger \$0.95, fries \$0.50, coke \$0.50
- burger \$0.80, fries \$0.35, coke \$0.80, and
- burger \$0.75, fries \$0.30, coke \$0.90.

69. Let x, y, and z be the number of pennies, nickels, and dimes, respectively. Then

$$\begin{aligned} x + y + z &= 232 \\ y + z &= x \\ 0.01x + 0.05y + 0.10z &= 10.36. \end{aligned}$$

Multiply first equation by -1 and add to 10 times the third equation. Also combine first two equations.

$$\begin{array}{rcr} -x - y - z & = & -232 \\ 0.1x + 0.5y + z & = & 103.6 \\ \hline -0.9x - 0.5y & = & -128.4 \end{array}$$

$$\begin{array}{rcr} -x - y - z & = & -232 \\ -x + y + z & = & 0 \\ \hline -2x & = & -232 \end{array}$$

Then $x = 116$. Substituting into $-0.9x - 0.5y = -128.4$, we obtain

$$\begin{aligned} -0.9(116) - 0.5y &= -128.4 \\ -104.4 - 0.5y &= -128.4 \\ 24 &= 0.5y \\ 48 &= y. \end{aligned}$$

Since $x + y + z = 232$, $116 + 48 + z = 232$ and $z = 68$. Emma used 116 pennies, 48 nickels, and 68 dimes.

70. Let x, y, and z be the number of male students, female students, and teachers, respectively. Then

$$\begin{aligned} x + y + z &= 564 \\ \frac{1}{4}x + \frac{1}{6}y + \frac{3}{4}z &= 128 \\ \frac{1}{8}x + \frac{1}{60}y + \frac{1}{4}z &= 41. \end{aligned}$$

Multiply first equation by -3 and add to 12 times the second equation. Also, multiply first equation by -15 and add to 120 times the third.

$$\begin{aligned} -3x - 3y - 3z &= -1692 \\ 3x + 2y + 9z &= 1536 \\ \hline -y + 6z &= -156 \end{aligned}$$

$$\begin{aligned} -15x - 15y - 15z &= -8460 \\ 15x + 2y + 30z &= 4920 \\ \hline -13y + 15z &= -3540 \end{aligned}$$

Multiply $-y + 6z = -156$ by -13 and add to $-13y + 15z = -3540$.

$$\begin{aligned} 13y - 78z &= 2028 \\ -13y + 15z &= -3540 \\ \hline -63z &= -1512 \end{aligned}$$

So $z = 24$. Substituting into $-y + 6z = -156$, we get $-y + 144 = -156$ and $y = 300$. Since $x+y+z = 564$, we find $x+300+24 = 564$ and $x = 240$. There are 240 male students, 300 female students, and 24 teachers.

71. Let x, y, and z be the prices of a carton of milk, a cup of coffee, and a doughnut, respectively. So

$$\begin{aligned} 3x + 4y + 7z &= 5.45 \\ 4x + 2y + 8z &= 5.30 \\ 2x + 5y + 6z &= 5.15. \end{aligned}$$

Multiply third equation by -2 and add to second equation. Also, multiply first equation by -4 and add to 3 times the second equation.

$$\begin{aligned} -4x - 10y - 12z &= -10.30 \\ 4x + 2y + 8z &= 5.30 \\ \hline -8y - 4z &= -5 \end{aligned}$$

$$\begin{aligned} -12x - 16y - 28z &= -21.80 \\ 12x + 6y + 24z &= 15.90 \\ \hline -10y - 4z &= -5.90 \end{aligned}$$

Multiply $-8y - 4z = -5$ by -1 and add to $-10y - 4z = -5.90$.

$$\begin{aligned} 8y + 4z &= 5 \\ -10y - 4z &= -5.90 \\ \hline -2y &= -0.90 \end{aligned}$$

Then $y = 0.45$. Substitute into $-8y - 4z = -5$ to get $-3.60 - 4z = -5$ and $z = 0.35$. Since $3x + 4y + 7z = 5.45$, we have $3x + 1.80 + 2.45 = 5.45$ and $x = 0.40$.

Alphonse's bill was $5(0.40) + 2(0.45) + 9(0.35) = \$6.05$. His change is \$3.95.

72. Let x, y, and z be the present ages of the Toyota, Ford, and Buick. So

$$\begin{aligned} x + y + z &= 24 \\ x - 3 &= 2(y - 3) \\ (y - 2) + (z - 2) &= x - 2. \end{aligned}$$

Rewrite third equation as $-x + y + z = 2$. Multiply by -1 and add to the first equation.

$$\begin{aligned} x - y - z &= -2 \\ x + y + z &= 24 \\ \hline 2x &= 22 \end{aligned}$$

So $x = 11$. Substituting into $x - 3 = 2(y - 3)$, $8 = 2(y - 3)$ and $y = 7$. Since $x+y+z = 24$, $11+7+z = 24$ and $z = 6$.

The Buick is 6 years old, the Ford 7 years old, and the Toyota 11 years old.

73. A system of equations is

$$\begin{aligned} 4x &= 6y \\ 2(x+y) &= 15(8) \\ 6(x+y+15+10) &= 10z. \end{aligned}$$

Rewriting the first two equations, one obtains

$$\begin{aligned} 2x-3y &= 0 \\ x+y &= 60. \end{aligned}$$

Solving this smaller system, one finds $x = 36$ lb, $y = 24$ lb. Substituting into the third equation, one finds

$$z = \frac{6(x+y+25)}{10} = \frac{6(60+25)}{10} = 51 \text{ lb.}$$

74. Substitute $(12, 0.18)$, $(22, 0.23)$, $(30, 0.14)$ into $E = as^2 + bs + c$. This gives the following system:

$$\begin{aligned} 144a+12b+c &= 0.18 \\ 484a+22b+c &= 0.23 \\ 900a+30b+c &= 0.14. \end{aligned}$$

Subtract the first equation from the second equation. Also subtract the second from the third. Then

$$\begin{aligned} 340a+10b &= 0.05 \\ 416a+8b &= -0.09. \end{aligned}$$

Multiply $416a + 8b = -0.09$ by 5 and add to -4 times $340a + 10b = 0.05$.

$$\begin{array}{rcr} -1360a-40b &=& -0.20 \\ 2080a+40b &=& -0.45 \\ \hline 720a &=& -0.65 \\ a &=& -0.65/720 \\ a &\approx& -0.0009 \end{array}$$

Since $416a + 8b = -0.09$, we find

$$\begin{aligned} 8b &= -0.09-416a \\ b &= \frac{-0.09-416(-0.65/720)}{8} \\ b &\approx 0.035694 \\ b &\approx 0.0357. \end{aligned}$$

Since $144a + 12b + c = 0.18$, we obtain

$$\begin{aligned} c &\approx 0.18-144(-0.65/720)-12(0.035694) \\ c &\approx -0.1183. \end{aligned}$$

So $E = -0.0009s^2 + 0.0357s - 0.1183$.
The speed that maximizes efficiency is
$-\dfrac{b}{2a} \approx -\dfrac{0.0357}{2(-0.0009)} \approx 19.8$ mph.

75.

a) Substitute $(0, 0)$, $(10, 40)$, $(20, 70)$ into $y = ax^2+bx+c$. Consequently, we obtain the
following system of equations:

$$\begin{aligned} c &= 0 \\ 100a+10b+c &= 40 \\ 400a+20b+c &= 70. \end{aligned}$$

Multiply the second equation by -2 and add to third equation.

$$\begin{array}{rcr} -200a-20b-2c &=& -80 \\ 400a+20b+c &=& 70 \\ \hline 200a-c &=& -10 \end{array}$$

Substitute $c = 0$ into $200a - c = -10$. Then $a = -\dfrac{1}{20}$. From $100a+10b+c = 40$, we obtain $-5 + 10b = 40$ and $b = \dfrac{9}{2}$.
The parabola is $y = -\dfrac{1}{20}x^2 + \dfrac{9}{2}x$.

b) Since $-b/(2a) = 45$, the maximum height is

$$-\frac{1}{20}(45)^2 + \frac{9}{2}(45) = 101.25 \text{ m.}$$

c) Since the zeros of $y = -\dfrac{x}{20}(x-90)$ are $x = 0, 90$, the missile will strike 90 m from the origin.

76. Applying a quadratic regression on the data below

x	12	22	30
y	0.18	0.23	0.14

we find

$$y = -0.0009x^2 + 0.0357x - 0.1183.$$

Using the maximimum feature of a graphing calculator, we obtain that the x-coordinate that gives the maximum y-coordinate is

$$x \approx 19.8.$$

79. Multiply the second equation by -2 and add the result to the first equation.

$$\begin{array}{rcr} 2x - 3y & = & 20 \\ -2x - 8y & = & 2 \\ \hline -11y & = & 22 \end{array}$$

Then $y = -2$ and $x = -4y - 1 = -4(-2) - 1 = 7$. The solution set is $\{(7, -2)\}$.

80. Since the equations represent parallel lines with slopes $m = 3$ and with different y-intercepts, the line do not have a point of intersection. The system is inconsistent.

81. $10,000\left(1 + \dfrac{0.035}{4}\right)^{24} = \$12,325.52$

82. **a)** $\log_2\left(\dfrac{1}{8}\right) = -3$ since $2^{-3} = \dfrac{1}{8}$

b) Since $\log_2 x = 4$, we find $x = 2^4 = 16$

83. No, the Vertical Line Test fails for a circle.

84. $\dfrac{f(x+h) - f(x)}{h} =$

$\dfrac{(x+h)^2 + (x+h) - (x^2 + x)}{h} =$

$\dfrac{2xh + h^2 + h}{h} = 2x + h + 1$

Thinking Outside the Box LXXI

Let h, f, and c be the costs of a hamburger, an order of french fries, and a coke, respectively.

$$\begin{array}{rcr} 8h + 5f + 2c & = & 14.25 \\ 5h + 3f + c & = & 8.51 \end{array}$$

If you subtract the 2nd equation from the 1st equation, we obtain

$$3h + 2f + c = 5.74.$$

If you multiply the 2nd equation by -2 and add the result to the first equation, we find

$$-2h - f = -2.77.$$

When we add the last two equations, we obtain

$$h + f + c = \$2.97$$

which is the cost of 1 hamburger, 1 order of french fries, and a coke.

8.2 Pop Quiz

1. Add the first two equations, and add the last two equations. Then we obtain

$$\begin{array}{rcl} 2x + 3y & = & 33 \\ 2x + y & = & 19. \end{array}$$

If we subtract the last equation from the previous equation, we obtain

$$2y = 14.$$

Then $y = 7$. Since $2x + y = 19$, we find $2x + 7 = 19$ or $x = 6$. Since $x + y + z = 22$, we find $6 + 7 + z = 22$ or $z = 9$. The solution set is $\{(6, 7, 9)\}$.

2. Adding both equations, we obtain

$$\begin{array}{rcl} 2x - y + 2z & = & 4 \\ x + y - z & = & 2 \\ \hline 3x \qquad + z & = & 6 \end{array}$$

We have a dependent system of equations.

Solving for z in the last equation, we find

$$z = 6 - 3x.$$

Since $x + y - z = 2$, we obtain

$$y = 2 - x + z = 2 - x + (6 - 3x) = 8 - 4x.$$

The solution set is

$$\{(x, 8 - 4x, 6 - 3x) : x \text{ is a real number}\}.$$

8.2 Linking Concepts

a) Let s and f be the state and federal taxes. Then

$$s = 0.05(200,000 - f).$$

b) Using the notation in part a),

$$f = 0.3(200,000 - s).$$

c) Rewriting the equations in parts a) and b), one obtains

$$\begin{aligned} 0.05f + s &= 10,000 \\ f + 0.30s &= 60,000. \end{aligned}$$

Solve the second equation for f and substitute in the first equation.

$$\begin{aligned} 0.05(60,000 - 0.30s) + s &= 10,000 \\ 0.985s &= 7,000 \\ s &= \$7,106.60 \end{aligned}$$

The state tax is \$7,106.60 and the federal tax is $f = 60,000 - 0.30(7106.60) = \$57,868.02$.

d) If b is the bonus, then

$$\begin{aligned} b &= 0.2(200,000 - s - f - b) \\ f &= 0.3(200,000 - s - b) \\ s &= 0.05(200,000 - f - b). \end{aligned}$$

Solving, one finds the bonus is $b = \$23,792.49$, the federal tax is $f = \$50,983.90$, and the state tax is $s = \$6261.18$.

For Thought

1. True, since the line $y = x$ passes through the center of the circle.

2. False, when a line is tangent to a circle it intersects the circle at only one point.

3. True, the parabola $y = 2x^2 - 4$ intersects the circle $x^2 + y^2 = 16$ at three points.

4. False, they intersect at $(\pm 1, 0)$.

5. False, they intersect at $(1, 1)$ and $(-1, -1)$.

6. False, since three noncollinear points determine a unique circle through the points.

7. True, since either leg can serve as base and the other leg as altitude.

8. True **9.** True

10. False, two such numbers are $\frac{1}{2}\left(7 \pm \sqrt{45}\right)$.

8.3 Exercises

1. Since $x = -1$ and $y = 4$ satisfy both equations in the system , $(-1, 4)$ is a solution.

2. Since $x = 2$ and $y = -3$ satisfy both equations in the system, $(2, -3)$ is a solution.

3. Since $x = 4$ and $y = -5$ does not satisfy $x - y = 1$, $(4, -5)$ is not a solution.

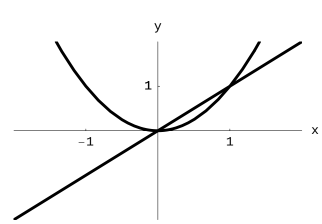
4. Since $x = -3$ and $y = -4$ does not satisfy $2x - 3y = 18$, $(-3, -4)$ is not a solution.

5. Since $y = x$ and $y = x^2$, we obtain

$$\begin{aligned} x &= x^2 \\ x - x^2 &= 0 \\ x(x - 1) &= 0. \end{aligned}$$

Then $x = 0, 1$ and the solution set is

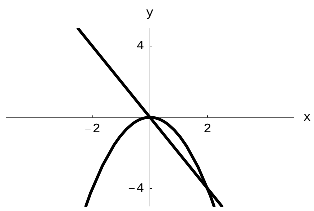
$$\{(0, 0),\ (1, 1)\}.$$



6. Substitute $y = -2x$ into $y = -x^2$.

$$\begin{aligned} -2x &= -x^2 \\ x^2 - 2x &= 0 \\ x(x-2) &= 0 \end{aligned}$$

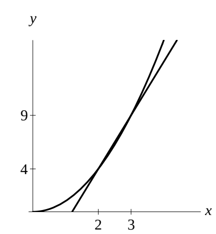
Since $x = 0, 2$, we get $y = -2(0) = 0$ and $y = -2(2) = -4$. The solution set is $\{(0,0), (2,-4)\}$.



7. Substitute $y = x^2$ into $5x - y = 6$.

$$\begin{aligned} 5x - x^2 &= 6 \\ x^2 - 5x &= -6 \\ x^2 - 5x + 6 &= 0 \\ (x-3)(x-2) &= 0 \end{aligned}$$

If $x = 3, 2$ in $y = x^2$, then $y = 9, 4$. The solution set is $\{(2,4),\ (3,9)\}$.



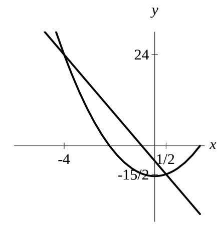
8. Substitute $y = -7x - 4$ into $2x^2 - y = 8$.

$$\begin{aligned} 2x^2 + 7x + 4 &= 8 \\ 2x^2 + 7x - 4 &= 0 \\ (2x-1)(x+4) &= 0 \end{aligned}$$

If $x = \dfrac{1}{2}, -4$ in $y = -7x - 4$, then

$y = -\dfrac{15}{2}, 24$, respectively.

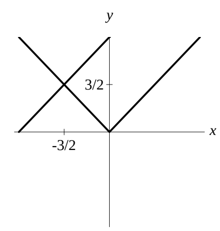
The solution set is $\left\{\left(\dfrac{1}{2}, -\dfrac{15}{2}\right), (-4, 24)\right\}$.



9. Substitute $y = |x|$ into $y = x + 3$.

$$\begin{aligned} |x| &= x + 3 \\ x = x + 3 \quad &\text{or} \quad -x = x + 3 \\ 0 = 3 \quad &\text{or} \quad -2x = 3 \end{aligned}$$

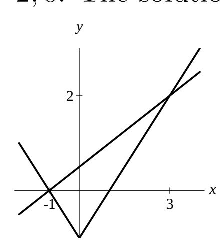
Then $x = -3/2$. Since $y = |x|$, $y = 3/2$. The solution set is $\{(-3/2, 3/2)\}$.



10. Substitute $y = \dfrac{x+1}{2}$ into $y = |x| - 1$.

$$\begin{aligned} \frac{x+1}{2} &= |x| - 1 \\ \frac{x}{2} + \frac{3}{2} &= |x| \\ \frac{x}{2} + \frac{3}{2} = x \quad &\text{or} \quad \frac{x}{2} + \frac{3}{2} = -x \\ \frac{3}{2} = \frac{x}{2} \quad &\text{or} \quad \frac{3x}{2} = -\frac{3}{2} \\ x = 3 \quad &\text{or} \quad x = -1 \end{aligned}$$

Using $x = 3, -1$ in $y = \dfrac{x+1}{2}$, one finds $y = 2, 0$. The solution set is $\{(3,2),\ (-1,0)\}$.

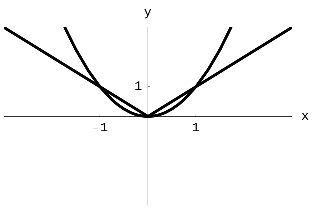


11. Substitute $y = x^2$ into $y = |x|$.

$$\begin{aligned} x^2 &= |x| \\ x^2 = x \quad &\text{or} \quad x^2 = -x \end{aligned}$$

$$\begin{array}{rcl} x^2 - x = 0 & \text{or} & x^2 + x = 0 \\ x(x-1) = 0 & \text{or} & x(x+1) = 0 \\ x = 0, -1 & \text{or} & x = 0, -1 \end{array}$$

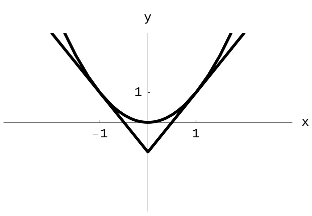
Using $x = 0, \pm 1$ in $y = x^2$, one finds $y = 0, 1$. The solution set is $\{(-1,1), (0,0), (1,1)\}$.



12. Substitute $y = x^2$ into $y = 2|x| - 1$. Then

$$\begin{array}{rcl} x^2 + 1 & = & 2|x| \\ x^2 + 1 = 2x & \text{or} & x^2 + 1 = -2x \\ x^2 - 2x + 1 = 0 & \text{or} & x^2 + 2x + 1 = 0 \\ (x-1)^2 = 0 & \text{or} & (x+1)^2 = 0. \end{array}$$

Using $x = \pm 1$ in $y = x^2$, one finds $y = 1$. The solution set is $\{(1,1), (-1,1)\}$.



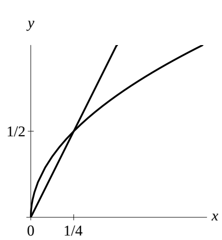
13. Substitute $y = \sqrt{x}$ into $y = 2x$ to obtain

$$\begin{aligned} \sqrt{x} &= 2x \\ x &= 4x^2 \\ x - 4x^2 &= 0 \\ x(1-4x) &= 0. \end{aligned}$$

Using $x = 0, 1/4$ in $y = 2x$, $y = 0, 1/2$.

The solution set is

$$\{(0,0), (1/4, 1/2)\}.$$

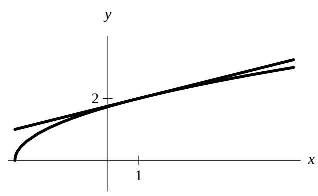


14. Substitute $y = \dfrac{x+7}{4}$ into $y = \sqrt{x+3}$.

$$\begin{aligned} \frac{x+7}{4} &= \sqrt{x+3} \\ \frac{x^2+14x+49}{16} &= x+3 \\ x^2+14x+49 &= 16x+48 \\ x^2-2x+1 &= 0 \\ (x-1)^2 &= 0 \end{aligned}$$

Using $x = 1$ in $y = \dfrac{x+7}{4}$, one finds $y = 2$.

The solution set is $\{(1,2)\}$.

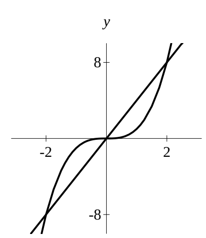


15. Substitute $y = x^3$ into $y = 4x$ to get

$$\begin{aligned} x^3 &= 4x \\ x^3 - 4x &= 0 \\ x(x^2-4) &= 0 \\ x &= 0, \pm 2. \end{aligned}$$

Substituting $x = 0, 2, -2$ into $y = 4x$, we get $y = 0, 8, -8$.
The solution set is

$$\{(0,0), (2,8), (-2,-8).$$

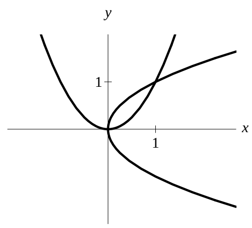


16. Substitute $y = x^2$ into $x = y^2$.

$$\begin{aligned} x &= x^4 \\ x - x^4 &= 0 \\ x(1-x^3) &= 0 \\ x &= 0, 1 \end{aligned}$$

Using $x = 0, 1$ in $y = x^2$, we get $y = 0, 1$. The solution set is

$$\{(0,0), (1,1)\}.$$

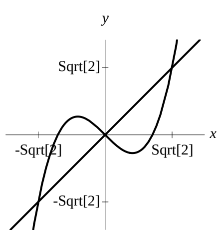


17. Substitute $y = x$ into $y = x^3 - x$.

$$\begin{aligned} x &= x^3 - x \\ 2x - x^3 &= 0 \\ x(2 - x^2) &= 0 \\ x &= 0, \sqrt{2}, -\sqrt{2} \end{aligned}$$

Since $y = x$, the solution set is

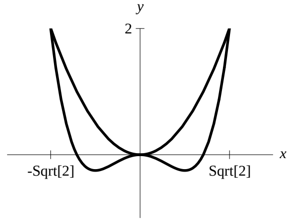
$$\{(0,0), (\sqrt{2}, \sqrt{2}), (-\sqrt{2}, -\sqrt{2})\}.$$



18. Substitute $y = x^2$ into $y = x^4 - x^2$.

$$\begin{aligned} x^2 &= x^4 - x^2 \\ 2x^2 - x^4 &= 0 \\ x^2(2 - x^2) &= 0 \\ x &= 0, \sqrt{2}, -\sqrt{2}. \end{aligned}$$

Using $x = 0, \sqrt{2}, -\sqrt{2}$ in $y = x^2$, we obtain $y = 0, 2, 2$. The solution set is $\{(0,0), (\sqrt{2}, 2), (-\sqrt{2}, 2)\}$.

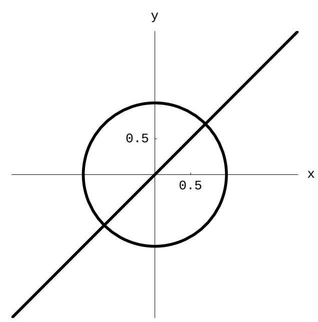


19. Substitute $y = x$ into $x^2 + y^2 = 1$.

$$\begin{aligned} x^2 + x^2 &= 1 \\ 2x^2 &= 1 \\ x^2 &= \frac{1}{2} \\ x &= \pm\frac{\sqrt{2}}{2} \end{aligned}$$

Since $y = x$, the solution set is

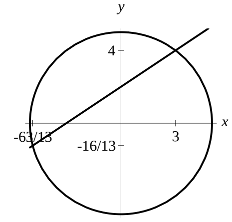
$$\left\{\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right), \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)\right\}.$$



20. Substitute $x = \dfrac{3y-6}{2}$ into $x^2 + y^2 = 25$.

$$\begin{aligned} \left(\frac{3y-6}{2}\right)^2 + y^2 &= 25 \\ \frac{9y^2 - 36y + 36}{4} + y^2 &= 25 \\ 9y^2 - 36y + 36 + 4y^2 &= 100 \\ 13y^2 - 36y - 64 &= 0 \\ (y-4)(13y+16) &= 0 \\ y &= 4, -\frac{16}{13} \end{aligned}$$

Substitute $y = 4$ and $y = -\dfrac{16}{13}$ into $x = \dfrac{3y-6}{2}$. Then we get $x = 3, -\dfrac{63}{13}$.
The solution set is $\left\{(3,4), \left(-\dfrac{63}{13}, -\dfrac{16}{13}\right)\right\}$.



21. Substitute $y = -x - 4$ into $xy = 1$.

$$\begin{aligned} x(-x-4) &= 1 \\ -x^2 - 4x &= 1 \\ x^2 + 4x &= -1 \\ x^2 + 4x + 4 &= 3 \\ (x+2)^2 &= 3 \\ x &= -2 \pm \sqrt{3} \end{aligned}$$

Using $x = -2 + \sqrt{3}, -2 - \sqrt{3}$ in $y = -x - 4$, we find $y = -2 - \sqrt{3}, -2 + \sqrt{3}$.
The solution set is
$\left\{(-2+\sqrt{3}, -2-\sqrt{3}), (-2-\sqrt{3}, -2+\sqrt{3})\right\}$.

22. Substituting $y = 10 - x$ into $xy = 21$, we get

$$\begin{aligned} x(10-x) &= 21 \\ 10x - x^2 &= 21 \\ x^2 - 10x &= -21 \\ x^2 - 10x + 25 &= 4 \\ (x-5)^2 &= 4 \\ x &= 5 \pm 2 \\ x &= 7, 3. \end{aligned}$$

Using $x = 7, 3$ in $y = 10 - x$, we get $y = 3, 7$.
The solution set is $\{(7,3), (3,7)\}$.

23. Substitute $x^2 = 2y^2 - 1$ into $2x^2 - y^2 = 1$.

$$\begin{aligned} 2(2y^2 - 1) - y^2 &= 1 \\ 3y^2 - 2 &= 1 \\ 3y^2 &= 3 \\ y^2 &= 1 \\ y &= \pm 1 \end{aligned}$$

Using $y = 1$ in $x^2 = 2y^2 - 1$, we get $x^2 = 1$ or $x = \pm 1$. Also, if $y = -1$ then $x = \pm 1$.
The solution set is $\{(1, \pm 1), (-1, \pm 1)\}$.

24. Multiply $x^2 + y^2 = 5$ by -1 and add to the second equation.

$$\begin{array}{rcr} -x^2 - y^2 &=& -5 \\ x^2 + 4y^2 &=& 14 \\ \hline 3y^2 &=& 9 \\ y^2 &=& 3 \\ y &=& \pm\sqrt{3} \end{array}$$

Since $x^2 + y^2 = 5$, we get $x^2 + 3 = 5$ or $x = \pm\sqrt{2}$. The solution set is
$\{(\sqrt{2}, \pm\sqrt{3}), (-\sqrt{2}, \pm\sqrt{3})\}$.

25. Since $2x - y = 1$, we obtain $y = 2x - 1$.
Then substitute into $xy - 2x = 2$.

$$\begin{aligned} x(2x-1) - 2x &= 2 \\ 2x^2 - 3x - 2 &= 0 \\ (2x+1)(x-2) &= 0 \\ x &= -\frac{1}{2}, 2 \end{aligned}$$

Since $y = 2x - 1$, we get $y = 2 \cdot \left(-\frac{1}{2}\right) - 1 = -2$ and $x = 2(2) - 1 = 3$. The solution set is $\left\{\left(-\frac{1}{2}, -2\right), (2,3)\right\}$.

26. Since $x - 2y = -7$, we obtain $x = 2y - 7$.
Then substitute into $y - xy = -10$.

$$\begin{aligned} y - (2y-7)y &= -10 \\ 2y^2 - 8y - 10 &= 0 \\ y^2 - 4y - 5 &= 0 \\ (y-5)(y+1) &= 0 \\ y &= -1, 5 \end{aligned}$$

Since $x = 2y - 7$, we get $x = -9, 3$.
The solution set is $\{(-9,-1), (3,5)\}$.

27. Multiply the first equation by 2 and add to the second.

$$\begin{array}{rcl} \frac{6}{x} - \frac{2}{y} &=& \frac{26}{10} \\ \frac{1}{x} + \frac{2}{y} &=& \frac{9}{10} \\ \hline \frac{7}{x} &=& \frac{35}{10} \end{array}$$

So $70 = 35x$ and $x = 2$.
Substituting into $\frac{3}{x} - \frac{1}{y} = \frac{13}{10}$, we obtain

$$\begin{aligned} \frac{3}{2} - \frac{1}{y} &= \frac{13}{10} \\ -\frac{1}{y} &= \frac{13}{10} - \frac{15}{10} \\ -\frac{1}{y} &= -\frac{2}{10} \\ y &= 5. \end{aligned}$$

The solution set is $\{(2,5)\}$.

28. Multiply first equation by 4 and the second equation by 2. Then

$$\begin{aligned} \frac{8}{x}+\frac{6}{y} &= 11 \\ \frac{5}{x}-\frac{4}{y} &= 3. \end{aligned}$$

Multiply $\frac{8}{x}+\frac{6}{y}=11$ by 2 and multiply $\frac{5}{x}-\frac{4}{y}=3$ by 3. Then

$$\begin{aligned} \frac{16}{x}+\frac{12}{y} &= 22 \\ \frac{15}{x}-\frac{12}{y} &= 9 \\ \hline \frac{31}{x} &= 31 \\ x &= 1. \end{aligned}$$

Using $x=1$ in $\frac{8}{x}+\frac{6}{y}=11$, we get

$$\begin{aligned} \frac{8}{x}+\frac{6}{y} &= 11 \\ 8+\frac{6}{y} &= 11 \\ \frac{6}{y} &= 3 \\ y &= 2. \end{aligned}$$

The solution set is $\{(1,2)\}$.

29. Substitute $y=1-x$ into $x^2+xy-y^2=-5$.

$$\begin{aligned} x^2+x(1-x)-(1-x)^2 &= -5 \\ x^2+x-x^2-(1-2x+x^2) &= -5 \\ -x^2+3x-1 &= -5 \\ x^2-3x+1 &= 5 \\ x^2-3x-4 &= 0 \\ (x-4)(x+1) &= 0 \\ x &= 4,-1 \end{aligned}$$

Using $x=4,-1$ in $y=1-x$, we get $y=-3,2$. The solution set is $\{(4,-3),(-1,2)\}$.

30. Substitute $y=2-x$ into $x^2+xy+y^2=12$.

$$\begin{aligned} x^2+x(2-x)+(2-x)^2 &= 12 \\ x^2+2x-x^2+(4-4x+x^2) &= 12 \\ x^2-2x-8 &= 0 \\ (x-4)(x+2) &= 0 \\ x &= 4,-2 \end{aligned}$$

Using $x=4,-2$ in $y=2-x$, we have $y=-2,4$. The solution set is $\{(4,-2),(-2,4)\}$.

31. Add the two given equations to obtain $xy=-2$. Substitute $y=-2/x$ into $x^2+2xy-2y^2=-11$.

$$\begin{aligned} x^2+2x\left(-\frac{2}{x}\right)-2\cdot\frac{4}{x^2} &= -11 \\ x^2-4-\frac{8}{x^2} &= -11 \\ x^4+7x^2-8 &= 0 \\ (x^2+8)(x^2-1) &= 0 \\ x &= \pm 1 \end{aligned}$$

Using $x=1,-1$ in $y=-2/x$, $y=-2,2$. The solution set is $\{(1,-2),(-1,2)\}$.

32. Add the two given equations to obtain $xy=6$. Substitute $y=6/x$ into $3x^2-xy+y^2=15$.

$$\begin{aligned} 3x^2-x\left(\frac{6}{x}\right)+\frac{36}{x^2} &= 15 \\ 3x^2-6+\frac{36}{x^2} &= 15 \\ 3x^2-21+\frac{36}{x^2} &= 0 \\ 3x^4-21x^2+36 &= 0 \\ x^4-7x^2+12 &= 0 \\ (x^2-4)(x^2-3) &= 0 \\ x &= \pm 2, \pm\sqrt{3} \end{aligned}$$

Since $y=6/x$ and $\frac{6}{\sqrt{3}}=2\sqrt{3}$, the solution set is

$\left\{(2,3),(-2,-3),(\sqrt{3},2\sqrt{3}),(-\sqrt{3},-2\sqrt{3})\right\}$.

33. Multiply the first equation by 7 and multiply the second equation by -5.

$$\begin{aligned} \frac{28}{x}+\frac{35}{y^2} &= 84 \\ -\frac{15}{x}-\frac{35}{y^2} &= -110 \\ \hline \frac{13}{x} &= -26 \\ x &= -\frac{1}{2} \end{aligned}$$

Then substitute into $\frac{4}{x}+\frac{5}{y^2}=12$.

$$\begin{aligned} -8+\frac{5}{y^2} &= 12 \\ \frac{5}{y^2} &= 20 \\ y^2 &= \frac{1}{4} \\ y &= \pm\frac{1}{2}. \end{aligned}$$

The solution set is $\left\{\left(-\frac{1}{2},\frac{1}{2}\right),\left(-\frac{1}{2},-\frac{1}{2}\right)\right\}$.

34. Multiply the second equation by 5.

$$\begin{aligned} \frac{3}{x^2}-\frac{5}{y} &= 33 \\ \frac{25}{x^2}+\frac{5}{y} &= 415 \\ \hline \frac{28}{x^2} &= 448 \\ x &= \pm\frac{1}{4} \end{aligned}$$

Then substitute into $\frac{3}{x^2}-\frac{5}{y}=33$. If $x=\pm 1/4$, then

$$\begin{aligned} 48-\frac{5}{y} &= 33 \\ -\frac{5}{y} &= -15 \\ y &= \frac{1}{3}. \end{aligned}$$

The solution set is $\left\{\left(\frac{1}{4},\frac{1}{3}\right),\left(-\frac{1}{4},\frac{1}{3}\right)\right\}$.

35. Since $x=\frac{10^{11}}{y^2}$ and $\frac{x^3}{y}=10^{12}$, we find

$$\begin{aligned} \frac{10^{33}}{y^7} &= 10^{12} \\ 10^{21} &= y^7 \\ y &= 10^3. \end{aligned}$$

Substitute into $x=\frac{10^{11}}{y^2}$. Then

$$x=\frac{10^{11}}{10^6}=10^5.$$

The solution set is $\{(10^5,10^3)\}$.

36. Since $x^2=\frac{10^{23}}{y^3}$ and $\frac{x^4}{y^2}=10^6$, we obtain

$$\begin{aligned} \frac{10^{46}}{y^8} &= 10^6 \\ 10^{40} &= y^8 \\ y &= \pm 10^5. \end{aligned}$$

But $y=-10^5$ is an extraneous root.

Substitute $y=10^5$ into $x^2=\frac{10^{23}}{y^3}$.

$$\begin{aligned} x^2 &= \frac{10^{23}}{10^{15}} = 10^8 \\ x &= \pm 10^4. \end{aligned}$$

The solution set is $\{(10^4,10^5),(-10^4,10^5)\}$.

37. Substitute $y=2^{x+1}$ into $y=4^{-x}$.

$$\begin{aligned} 2^{x+1} &= (2^2)^{-x} \\ 2^{x+1} &= 2^{-2x} \\ x+1 &= -2x \\ 3x &= -1 \end{aligned}$$

Using $x = -1/3$ in $y = 2^{x+1}$, we get $y = 2^{2/3}$. The solution set is $\left\{\left(-\frac{1}{3}, 2^{2/3}\right)\right\}$.

38. Substitute $y = 3^{2x+1}$ into $y = 9^{-x}$

$$\begin{aligned} 3^{2x+1} &= (3^2)^{-x} \\ 3^{2x+1} &= 3^{-2x} \\ 2x+1 &= -2x \\ 4x &= -1 \end{aligned}$$

Using $x = -1/4$ in $y = 9^{-x}$, $y = 9^{1/4} = \sqrt{3}$. The solution set is $\left\{\left(-\frac{1}{4}, \sqrt{3}\right)\right\}$.

39. Substitute $y = \log_2(x)$ into $y = \log_4(x+2)$ and use the base-changing formula.

$$\begin{aligned} \log_2(x) &= \log_4(x+2) \\ \log_2(x) &= \frac{\log_2(x+2)}{\log_2(4)} \\ \log_2(x) &= \frac{\log_2(x+2)}{2} \\ 2\cdot\log_2(x) &= \log_2(x+2) \\ 2\cdot\log_2(x) - \log_2(x+2) &= 0 \\ \log_2\left(\frac{x^2}{x+2}\right) &= 0 \\ \frac{x^2}{x+2} &= 1 \\ x^2 &= x+2 \\ x^2 - x - 2 &= 0 \\ (x-2)(x+1) &= 0 \\ x &= 2, -1 \end{aligned}$$

But $x = -1$ is an extraneous root since $\log_2(-1)$ is undefined. Using $x = 2$ in $y = \log_2(x)$, we have $y = 1$. The solution set is $\{(2,1)\}$.

40. Substitute $y = \log_2(-x)$ into $y = \log_2(x+4)$ and use the fact that $y = \log_2(x)$ is a one-to-one function.

$$\begin{aligned} \log_2(-x) &= \log_2(x+4) \\ -x &= x+4 \\ -2x &= 4 \\ x &= -2 \end{aligned}$$

Use $x = -2$ in $y = \log_2(-x)$ to obtain $y = \log_2(2) = 1$. The solution set is $\{(-2,1)\}$.

41. Substitute $y = \log_2(x+2)$ into $y = 3 - \log_2(x)$.

$$\begin{aligned} \log_2(x+2) &= 3 - \log_2(x) \\ \log_2(x+2) + \log_2(x) &= 3 \\ \log_2(x^2+2x) &= 3 \\ x^2+2x &= 2^3 \\ x^2+2x-8 &= 0 \\ (x+4)(x-2) &= 0 \\ x &= -4, 2 \end{aligned}$$

But $x = -4$ is an extraneous root since $\log_2(-4)$ is undefined. Using $x = 2$ in $y = \log_2(x+2)$, we get $y = \log_2(4) = 2$. The solution set is $\{(2,2)\}$.

42. Substitute $y = \log(2x+4)$ into $y = 1 + \log(x-2)$.

$$\begin{aligned} \log(2x+4) &= 1 + \log(x-2) \\ \log(2x+4) - \log(x-2) &= 1 \\ \log\left(\frac{2x+4}{x-2}\right) &= 1 \\ \frac{2x+4}{x-2} &= 10 \\ 2x+4 &= 10x - 20 \\ 24 &= 8x \\ x &= 3 \end{aligned}$$

Using $x = 3$ in $y = \log(2x+4)$, we have $y = \log(10) = 1$. The solution set is $\{(3,1)\}$.

43. Substitute $y = 3^x$ into $y = 2^x$.

$$\begin{aligned} 3^x &= 2^x \\ \log(3^x) &= \log(2^x) \\ x\cdot\log(3) &= x\cdot\log(2) \\ x\left[\log(3) - \log(2)\right] &= 0 \\ x &= 0 \end{aligned}$$

Using $x = 0$ in $y = 3^x$, we obtain $y = 3^0 = 1$. The solution set is $\{(0,1)\}$.

44. Equate $y = 6^{x-1}$ to $y = 2^{x+1}$ and take the natural logarithm of both sides.

$$\ln(6^{x-1}) = \ln(2^{x+1})$$

$$\begin{aligned}(x-1)\cdot\ln(6) &= (x+1)\cdot\ln(2)\\ x\cdot\ln(6)-\ln(6) &= x\cdot\ln(2)+\ln(2)\\ x\,[\ln(6)-\ln(2)] &= \ln(6)+\ln(2)\\ x\cdot\ln(3) &= \ln(12)\\ x &= \frac{\ln(12)}{\ln(3)}\end{aligned}$$

Using $x=\dfrac{\ln(12)}{\ln(3)}$ in $y=2^{x+1}$, the solution set is $\left\{\left(\dfrac{\ln(12)}{\ln(3)}, 2^{\ln(12)/\ln(3)+1}\right)\right\}$.

45. Substitute $y=2^x$ into $x=\log_4(y)$.

$$x=\log_4\left(2^x\right)=x\log_4(2)=x\cdot\frac{1}{2}$$

SInce $x=\frac{x}{2}$, we find $x=0$. Then

$$y=2^x=2^0=1.$$

The solution set is $\{(0,1)\}$.

46. Substitute $y=8^x$ into $x=\log_2(2y)$.

$$\begin{aligned}x &= \log_2(2)+\log_2(y)\\ x &= 1+\log_2(8^x)\\ x &= 1+x\log_2 8\\ x &= 1+3x\\ x &= -\frac{1}{2}\end{aligned}$$

Then

$$y=8^x=8^{-1/2}=\frac{1}{\sqrt{8}}=\frac{\sqrt{2}}{4}.$$

The solution set is $\left\{\left(-\dfrac{1}{2},\dfrac{\sqrt{2}}{4}\right)\right\}$.

47. If we subtract the second equation from the first, we find

$$\begin{aligned}x+\log_{16}(y+1) &= \frac{1}{2}\\ x+\log_{16}(y) &= \frac{1}{4}\\ \hline \log_{16}\left(\frac{y+1}{y}\right) &= \frac{1}{4}\\ \frac{y+1}{y} &= 2\\ y &= 1\end{aligned}$$

Substitute into $x+\log_{16}y=\dfrac{1}{4}$. Then

$$x+0=\frac{1}{4}.$$

The solution set is $\left\{\left(\dfrac{1}{4},1\right)\right\}$.

48. If we subtract the second equation from the first, we obtain

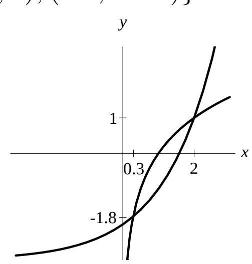
$$\begin{aligned}\log_2 x-\log_8 x &= 2\\ \log_2 x-\frac{\log_2 x}{\log_2 8} &= 2\\ \log_2 x-\frac{\log_2 x}{3} &= 2\\ \frac{2}{3}\log_2 x &= 2\\ \log_2 x &= 3\\ x &= 8\end{aligned}$$

Substitute into $y+\log_8 x=2$. Then

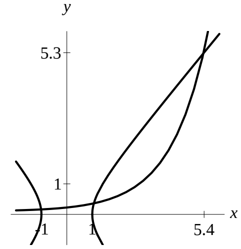
$$y+1=2$$

Since $y=1$, the solution set is $\{(8,1)\}$.

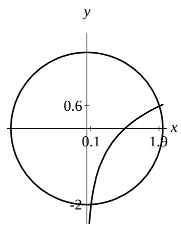
49. From the graphs, the solution set is $\{(2,1),(0.3,-1.8)\}$.



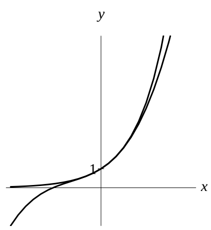
50. From the graphs, the solution set is $\{(5.4,5.3),(1.0,0.3),(-1.0,0.1\}$.



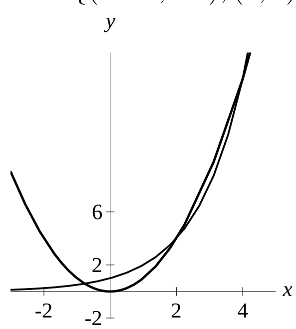
51. From the graphs, the solution set is $\{(1.9, 0.6), (0.1, -2.0)\}$.



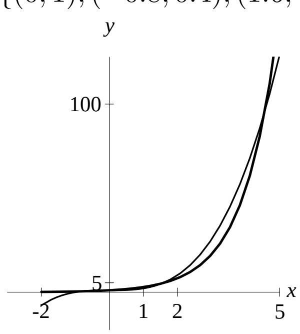
52. From the graphs, the solution set is $\{(0, 1)\}$.



53. From the graphs, the solution set is $\{(-0.8, 0.6), (2, 4), (4, 16)\}$.



54. From the graphs, the solution set is $\{(0, 1), (-0.8, 0.4), (1.6, 4.7), (4.6, 96.3)\}$.



55. From the first equation, we find

$$x + 2y = z + 5.$$

Substitute into the third equation:

$$\begin{aligned} (x + 2y)^2 - z^2 &= 15 \\ (z + 5)^2 - z^2 &= 15 \end{aligned}$$

Solving for z, we find $z = -1$. Then substitute $z = -1$ into the first two equations, and subtract as follows:

$$\begin{aligned} x + 2y &= 4 \\ 3x + 2y &= 12 \\ \hline -2x &= -8 \\ x &= 4 \end{aligned}$$

Since $x + 2y = 4$ and $x = 4$, we find $y = 0$. The solution set is $\{(4, 0)\}$.

56. From the first equation, we find

$$x + y = z + 10.$$

Substitute into the third equation:

$$\begin{aligned} (x + y)^2 + z^2 &= 122 \\ (z + 10)^2 + z^2 &= 122 \end{aligned}$$

Solving for z, we find $z = -11$ or $z = 1$. Rewrite the first two equation as follows:

$$\begin{aligned} x + y &= z + 10 \\ 2x - y &= 13 - 3z \end{aligned}$$

If we substitute $z = -11$ into the above system of two equations, the corresponding solutions are $x = 15$ and $y = -16$.

If we substitute $z = 1$ into the above system of two equations, the corresponding solutions are $x = 7$ and $y = 4$,

The solution set is $\{(7, 4, 1), (15, -16, -11)\}$.

57. If we square the second equation and use the third equation

$$x^2 + y^2 + z^2 = 133$$

we obtain

$$\begin{aligned} (x + y + z)^2 &= 49 \\ 133 + 2(xy + yz + xz) &= 49 \\ 2(xy + yz + xz) &= -84 \end{aligned}$$

Using the first equation $xy = z^2$, we find

$$\begin{aligned} 2(z^2 + yz + xz) &= -84 \\ (z + y + x)z &= -42 \\ 7z &= -42 \\ z &= -6 \end{aligned}$$

for $x + y + z = 7$.

Since $z = -6$, we may rewrite the first two given equations as follows:

$$\begin{aligned} xy &= 36 \\ x + y &= 13 \end{aligned}$$

Since $y = \dfrac{36}{x}$, we substitute and find:

$$\begin{aligned} x + \frac{36}{x} &= 13 \\ x^2 - 13x + 36z &= 0 \\ (x-4)(x-9) &= 0 \\ x &= 4, 9 \end{aligned}$$

If $x = 4$, then $y = \dfrac{36}{x} = \dfrac{36}{4} = 9$.

If $x = 9$, then $y = \dfrac{36}{x} = \dfrac{36}{9} = 4$.

The solution set is $\{(4, 9, -6), (9, 4, -6)\}$.

58. If we square the second equation and use the third equation

$$2xy = z^2$$

we obtain

$$\begin{aligned} (x - y + z)^2 &= 1 \\ x^2 + y^2 + z^2 - 2xy - 2yz + 2xz &= 1 \\ x^2 + y^2 - 2yz + 2xz &= 1 \\ x^2 + y^2 + 2z(x - y) &= 1. \end{aligned}$$

From the second given equation, we obtain

$$x - y = -1 - z.$$

From the third given equation, we obtain

$$x^2 + y^2 = 2z^2 + 13.$$

Thus, we obtain

$$2z^2 + 13 + 2z(-1 - z) = 1.$$

Solving for z, we find $z = 6$.

Since $z = -6$, we may rewrite the first two given equations as follows:

$$\begin{aligned} 2xy &= 36 \\ x - y &= -7 \end{aligned}$$

The solutions are $x = -9$ and $y = -2$, and $x = 2$ and $y = 9$.

Thus, the solution set is

$$\{(2, 9, 6), (-9, -2, 6)\}.$$

59. Let x and y be the base and height of a 42-inch LCD TV, respectively.

$$\begin{aligned} \frac{y}{x} &= \frac{9}{15} \\ x^2 + y^2 &= 42^2 \end{aligned}$$

Then we find $x \approx 36.0$ in. and $y \approx 21.6$ in.

60. Let x and y be the base and height of a 4G iPod, respectively.

$$\begin{aligned} \frac{y}{x} &= \frac{5}{4} \\ x^2 + y^2 &= 51.83^2 \end{aligned}$$

Then we find $x \approx 32.4$ mm and $y \approx 40.5$ mm.

61. Let x and $6 - x$ be the two numbers.

$$\begin{aligned} x(6 - x) &= -16 \\ x^2 - 6x &= 16 \\ x^2 - 6x + 9 &= 25 \\ (x - 3)^2 &= 25 \\ x - 3 &= \pm 5 \\ x &= 3 \pm 5 \\ x &= -2, 8 \end{aligned}$$

The numbers are -2 and 8.

62. Let x and $-8 - x$ be the two numbers.

$$\begin{aligned} x(-8 - x) &= -20 \\ x^2 + 8x &= 20 \\ x^2 + 8x + 16 &= 36 \\ (x + 4)^2 &= 36 \\ x + 4 &= \pm 6 \\ x &= -4 \pm 6 \\ x &= -10, 2 \end{aligned}$$

The numbers are -10 and 2.

63. Let x and y be the lengths of the legs of the triangle.

$$\begin{aligned} x^2 + y^2 &= 15^2 \\ \frac{1}{2}xy &= 54 \end{aligned}$$

Substitute $y = \dfrac{108}{x}$ into $x^2 + y^2 = 225$ and use the quadratic formula.

$$\begin{aligned} x^2 + \frac{11664}{x^2} &= 225 \\ x^4 - 225x^2 + 11,664 &= 0 \end{aligned}$$

$$x^2 = \frac{225 \pm \sqrt{225^2 - 4(11,664)}}{2}$$

$$\begin{aligned} x^2 &= \frac{225 \pm 63}{2} \\ x^2 &= 144, 81 \\ x &= 12, 9 \end{aligned}$$

Using $x = 12, 9$ in $y = \dfrac{108}{x}$, we get $y = 9, 12$. The sides are 9 m and 12 m.

64. Let w and l be the width and length of the rectangle.

$$\begin{aligned} w^2 + l^2 &= 35^2 \\ 2w + 2l &= 98 \end{aligned}$$

Substitute $w = 49 - l$ into $w^2 + l^2 = 1225$ and use the quadratic formula.

$$\begin{aligned} (49 - l)^2 + l^2 &= 1225 \\ (2401 - 98l + l^2) + l^2 &= 1225 \\ 2l^2 - 98l + 1176 &= 0 \\ l^2 - 49l + 588 &= 0 \end{aligned}$$

$$l = \frac{49 \pm \sqrt{49^2 - 4(588)}}{2}$$

$$\begin{aligned} l &= \frac{49 \pm 7}{2} \\ l &= 28, 21 \end{aligned}$$

Using $l = 28, 21$ in $w = 49 - l$, we get $y = 21, 28$. Dimensions are 21 cm and 28 cm.

65. If x is the length of the hypotenuse, then $\dfrac{x}{2}$ and $\dfrac{x\sqrt{3}}{2}$ are the lengths of the sides opposite the 30^o and 60^o angles.

$$\begin{aligned} x + \frac{x}{2} + \frac{x\sqrt{3}}{2} &= 12 \\ 2x + x + \sqrt{3}x &= 24 \\ (3 + \sqrt{3})x &= 24 \\ x &= \frac{24}{3 + \sqrt{3}} \cdot \frac{3 - \sqrt{3}}{3 - \sqrt{3}} \\ x &= 12 - 4\sqrt{3} \end{aligned}$$

Substituting $x = 12 - 4\sqrt{3}$ in $\dfrac{x}{2}$ and $\dfrac{x\sqrt{3}}{2}$, we find $6 - 2\sqrt{3}$ ft and $6\sqrt{3} - 6$ ft are the lengths of the two sides and the hypotenuse is $12 - 4\sqrt{3}$ ft.

66. Let x and y be the base and height of the triangular vent. By similar triangles, $\dfrac{y}{x/2} = \dfrac{1}{2}$. So $y = \dfrac{x}{4}$. From the area, we have $4.5 = \dfrac{1}{2} \cdot x \cdot \dfrac{x}{4}$ and $x^2 = 36$. The base is $x = 6$ ft and the height is $y = \dfrac{6}{4} = 1.5$ ft.

67. The values of x and y must satisfy

$$\begin{aligned} 6y &= 6x \\ x(6 + y) &= 7(4 + 12). \end{aligned}$$

Since $x = y$ as seen from the first equation, upon substitution into the second equation one obtains

$$\begin{aligned} 6x + x^2 &= 112 \\ x^2 + 6x - 112 &= 0 \\ (x + 14)(x - 8) &= 0. \end{aligned}$$

Then $x = 8$ in. and $y = 8$ oz. One must exclude the negative value $x = -14$.

68. Let w and $\dfrac{3}{2}w$ be the width and height of the window, respectively. By the Pythagorean

Theorem, one finds a relationship between the 9-ft radius, the width, and the height. Namely,

$$\begin{aligned} \left(\frac{w}{2}\right)^2 + \left(\frac{3w}{2}\right)^2 &= 9^2 \\ \frac{w^2}{4} + \frac{9w^2}{4} &= 81 \\ 10w^2 &= 4(81) \\ w &= \frac{2(9)}{\sqrt{10}} \\ w &= \frac{9\sqrt{10}}{5}. \end{aligned}$$

The width is $w = \dfrac{9\sqrt{10}}{5} \approx 5.7$ ft and the height is $\dfrac{3}{2} \cdot \dfrac{9\sqrt{10}}{5} = \dfrac{27\sqrt{10}}{10} \approx 8.5$ ft.

69. Let x and y be the number of minutes it takes for pump A and pump B, respectively, to fill the vat.

$$\begin{aligned} \frac{1}{x} + \frac{1}{y} &= \frac{1}{8} \\ \frac{1}{x} - \frac{1}{y} &= \frac{1}{12} \end{aligned}$$

Adding the two equations, we get

$$\begin{aligned} \frac{2}{x} &= \frac{3}{24} + \frac{2}{24} \\ \frac{2}{x} &= \frac{5}{24} \\ 5x &= 48 \\ x &= 9.6. \end{aligned}$$

Substituting $x = \dfrac{48}{5}$ into $\dfrac{1}{x} + \dfrac{1}{y} = \dfrac{1}{8}$, we find

$$\begin{aligned} \frac{5}{48} + \frac{1}{y} &= \frac{1}{8} \\ \frac{1}{y} &= \frac{6}{48} - \frac{5}{48} \\ \frac{1}{y} &= \frac{1}{48} \\ y &= 48. \end{aligned}$$

Pump A can fill the vat by itself in $x = 9.6$ min while Pump B will take $y = 48$ min.

70. Let x be the number of hours it takes Morris to plant an acre by himself. So $x - 2$ is Blanche's time.

$$\begin{aligned} \frac{1}{x} + \frac{1}{x-2} &= \frac{1}{8} \\ (8x - 16) + 8x &= x^2 - 2x \\ -16 &= x^2 - 18x \\ 65 &= x^2 - 18x + 81 \\ 65 &= (x-9)^2 \\ 9 \pm \sqrt{65} &= x \end{aligned}$$

But $x = 9 - \sqrt{65} \approx 0.9$ is not possible because $x - 2$ would be negative, so $x = 9 + \sqrt{65}$ hr.

71. Let x and y be two numbers satisfying

$$\begin{aligned} x + y &= 6 \\ xy &= 10. \end{aligned}$$

Substituting $y = 6 - x$ into $xy = 10$,

$$\begin{aligned} x(6 - x) &= 10 \\ 6x - x^2 &= 10 \\ x^2 - 6x &= -10 \\ x^2 - 6x + 9 &= -1 \\ (x-3)^2 &= -1 \\ x &= 3 \pm i. \end{aligned}$$

If $x = 3 + i$, then $y = 6 - (3 + i) = 3 - i$.
If $x = 3 - i$, then $y = 6 - (3 - i) = 3 + i$.
The two numbers are $3 + i$ and $3 - i$.

72. Let x and y be the two numbers.

$$\begin{aligned} x + y &= 1 \\ xy &= 5 \end{aligned}$$

Substitute $y = 1 - x$ into $xy = 5$ and use the quadratic formula.

$$\begin{aligned} x(1 - x) &= 5 \\ x - x^2 &= 5 \\ x^2 - x &= -5 \\ x^2 - x + 5 &= 0 \\ x &= \frac{1 \pm \sqrt{1 - 4(5)}}{2} \\ x &= \frac{1 \pm i\sqrt{19}}{2}. \end{aligned}$$

Using $x = \frac{1+i\sqrt{19}}{2}, \frac{1-i\sqrt{19}}{2}$ in $y = 1 - x$, we get $y = \frac{1-i\sqrt{19}}{2}, \frac{1+i\sqrt{19}}{2}$.

The two numbers are $\frac{1+i\sqrt{19}}{2}$ and $\frac{1-i\sqrt{19}}{2}$.

73. Let x and y be the length and width.

$$\begin{aligned} 20xy &= 36,000 \\ 40x + 40y + 2xy &= 7200 \end{aligned}$$

Substitute $x = \frac{1800}{y}$ into $40x + 40y + 2xy = 7200$.

$$\begin{aligned} 40 \cdot \frac{1800}{y} + 40y + 2 \cdot \frac{1800}{y} \cdot y &= 7200 \\ \frac{72,000}{y} + 40y + 3600 &= 7200 \\ \frac{72,000}{y} + 40y &= 3600 \\ 40y^2 - 3600y + 72,000 &= 0 \\ y^2 - 90y + 1800 &= 0 \\ (y-60)(y-30) &= 0 \end{aligned}$$

Using $y = 30$ and $y = 60$ in $x = \frac{1800}{w}$, we get $x = 60$ and $x = 30$. Thus, the length is 60 ft and the width is 30 ft.

74. Let x and y be the original length and width, respectively. Then we obtain

$$\begin{aligned} (y+1)(x+2) &= xy + 30 \\ (y+2)(x-3) &= xy - 6. \end{aligned}$$

Simplify both equations.

$$\begin{aligned} xy + 2y + x + 2 &= xy + 30 \\ 2y + x &= 28 \end{aligned}$$

$$\begin{aligned} xy - 3y + 2x - 6 &= xy - 6 \\ -3y + 2x &= 0 \end{aligned}$$

Substitute $x = 28 - 2y$ into $-3y + 2x = 0$.

$$\begin{aligned} -3y + 2(28 - 2y) &= 0 \\ -7y + 56 &= 0 \\ y &= 8 \end{aligned}$$

Using $y = 8$ in $x = 28 - 2y$, $x = 12$. Thus, the original dimensions are $y = 8$ ft and $x = 12$ ft.

75. From the graphs, the two models give the same population for $t = 0$ and $t = 9.65$ years. The exponential population model is twice the linear model when $t \approx 29.5$ years.

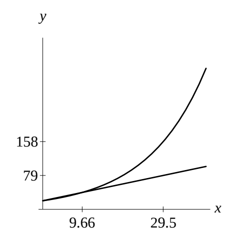
76. Let t be the number of years before Harvey makes a payment.

$$e^{0.05t} = 2(1 + 0.05t)$$

Using a calculator, we find

$$t \approx 33.5669398$$

or equivalently $t \approx 33$ years, 207 days.



77. Let x be the time of the sunrise or the number of hours since midnight. Let S and B be Sally's and Bob's speeds. The number of miles Sally and Bob drove are $(16 - x)S$ and $(21 - x)B$, respectively. Since they met at noon, the distance between Sally's house and Bob's house is $(12 - x)S + (12 - x)B$; or equivalently $(12 - x)(S + B)$. Then

$$\begin{aligned} (16-x)S &= (12-x)(S+B) \\ (16-x)S &= (16-x)(S+B) - 4(S+B) \\ 0 &= (16-x)B - 4(S+B) \\ (16-x)B &= 4(S+B). \end{aligned}$$

Likewise,

$$\begin{aligned} (21-x)B &= (21-x)(S+B) - 9(S+B) \\ (21-x)S &= 9(S+B). \end{aligned}$$

Combining, one obtains $\frac{(16-x)B}{(21-x)S} = \frac{4(S+B)}{9(S+B)}$. So, $\frac{(16-x)9B}{(21-x)4S} = 1$.

Furthermore, since $(16-x)S = (21-x)B$ one finds $\dfrac{B}{S} = \dfrac{16-x}{21-x}$. Then

$$\begin{aligned} \frac{9(16-x)}{4(21-x)} \cdot \frac{B}{S} &= 1 \\ \frac{9}{4}\left(\frac{16-x}{21-x}\right)^2 &= 1 \\ \frac{16-x}{21-x} &= \frac{2}{3} \text{ since } 16-x>0 \\ 48-3x &= 42-2x \\ x &= 6. \end{aligned}$$

The sunrise was at 6:00 A.M.

78. Let r_M and r_T be the wagon master's speed and the wagon trains' speed, respectively. Since the wagon master rides back x miles (or fraction thereof) in the same time the wagon train goes $1-x$ miles forward, then

$$\frac{x}{r_M} = \frac{1-x}{r_T}.$$

Rewriting, one has $\dfrac{x}{1-x} = \dfrac{r_M}{r_T}$. Going forward, the wagon master rides $1+x$ miles in the same time the wagon train advances x miles. Thus,

$$\frac{1+x}{r_M} = \frac{x}{r_T}.$$

Equivalently, $\dfrac{1+x}{x} = \dfrac{r_M}{r_T}$. Combining the equations, one obtains

$$\begin{aligned} \frac{x}{1-x} &= \frac{1+x}{x} \\ x^2 &= 1-x^2 \\ 2x^2 &= 1 \\ x &= \frac{\sqrt{2}}{2}. \end{aligned}$$

So the wagon master rides $1+\sqrt{2}$ $(= 2x+1)$ miles.

79. By the time the mouse reaches the northeast corner (50 feet from the southwest corner) of the train, the train would have traveled 20 feet which is one-half the distance the train travels before the mouse returns to the southeast corner. So, when the mouse reaches the northwest corner (40 feet from the southwest corner) the train (in proportion to 20 feet) would have traveled 16 feet.

As the distance traveled by the train ranges from 16 feet to 20 feet, the path the mouse takes will be the hypotenuse of a right triangle whose sides are 10 feet and 4 feet. So, in this range, the mouse travels a distance of $\sqrt{10^2+4^2}$ feet on the ground, or equivalently $2\sqrt{29}$ feet on the ground.

Thus, the total diagonal ground distance (including the diagonal ground distance as the mouse moves from the southeast corner to the southwest corner) traveled by the mouse is twice of $2\sqrt{29}$ feet, or $4\sqrt{29}$ feet. Since the north-south distance traveled by the mouse is 80 feet on the ground, the total distance on the ground traveled by the mouse is $80+4\sqrt{29}$ feet, or about 101.54 ft.

80. Let m and w be the ages of the man and woman, respectively.

$$\begin{aligned} m-w &= 20 \\ m^2-w^2 &= 1160 \end{aligned}$$

If you divide the second equation by the first equation, we obtain

$$m+w = 58.$$

Solving the above equation with $m-w=20$, we obtain $m=39$ and $w=19$. The man is 39 years old and the woman is 19 years old.

83. If we add the first and third equations, we obtain

$$2x+3y = 51$$

If we add the second equation to three times the third equation, we find

$$5x+5y = 100.$$

If we multiply the above equation by $-2/5$, we obtain

$$-2x-2y = -40.$$

If we add the last equation to $2x + 3y = 51$, we obtain $y = 11$. Working backwards, we find $x = 9$ and $z = 13$. The solution set is $\{(9, 11, 13)\}$.

84. Note, $6x-2 = 10y$ is equivalent to $6x-10y = 2$ or $3x-5y = 1$ which is the first equation. The system is dependent. The solution set is

$$\{(x, y)|3x - 5y = 1\}.$$

85. Combine the logarithms as follows:

$$\begin{aligned} \log_3((x+1)(x-5)) &= 3 \\ \log_3(x^2-4x-5) &= 3 \\ x^2-4x-5 &= 27 \\ x^2-4x-32 &= \\ (x-8)(x+4) &= 0 \\ x &= 8, -4 \end{aligned}$$

Note, -4 is an extraneous root. The solution set is $\{8\}$.

86. Since $e^{x^2-x} = 1$, we have $x^2-x = 0$. However, $x^2 - x = x(x-1)$. Then the solution set is $\{0, 1\}$.

87. Let $p(x) = 2x^3 + x^2 - 41x + 20$.

4	2	1	-41	20
		8	36	-20
	2	9	-5	0

Since the quotient factors as

$$2x^2 + 9x - 5 = (2x-1)(x+5)$$

the solution set is $\left\{\left(4, \frac{1}{2}, -5\right)\right\}$

88. To find the domain, we require $3x - 1 \geq 0$. Then the domain is $[\frac{1}{3}, \infty)$.

The range is $[4, \infty)$.

Thinking Outside the Box

LXXII Since $x + \sqrt{y} = 32$ and $y + \sqrt{x} = 54$, we obtain $\sqrt{y} = 32 - x$ and $x^2 = (54-y)^2$. Then

$$y = (32 - (54 - y))^2.$$

Using a graphing calculator, we find $y = 49$, $y \approx 47.8$, $y \approx 58.9$, or $y \approx 60.3$

Note, $y \approx 58.9$ and $y \approx 60.3$ cannot satisfy $y + \sqrt{x} = 54$ for any real number x.

If $y \approx 47.8$, then $\sqrt{x} \approx 54 - 47.8 = 6.2$ or $x \approx 38.44$. Note, $x \approx 38.44$ does not satisfy $x + \sqrt{y} = 32$ for any real number y.

However, if $y = 49$ then $\sqrt{x} = 54 - 49 = 5$ or $x = 25$. Thus, the solution is $(x, y) = (25, 49)$.

LXXIII Since $(142, 857)(3) = 428, 571$ and no number smaller than 142, 857 satisfies the condition described in the problem, we find that the smallest such number is 142, 857.

8.3 Pop Quiz

1. Since $y = x^2$ and $y = x$, we find

$$\begin{aligned} x^2 &= x \\ x^2 - x &= 0 \\ x(x-1) &= 0 \\ x &= 0, 1. \end{aligned}$$

Since $y = x$, the solution set is $\{(0, 0), (1, 1)\}$.

2. Since $y - 1 = |x|$, we have

$$x = y - 1 \text{ or } x = -y + 1.$$

Since $y = \frac{1}{2}x + 4$, we obtain

$$\begin{aligned} y = \frac{1}{2}(y-1) + 4 \quad &\text{or} \quad y = \frac{1}{2}(-y+1) + 4 \\ 2y = (y-1) + 8 \quad &\text{or} \quad 2y = (-y+1) + 8 \\ y = 7 \quad &\text{or} \quad 3y = 9 \\ y = 7 \quad &\text{or} \quad y = 3 \end{aligned}$$

Solving for x in $y = \frac{1}{2}x + 4$, we find

$$x = 2(y - 4).$$

If $y = 3$, then $x = 2(3-4) = -2$.
If $y = 7$, then $x = 2(7-4) = 6$.
The solution set is $\{(-2,3),(6,7)\}$.

3. Adding both equations, we find

$$\begin{array}{rcr} x^2 - y^2 &=& 5 \\ x^2 + y^2 &=& 13 \\ \hline 2x^2 &=& 18 \\ x^2 &=& 9 \\ x &=& \pm 3. \end{array}$$

Since $y = \pm\sqrt{13 - x^2}$ and $x = \pm 3$, we find $y = \pm\sqrt{13-9} = \pm 2$. The solution set is $\{(3,\pm 2),(-3,\pm 2)\}$.

8.3 Linking Concepts

a) Since $2d_1 = 0.280v$, $d_1 = 0.140v$.

b) Since $\frac{d_2}{v} + \frac{d_3}{v} = 0.446$, we find $2 \cdot \frac{d_2}{v} = 0.446$ or $d_2 = 0.223v$.

c) By using the Pythagorean Theorem, we find

$$d_1 = \sqrt{d_2^2 - 250^2}.$$

d) Note, $d_1 = 0.140v = 0.140\frac{d_2}{0.223}$ and $d_1 = \frac{140d_2}{223}$. By the Pythagorean Theorem, we obtain

$$\begin{array}{rcl} d_2^2 &=& d_1^2 + 250^2 \\ d_2^2 &=& \left(\frac{140d_2}{223}\right)^2 + 250^2 \\ d_2^2\left(1 - \left(\frac{140}{223}\right)^2\right) &=& 250^2 \\ d_2 &=& \sqrt{\frac{250^2}{\left(1-\left(\frac{140}{223}\right)^2\right)}}. \end{array}$$

Substituting into $d_1 = \frac{140d_2}{223}$, we get $d_1 \approx 201.6$ meters.

For Thought

1. True, $\frac{1}{x} + \frac{3}{x+1} = \frac{(x+1)+3x}{x(x+1)} = \frac{4x+1}{x(x+1)}$.

2. True, $x + \frac{3x}{x^2-1} = \frac{x(x^2-1)+3x}{x^2-1} = \frac{x^3+2x}{x^2-1}$.

3. False, by using long division we obtain $\frac{x^2}{x^2-9} = 1 + \frac{9}{x^2-9} = 1 + \frac{A}{x-3} + \frac{B}{x+3}$

4. True, since $\frac{1}{2} + \frac{1}{2^3} = \frac{2^2+1}{2^3} = \frac{5}{8}$.

5. False, since $\frac{3x-1}{x^3+x} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$.

6. False, since $\frac{1}{x^2-1} = \frac{1/2}{x-1} - \frac{1/2}{x+1}$.

7. True, by using long division we get

$$\begin{array}{r|l} & x - 1 \\ \hline x^2+x-2 & x^3 + 0x^2 + 0x + 1 \\ & x^3 + x^2 - 2x \\ \hline & \quad -x^2 + 2x + 1 \\ & \quad -x^2 - x + 2 \\ \hline & \qquad 3x - 1 \end{array}$$

So $\frac{x^3+1}{x^2+x-2} = x - 1 + \frac{3x-1}{x^2+x-2}$.

8. False, since $x^3 - 8 = (x-2)(x^2+2x+4)$.

9. True, since $\frac{1}{x-1} + \frac{1}{x^2+x+1} = \frac{(x^2+x+1)+(x-1)}{x^3-1} = \frac{x^2+2x}{x^3-1}$.

10. True, it is already in the form $\frac{Ax+B}{x^2+9}$.

8.4 Exercises

1. $\frac{3(x+1)+4(x-2)}{(x-2)(x+1)} = \frac{7x-5}{(x-2)(x+1)}$

2. $\frac{-(x-4)-3(x+5)}{(x+5)(x-4)} = \frac{-4x-11}{(x+5)(x-4)}$

3. $\dfrac{(x^2+2)-3(x-1)}{(x-1)(x^2+2)} = \dfrac{x^2-3x+5}{(x-1)(x^2+2)}$

4.

$$\frac{(x+3)(x-1)+(x^2+x+1)}{(x-1)(x^2+x+1)} =$$

$$\frac{(x^2+2x-3)+(x^2+x+1)}{(x-1)(x^2+x+1)} =$$

$$\frac{2x^2+3x-2}{(x-1)(x^2+x+1)}$$

5.

$$\frac{(2x+1)(x^2+3)+(x^3+2x+2)}{(x^2+3)^2} =$$

$$\frac{(2x^3+x^2+6x+3)+(x^3+2x+2)}{(x^2+3)^2} =$$

$$\frac{3x^3+x^2+8x+5}{(x^2+3)^2}$$

6.

$$\frac{(3x-1)(x^2+x-3)+(x^3+x-1)}{(x^2+x-3)^2} =$$

$$\frac{(3x^3+2x^2-10x+3)+(x^3+x-1)}{(x^2+x-3)^2} =$$

$$\frac{4x^3+2x^2-9x+2}{(x^2+x-3)^2}$$

7.

$$\frac{(x-1)^2+(2x+3)(x-1)+(x^2+1)}{(x-1)^3} =$$

$$\frac{(x^2-2x+1)+(2x^2+x-3)+(x^2+1)}{(x-1)^3} =$$

$$\frac{4x^2-x-1}{(x-1)^3}$$

8.

$$\frac{3(x+2)(x^2+2)+(x-1)(x^2+2)+(x+2)^2}{(x+2)^2(x^2+2)} =$$

$$\frac{(3x^3+6x^2+6x+12)+(x^3-x^2+2x-2)}{(x+2)^2(x^2+2)} +$$

$$\frac{(x^2+4x+4)}{(x+2)^2(x^2+2)} =$$

$$\frac{4x^3+6x^2+12x+14}{(x+2)^2(x^2+2)}$$

9. Multiply the equation by $(x-3)(x+3)$.

$$\begin{aligned} 12 &= A(x+3)+B(x-3) \\ 12 &= (A+B)x+(3A-3B) \end{aligned}$$

$$A+B=0 \quad \text{and} \quad 3A-3B=12$$

Divide $3A-3B=12$ by 3 and add to $A+B=0$.

$$\begin{array}{rcl} A-B &=& 4 \\ A+B &=& 0 \\ \hline 2A &=& 4 \end{array}$$

Using $A=2$ in $A+B=0$, $B=-2$.
Then $A=2$ and $B=-2$.

10. Multiply the equation by $(x-2)(x+2)$.

$$\begin{aligned} 5x+2 &= A(x+2)+B(x-2) \\ 5x+2 &= (A+B)x+(2A-2B) \end{aligned}$$

$$A+B=5 \quad \text{and} \quad 2A-2B=2$$

Divide $2A-2B=2$ by 2 and add to $A+B=5$.

$$\begin{array}{rcl} A-B &=& 1 \\ A+B &=& 5 \\ \hline 2A &=& 6 \end{array}$$

Use $A=3$ in $A+B=5$ to get $B=2$.
So $A=3$ and $B=2$.

11. $\dfrac{5x-1}{(x+1)(x-2)} = \dfrac{A}{x+1} + \dfrac{B}{x-2}$

$$\begin{aligned} 5x-1 &= A(x-2)+B(x+1) \\ 5x-1 &= (A+B)x+(-2A+B) \end{aligned}$$

$$A+B=5 \quad \text{and} \quad -2A+B=-1$$

Multiply $-2A+B=-1$ by -1 and add to $A+B=5$.

$$\begin{array}{rcl} 2A-B &=& 1 \\ A+B &=& 5 \\ \hline 3A &=& 6 \end{array}$$

Using $A=2$ in $A+B=5$, $B=3$.

The answer is $\dfrac{2}{x+1} + \dfrac{3}{x-2}$.

12. $\dfrac{-3x-5}{(x+3)(x-1)} = \dfrac{A}{x+3} + \dfrac{B}{x-1}$

$$\begin{aligned} -3x-5 &= A(x-1)+B(x+3) \\ -3x-5 &= (A+B)x+(-A+3B) \end{aligned}$$

$A+B=-3$ and $-A+3B=-5$

Adding $A+B=-3$ to $-A+3B=-5$, we get $4B=-8$. Using $B=-2$ in $A+B=-3$, we find $A=-1$. The answer is $\dfrac{-1}{x+3} + \dfrac{-2}{x-1}$.

13. $\dfrac{2x+5}{(x+4)(x+2)} = \dfrac{A}{x+4} + \dfrac{B}{x+2}$

$$\begin{aligned} 2x+5 &= A(x+2)+B(x+4) \\ 2x+5 &= (A+B)x+(2A+4B) \end{aligned}$$

$A+B=2$ and $2A+4B=5$

Multiply $A+B=2$ by -2 and add to $2A+4B=5$.

$$\begin{array}{rcr} -2A-2B &=& -4 \\ 2A+4B &=& 5 \\ \hline 2B &=& 1 \end{array}$$

Using $B=1/2$ in $A+B=2$, we find $A=3/2$.

The answer is $\dfrac{3/2}{x+4} + \dfrac{1/2}{x+2}$.

14. $\dfrac{x+2}{(x+8)(x+4)} = \dfrac{A}{x+8} + \dfrac{B}{x+4}$

$$\begin{aligned} x+2 &= A(x+4)+B(x+8) \\ x+2 &= (A+B)x+(4A+8B) \end{aligned}$$

$A+B=1$ and $4A+8B=2$

Multiply $A+B=1$ by -4 and add to $4A+8B=2$.

$$\begin{array}{rcr} -4A-4B &=& -4 \\ 4A+8B &=& 2 \\ \hline 4B &=& -2 \end{array}$$

Using $B=-1/2$ in $A+B=1$, we obtain $A=3/2$. The answer is $\dfrac{3/2}{x+8} + \dfrac{-1/2}{x+4}$.

15. $\dfrac{2}{(x-3)(x+3)} = \dfrac{A}{x-3} + \dfrac{B}{x+3}$

$$\begin{aligned} 2 &= A(x+3)+B(x-3) \\ 2 &= (A+B)x+(3A-3B) \end{aligned}$$

$A+B=0$ and $3A-3B=2$

Multiply $A+B=0$ by 3 and add to $3A-3B=2$.

$$\begin{array}{rcr} 3A+3B &=& 0 \\ 3A-3B &=& 2 \\ \hline 6A &=& 2 \end{array}$$

Using $A=1/3$ in $A+B=0$, we get $B=-1/3$.

The answer is $\dfrac{1/3}{x-3} + \dfrac{-1/3}{x+3}$.

16. $\dfrac{1}{(3x-1)(3x+1)} = \dfrac{A}{3x-1} + \dfrac{B}{3x+1}$

$$\begin{aligned} 1 &= A(3x+1)+B(3x-1) \\ 1 &= (3A+3B)x+(A-B) \end{aligned}$$

$3A+3B=0$ and $A-B=1$

Multiply $A-B=1$ by 3 and add to $3A+3B=0$.

$$\begin{array}{rcr} 3A-3B &=& 3 \\ 3A+3B &=& 0 \\ \hline 6A &=& 3 \end{array}$$

Using $A=1/2$ in $A-B=1$, we find $B=-1/2$. The answer is $\dfrac{1/2}{3x-1} + \dfrac{-1/2}{3x+1}$.

17.

$$\begin{aligned} \frac{1}{x(x-1)} &= \frac{A}{x} + \frac{B}{x-1} \\ 1 &= A(x-1)+Bx \\ 1 &= (A+B)x-A \end{aligned}$$

$A+B=0$ and $-A=1$

Using $A=-1$ in $A+B=0$, we find $B=1$.

The answer is $\dfrac{-1}{x} + \dfrac{1}{x-1}$.

18. $\dfrac{2}{x(x-2)} = \dfrac{A}{x} + \dfrac{B}{x-2}$

$$\begin{aligned} 2 &= A(x-2) + Bx \\ 2 &= (A+B)x - 2A \end{aligned}$$

$$A + B = 0 \quad \text{and} \quad -2A = 2$$

Using $A = -1$ in $A + B = 0$, we find $B = 1$.

The answer is $\dfrac{-1}{x} + \dfrac{1}{x-2}$.

19. Multiplying the equation by $(x+3)^2(x-2)$, we obtain $x^2 + x - 31 =$

$= A(x+3)(x-2) + B(x-2) + C(x+3)^2$

$= A(x^2 + x - 6) + B(x-2) + C(x^2 + 6x + 9)$

$= (A+C)x^2 + (A+B+6C)x + (-6A-2B+9C)$.

Equate the coefficients and solve the system.

$$\begin{aligned} A + C &= 1 \\ A + B + 6C &= 1 \\ -6A - 2B + 9C &= -31 \end{aligned}$$

Multiply $A + B + 6C = 1$ by 2 and add to $-6A - 2B + 9C = -31$.

$$\begin{aligned} 2A + 2B + 12C &= 2 \\ -6A - 2B + 9C &= -31 \\ \hline -4A + 21C &= -29 \end{aligned}$$

Multiply $A + C = 1$ by 4 and add to $-4A + 21C = -29$.

$$\begin{aligned} 4A + 4C &= 4 \\ -4A + 21C &= -29 \\ \hline 25C &= -25 \end{aligned}$$

Using $C = -1$ in $A + C = 1$, we obtain $A = 2$. From $A + B + 6C = 1$, $2 + B - 6 = 1$ and $B = 5$. So $A = 2, B = 5$, and $C = -1$.

20. Multiplying the equation by $(x+1)^2(x-3)$, we obtain $-x^2 - 9x - 12 =$

$= A(x+1)(x-3) + B(x-3) + C(x+1)^2$

$= A(x^2 - 2x - 3) + B(x-3) + C(x^2 + 2x + 1)$

$= (A+C)x^2 + (-2A + B + 2C)x + (-3A - 3B + C)$.

Equate the coefficients and solve the system.

$$\begin{aligned} A + C &= -1 \\ -2A + B + 2C &= -9 \\ -3A - 3B + C &= -12 \end{aligned}$$

Multiply $-2A + B + 2C = -9$ by 3 and add to $-3A - 3B + C = -12$.

$$\begin{aligned} -6A + 3B + 6C &= -27 \\ -3A - 3B + C &= -12 \\ \hline -9A + 7C &= -39 \end{aligned}$$

Multiply $A + C = -1$ by 9 and add to $-9A + 7C = -39$.

$$\begin{aligned} 9A + 9C &= -9 \\ -9A + 7C &= -39 \\ \hline 16C &= -48 \end{aligned}$$

Using $C = -3$ in $A + C = -1$, we obtain $A = 2$. From $-2A + B + 2C = -9$, we get $-4 + B - 6 = -9$ and $B = 1$. Thus, we have $A = 2, B = 1$, and $C = -3$.

21. $\dfrac{4x-1}{(x-1)^2(x+2)} = \dfrac{A}{x-1} + \dfrac{B}{(x-1)^2} + \dfrac{C}{x+2}$

$$\begin{aligned} 4x - 1 &= A(x-1)(x+2) + B(x+2) + C(x-1)^2 \\ 4x - 1 &= A(x^2 + x - 2) + B(x+2) + C(x^2 - 2x + 1) \\ 4x - 1 &= (A+C)x^2 + (A + B - 2C)x + \\ &\quad (-2A + 2B + C) \end{aligned}$$

If we equate the coefficients of x, we obtain

$$\begin{aligned} A + C &= 0 \\ A + B - 2C &= 4 \\ -2A + 2B + C &= -1. \end{aligned}$$

Solving the system, we get $A = 1$, $B = 1$, and $C = -1$. The answer is

$$\frac{1}{x-1} + \frac{1}{(x-1)^2} + \frac{-1}{x+2}.$$

22. $\dfrac{5x^2 - 15x + 7}{(x-2)^2(x+1)} = \dfrac{A}{x-2} + \dfrac{B}{(x-2)^2} + \dfrac{C}{x+1}$

$$5x^2 - 15x + 7 = A(x-2)(x+1) + B(x+1) +$$

$$\begin{aligned} & C(x-2)^2 \\ 5x^2-15x+7 &= A(x^2-x-2)+B(x+1)+ \\ & C(x^2-4x+4) \\ 5x^2-15x+7 &= (A+C)x^2+(-A+B-4C)x+ \\ & (-2A+B+4C) \end{aligned}$$

If we equate the coefficients of x, we obtain

$$\begin{aligned} A+C &= 5 \\ -A+B-4C &= -15 \\ -2A+B+4C &= 7 \end{aligned}$$

Solving the system, we get $A=2$, $B=-1$, and $C=3$. The answer is

$$\frac{2}{x-2}+\frac{-1}{(x-2)^2}+\frac{3}{x+1}.$$

23. $\dfrac{20-4x}{(x-2)^2(x+4)}=\dfrac{A}{x-2}+\dfrac{B}{(x-2)^2}+\dfrac{C}{x+4}$

$$\begin{aligned} 20-4x &= A(x-2)(x+4)+B(x+4)+ \\ & C(x-2)^2 \\ 20-4x &= A(x^2+2x-8)+B(x+4)+ \\ & C(x^2-4x+4) \\ 20-4x &= (A+C)x^2+(2A+B-4C)x+ \\ & (-8A+4B+4C) \end{aligned}$$

If we equate the coefficients of x, we obtain

$$\begin{aligned} A+C &= 0 \\ 2A+B-4C &= -4 \\ -8A+4B+4C &= 20 \end{aligned}$$

Solving the system, we get $A=-1$, $B=2$, and $C=1$. The answer is

$$\frac{-1}{x-2}+\frac{2}{(x-2)^2}+\frac{1}{x+4}.$$

24. $\dfrac{x^2-3x+14}{(x-1)^2(x+3)}=\dfrac{A}{x-1}+\dfrac{B}{(x-1)^2}+\dfrac{C}{x+3}$

$$\begin{aligned} x^2-3x+14 &= A(x-1)(x+3)+B(x+3)+ \\ & C(x-1)^2 \\ x^2-3x+14 &= A(x^2+2x-3)+B(x+3)+ \\ & C(x^2-2x+1) \\ x^2-3x+14 &= (A+C)x^2+(2A+B-2C)x+ \\ & (-3A+3B+C) \end{aligned}$$

If we equate the coefficients of x, we obtain

$$\begin{aligned} A+C &= 1 \\ 2A+B-2C &= -3 \\ -3A+3B+C &= 14. \end{aligned}$$

Solving the system, we get $A=-1$, $B=3$, and $C=2$. The answer is

$$\frac{-1}{x-1}+\frac{3}{(x-1)^2}+\frac{2}{x+3}.$$

25. Note, $\dfrac{3x^2+3x-2}{(x+1)^2(x-1)}=\dfrac{A}{x+1}+\dfrac{B}{(x+1)^2}+\dfrac{C}{x-1}$

$$\begin{aligned} 3x^2+3x-2 &= A(x+1)(x-1)+B(x-1)+ \\ & C(x+1)^2 \\ 3x^2+3x-2 &= A(x^2-1)+B(x-1)+ \\ & C(x^2+2x+1) \\ 3x^2+3x-2 &= (A+C)x^2+(B+2C)x+ \\ & (-A-B+C) \end{aligned}$$

If we equate the coefficients of x, we obtain

$$\begin{aligned} A+C &= 3 \\ B+2C &= 3 \\ -A-B+C &= -2. \end{aligned}$$

Solving the system, we get $A=2$, $B=1$, and $C=1$. The answer is $\dfrac{2}{x+1}+\dfrac{1}{(x+1)^2}+\dfrac{1}{x-1}$.

26. Note, $\dfrac{-2x^2+8x+6}{(x-3)^2(x+3)}=\dfrac{A}{x-3}+\dfrac{B}{(x-3)^2}+\dfrac{C}{x+3}$

$$\begin{aligned} -2x^2+8x+6 &= A(x-3)(x+3)+B(x+3)+ \\ & C(x-3)^2 \\ -2x^2+8x+6 &= A(x^2-9)+B(x+3)+ \\ & C(x^2-6x+9) \\ -2x^2+8x+6 &= (A+C)x^2+(B-6C)x+ \\ & (-9A+3B+9C) \end{aligned}$$

If we equate the coefficients of x, we obtain

$$\begin{aligned} A + C &= -2 \\ B - 6C &= 8 \\ -9A + 3B + 9C &= 6 \end{aligned}$$

Solving the system, we get $A = -1$, $B = 2$, and $C = -1$. The answer is

$$\frac{-1}{x-3} + \frac{2}{(x-3)^2} + \frac{-1}{x+3}.$$

27. Multiplying the equation by $(x+1)(x^2+4)$, we get

$$\begin{aligned} x^2 - x - 7 &= A(x^2+4) + (Bx+C)(x+1) \\ x^2 - x - 7 &= (A+B)x^2 + (B+C)x + (4A+C). \end{aligned}$$

Equating the coefficients, we have

$$\begin{aligned} A + B &= 1 \\ B + C &= -1 \\ 4A + C &= -7. \end{aligned}$$

Multiply $A + B = 1$ by -1 and add to $B + C = -1$.

$$\begin{array}{rcr} -A - B &=& -1 \\ B + C &=& -1 \\ \hline -A + C &=& -2 \end{array}$$

Multiply $4A + C = -7$ by -1 and add to $-A + C = -2$.

$$\begin{array}{rcr} -A + C &=& -2 \\ -4A - C &=& 7 \\ \hline -5A &=& 5 \end{array}$$

Using $A = -1$ in $A + B = 1$, $B = 2$.
Using $B = 2$ in $B + C = -1$, $C = -3$.
So $A = -1, B = 2$, and $C = -3$.

28. Multiplying the equation by $(x-1)(x^2+3)$, we get $4x^2-3x+7 = A(x^2+3)+(Bx+C)(x-1)$ $(A+B)x^2+(-B+C)x+(3A-C)$. Equating the coefficients, we have

$$\begin{aligned} A + B &= 4 \\ -B + C &= -3 \\ 3A - C &= 7. \end{aligned}$$

Add $A + B = 4$ and $-B + C = -3$.

$$\begin{array}{rcr} A + B &=& 4 \\ -B + C &=& -3 \\ \hline A + C &=& 1 \end{array}$$

Add $3A - C = 7$ to $A + C = 1$.

$$\begin{array}{rcr} 3A - C &=& 7 \\ A + C &=& 1 \\ \hline 4A &=& 8 \end{array}$$

Using $A = 2$ in $A + B = 4$, $B = 2$.
Using $B = 2$ in $-B + C = -3$, $C = -1$.
Thus, $A = 2, B = 2$, and $C = -1$.

29. Note, $\dfrac{5x^2+5x}{(x+2)(x^2+1)} = \dfrac{A}{x+2} + \dfrac{Bx+C}{x^2+1}$.

$$\begin{aligned} 5x^2 + 5x &= A(x^2+1) + (Bx+C)(x+2) \\ 5x^2 + 5x &= (A+B)x^2 + (2B+C)x + \\ &\quad (A+2C) \end{aligned}$$

If we equate the coefficients of x, we obtain

$$\begin{aligned} A + B &= 5 \\ 2B + C &= 5 \\ A + 2C &= 0 \end{aligned}$$

Solving the system, we obtain $A = 2$, $B = 3$, and $C = -1$. The answer is $\dfrac{2}{x+2} + \dfrac{3x-1}{x^2+1}$.

30. Note, $\dfrac{3x^2-2x+11}{(x-1)(x^2+5)} = \dfrac{A}{x-1} + \dfrac{Bx+C}{x^2+5}$.

$$\begin{aligned} 3x^2 - 2x + 11 &= A(x^2+5) + (Bx+C)(x-1) \\ 3x^2 - 2x + 11 &= (A+B)x^2 + (-B+C)x + \\ &\quad (5A-C) \end{aligned}$$

If we equate the coefficients of x, we obtain

$$\begin{aligned} A + B &= 3 \\ -B + C &= -2 \\ 5A - C &= 11 \end{aligned}$$

Solving the system, we obtain $A = 2$, $B = 1$, and $C = -1$. The answer is $\dfrac{2}{x-1} + \dfrac{x-1}{x^2+5}$.

31. Note,

$$\frac{x^2-2}{(x+1)(x^2+x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+x+1}.$$

$$\begin{aligned} x^2-2 &= A(x^2+x+1)+(Bx+C)(x+1) \\ x^2-2 &= (A+B)x^2+(A+B+C)x+ \\ &\quad (A+C) \end{aligned}$$

If we equate the coefficients of x, we obtain

$$\begin{aligned} A+B &= 1 \\ A+B+C &= 0 \\ A+C &= -2. \end{aligned}$$

Solving the system, we obtain $A=-1$, $B=2$, and $C=-1$. The answer is

$$\frac{-1}{x+1} + \frac{2x-1}{x^2+x+1}.$$

32. Note,

$$\frac{3x^2+7x-4}{(x-2)(x^2+3x+1)} = \frac{A}{x-2} + \frac{Bx+C}{x^2+3x+1}.$$

$$\begin{aligned} 3x^2+7x-4 &= A(x^2+3x+1)+(Bx+C)(x-2) \\ 3x^2+7x-4 &= (A+B)x^2+(3A-2B+C)x+ \\ &\quad (A-2C) \end{aligned}$$

If we equate the coefficients of x, we obtain

$$\begin{aligned} A+B &= 3 \\ 3A-2B+C &= 7 \\ A-2C &= -4 \end{aligned}$$

Solving the system, we obtain $A=2$, $B=1$, and $C=3$. The answer is

$$\frac{2}{x-2} + \frac{x+3}{x^2+3x+1}.$$

33. $\dfrac{-2x-7}{(x+2)^2} = \dfrac{A}{x+2} + \dfrac{B}{(x+2)^2}$

$$\begin{aligned} -2x-7 &= A(x+2)+B \\ -2x-7 &= Ax+(2A+B) \\ A=-2 &\quad \text{and} \quad 2A+B=-7 \end{aligned}$$

Using $A=-2$ in $2A+B=-7$, we get $B=-3$. The answer is

$$\frac{-3}{(x+2)^2} + \frac{-2}{x+2}.$$

34.

$$\begin{aligned} \frac{-3x+2}{(x-1)^2} &= \frac{A}{x-1} + \frac{B}{(x-1)^2} \\ -3x+2 &= A(x-1)+B \\ -3x+2 &= Ax+(-A+B) \\ A=-3 &\quad \text{and} \quad -A+B=2 \end{aligned}$$

Using $A=-3$ in $-A+B=2$, we find $B=-1$.

The answer is $\dfrac{-1}{(x-1)^2} + \dfrac{-3}{x-1}$.

35. Note that $x^3+x^2+x+1 =$ $x^2(x+1)+(x+1) = (x^2+1)(x+1)$. Then we obtain

$$\frac{6x^2-x+1}{(x^2+1)(x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}$$

$$\begin{aligned} 6x^2-x+1 &= A(x^2+1)+(Bx+C)(x+1) \\ 6x^2-x+1 &= (A+B)x^2+(B+C)x+(A+C). \end{aligned}$$

Equating the coefficients, we get

$$\begin{aligned} A+B &= 6 \\ B+C &= -1 \\ A+C &= 1. \end{aligned}$$

Multiply $A+B=6$ by -1 and add to $B+C=-1$.

$$\begin{aligned} -A-B &= -6 \\ B+C &= -1 \\ \hline -A+C &= -7 \end{aligned}$$

Adding $-A+C=-7$ and $A+C=1$, $2C=-6$. Using $C=-3$ in $B+C=-1$ and $A+C=1$, we obtain $B=2$ and $A=4$.

The answer is $\dfrac{4}{x+1} + \dfrac{2x-3}{x^2+1}$.

36. Note that $x^3+2x^2+4x+8 =$ $x^2(x+2)+4(x+2) = (x^2+4)(x+2)$.

From $\dfrac{3x^2-2x+8}{(x^2+4)(x+2)} = \dfrac{A}{x+2} + \dfrac{Bx+C}{x^2+4}$,

we get $3x^2-2x+8 = A(x^2+4)+(Bx+C)(x+2)$ $= (A+B)x^2+(2B+C)x+(4A+2C)$. Equating the coefficients, we obtain

$$\begin{aligned} A+B &= 3 \\ 2B+C &= -2 \\ 4A+2C &= 8. \end{aligned}$$

Multiply $A+B=3$ by -2 and add to $2B+C=-2$.

$$\begin{aligned} -2A-2B &= -6 \\ 2B+C &= -2 \\ \hline -2A+C &= -8 \end{aligned}$$

Multiply $-2A+C=-8$ by 2 and add to $4A+2C=8$.

$$\begin{aligned} -4A+2C &= -16 \\ 4A+2C &= 8 \\ \hline 4C &= -8 \end{aligned}$$

Using $C=-2$ in $2B+C=-2$ and $4A+2C=8$,we have $B=0$ and $A=3$. The answer is

$$\frac{3}{x+2}+\frac{-2}{x^2+4}.$$

37. Note,

$$\frac{3x^3-x^2+19x-9}{(x^2+9)^2}=\frac{Ax+B}{x^2+9}+\frac{Cx+D}{(x^2+9)^2}.$$

So $3x^3-x^2+19x-9=$
$(Ax+B)(x^2+9)+(Cx+D)=$
$Ax^3+Bx^2+(9A+C)x+(9B+D)$.

Then $A=3$ and $B=-1$. Since $9A+C=19$ and $9B+D=-9$, we get $C=-8$ and $D=0$.

The answer is $\dfrac{-8x}{(x^2+9)^2}+\dfrac{3x-1}{x^2+9}$.

38. Note $\dfrac{-x^3-10x-3}{(x^2+5)^2}=\dfrac{Ax+B}{x^2+5}+\dfrac{Cx+D}{(x^2+5)^2}$.

Then $-x^3-10x-3=(Ax+B)(x^2+5)+(Cx+D)$
$=Ax^3+Bx^2+(5A+C)x+(5B+D)$.

So $A=-1$, $B=0$. From $5A+C=-10$ and $5B+D=-3$, we find $C=-5$ and $D=-3$.

The answer is $\dfrac{-x}{x^2+5}+\dfrac{-5x-3}{(x^2+5)^2}$.

39. Observe that

$$\frac{3x^2+17x+14}{(x-2)(x^2+2x+4)}=\frac{A}{x-2}+\frac{Bx+C}{x^2+2x+4}.$$

Then $3x^2+17x+14=$
$A(x^2+2x+4)+(Bx+C)(x-2)=$
$(A+B)x^2+(2A-2B+C)x+(4A-2C)$

Equating the coefficients, we obtain

$$\begin{aligned} A+B &= 3 \\ 2A-2B+C &= 17 \\ 4A-2C &= 14. \end{aligned}$$

Multiply $A+B=3$ by 2 and add to $2A-2B+C=17$.

$$\begin{aligned} 2A+2B &= 6 \\ 2A-2B+C &= 17 \\ \hline 4A+C &= 23 \end{aligned}$$

Multiplying $4A-2C=14$ by -1 and adding to $4A+C=23$, $3C=9$. So $C=3$ and from $4A-2C=14$, $A=5$. Using these values in $2A-2B+C=17$, we get $B=-2$. The answer is $\dfrac{5}{x-2}+\dfrac{-2x+3}{x^2+2x+4}$.

40. By factoring a sum of two cubes, we obtain

$$\frac{2x^2+17x-21}{(x+3)(x^2-3x+9)}=\frac{A}{x+3}+\frac{Bx+C}{x^2-3x+9}.$$

Then $2x^2+17x-21=$
$A(x^2-3x+9)+(Bx+C)(x+3)=$
$(A+B)x^2+(-3A+3B+C)x+(9A+3C)$.

Equating the coefficients, we obtain

$$\begin{aligned} A+B &= 2 \\ -3A+3B+C &= 17 \\ 9A+3C &= -21. \end{aligned}$$

Multiply $-3A+3B+C=17$ by -3 and add to $9A+3C=-21$.

$$\begin{aligned} 9A-9B-3C &= -51 \\ 9A+3C &= -21 \\ \hline 18A-9B &= -72 \end{aligned}$$

Multiplying $A+B=2$ by 9 and adding $18A-9B=-72$, we get $27A=-54$.

Using $A = -2$ in $A + B = 2$ and $9A + 3C = -21$, $B = 4$ and $C = -1$.

The answer is $\dfrac{4x-1}{x^2-3x+9} + \dfrac{-2}{x+3}$.

41. Divide $2x^3 + x^2 + 3x - 2$ by $x^2 - 1$ by long division.

$$\begin{array}{r|l} & 2x+1 \\ x^2-1 & 2x^3+x^2+3x-2 \\ & \underline{2x^3+0x^2-2x} \\ & \quad x^2+5x-2 \\ & \quad \underline{x^2+0x-1} \\ & \qquad 5x-1 \end{array}$$

Then $\dfrac{2x^3+x^2+3x-2}{x^2-1} = 2x+1+\dfrac{5x-1}{x^2-1}$.

Decompose $\dfrac{5x-1}{x^2-1} = \dfrac{A}{x-1} + \dfrac{B}{x+1}$.

$$\begin{aligned} 5x-1 &= A(x+1)+B(x-1) \\ 5x-1 &= (A+B)x+(A-B) \end{aligned}$$

So $A+B=5$ and $A-B=-1$.

Adding $A+B=5$ and $A-B=-1$, $2A=4$. Using $A=2$ in $A+B=5$, we find $B=3$.

The answer is $2x+1+\dfrac{2}{x-1}+\dfrac{3}{x+1}$.

42. Divide $2x^3-19x-9$ by x^2-9 by long division.

$$\begin{array}{r|l} & 2x \\ x^2-9 & 2x^3-19x-9 \\ & \underline{2x^3-18x} \\ & \qquad -x-9 \end{array}$$

Then $\dfrac{2x^3-19x-9}{x^2-9} = 2x+\dfrac{-x-9}{x^2-9}$.

Decompose $\dfrac{-x-9}{x^2-9} = \dfrac{A}{x-3}+\dfrac{B}{x+3}$.

$$\begin{aligned} -x-9 &= A(x+3)+B(x-3) \\ -x-9 &= (A+B)x+(3A-3B) \end{aligned}$$

So $A+B=-1$ and $3A-3B=-9$.

Divide $3A-3B=-9$ by 3 and add to $A+B=-1$.

$$\begin{aligned} A-B &= -3 \\ A+B &= -1 \\ \hline 2A &= -4 \end{aligned}$$

Using $A=-2$ in $A+B=-1$, $B=1$.

The answer is $2x+\dfrac{-2}{x-3}+\dfrac{1}{x+3}$.

43.

Since $\dfrac{3x^3-2x^2+x-2}{(x^2+x+1)^2} = \dfrac{Ax+B}{x^2+x+1}+\dfrac{Cx+D}{(x^2+x+1)^2}$, we get

$3x^3-2x^2+x-2 = (Ax+B)(x^2+x+1)+(Cx+D)$
$= Ax^3+(A+B)x^2+(A+B+C)x+(B+D)$.

Equating the coefficients, we find $A=3$. Since $A+B=-2$, we get $B=-5$. From $A+B+C=1$ and $B+D=-2$, we have $C=3$ and $D=3$.

The answer is $\dfrac{3x-5}{x^2+x+1}+\dfrac{3x+3}{(x^2+x+1)^2}$.

44.

Since $\dfrac{x^3-8x^2-5x-33}{(x^2+x+4)^2} = \dfrac{Ax+B}{x^2+x+4}+\dfrac{Cx+D}{(x^2+x+4)^2}$,

$x^3-8x^2-5x-33 =$
$= (Ax+B)(x^2+x+4)+(Cx+D)$
$= Ax^3+(A+B)x^2+(4A+B+C)x+(4B+D)$.

Equating the coefficients, we obtain $A=1$. Since $A+B=-8$, $B=-9$. From $4A+B+C=-5$ and $4B+D=-33$, we get $C=0$ and $D=3$.

Answer is $\dfrac{x-9}{x^2+x+4}+\dfrac{3}{(x^2+x+4)^2}$.

45.

Since $\dfrac{3x^3+4x^2-12x+16}{(x-2)(x+2)(x^2+4)} = \dfrac{A}{x-2}+\dfrac{B}{x+2}+\dfrac{Cx+D}{x^2+4}$, we obtain

$3x^3+4x^2-12x+16 =$
$= A(x+2)(x^2+4)+B(x-2)(x^2+4)+(Cx+D)(x^2-4)$
$= (A+B+C)x^3+(2A-2B+D)x^2+(4A+4B-4C)x+(8A-8B-4D)$.

Equating the coefficients, we get

$$\begin{aligned} A+B+C &= 3 \\ 2A-2B+D &= 4 \\ 4A+4B-4C &= -12 \\ 8A-8B-4D &= 16. \end{aligned}$$

Multiply first equation by -4 and add to the third. Multiply second equation by -4 and add to the fourth. Also multiply first equation by 2 and add to the second.

$$\begin{array}{rcr} -4A-4B-4C & = & -12 \\ 4A+4B-4C & = & -12 \\ \hline -8C & = & -24 \end{array}$$

$$\begin{array}{rcr} -8A+8B-4D & = & -16 \\ 8A-8B-4D & = & 16 \\ \hline -8D & = & 0 \end{array}$$

$$\begin{array}{rcr} 2A+2B+2C & = & 6 \\ 2A-2B+D & = & 4 \\ \hline 4A+2C+D & = & 10 \end{array}$$

So $C=3$ and $D=0$. From $4A+2C+D=10$, we find $A=1$ and from $2A-2B+D=4$, we get $B=-1$.

The answer is $\dfrac{1}{x-2}+\dfrac{-1}{x+2}+\dfrac{3x}{x^2+4}$.

46. Since $\dfrac{9x^2+3}{(9x^2+1)(3x-1)(3x+1)}=\dfrac{A}{3x-1}+\dfrac{B}{3x+1}+\dfrac{Cx+D}{9x^2+1}$, we obtain

$$\begin{aligned} 9x^2+3 &= \\ &= A(3x+1)(9x^2+1)+B(3x-1)(9x^2+1)+(Cx+D)(9x^2-1) \\ &= (27A+27B+9C)x^3+(9A-9B+9D)x^2+(3A+3B-C)x+(A-B-D). \end{aligned}$$

Set the coefficients equal to each other. Divide $27A+27B+9C=0$ by -9 and add to $3A+3B-C=0$.

$$\begin{array}{rcr} -3A-3B-C & = & 0 \\ 3A+3B-C & = & 0 \\ \hline -2C & = & 0 \\ C & = & 0 \end{array}$$

Divide $9A-9B+9D=9$ by -9 and add to $A-B-D=3$.

$$\begin{array}{rcr} -A+B-D & = & -1 \\ A-B-D & = & 3 \\ \hline -2D & = & 2 \\ D & = & -1 \end{array}$$

Multiply $A-B-D=3$ by -3 and add to $3A+3B-C=0$.

$$\begin{array}{rcr} -3A+3B+3D & = & -9 \\ 3A+3B-C & = & 0 \\ \hline 6B+3D-C & = & -9 \\ 6B-3-0 & = & -9 \\ B & = & -1 \end{array}$$

Using $B=-1$ and $C=0$ in $3A+3B-C=0$, we obtain $A=1$.

Answer is $\dfrac{1}{3x-1}+\dfrac{-1}{3x+1}+\dfrac{-1}{9x^2+1}$.

47.

$$\frac{5x^3+x^2+x-3}{x^3(x-1)}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x^3}+\frac{D}{x-1},$$

$$\begin{aligned} 5x^3+x^2+x-3 &= \\ &= Ax^2(x-1)+Bx(x-1)+C(x-1)+Dx^3 \\ &= (A+D)x^3+(-A+B)x^2+(-B+C)x-C \end{aligned}$$

Equating the coefficients, we get $C=3$.
From $-B+C=1$, we find $B=2$.
From $-A+B=1$, we obtain $A=1$.
From $A+D=5$, we have $D=4$.

The answer is $\dfrac{1}{x}+\dfrac{2}{x^2}+\dfrac{3}{x^3}+\dfrac{4}{x-1}$.

48. Note,

$$\frac{2x^3+3x^2-8x+4}{x^2(x-2)(x+2)}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x-2}+\frac{D}{x+2}.$$

Then $2x^3+3x^2-8x+4=$

$$\begin{aligned} &= Ax(x^2-4)+B(x^2-4)+Cx^2(x+2)+Dx^2(x-2) \\ &= (A+C+D)x^3+(B+2C-2D)x^2-4Ax-4B. \end{aligned}$$

Since $-4A=-8$ and $-4B=4$, we find $A=2$ and $B=-1$. Using these values in $A+C+D=2$ and $B+2C-2D=3$, we obtain

$$\begin{array}{rcl} C+D & = & 0 \\ 2C-2D & = & 4. \end{array}$$

Multiply first equation by -2 and add to the second.

$$\begin{array}{rcr} -2C-2D & = & 0 \\ 2C-2D & = & 4 \\ \hline -4D & = & 4 \end{array}$$

Using $D=-1$ in $C+D=0$, we get $C=1$.

The answer is $\dfrac{2}{x}+\dfrac{-1}{x^2}+\dfrac{1}{x-2}+\dfrac{-1}{x+2}$.

49. Note,
$$\frac{6x^2-28x+33}{(x-2)^2(x-3)} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{x-3}.$$
Then $6x^2-28x+33 =$
$$= A(x-2)(x-3) + B(x-3) + C(x-2)^2$$
$$= (A+C)x^2 + (-5A+B-4C)x + (6A-3B+4C).$$

Equating the coefficients, we obtain

$$\begin{aligned} A+C &= 6 \\ -5A+B-4C &= -28 \\ 6A-3B+4C &= 33. \end{aligned}$$

Multiply second equation by 3 and add to the third.

$$\begin{array}{rcr} -15A+3B-12C &=& -84 \\ 6A-3B+4C &=& 33 \\ \hline -9A-8C &=& -51 \end{array}$$

Multiply $A+C=6$ by 8 and add to $-9A-8C=-51$

$$\begin{array}{rcr} -9A-8C &=& -51 \\ 8A+8C &=& 48 \\ \hline -A &=& -3 \end{array}$$

Using $A=3$ in $A+C=6$, we find $C=3$. From $6A-3B+4C=33$, we obtain $B=-1$.

The answer is $\frac{3}{x-2} + \frac{-1}{(x-2)^2} + \frac{3}{x-3}$.

50. Observe that
$$\frac{7x^2+45x+58}{(x+3)^2(x+1)} = \frac{A}{x+3} + \frac{B}{(x+3)^2} + \frac{C}{x+1}.$$
Thus, $7x^2+45x+58 =$
$$= A(x+1)(x+3) + B(x+1) + C(x+3)^2$$
$$= (A+C)x^2 + (4A+B+6C)x + (3A+B+9C).$$

Equating the coefficients, we have

$$\begin{aligned} A+C &= 7 \\ 4A+B+6C &= 45 \\ 3A+B+9C &= 58. \end{aligned}$$

Multiply second equation by -1 and add to the third equation.

$$\begin{array}{rcr} -4A-B-6C &=& -45 \\ 3A+B+9C &=& 58 \\ \hline -A+3C &=& 13 \end{array}$$

Adding $A+C=7$ and $-A+3C=13$, we get $4C=20$ or $C=5$. From $A+C=7$, we have $A=2$, and from $3A+B+9C=58$, $B=7$.

Answer is $\frac{2}{x+3} + \frac{7}{(x+3)^2} + \frac{5}{x+1}$.

51. Use synthetic division to factor the denominator.

-5	1	4	-11	-30
		-5	5	30
	1	-1	-6	0

$$\begin{aligned} x^3+4x^2-11x-30 &= (x+5)(x^2-x-6) \\ &= (x+5)(x+2)(x-3) \end{aligned}$$

Decomposing, $\frac{9x^2+21x-24}{(x+5)(x+2)(x-3)} =$
$$= \frac{A}{x+5} + \frac{B}{x+2} + \frac{C}{x-3}.$$
Then $9x^2+21x-24 =$
$$= A(x+2)(x-3) + B(x+5)(x-3) + C(x+5)(x+2)$$
and substituting $x=-2,3,-5$, we obtain

$$\begin{aligned} -30 &= -15B \\ 2 &= B \end{aligned}$$

$$\begin{aligned} 120 &= 40C \\ 3 &= C \end{aligned}$$

$$\begin{aligned} 96 &= -24A \\ 4 &= A. \end{aligned}$$

The answer is $\frac{4}{x+5} + \frac{2}{x+2} + \frac{3}{x-3}$.

52. Use synthetic division to factor the denominator.

3	2	-1	-13	-6
		6	15	6
	2	5	2	0

$$\begin{aligned} 2x^3-x^2-13x-6 &= (x-3)(2x^2+5x+2) \\ &= (x-3)(2x+1)(x+2) \end{aligned}$$

Decomposing, $\dfrac{3x^2+24x+6}{(x-3)(2x+1)(x+2)} =$

$$= \frac{A}{x-3} + \frac{B}{2x+1} + \frac{C}{x+2}.$$

So $3x^2+24x+6 =$

$$= A(2x+1)(x+2) + B(x-3)(x+2) + C(x-3)(2x+1)$$

and substituting $x = -2, 3, -1/2$ we get

$$\begin{aligned} -30 &= 15C \\ -2 &= C \end{aligned}$$

$$\begin{aligned} 105 &= 35A \\ 3 &= A \end{aligned}$$

$$\begin{aligned} \frac{3}{4} - 12 + 6 &= B\left(-\frac{7}{2}\right)\left(\frac{3}{2}\right) \\ \frac{3}{4} - 6 &= -\frac{21B}{4} \\ 1 &= B. \end{aligned}$$

Answer is $\dfrac{3}{x-3} + \dfrac{1}{2x+1} + \dfrac{-2}{x+2}$.

53. Note that $x^3 - 3x^2 + 3x - 1 = (x-1)^3$.

Then $\dfrac{x^2-2}{(x-1)^3} = \dfrac{A}{x-1} + \dfrac{B}{(x-1)^2} + \dfrac{C}{(x-1)^3}$.

$$\begin{aligned} x^2 - 2 &= A(x-1)^2 + B(x-1) + C \\ &= Ax^2 + (-2A+B)x + (A-B+C) \end{aligned}$$

Then $A = 1$. Since $-2A + B = 0$, $B = 2$. Since $A - B + C = -2$, we find $1 - 2 + C = -2$ and $C = -1$.

The answer is $\dfrac{1}{x-1} + \dfrac{2}{(x-1)^2} + \dfrac{-1}{(x-1)^3}$.

54. Note that $x^3 + 3x^2 + 3x + 1 = (x+1)^3$.

$$\begin{aligned} \frac{2x^2}{(x+1)^3} &= \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3} \\ 2x^2 &= A(x+1)^2 + B(x+1) + C \\ 2x^2 &= Ax^2 + (2A+B)x + (A+B+C) \end{aligned}$$

So $A = 2$. Since $2A + B = 0$, $B = -4$. Since $A + B + C = 0$, we get $2 - 4 + C = 0$ and $C = 2$.

Answer is $\dfrac{2}{(x+1)^3} + \dfrac{-4}{(x+1)^2} + \dfrac{2}{x+1}$.

55.

$$\begin{aligned} \frac{x}{(ax+b)^2} &= \frac{A}{ax+b} + \frac{B}{(ax+b)^2} \\ x &= A(ax+b) + B \\ &= aAx + (bA + B) \end{aligned}$$

So $aA = 1$ and $A = 1/a$. Since $bA + B = 0$, we obtain $\dfrac{b}{a} + B = 0$ and $B = -b/a$.

The answer is $\dfrac{-b/a}{(ax+b)^2} + \dfrac{1/a}{ax+b}$.

56. $\dfrac{x^2}{(ax+b)^3} = \dfrac{A}{ax+b} + \dfrac{B}{(ax+b)^2} + \dfrac{C}{(ax+b)^3}$

$x^2 = A(ax+b)^2 + B(ax+b) + C$

$x^2 = a^2Ax^2 + (2abA + aB)x + (b^2A + bB + C)$.

So $a^2A = 1$ and $A = \dfrac{1}{a^2}$. Since $2abA + aB = 0$, we get $\dfrac{2b}{a} + aB = 0$ and $B = -\dfrac{2b}{a^2}$. From $b^2A + bB + C = 0$, we have $\dfrac{b^2}{a^2} - \dfrac{2b^2}{a^2} + C = 0$ and $C = \dfrac{b^2}{a^2}$.

Answer is $\dfrac{1/a^2}{ax+b} + \dfrac{-2b/a^2}{(ax+b)^2} + \dfrac{b^2/a^2}{(ax+b)^3}$.

57. Since $\dfrac{x+c}{x(ax+b)} = \dfrac{A}{x} + \dfrac{B}{ax+b}$, we have

$$\begin{aligned} x + c &= A(ax+b) + Bx \\ &= (aA + B)x + bA. \end{aligned}$$

So $bA = c$ and $A = \dfrac{c}{b}$. Since $aA + B = 1$, we have $\dfrac{ac}{b} + B = 1$ and $B = 1 - \dfrac{ac}{b}$.

Answer is $\dfrac{c/b}{x} + \dfrac{1-ac/b}{ax+b}$.

58. Since $\dfrac{1}{x^3(ax+b)} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x^3} + \dfrac{D}{ax+b}$,

$1 = Ax^2(ax+b) + Bx(ax+b) + C(ax+b) + Dx^3$

$1 = (aA+D)x^3 + (bA+aB)x^2 + (bB+aC)x + bC$.

If we equate the coefficients, we find $bC = 1$

and $C = \frac{1}{b}$. Since $bB + aC = 0$, $B = -\frac{a}{b^2}$.
Since $bA + aB = 0$, we get $A = \frac{a^2}{b^3}$.

Since $aA + D = 0$, we obtain $D = -\frac{a^3}{b^3}$.

The answer is $\frac{a^2/b^3}{x} + \frac{-a/b^2}{x^2} + \frac{1/b}{x^3} + \frac{-a^3/b^3}{ax+b}$.

59. Since $\frac{1}{x^2(ax+b)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{ax+b}$, we get
$1 = Ax(ax+b) + B(ax+b) + Cx^2$
$1 = (aA+C)x^2 + (bA+aB)x + bB$.

So $bB = 1$ and $B = \frac{1}{b}$. Since $bA + aB = 0$, we obtain $bA + \frac{a}{b} = 0$ and $A = -\frac{a}{b^2}$.

Since $aA + C = 0$, $-\frac{a^2}{b^2} + C = 0$ and $C = \frac{a^2}{b^2}$.
The answer is

$$\frac{-a/b^2}{x} + \frac{1/b}{x^2} + \frac{a^2/b^2}{ax+b}.$$

60.

$$\frac{1}{x^2(ax+b)^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{ax+b} + \frac{D}{(ax+b)^2}$$

Then $1 = Ax(ax+b)^2 + B(ax+b)^2 + Cx^2(ax+b) + Dx^2$. Simplifying, $1 = (a^2A + aC)x^3 + (2abA + a^2B + bC + D)x^2 + (b^2A + 2abB)x + b^2B$.

So $b^2B = 1$ and $B = \frac{1}{b^2}$. Since $b^2A + 2abB = 0$, we obtain $b^2A + \frac{2a}{b} = 0$ and $A = -\frac{2a}{b^3}$.

Since $a^2A + aC = 0$, we find $-\frac{2a^3}{b^3} + aC = 0$ and $C = \frac{2a^2}{b^3}$. Since $2abA + a^2B + bC + D = 0$, we get $-\frac{4a^2}{b^2} + \frac{a^2}{b^2} + \frac{2a^2}{b^2} + D = 0$ and $D = \frac{a^2}{b^2}$.
Answer is $\frac{-2a/b^3}{x} + \frac{1/b^2}{x^2} + \frac{2a^2/b^3}{ax+b} + \frac{a^2/b^2}{(ax+b)^2}$.

61. If we substitute $y = x + 2$ into the other equation, we find

$$\begin{aligned} x^2 + (x+2)^2 &= 34 \\ 2x^2 + 4x - 30 &= 0 \\ x^2 + 2x - 15 &= 0 \\ (x+5)(x-3) &= 0 \\ x &= -5, 3 \end{aligned}$$

If $x = -5$, then $y = x + 2 = -5 + 2 = -3$. Similarly, if $x = 3$ then $y = 5$.

The solution set is $\{(-5, -3), (3, 5)\}$.

62. If we subtract the first equation from the third equation, we get the second equation as shown below:

$$\begin{array}{rcr} 3x - 2y + z &=& 15 \\ x - y - z &=& 5 \\ \hline 2x - y + 2z &=& 10 \end{array}$$

Then the system of equations is dependent.

If we multiply the first equation by two, and add the result to the negative of the third equation, we find

$$\begin{array}{rcr} 2x - 2y - 2z &=& 10 \\ -3x + 2y - z &=& -15 \\ \hline -x - 3z &=& -5 \end{array}$$

Equivalently, the result is $x = 5 - 3z$.

Similarly, if we eliminate x, we find $y = -4z$.

Thus, the solution set is

$$\{(5 - 3z, -4z, z) | z \text{ is a real number}\}.$$

63. We may rewrite $2y = 10x - 6$ as $16 = 10x - 2y$ or $8 = 5x - y$. Then the system of two equations are dependent. Since $y = 5x - 8$, the solution set is

$$\{(x, 5x - 8) | x \text{ is a real number}\}.$$

64. $y = -(x+6)^2 + 5$

65. Apply the method of completing the square.

$$\begin{aligned} y &= \frac{1}{2}\left(x^2 + 8x\right) - 9 \\ y &= \frac{1}{2}\left(x^2 + 8x + 16\right) - 9 - 8 \\ y &= \frac{1}{2}(x+4)^2 - 17 \end{aligned}$$

66. From Exercise 65, we equivalently have

$$y = \frac{1}{2}(x+4)^2 - 17$$

and the graph of this equation is a parabola. The domain is $(-\infty, \infty)$ and the range is $[-17, \infty)$.

Thinking Outside the Box LXXIV

Open the cylinder to form a 4ft-by-6ft rectangle in such a way that the lizard is on the left side and 1 ft from the base of the rectangle, and the fly is 1 ft from the top of the rectangle and 3 ft from the left side.

If we imagine the rectangle as a piece of paper, then the lizard is on the front page of the paper and the fly in on the back page. Cut the rectangle vertically in the middle into two pieces, then hold the right piece and turn it over. Now, the lizard and fly are on the same side of the pages. Next, move the right piece up by 4 ft.

At this point, we have two 4 ft-by-3 ft rectangles that are joined together at the right top corner of one rectangle (with the lizard) and the left bottom corner of the other rectangle (with the fly). Recall, the lizard and fly are on the same side of the pages.

If the lizard moves 3ft to the right and 4ft up, then the lizard would have reached the fly. However, the shortest path for the lizard is the path along the hypotenuse of a right triangle whose sides are the 3 ft horizontal side and the 4 ft vertical side. The length of the hypotenuse is 5 ft, which is the shortest path from the lizard to the fly.

8.4 Pop Quiz

1. From the equation

$$\frac{6x-10}{x^2-25} = \frac{A}{x-5} + \frac{B}{x+5}$$

we find

$$\begin{aligned} 6x - 10 &= A(x+5) + B(x-5) \\ 6x - 10 &= (A+B)x + (5A - 5B). \end{aligned}$$

Equating the coefficients, we obtain a system of equations.

$$\begin{aligned} A + B &= 6 \\ 5A - 5B &= -10. \end{aligned}$$

The solutions of the above system of equations are $A = 2$ and $B = 4$.

The partial fraction decomposition is

$$\frac{2}{x-5} + \frac{4}{x+5}.$$

2. Using the equation

$$\frac{4x^2+2x+6}{(x+1)(x^2+3)} = \frac{Ax+B}{x^2+3} + \frac{C}{x+1}$$

we obtain

$$\begin{aligned} 4x^2 + 2x + 6 &= (Ax+B)(x+1) + C(x^2+3) \\ 4x^2 + 2x + 6 &= (A+C)x^2 + (A+B)x + (B+3C). \end{aligned}$$

Equating the coefficients, we obtain a system of equations.

$$\begin{aligned} A \quad\quad + C &= 4 \\ A + B \quad\quad &= 2 \\ B + 3C &= 6 \end{aligned}$$

The solutions of the above system of equations are $A = 2$, $B = 0$, and $C = 2$.

The partial fraction decomposition is

$$\frac{2x}{x^2+3} + \frac{2}{x+1}.$$

8.4 Linking Concepts

a) Since 2 hours and 24 minutes is $\frac{12}{5}$ hours, we obtain

$$\frac{1}{a} + \frac{1}{b} = \frac{5}{12}.$$

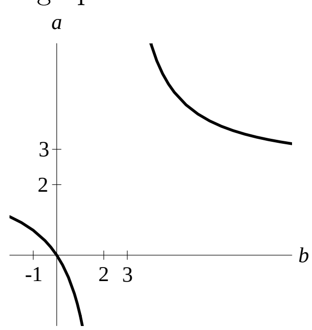
The sum above represents the part of

the garage painted in 1 hour.

b) Solving for a, one finds

$$\begin{aligned} \frac{1}{a}+\frac{1}{b} &= \frac{5}{12} \\ b+a &= \frac{5ab}{12} \\ a\left(\frac{5b}{12}-1\right) &= b \\ a &= \frac{b}{\frac{5b}{12}-1} \\ a &= \frac{12b}{5b-12}. \end{aligned}$$

A graph of a as a function of b is given below.



c) The horizontal and vertical asymptotes are $a=\frac{12}{5}$ and $b=\frac{12}{5}$, respectively. If a is large, then b is approximately 2 hours, 24 minutes. If b is large, then a is approximately 2 hours, 24 minutes.

d) With a graphing calculator, the possible values of a and b are tabulated as follows.

a	3	4	6	12
b	12	6	4	3

e) Since 1 hour and 15 minutes is $\frac{5}{4}$ hours, then $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=\frac{4}{5}$. Suppose $2 \le a \le b \le c$. Solving for c, one finds

$$\begin{aligned} 5(bc+ac+ab) &= 4abc \\ 4abc-5bc-5ac &= 5ab \\ c &= \frac{5ab}{4ab-5b-5a}. \end{aligned}$$

If $b \ge 7$, then $10-4b$ is negative and one finds

$$\begin{aligned} \frac{5ab}{4ab-5b-5a} &\ge b \\ 5ab &\ge 4ab^2-5ab-5b^2 \\ 10ab &\ge 4ab^2-5b^2 \\ 10a &\ge 4ab-5b \\ a(10-4b) &\ge -5b \\ a &\le \frac{-5b}{10-4b} \\ a &\le \frac{5b}{4b-10}. \end{aligned}$$

From the graph of $f(b)=\frac{5b}{4b-10}$, one finds for $b \ge 7$ that

$$a \le \frac{5b}{4b-10} \le f(7) = \frac{35}{18} < 2.$$

This contradicts $a \ge 2$, thus, $2 \le b \le 6$.

Note, c is uniquely determined by a and b. To find all the solutions with $a \le b \le 6$, we list the finitely many pairs (a,b), $a \le b \le 6$, where each such pair determines an integer value for c. Doing this, we find

$$(a,b,c)=(2,4,20)$$

and

$$(2,5,10).$$

If we drop the requirement that $a \le b \le c$, then we find 12 solutions. Namely,

a	2	2	4	4	20	20
b	4	20	2	20	2	4
c	20	4	20	2	4	2

and

a	2	2	5	5	10	10
b	5	10	2	10	2	5
c	10	5	10	2	5	2

.

For Thought

1. False, since $3 > 1+2$ is false.

2. False **3.** True

4. False, because $x^2 + y^2 > 5$ is the region outside of a circle of radius $\sqrt{5}$.

5. True, since $(-2, 1)$ satisfies both equations in system (a).

6. True

7. False, $(-2, 0)$ does not satisfy $y < x + 2$.

8. False, $(-1, 2)$ lies on the line $y - 3x = 5$.

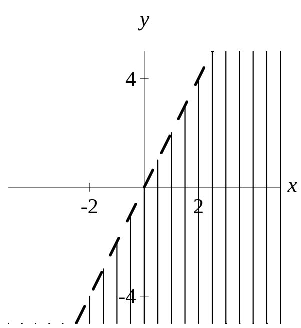
9. True

10. True

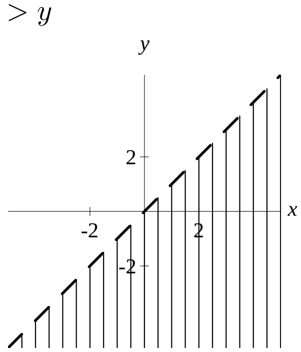
8.5 Exercises

1. c **2.** b **3.** d **4.** a

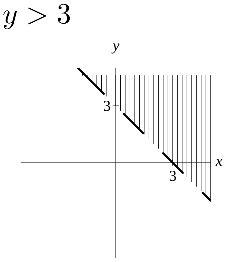
5. $y < 2x$



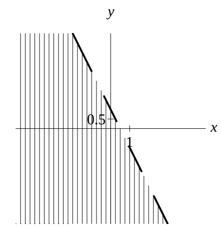
6. $x > y$



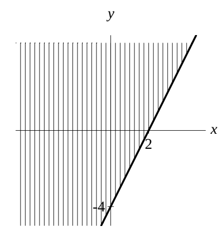
7. $x + y > 3$



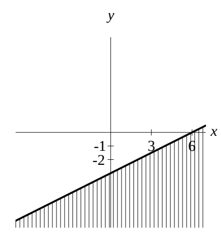
8. $2x + y < 1$



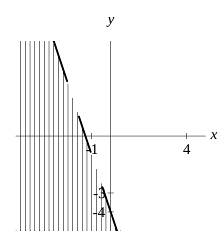
9. $2x - y \leq 4$



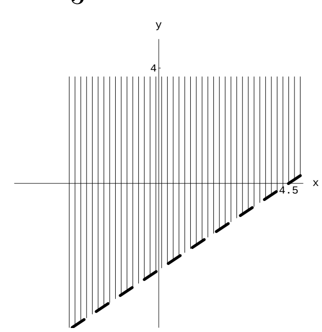
10. $x - 2y \geq 6$



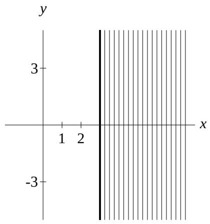
11. $y < -3x - 4$



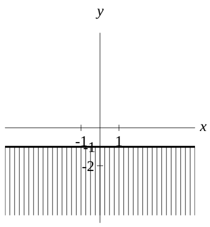
12. $y > \dfrac{2}{3}x - 3$



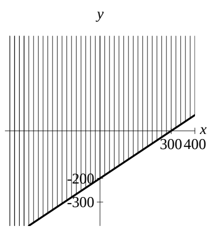
13. $x - 3 \geq 0$



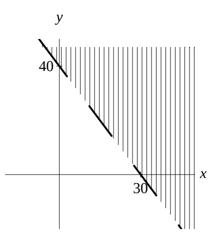
14. $y + 1 \leq 0$



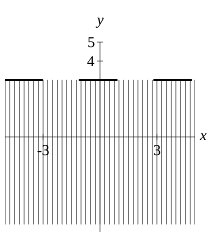
15. $20x - 30y \leq 6000$



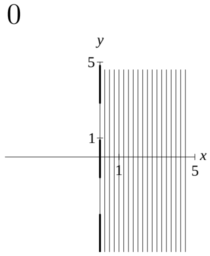
16. $30y + 40x > 1200$



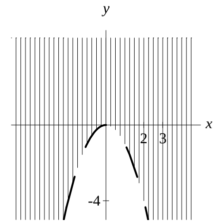
17. $y < 3$



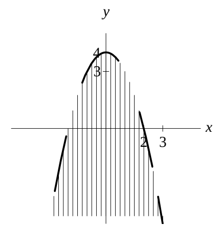
18. $x > 0$



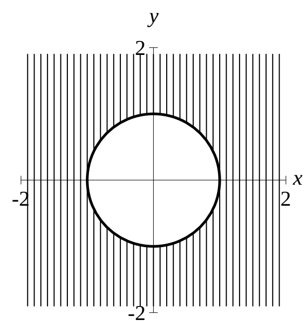
19. $y > -x^2$



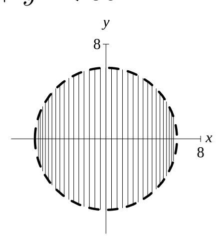
20. $y < 4 - x^2$



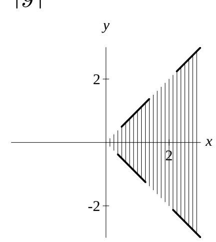
21. $x^2 + y^2 \geq 1$



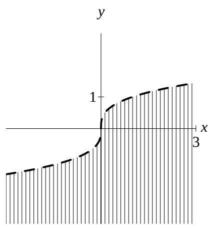
22. $x^2 + y^2 < 36$



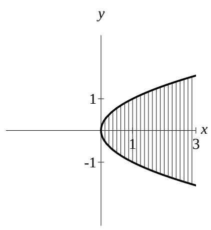
23. $x > |y|$



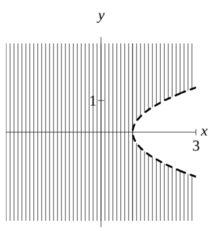
24. $y < x^{1/3}$



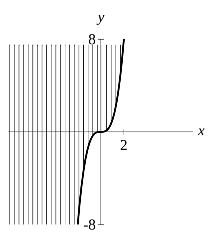
25. $x \geq y^2$



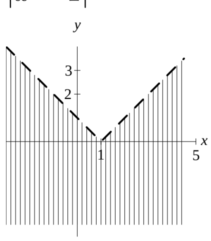
26. $y^2 > x - 1$



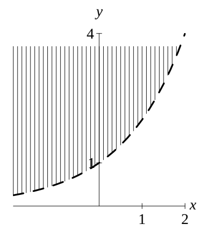
27. $y \geq x^3$



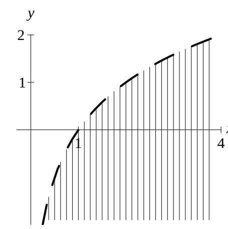
28. $y < |x - 1|$



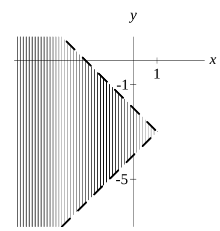
29. $y > 2^x$



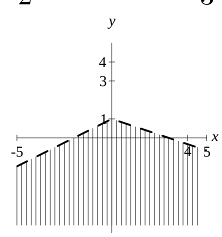
30. $y < \log_2(x)$



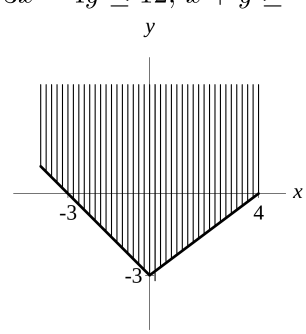
31. $y > x - 4$, $y < -x - 2$



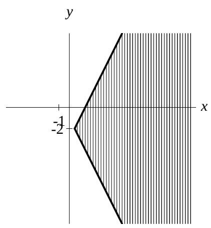
32. $y < \frac{1}{2}x + 1$, $y < -\frac{1}{3}x + 1$



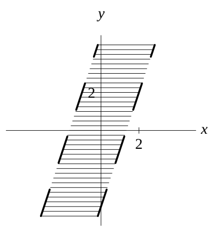
33. $3x - 4y \leq 12$, $x + y \geq -3$



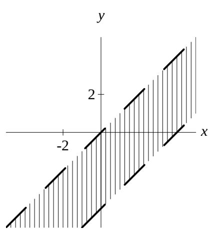
34. $2x + y \geq -1$, $y - 2x \leq -3$



35. $3x - y < 4$, $y < 3x + 5$



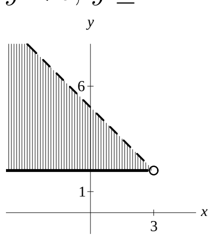
36. $x - y > 0$, $y + 4 > x$



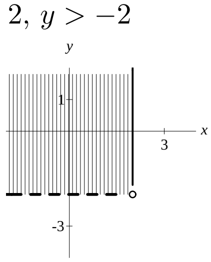
37. No solution since the graphs of $y + x < 0$ and $y > 3 - x$ do not overlap.

38. No solution since the graphs of $3x - 2y \leq 6$ and $2y - 3x \leq -8$ do not overlap.

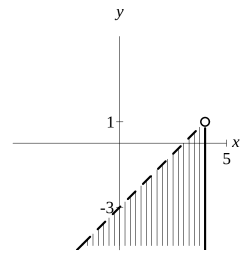
39. $x + y < 5$, $y \geq 2$



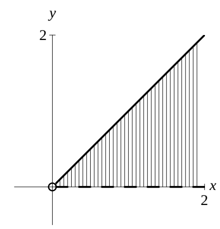
40. $x \leq 2$, $y > -2$



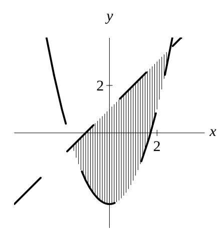
41. $y < x - 3$, $x \leq 4$



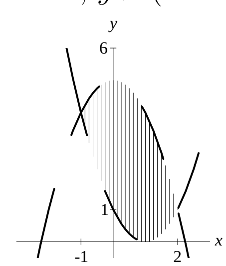
42. $y > 0$, $y \leq x$



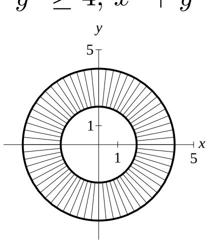
43. $y > x^2 - 3$, $y < x + 1$



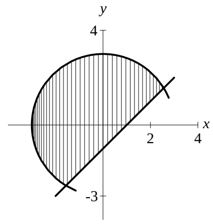
44. $y < 5 - x^2$, $y > (x - 1)^2$



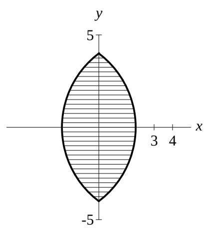
45. $x^2 + y^2 \geq 4$, $x^2 + y^2 \leq 16$



46. $x^2 + y^2 \leq 9,\ y \geq x - 1$

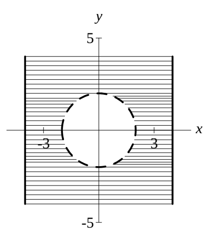


47. $(x-3)^2 + y^2 \leq 25,\ (x+3)^2 + y^2 \leq 25$

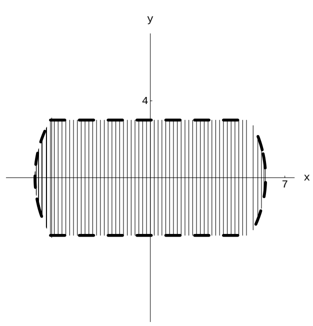


48. No solution since the graphs of $x^2 + y^2 \geq 64$ and $x^2 + y^2 \leq 16$ do not overlap.

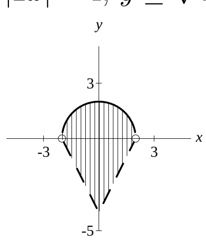
49. $x^2 + y^2 > 4,\ |x| \leq 4$



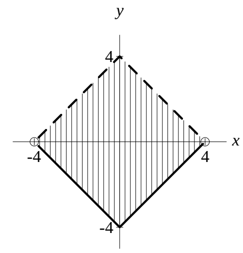
50. $x^2 + y^2 < 36,\ |y| < 3$



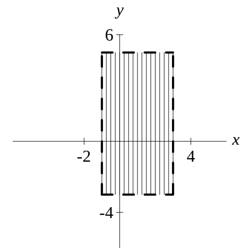
51. $y > |2x| - 4,\ y \leq \sqrt{4 - x^2}$



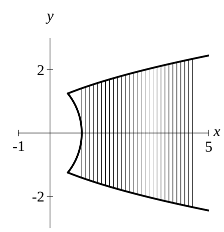
52. $y < 4 - |x|,\ y \geq |x| - 4$



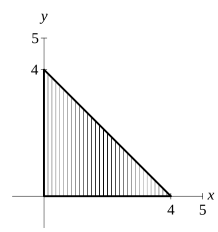
53. $|x - 1| < 2,\ |y - 1| < 4$



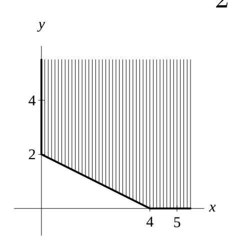
54. $x \geq y^2 - 1,\ (x+1)^2 + y^2 \geq 4$



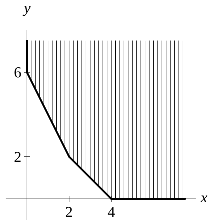
55. $x \geq 0,\ y \geq 0,\ x + y \leq 4$



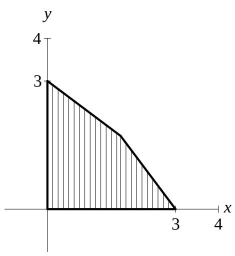
56. $x \geq 0,\ y \geq 0,\ y \geq -\dfrac{1}{2}x + 2$



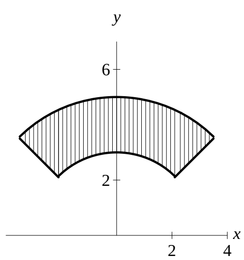
57. $x \geq 0$, $y \geq 0$, $x + y \geq 4$, $y \geq -2x + 6$



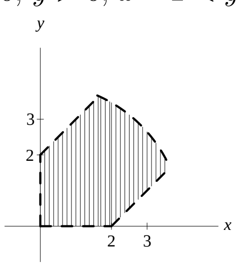
58. $x \geq 0$, $y \geq 0$, $4x + 3y \leq 12$, $3x + 4y \leq 12$



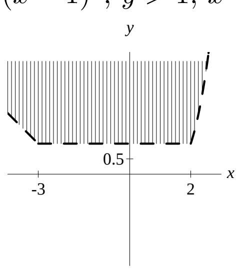
59. $x^2 + y^2 \geq 9$, $x^2 + y^2 \leq 25$, $y \geq |x|$



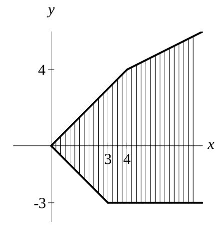
60. $x > 0$, $y > 0$, $x - 2 < y < x + 2$, $x^2 + y^2 < 16$



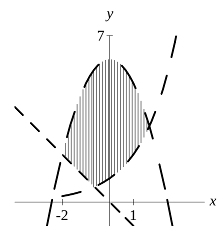
61. $y > (x - 1)^3$, $y > 1$, $x + y > -2$



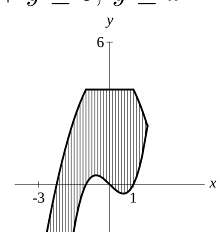
62. $x \geq |y|$, $y \geq -3$, $2y - x \leq 4$



63. $y > 2^x$, $y < 6 - x^2$, $x + y > 0$



64. $x^2 + y \leq 5$, $y \geq x^3 - x$, $y \leq 4$



65. $x \geq 0$, $y \geq 0$, $y \leq -\dfrac{2}{3}x + 5$, $y \leq -3x + 12$

66. $x \geq 0$, $y \geq 0$, $y \leq 3$, $y \leq -x + 6$

67. $x \geq 0$, $y \geq 0$, $y \geq -\dfrac{1}{2}x + 3$, $y \geq -\dfrac{3}{2}x + 5$

68. $x \geq 0$, $y \geq 2$, $y \geq -x + 5$

69. The system is

$$\begin{aligned} |x| &< 2 \\ |y| &< 2. \end{aligned}$$

70. Since the line through $(0, 6)$ and $(3, 0)$ is given by $y = -2x + 6$, the system is

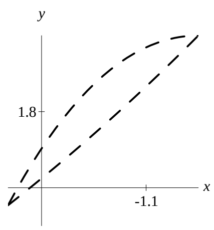
$$\begin{aligned} y &< -2x + 6 \\ x &> 0 \\ y &> 0. \end{aligned}$$

71. Since a circle of radius 9 with center at the origin is given by $x^2 + y^2 = 81$, the system is

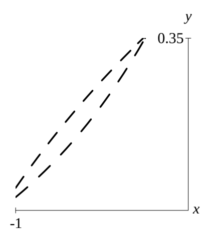
$$\begin{aligned} x^2 + y^2 &< 81 \\ x &> 0 \\ y &> 0. \end{aligned}$$

72. The solution set contains the points between the graph of $y = x$ and the x-axis. This is given by the inequality $|y| < |x|$.

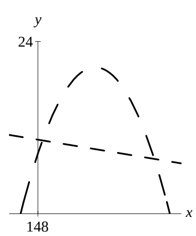
73. $(-1.17,\ 1.84)$ is a solution.



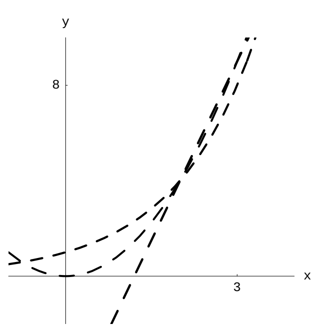
74. $(-0.65,\ 0.25)$ is a solution.



75. $(150,\ 22.4)$ is a solution.

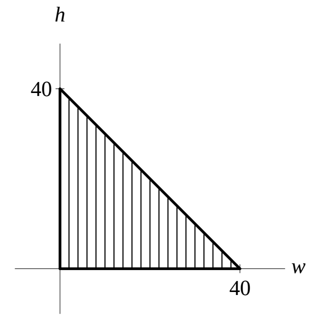


76. $(3, 8.5)$ is a solution.



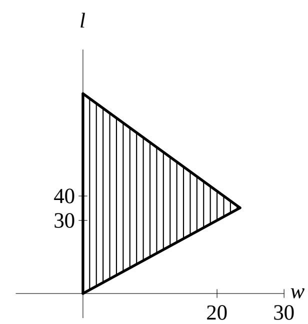
77. Let w and h be the width and height, respectively. Then $50 + 2w + 2h \leq 130$. The system is

$$\begin{aligned} w + h &\leq 40 \\ w, h &\geq 0. \end{aligned}$$



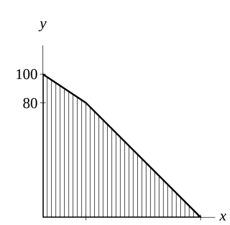
78. Let w and l be the width and length, respectively. Then $l + 2w + 48 \leq 130$ and $w \leq \frac{2}{3}l$. The system is

$$\begin{aligned} l + 2w &\leq 82 \\ 3w - 2l &\leq 0 \\ w, l &\geq 0. \end{aligned}$$



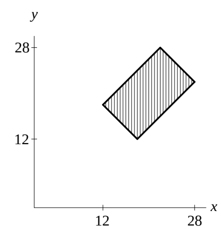
79. Let x and y be the number of mid-size and full-size cars, respectively. Divide $10,000x + 15,000y \leq 1,500,000$ by 1000. The system is

$$\begin{aligned} x + y &\leq 110 \\ x + 1.5y &\leq 150 \\ x &\geq 0 \\ y &\geq 0. \end{aligned}$$



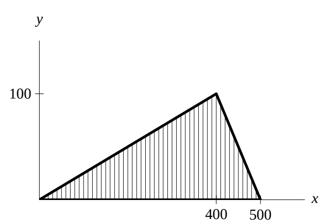
80. Let x and y be the number of male and female employees, respectively. The system is

$$\begin{aligned} 30 \le x + y &\le 50 \\ x - y &\le 6 \\ y - x &\le 6 \\ x \ge 0, \quad & y \ge 0 \end{aligned}$$



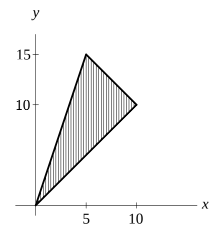
81. Let x and y be the number of \$50 tickets and \$100 tickets, respectively. Simplifying $y \le 0.2(x+y)$, we get $0.8y \le 0.2x$. Then $y \le \frac{1}{4}x$. The system is

$$\begin{aligned} y &\le \frac{1}{4}x \\ x + y &\le 500 \\ x \ge 0, y &\ge 0. \end{aligned}$$



82. Let x and y be the number of ounces of alloy A and alloy B, respectively. Simplifying $0.2x + 0.6y \le 0.5(x+y)$, we get $0.1y \le 0.3x$. So $y \le 3x$. Simplifying $0.8x + 0.4y \le 0.6(x+y)$, we obtain $0.2x \le 0.2y$. Then $x \le y$. The system is

$$\begin{aligned} y &\le 3x \\ x &\le y \\ x + y &\le 20 \\ x \ge 0, y &\ge 0. \end{aligned}$$



83. From the equation

$$\frac{x^2+5x+3}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$$

we find

$$\begin{aligned} x^2+5x+3 &= Ax(x+1) + B(x+1) + Cx^2 \\ x^2+5x+3 &= (A+C)x^2 + (A+B)x + B. \end{aligned}$$

Equating the coefficients, we obtain a system of equations.

$$\begin{aligned} B &= 3 \\ A + B &= 5 \\ A + C &= 1 \end{aligned}$$

The solutions of the above system of equations are $B = 3$, $A = 2$, and $C = -1$.

The partial fraction decomposition is

$$\frac{2}{x} + \frac{3}{x^2} + \frac{-1}{x+1}.$$

84. If we add the first two equations, we find

$$\begin{array}{rcr} x - y - 3z &=& 9 \\ 2x + y - 4z &=& 12 \\ \hline 3x - 7z &=& 21 \end{array}$$

If we multiply the second equation by two, and add the result to the third equation, we obtain

$$\begin{array}{rcr} 4x + 2y - 8z &=& 24 \\ 2x - 2y - 6z &=& 8 \\ \hline 6x - 14z &=& 32 \end{array}$$

The above equation simplifies to $3x - 7z = 16$; and contradicts $3x - 7z = 21$. The solution set is $\emptyset$.

85. The second equation $18y - 10x = 20$ is equivalent to $9y - 5x = 10$ or $5x - 9y = -10$. This contradicts the first equation $5x - 9y = 12$. The solution set is $\emptyset$.

86. Set the equations equal as follows:

$$\begin{aligned} x^2+1 &= x+1 \\ x^2-x &= 0 \\ x(x-1) &= 0 \\ x &= 0,1 \end{aligned}$$

If $x=0$, then $y=x+1=0+1=1$.

If $x=1$, then $y=x+1=1+1=2$.

The solution set is $\{(0,1),(1,2)\}$.

87. $\$20,000e^{0.05(3.5)} = \$23,824.92$

88. Let t be the number of years.

$$\begin{aligned} 8000 &= 5000e^{0.05t} \\ \ln(8/5) &= 0.05t \\ t &= \frac{\ln(8/5)}{0.05} \\ t &\approx 9.40007 \text{ years} \\ t &\approx 9 \text{ years}, 146 \text{ days} \end{aligned}$$

Thinking Outside the Box LXXV

A rower can either step over another rower or slide to an adjacent empty seat. The total number of moves is the number of slides plus the number of step overs. Each man in the front must step over a woman or be stepped over by a woman. So the minimum number of step overs is $5 \cdot 5$ or 25. Each rower must move a total of 6 spaces. So the total number of spaces moved is $10 \cdot 6$ or 60. Since each step over is two spaces, the number of slides is $60 - 2 \cdot 25$ or 10.

So the minimum number of moves is $10 + 25$ or 35. Represent the five women in the back and five men in the front as $BBBBB\_\_FFFFF$

B will only move to the right, F will only move to the left, and step overs will only occur between opposite types. The 35 moves can be represented as follows:

$$B, F, F, B, B, B, F, F, F, F, B, B, B, B, B,$$
$$F, F, F, F, F, B, B, B, B, B, F, F, F, F,$$
$$B, B, B, F, F, B$$

8.5 Pop Quiz

1. Since $(1,0)$ satisfies the inequality $x-2y>0$ and is a point below the line $x-2y=0$, the graph of $x-2y>0$ is the region below the line $x-2y=0$.

2. Since $(0,0)$ satisfies the inequality $x^2+y^2<9$ and is a point inside the circle $x^2+y^2=9$, the graph of $x^2+y^2<9$ is the region inside the circle $x^2+y^2=9$.

3. No, since $(0,0)$ does not satisfy $x+2y \le -5$.

4. Note, $x^2+y^2=25$ is an equation of a circle with radius 5 and center $(0,0)$. An inequality is

$$x<0,\ y>0,\ x^2+y^2<25.$$

5. The region is a 10-by-4 rectangle. Thus, the area of the region is 40 square units.

8.5 Linking Concepts

a) The vertices of the triangle are $A(0,1)$, $B(\sqrt{3}/2,-1/2)$, and $C(-\sqrt{3}/2,-1/2)$. From these we can derive equations for the lines or sides of the triangles. Namely, they are $y=-\sqrt{3}x+1$, $y=\sqrt{3}x+1$, and $y=-1/2$. A system of inequalities for the set of points on and inside the triangle is

$$\begin{aligned} y &\le -\sqrt{3}x+1 \\ y &\le \sqrt{3}x+1 \\ y &\ge -1/2. \end{aligned}$$

b) Suppose the sides of the square are vertical and horizontal. Then the four vertices of the square are $(\pm\sqrt{2}/2, \pm\sqrt{2}/2)$. A set of equations for the sides are $x=\pm\sqrt{2}/2$ and $y=\pm\sqrt{2}/2$. A system of inequalities for the set of points on and inside the square is

$$\begin{aligned} |x| &\le \sqrt{2}/2 \\ |y| &\le \sqrt{2}/2. \end{aligned}$$

c) Suppose one vertex of the pentagon is at $(0,1)$. Then the other four vertices are $(\cos 18^o, \sin 18^o)$, $(\cos 162^o, \sin 162^o)$, $(\cos 234^o, \sin 234^o)$, and $(\cos 306^o, \sin 306^o)$.

A set of equations for the sides are $y \approx -0.809$, $y \approx 3.078x - 2.618$, $y \approx -3.078x - 2.618$, $y \approx -0.727x + 1$, and $y \approx 0.727x + 1$.
A system of inequalities for the set of points on and inside the pentagon is

$$\begin{aligned} y &\geq -0.809 \\ y &\geq 3.078x - 2.618 \\ y &\geq -3.078x - 2.618 \\ y &\leq -0.727x + 1 \\ y &\leq 0.727x + 1. \end{aligned}$$

d) Suppose one vertex of the hexagon is at $(0, 1)$. Then the other five vertices are $(\cos 150^o, \sin 150^o)$, $(\cos 210^o, \sin 210^o)$, $(\cos 270^o, \sin 270^o)$, and $(\cos 330^o, \sin 330^o)$, and $(\cos 30^o, \sin 30^o)$. By using equations of the sides of the hexagon, a system of inequalities for the set of points on and inside the hexagon is

$$\begin{aligned} |x| &\leq \frac{\sqrt{3}}{2} \\ y &\leq -\frac{\sqrt{3}}{3}x + 1 \\ y &\leq \frac{\sqrt{3}}{3}x + 1 \\ y &\geq \frac{\sqrt{3}}{3}x - 1 \\ y &\geq -\frac{\sqrt{3}}{3}x - 1. \end{aligned}$$

For Thought

1. False, $x \geq 0$ include points on the x-axis and the first and fourth quadrants.

2. False, $y \geq 2$ include points on or above the line $y = 2$. **3.** False

4. False, since x-intercept is $(6, 0)$ and y-intercept is $(0, 4)$. **5.** True **6.** True **7.** False

8. True, since $R(1, 3) = 30(1) + 15(3) = 75$.

9. False, since $C(0, 5) = 7(0) + 9(5) + 3 = 48$.

10. True

8.6 Exercises

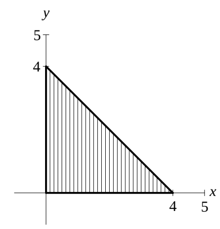
1. constraints

2. feasible

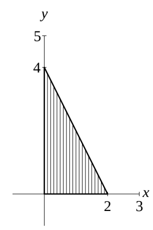
3. natural

4. objective

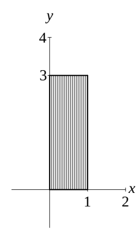
5. Vertices are $(0, 0), (0, 4), (4, 0)$



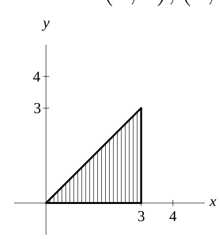
6. Vertices are $(0, 0), (0, 4), (2, 0)$



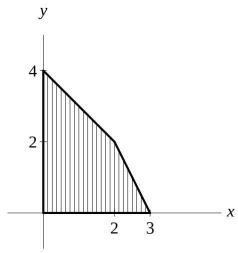
7. Vertices are $(0, 0), (1, 3), (1, 0), (0, 3)$



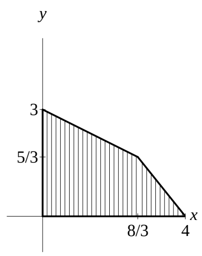
8. Vertices are $(0, 0), (3, 0), (3, 3)$



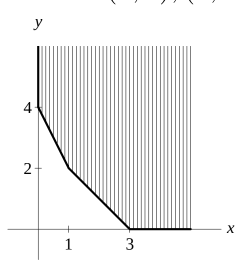
9. Vertices are $(0,0),(2,2),(0,4),(3,0)$



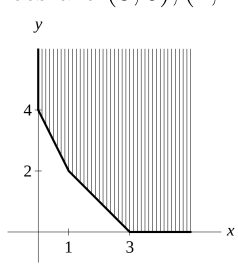
10. Vertices are $(0,0), \left(\frac{8}{3}, \frac{5}{3}\right), (4,0), (0,3)$



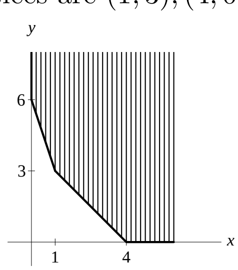
11. Vertices are $(3,0),(1,2),(0,4)$



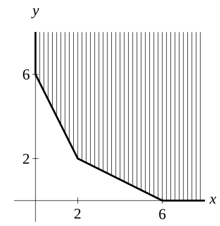
12. Vertices are $(3,0),(1,2),(0,4)$



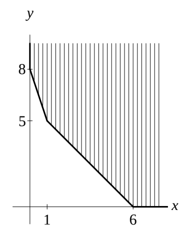
13. Vertices are $(1,3),(4,0),(0,6)$



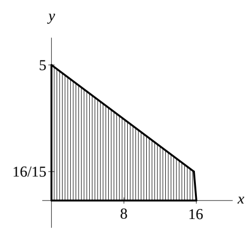
14. Vertices are $(2,2),(6,0),(0,6)$



15. Vertices are $(1,5),(6,0),(0,8)$



16. Vertices are $\left(\frac{236}{15}, \frac{16}{15}\right), (16,0), (0,5), (0,0)$



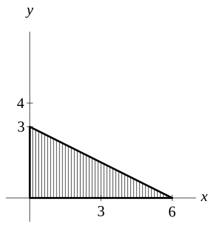
17. The values of $T(x,y) = 2x + 3y$ at the vertices are $T(0,0) = 0$, $T(0,4) = 12$, $T(3,3) = 15$, and $T(5,0) = 10$. The maximum value is 15.

18. The values of $T(x,y) = 2x + 3y$ at the vertices are $T(0,0) = 0$, $T(0,5) = 15$, $T(3,2) = 12$, and $T(4,0) = 8$. The maximum value is 15.

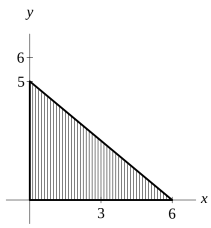
19. The values of $H(x,y) = 2x + 2y$ at the vertices are $T(0,6) = 12$, $T(2,2) = 8$, and $T(5,0) = 10$. The minimum value is 8.

20. The values of $H(x,y) = 2x + 2y$ at the vertices are $T(0,4) = 8$ and $T(3,2) = 10$.
The minimum value is 8.

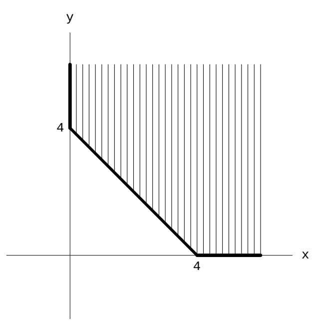
21. The values of $P(x,y) = 5x + 9y$ at the vertices are $P(0,0) = 0$, $P(6,0) = 30$, $P(0,3) = 27$.
Maximum value is 30.



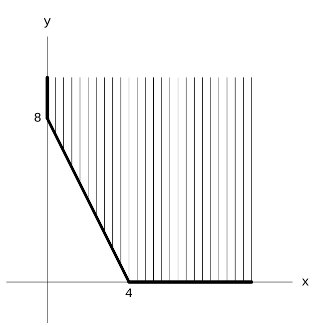
22. The values of $P(x, y) = 25x + 31y$ at the vertices are $P(0, 0) = 0$, $P(6, 0) = 150$, $P(0, 5) = 155$. Maximum value is 155.



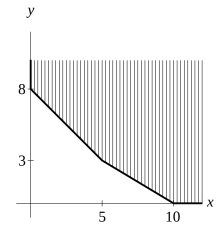
23. The values of $C(x, y) = 3x + 2y$ at the vertices are $C(0, 4) = 8$ and $C(4, 0) = 12$.
The minimum value is 8.



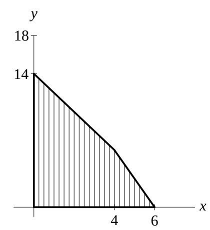
24. The values of $C(x, y) = 2x + 5y$ at the vertices are $C(4, 0) = 8$ and $C(0, 8) = 40$.
The minimum value is 8.



25. The values of $C(x, y) = 10x + 20y$ at the vertices are $C(0, 8) = 160$, $C(5, 3) = 110$, and $C(10, 0) = 100$. Minimum value is 100.



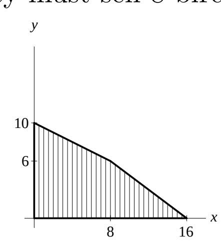
26. The values of $R(x, y) = 50x + 20y$ at the vertices are $R(0, 0) = 0$, $R(0, 14) = 280$, $R(4, 6) = 320$, and $R(6, 0) = 300$. Maximum value is 320.



27. Let x and y be the number of bird houses and mailboxes, respectively.

$$\begin{aligned} \text{Maximize} \quad & 12x + 20y \\ \text{subject to } 3x + 4y &\leq 48 \\ x + 2y &\leq 20 \\ x, y &\geq 0 \end{aligned}$$

The values of $R(x, y) = 12x + 20y$ at the vertices are $R(0, 0) = 0$, $R(0, 10) = 200$, $R(8, 6) = 216$, and $R(16, 0) = 192$. To maximize revenue, they must sell 8 bird houses and 6 mailboxes.

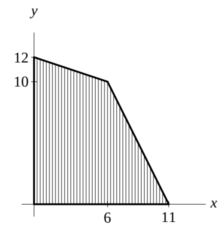


28. Let x and y be the number of tacos and burritos, respectively.

$$\begin{aligned} \text{Maximize} \quad & 0.40x + 0.65y \\ \text{subject to } 2x + y &\leq 22 \\ x + 3y &\leq 36 \\ x, y &\geq 0 \end{aligned}$$

The values of $R(x, y) = 0.40x + 0.65y$ at the vertices are $R(0, 0) = 0$, $R(0, 12) = 7.80$,

$R(6,10) = 8.90$, and $R(11,0) = 4.40$.
To maximize revenue, they must sell
6 tacos and 10 burritos.



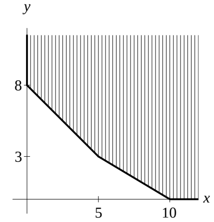
29. Let x and y be the number of bird houses and mailboxes, respectively. The values of $R(x,y) = 18x + 20y$ at the vertices are $R(0,0) = 0$, $R(0,10) = 200$, $R(8,6) = 264$, and $R(16,0) = 288$. To maximize revenue, they must sell 16 bird houses and 0 mailboxes.

30. Let x and y be the number of tacos and burritos, respectively. The values of $R(x,y) = 0.20x + 0.65y$ on the vertices are $R(0,0) = 0$, $R(0,12) = 7.80$, $R(6,10) = 7.70$, and $R(11,0) = 2.20$. To maximize revenue, they must sell 12 burritos and 0 tacos.

31. Let x and y be the number of small and large truck loads, respectively.

$$\begin{aligned} \text{Minimize } 70x+ & \ 60y \\ \text{subject to } 12x + 20y & \geq 120 \\ x + y & \geq 8 \\ x, y & \geq 0 \end{aligned}$$

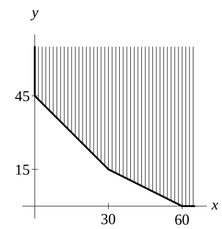
The values of $C(x,y) = 70x + 60y$ at the vertices are $C(0,8) = 480$, $C(10,0) = 700$, and $C(5,3) = 530$, To minimize costs, they must make 8 large truck loads and 0 small truck loads.



32. Let x and y be the number of part-time and full-time employees, respectively.

$$\begin{aligned} \text{Minimize } 120x+ & \ 320y \\ \text{subject to } 20x + 40y & \geq 1200 \\ x + y & \geq 45 \\ x, y & \geq 0 \end{aligned}$$

The values of $C(x,y) = 120x + 320y$ at the vertices are $C(0,45) = 14,400$, $C(30,15) = 8400$, and $C(60,0) = 7200$. To minimize costs, Tina must hire 60 part-time and 0 full-time employees.



33. Let x and y be the number of small and large truck loads, respectively. The values of $C(x,y) = 70x + 75y$ at the vertices are $C(0,8) = 600$, $C(10,0) = 700$, and $C(5,3) = 575$.

To minimize costs, they must make 5 small truck loads and 3 large truck loads.

34. Let x and y be the number of part-time and full-time employees, The values of $C(x,y) = 180x + 320y$ at the vertices are $C(0,45) = 14,400$, $C(30,15) = 10,200$, and $C(60,0) = 10,800$. To minimize costs, Tina must hire 30 part-time and 15 full-time employees.

35. The points (x,y) satisfying $y > x^2 - 2x$ lie in the region enclosed by the parabola $y = x^2 - 2x$. While the points (x,y) satisfying $y < -1 - x$ lie in the region below the line $y = -1 - x$. Since the line lies to the left of the parabola, the system $y > x^2 - 2x$ and $y < -1 - x$ has no solution. The solution set is $\emptyset$.

36. From the equation

$$\frac{11x - 3}{(x+2)(x-3)} = \frac{A}{x+2} + \frac{B}{x-3}$$

we find

$$\begin{aligned} 11x - 3 &= A(x-3) + B(x+2) \\ 11x - 3 &= (A+B)x + (-3A + 2B). \end{aligned}$$

Equating the coefficients, we obtain

$$\begin{aligned} A + B &= 11 \\ -3A + 2B &= -3 \end{aligned}$$

The solutions of the above system of equations are $A = 5$ and $B = 6$.

The partial fraction decomposition is

$$\frac{5}{x+2} + \frac{6}{x-3}.$$

37. Multiply by the LCD as follows:

$$\begin{aligned} \frac{1}{x} + \frac{1}{x-1} &= \frac{3}{2} \\ 2(x-1) + 2x &= 3x(x-1) \\ 4x - 2 &= 3x^2 - 3x \\ 0 &= 3x^2 - 7x + 2 \\ 0 &= (3x-1)(x-2) \end{aligned}$$

Then the solution set is $\{1/3, 2\}$.

38. Rewrite the inequality.

$$\begin{aligned} 5 - 4x + 8 &> 9 \\ -4x &> -4 \\ x &< 1 \end{aligned}$$

The solution set is $(-\infty, 1)$.

39. Rewrite the inequality.

$$\begin{aligned} \frac{3x-2}{x-2} - 1 &> 0 \\ \frac{2x}{x-2} &> 0 \end{aligned}$$

Let $f(x) = \dfrac{2x}{x-2} > 0$, and use test points.
If $x = -1$, then $f(-1) > 0$.
If $x = 1$, then $f(1) < 0$.
If $x = 3$, then $f(3) > 0$.

+	0	−	UD	+
-1	0	1	2	3

The solution set is $(-\infty, 0) \cup (2, \infty)$.

40. Use the formula $A = P\left(1 + \dfrac{r}{n}\right)^{nt}$.

$$\begin{aligned} 4\left(1 + \frac{r}{365}\right)^{365(500/365)} &= 6 \\ \left(1 + \frac{r}{365}\right)^{500} &= 1.5 \\ r &= 365\left(\sqrt[500]{1.5} - 1\right) \\ r &\approx 0.296 \end{aligned}$$

The annual interest rate is 29.6%.

Thinking Outside the Box LXXVI

Let x be the distance between B and the point P_1 of tangency along side BC of the circle on the left, and let y be the distance between D and P_1. Then the distance between B and D is $\overline{BD} = x + y$.

Let P_2 be the point of tangency along side AB of the circle on the left. Since $\overline{AB} = 7$, we find $\overline{AP_2} = 7 - x$.

Let Q_1 be the point of tangency along side AC of the circle on the right. Since $\overline{BC} = 10$, we find $\overline{CQ_1} = 10 - 2y - x$. Since $\overline{AC} = 12$, we obtain $\overline{AQ_1} = 2 + x + 2y$.

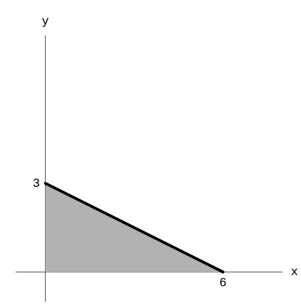
Since $\overline{AP_2} = \overline{AQ_1}$, we find

$$2 + x + 2y = 7 - x.$$

Then $2x + 2y = 5$ and $x + y = 2.5$. Thus, $\overline{BD} = 2.5$.

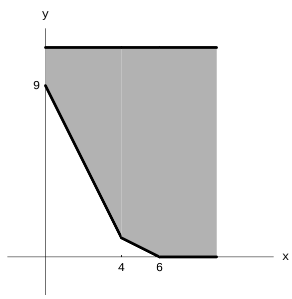
8.6 Pop Quiz

1. The x- and y-intercepts of the line $x + 2y = 6$ are $(6, 0)$ and $(0, 3)$, respectively. The region is a triangular region in the first quadrant with vertices $(0, 0)$, $(6, 0)$, and $(0, 3)$.



2. Since $P(x, y) = 20x + 50y - 100$, the values of P at the vertices $(0, 0)$, $(6, 0)$, and $(0, 3)$ are $P(0, 0) = -100$, $P(6, 0) = 20$, and $P(0, 3) = 50$. Thus, the maximum value of P is 50.

3. The vertices are $(0, 9)$, $(4, 1)$, and $(6, 0)$ as shown below.



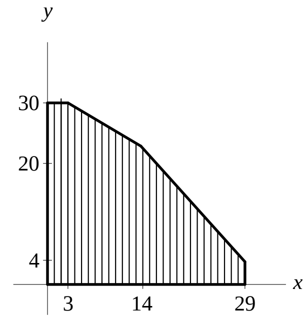
4. Since $C(x, y) = 5x + 4y$, the values of C at the vertices $(0, 9)$, $(4, 1)$, and $(6, 0)$ are $C(0, 9) = 36$, $C(4, 1) = 24$, and $C(6, 0) = 30$. Thus, the maximum value of C is 24.

8.6 Linking Concepts

a) Let x and y be the number of bookcases and desks, respectively. To maximize revenue, solve the optimization problem

$$\begin{array}{rcl} \text{Maximize } 400y + & 175x & \\ \text{subject to } 30y + 20x & \leq & 960 \\ 12y + 15x & \leq & 480 \\ \frac{1}{4}y + \frac{1}{8}x & \leq & 8 \\ \frac{3}{8}y + \frac{1}{4}x & \leq & 15 \\ 9y & \leq & 270 \\ 20x & \leq & 580 \\ 7y & \leq & 350 \\ 20y + 12x & \leq & 720 \\ \text{where } x, y & \geq & 0 \text{ are integers.} \end{array}$$

Several of the constraints are not necessary. For instance, this can be seen from the feasible region, as shown below.



The vertices of the region are $(0, 0)$, $(0, 30)$, $(3, 30)$, $\left(\frac{96}{7}, \frac{160}{7}\right)$, $(29, 3.75)$, and $(29, 0)$. One can show that the objective function $400y + 175x$ attains a maximum value at the vertex $(3, 30)$. Thus, to maximize revenue (which is \$12,525), Lucy must make 3 bookcases and 30 desks.

b) By making 3 bookcases and 30 desks, Lucy has used all her supply of plywood and drawer pulls; also, she has still some supplies left of the other items. Observe that $400y + 175x = c$ represents parallel lines for different values of c; the larger the value of c the farther is the line away from the origin.

If Lucy could increase the supply of plywood, then we can drop the constraint

$$30y + 20x \leq 960$$

from the optimization problem in part a). Then the vertices of the feasible region are $(0, 0)$, $(0, 30)$, $(4, 30)$ $\left(\frac{32}{3}, \frac{80}{3}\right)$, $(29, 3.75)$, and $(29, 0)$. By using the slope of the objective function, one can show that the maximum revenue is \$12,700 and is attained at the vertex $(4, 30)$.

If Lucy could increase the supply of drawer pulls, then we can drop the constraint

$$9y \leq 270$$

from the optimization problem in part a). Then the vertices of the feasible region are

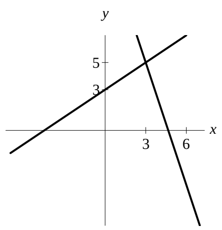
$(0, 0)$, $(0, 32)$, $\left(\frac{96}{7}, \frac{160}{7}\right)$, $(29, 3.75)$, and $(29, 0)$.

Similarly, by using the slope of the objective function, one can show that the maximum revenue is \$12,800 and is attained at the vertex $(0, 32)$.

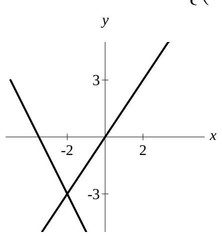
Hence, if Lucy could increase the supply of one item, she would increase her supply of drawer pulls and the maximum revenue will be \$12,800.

Review Exercises

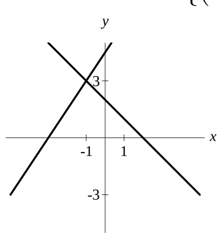
1. The solution set is $\{(3, 5)\}$.



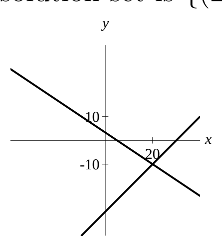
2. The solution set is $\{(-2, -3)\}$.



3. The solution set is $\{(-1, 3)\}$



4. The solution set is $\{(20, -10)\}$.



5. Substitute $y = x$ into $3x - 5y = 19$.

$$\begin{aligned} 3x - 5x &= 19 \\ -2x &= 19 \\ x &= -19/2 \end{aligned}$$

Independent and the solution set is $\{(-19/2, -19/2)\}$.

6. Substitute $y = x - 3$ into $x + y = 9$.

$$\begin{aligned} x + (x - 3) &= 9 \\ 2x &= 12 \end{aligned}$$

Using $x = 6$ in $y = x - 3$, we find $y = 3$. Independent and the solution set is $\{(6, 3)\}$.

7. Multiply $4x - 3y = 6$ by 2 and $3x + 2y = 9$ by 3, then add the equations.

$$\begin{aligned} 8x - 6y &= 12 \\ 9x + 6y &= 27 \\ \hline 17x &= 39 \end{aligned}$$

Substitute $x = 39/17$ into $4x - 3y = 6$.

$$\begin{aligned} \frac{156}{17} - 3y &= 6 \\ \frac{54}{17} &= 3y \\ \frac{18}{17} &= y \end{aligned}$$

Independent and the solution set is $\{(39/17, 18/17)\}$.

8. Multiply $3x - 2y = 4$ by -5 and $5x + 7y = 1$ by 3. Then add the equations.

$$\begin{aligned} -15x + 10y &= -20 \\ 15x + 21y &= 3 \\ \hline 31y &= -17 \end{aligned}$$

Using $y = -17/31$ in $3x - 2y = 4$, we find

$$\begin{aligned} 3x + \frac{34}{31} &= 4 \\ 3x &= \frac{90}{31} \\ x &= \frac{30}{31}. \end{aligned}$$

Independent and the solution set is $\{(30/31, -17/31)\}$.

9. Substitute $y = -3x + 1$ into $6x + 2y = 2$.

$$\begin{aligned} 6x + 2(-3x + 1) &= 2 \\ 6x - 6x + 2 &= 2 \\ 2 &= 2 \end{aligned}$$

Dependent and the solution set is $\{(x, y) \mid y = -3x + 1\}$.

10. Substitute $x = y + 9$ into $2y - 2x = -18$.

$$\begin{aligned} 2y - 2(y+9) &= -18 \\ 2y - 2y - 18 &= -18 \\ -18 &= -18 \end{aligned}$$

Dependent and the solution set is $\{(x, y) \mid x - y = 9\}$.

11. Multiply $3x - 4y = 12$ by 2 and add to the second equation.

$$\begin{array}{rcr} 6x - 8y &=& 24 \\ -6x + 8y &=& 9 \\ \hline 0 &=& 33 \end{array}$$

Inconsistent and the solution set is $\emptyset$.

12. Substitute $y = -5x + 3$ into $5x + y = 6$.

$$\begin{aligned} 5x + (-5x + 3) &= 6 \\ 3 &= 6 \end{aligned}$$

Inconsistent and the solution set is $\emptyset$.

13. Add the first and second equations. Multiply $2x + y + z = 1$ by -3 and add to the third equation.

$$\begin{array}{rcr} x + y - z &=& 8 \\ 2x + y + z &=& 1 \\ \hline 3x + 2y &=& 9 \end{array}$$

$$\begin{array}{rcr} -6x - 3y - 3z &=& -3 \\ x + 2y + 3z &=& -5 \\ \hline -5x - y &=& -8 \end{array}$$

Multiply $-5x - y = -8$ by 2 and add to $3x + 2y = 9$.

$$\begin{array}{rcr} -10x - 2y &=& -16 \\ 3x + 2y &=& 9 \\ \hline -7x &=& -7 \end{array}$$

Using $x = 1$ in $3x + 2y = 9$, $3 + 2y = 9$ or $y = 3$. From $x + y - z = 8$, $1 + 3 - z = 8$ or $z = -4$. The Solution set is $\{(1, 3, -4)\}$.

14. Multiply first equation by 2 and add to the second equation. Multiply third equation by 4 and add to the second equation.

$$\begin{array}{rcr} 4x + 6y - 4z &=& 16 \\ 3x - y + 4z &=& -20 \\ \hline 7x + 5y &=& -4 \end{array}$$

$$\begin{array}{rcr} 4x + 4y - 4z &=& 12 \\ 3x - y + 4z &=& -20 \\ \hline 7x + 3y &=& -8 \end{array}$$

Multiply $7x + 5y = -4$ by -1 and add to $7x + 3y = -8$.

$$\begin{array}{rcr} -7x - 5y &=& 4 \\ 7x + 3y &=& -8 \\ \hline -2y &=& -4 \end{array}$$

Using $y = 2$ in $7x + 5y = -4$, we get $7x + 10 = -4$ or $x = -2$. From $x + y - z = 3$, we find $-2 + 2 - z = 3$ and $z = -3$. Solution set is $\{(-2, 2, -3)\}$.

15. Multiply first equation by -2 and add to the second equation. Multiply first equation by -2 and add to the third one.

$$\begin{array}{rcr} -2x - 2y - 2z &=& -2 \\ 2x - y + 2z &=& 2 \\ \hline -3y &=& 0 \\ y &=& 0 \end{array}$$

$$\begin{array}{rcr} -2x - 2y - 2z &=& -2 \\ 2x + 2y + 2z &=& 2 \\ \hline 0 &=& 0 \end{array}$$

Using $y = 0$ in $x + y + z = 1$, $x + z = 1$ and $z = 1 - x$. The solution set is $\{(x, 0, 1 - x) \mid x \text{ is any real number}\}$.

16. Add first and second equations. Multiply first equation by 2 and add to the third one.

$$\begin{array}{rcr} x - y - z &=& 9 \\ x + y + 2z &=& -9 \\ \hline 2x + z &=& 0 \end{array}$$

$$\begin{array}{rcr} 2x - 2y - 2z &=& 18 \\ -2x + 2y + 2z &=& -18 \\ \hline 0 &=& 0 \end{array}$$

Substituting $z = -2x$ in $x - y - z = 9$, $x - y + 2x = 9$ or $y = 3x - 9$. Solution set is $\{(x, 3x - 9, -2x) \mid x \text{ is any real number}\}$.

17. Multiply first equation by −1 and add to the third equation.

$$\begin{array}{rcr} -x-y-z & = & -1 \\ x+y+z & = & 4 \\ \hline 0 & = & 3 \end{array}$$

Inconsistent and the solution set is $\emptyset$.

18. Add the second and third equations.

$$\begin{array}{rcr} x-y+z & = & -1 \\ -x+y-z & = & 0 \\ \hline 0 & = & -1 \end{array}$$

Inconsistent and the solution set is $\emptyset$.

19. Substitute $x = y^2$ into $x^2 + y^2 = 4$ and use the quadratic formula.

$$\begin{aligned} y^4 + y^2 &= 4 \\ y^4 + y^2 - 4 &= 0 \\ y^2 &= \frac{-1+\sqrt{17}}{2} \\ y &= \pm\sqrt{\frac{-1+\sqrt{17}}{2}} \end{aligned}$$

Thus, $x = y^2 = \dfrac{-1+\sqrt{17}}{2}$.

The solution set is

$$\left\{ \left(\frac{-1+\sqrt{17}}{2}, \pm\sqrt{\frac{-1+\sqrt{17}}{2}} \right) \right\}.$$

20. Adding $x^2 - y^2 = 9$ and $x^2 + y^2 = 7$, we have $2x^2 = 16$. So $x = \pm\sqrt{8}$.
Using $x = \pm\sqrt{8}$ in $x^2 - y^2 = 9$, $8 - y^2 = 9$ and so $y = \pm i$.
There are no real solutions.

21. Substitute $y = x^2$ into $y = |x|$.

$$\begin{aligned} x^2 &= \sqrt{x^2} \\ x^4 &= x^2 \\ x^2(x^2-1) &= 0 \\ x &= 0, \pm 1 \end{aligned}$$

Using $x = 0, 1, -1$ in $y = x^2$, we get $y = 0, 1, 1$, respectively. The solution set is $\{(0,0), (1,1), (-1,1)\}$.

22. Substitute $y = 12 - 6x$ into $y = 2x^2 + x - 3$.

$$\begin{aligned} 2x^2 + x - 3 &= 12 - 6x \\ 2x^2 + 7x - 15 &= 0 \\ (x+5)(2x-3) &= 0 \\ x &= -5, 3/2 \end{aligned}$$

Using $x = -5, 3/2$ in $y = 12 - 6x$, we find $y = 42, 3$. Solution set is $\{(-5, 42), (3/2, 3)\}$.

23. Note, $\dfrac{7x-7}{(x-3)(x+4)} = \dfrac{A}{x-3} + \dfrac{B}{x+4}$.
Then

$$\begin{aligned} 7x - 7 &= A(x+4) + B(x-3) \\ 7x - 7 &= (A+B)x + (4A - 3B). \end{aligned}$$

Equating the coefficients, we obtain

$$\begin{aligned} A + B &= 7 \\ 4A - 3B &= -7. \end{aligned}$$

The solution of this system is $A = 2$, $B = 5$.
The answer is $\dfrac{2}{x-3} + \dfrac{5}{x+4}$.

24. Note, $\dfrac{x-13}{(x-5)(x-1)} = \dfrac{A}{x-5} + \dfrac{B}{x-1}$.
Then

$$\begin{aligned} x - 13 &= A(x-1) + B(x-5) \\ x - 13 &= (A+B)x + (-A - 5B). \end{aligned}$$

Equating the coefficients, we obtain

$$\begin{aligned} A + B &= 1 \\ -A - 5B &= -13. \end{aligned}$$

The solution of this system is $A = -2$, $B = 3$.
The answer is $\dfrac{-2}{x-5} + \dfrac{3}{x-1}$.

25. Factoring the denominator, we obtain

$$\begin{aligned} x^3 - 3x^2 + 4x - 12 &= x^2(x-3) + 4(x-3) \\ &= (x^2+4)(x-3), \end{aligned}$$

and so $\dfrac{7x^2 - 7x + 23}{(x-3)(x^2+4)} = \dfrac{A}{x-3} + \dfrac{Bx+C}{x^2+4}$.
Then $7x^2 - 7x + 23 =$

$A(x^2+4)+(Bx+C)(x-3) =$
$= (A+B)x^2+(-3B+C)x+(4A-3C)$.
Equating the coefficients, we have

$$\begin{aligned} A+B &= 7 \\ -3B+C &= -7 \\ 4A-3C &= 23. \end{aligned}$$

The solution of this system is $A=5$, $B=2$, and $C=-1$. The answer is $\dfrac{5}{x-3}+\dfrac{2x-1}{x^2+4}$.

26. Note,

$$\frac{10x^2-6x+2}{(x-1)^2(x+2)} = \frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{x+2}.$$

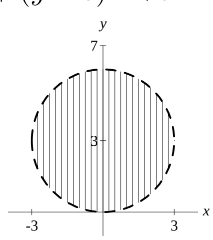
Then $10x^2-6x+2 =$
$= A(x-1)(x+2)+B(x+2)+C(x-1)^2$
$= (A+C)x^2+(A+B-2C)x+(-2A+2B+C)$.

Equating the coefficients, we find

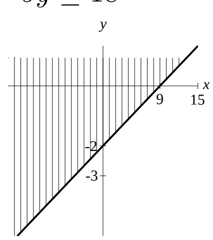
$$\begin{aligned} A+C &= 10 \\ A+B-2C &= -6 \\ -2A+2B+C &= 2. \end{aligned}$$

The solution of this system is $A=4$, $B=2$, $C=6$. The answer is $\dfrac{4}{x-1}+\dfrac{2}{(x-1)^2}+\dfrac{6}{x+2}$.

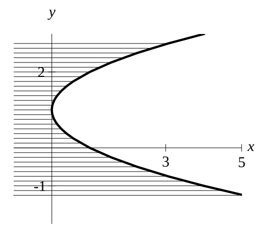
27. $x^2+(y-3)^2<9$



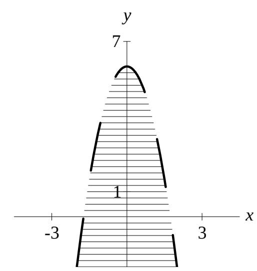
28. $2x-9y\le 18$



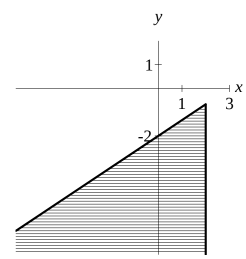
29. $x\le (y-1)^2$



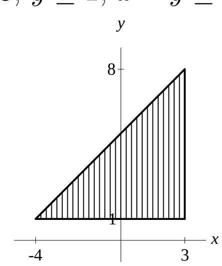
30. $y<6-2x^2$



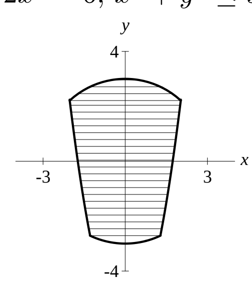
31. $2x-3y\ge 6$, $x\le 2$



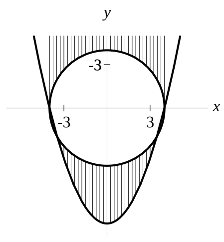
32. $x\le 3$, $y\ge 1$, $x-y\ge -5$



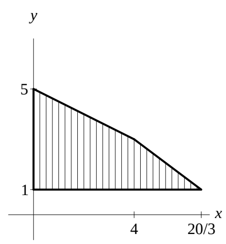
33. $y\ge 2x^2-6$, $x^2+y^2\le 9$



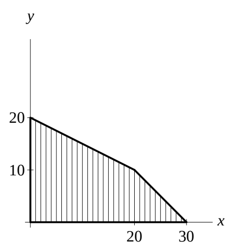
34. $x^2 + y^2 \geq 16$, $2y \geq x^2 - 16$



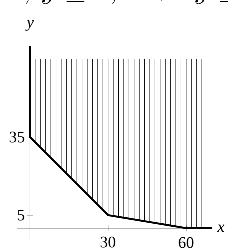
35. $x \geq 0$, $y \geq 1$, $x + 2y \leq 10$, $3x + 4y \leq 24$



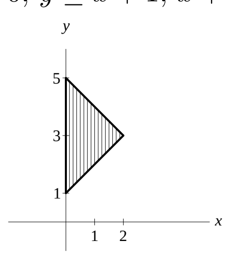
36. $x \geq 0$, $y \geq 0$, $30x + 60y \leq 1200$, $x + y \leq 30$



37. $x \geq 0$, $y \geq 0$, $x + 6y \geq 60$, $x + y \geq 35$



38. $x \geq 0$, $y \geq x + 1$, $x + y \leq 5$



39. Substitute $(-2, 3)$ and $(4, -1)$ into $y = mx + b$.

$$\begin{aligned} -2m + b &= 3 \\ 4m + b &= -1 \end{aligned}$$

Multiply first equation by -1 and add to the second equation.

$$\begin{aligned} 2m - b &= -3 \\ 4m + b &= -1 \\ \hline 6m &= -4 \end{aligned}$$

Using $m = -2/3$ in $-2m + b = 3$, $4/3 + b = 3$ and $b = 5/3$. Equation of line is $y = -\dfrac{2}{3}x + \dfrac{5}{3}$.

40. Substitute $(4, 7)$ and $(-2, -3)$ into $y = mx + b$.

$$\begin{aligned} 4m + b &= 7 \\ -2m + b &= -3 \end{aligned}$$

Multiply first equation by -1 and add to the second equation.

$$\begin{aligned} -4m - b &= -7 \\ -2m + b &= -3 \\ \hline -6m &= -10 \end{aligned}$$

Using $m = 5/3$ in $-2m + b = -3$, we obtain $-10/3 + b = -3$ and $b = 1/3$.
An equation of the line is $y = \dfrac{5}{3}x + \dfrac{1}{3}$.

41. Substitute $(1, 4)$, $(3, 20)$, and $(-2, 25)$ into $y = ax^2 + bx + c$.

$$\begin{aligned} a + b + c &= 4 \\ 9a + 3b + c &= 20 \\ 4a - 2b + c &= 25 \end{aligned}$$

The solution of the above system is $a = 3$, $b = -4$, $c = 5$. The parabola is given by $y = 3x^2 - 4x + 5$.

42. Substitute $(-1, 10)$, $(2, -5)$, and $(3, -18)$ into $y = ax^2 + bx + c$.

$$\begin{aligned} a - b + c &= 10 \\ 4a + 2b + c &= -5 \\ 9a + 3b + c &= -18 \end{aligned}$$

The solution of the above system is $a = -2$, $b = -3$, $c = 9$.
The parabola is given by $y = -2x^2 - 3x + 9$.

43. Let x and y be the number of tacos and burritos, respectively.

$$\begin{aligned} x + 2y &= 181 \\ 2x + 3y &= 300 \end{aligned}$$

Solving the above system, we find $x = 57$ tacos and $y = 62$ burritos.

44. Let x and y be the number of cars originally in Nicholas' and Seymour's lot, respectively.

$$\begin{aligned} x + y &= 300 \\ 0.10x + 0.30y &= 300(0.22) \end{aligned}$$

Solving the above system, we get $x = 120$ and $y = 180$. Nicholas had originally 120 cars and Seymour had 180 cars.

45. Let x, y and z be the selling price of a daisy, carnation, and a rose, respectively. Then

$$\begin{aligned} 5x + 3y + 2z &= 3.05 \\ 3x + y + 4z &= 2.75 \\ 4x + 2y + z &= 2.10. \end{aligned}$$

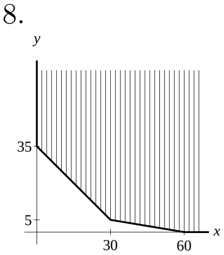
Solving the above system, we find $x = 0.30$, $y = 0.25$, and $z = 0.40$. Esther's economy special sells for $x + y + z = \$0.95$.

46. Let x, y and z be the number of peppers, tomatoes, and eggplants, respectively. Then

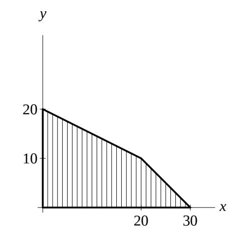
$$\begin{aligned} x + y + z &= 81 \\ 0.20x + 0.35y + 0.30z &= 23.85 \\ 2(x + y) &= z. \end{aligned}$$

Solving the above system, we get $x = 12$ peppers, $y = 15$ tomatoes, and $z = 54$ eggplants.

47. The values of $C(x, y) = 0.42x + 0.84y$ at the vertices are $C(0, 35) = 29.4$, $C(30, 5) = 16.8$, and $C(60, 0) = 25.2$. The minimum value is 16.8.



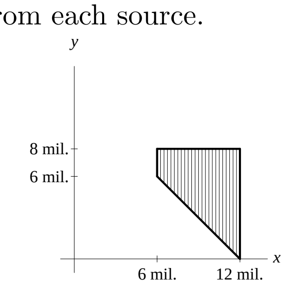
48. The values of $P(x, y) = 1.23x + 1.64y$ at the vertices are $P(0, 0) = 0$, $P(0, 20) = 32.8$, $P(20, 10) = 41$, and $P(30, 0) = 36.9$.
The maximum value is 41.



49. Let x and y be the number of barrels of oil obtained through the pipeline and barges, respectively.

$$\begin{aligned} \text{Minimize } 100x + 90y & \\ \text{subject to } x + y &\geq 12,000,000 \\ x &\leq 12,000,000 \\ x &\geq 6,000,000 \\ y &\leq 8,000,000 \\ x, y &\geq 0 \end{aligned}$$

The values of $C(x, y) = 20x + 18y$ at the vertices are
$C(12 \text{ million}, 0) = 1200$ million,
$C(12 \text{ million}, 8 \text{ million}) = 1920$ million,
$C(6 \text{ million}, 8 \text{ million}) = 1320$ million, and
$C(6 \text{ million}, 6 \text{ million}) = 1140$ million.
To minimize cost, purchase 6 million barrels from each source.



50. The values of $C(x, y) = 100x + 105y$ at the vertices are
$C(12 \text{ million}, 0) = 1200$ million,
$C(12 \text{ million}, 8 \text{ million}) = 2040$ million,
$C(6 \text{ million}, 8 \text{ million}) = 1440$ million, and
$C(6 \text{ million}, 6 \text{ million}) = 1230$ million.
To minimize cost, purchase 12 million barrels from the pipeline and 0 barrels from the barges.

Thinking Outside the Box

LXXVII. Dan can get 30,000 miles by rotating the tires every 6000 miles as follows:

$$S \to LR \to LF \to RF \to RR \to S$$

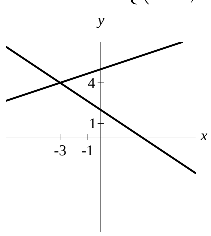
where S is the spare tire.

LXXVIII. Plant 101 trees on the 1000 foot edge, Then offset the next row so that each tree is the top vertex of an equilateral triangle with side of 10 feet. This gives 174 rows with 101 trees in each row plus 173 rows with 100 trees in each row for a total of 34,874 trees.

This *seems* to be the maximum number of trees that can be planted. I could not find a way of planting more trees.

Chapter 8 Test

1. Solution set is $\{(-3, 4)\}$.



2. Substitute $y = 4 - 2x$ into $3x - 4y = 9$.

$$\begin{aligned} 3x - 4(4 - 2x) &= 9 \\ 3x - 16 + 8x &= 9 \\ 11x &= 25 \\ x &= 25/11 \end{aligned}$$

Using $x = 25/11$ in $y = 4 - 2x$, we have $y = 4 - 50/11 = -6/11$. Solution set is $\{(25/11, -6/11)\}$.

3. Multiply $10x - 3y = 22$ by 2 and $7x + 2y = 40$ by 3. Then add the equations.

$$\begin{aligned} 20x - 6y &= 44 \\ 21x + 6y &= 120 \\ \hline 41x &= 164 \\ x &= 4 \end{aligned}$$

Using $x = 4$ in $10x - 3y = 22$, we get $40 - 3y = 22$ and $y = 6$. The solution set is $\{(4, 6)\}$.

4. Substitute $x = 6 - y$ into $3x + 3y = 4$.

$$\begin{aligned} 3(6 - y) + 3y &= 4 \\ 18 - 3y + 3y &= 4 \\ 18 &= 4 \end{aligned}$$

The system is inconsistent.

5. Substitute $y = \frac{1}{2}x + 3$ into $x - 2y = -6$.

$$\begin{aligned} x - 2\left(\frac{1}{2}x + 3\right) &= -6 \\ x - x - 6 &= -6 \\ -6 &= -6 \end{aligned}$$

The system is dependent.

6. Substitute $y = 2x - 1$ into $y = 3x + 20$.

$$\begin{aligned} 2x - 1 &= 3x + 20 \\ -21 &= x \end{aligned}$$

Using $x = -21$ in $y = 2x - 1$, we get $y = -43$. The system is independent.

7. Substitute $y = -x + 2$ into $y = -x + 5$.

$$\begin{aligned} -x + 2 &= -x + 5 \\ 2 &= 5 \end{aligned}$$

The system is inconsistent.

8. Add the two equations.

$$\begin{aligned} 2x - y + z &= 4 \\ -x + 2y - z &= 6 \\ \hline x + y &= 10 \end{aligned}$$

Using $y = 10 - x$ in $2x - y + z = 4$, we find $2x - (10 - x) + z = 4$ and $z = 14 - 3x$. The solution set is $\{(x, 10 - x, 14 - 3x) \mid x \text{ is any real number}\}$.

9. Add the first two equations. Also, multiply second equation by 3 and add to the third.

$$\begin{aligned} x - 2y - z &= 2 \\ 2x + 3y + z &= -1 \\ \hline 3x + y &= 1 \end{aligned}$$

$$\begin{array}{rcl} 6x+9y+3z &=& -3 \\ 3x-y-3z &=& -4 \\ \hline 9x+8y &=& -7 \end{array}$$

Multiply $3x+y=1$ by -3 and add to $9x+8y=-7$.

$$\begin{array}{rcl} -9x-3y &=& -3 \\ 9x+8y &=& -7 \\ \hline 5y &=& -10 \end{array}$$

Using $y=-2$ in $3x+y=1$, we get $3x-2=1$ or $x=1$. From $x-2y-z=2$, we have $1+4-z=2$ or $z=3$. The solution set is $\{(1,-2,3)\}$.

10. Add the second and third equations.

$$\begin{array}{rcl} x+y-z &=& 4 \\ -x-y+z &=& 2 \\ \hline 0 &=& 6 \end{array}$$

Inconsistent and the solution set is $\emptyset$.

11. Multiply $x^2+y^2=16$ by -1 and add to $x^2-4y^2=16$.

$$\begin{array}{rcl} -x^2-y^2 &=& -16 \\ x^2-4y^2 &=& 16 \\ \hline -5y^2 &=& 0 \\ y &=& 0 \end{array}$$

Using $y=0$ in $x^2+y^2=16$, we find $x^2=16$ and $x=\pm 4$. Solution set is $\{(4,0),(-4,0)\}$.

12. Substitute $y=x^2-5x$ into $x+y=-2$.

$$\begin{aligned} x+(x^2-5x) &= -2 \\ x^2-4x &= -2 \\ x^2-4x+4 &= -2+4 \\ (x-2)^2 &= 2 \\ x &= 2\pm\sqrt{2} \end{aligned}$$

Using $x=2+\sqrt{2}$ and $x=2-\sqrt{2}$ in $y=-2-x$, we have $y=-4-\sqrt{2}$ and $y=-4+\sqrt{2}$, respectively. The solution set is

$$\{(2+\sqrt{2},-4-\sqrt{2}),(2-\sqrt{2},-4+\sqrt{2})\}.$$

13.

$$\begin{aligned} \frac{2x+10}{(x-4)(x+2)} &= \frac{A}{x-4}+\frac{B}{x+2} \\ 2x+10 &= A(x+2)+B(x-4) \\ 2x+10 &= (A+B)x+(2A-4B) \end{aligned}$$

Equating the coefficients, we obtain

$$\begin{aligned} A+B &= 2 \\ 2A-4B &= 10. \end{aligned}$$

The solution of this system is $A=3, B=-1$. The answer is $\dfrac{3}{x-4}+\dfrac{-1}{x+2}$.

14. Note, $\dfrac{4x^2+x-2}{x^2(x-1)}=\dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{C}{x-1}$. Then

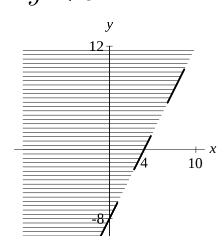
$$\begin{aligned} 4x^2+x-2 &= Ax(x-1)+B(x-1)+Cx^2 \\ &= (A+C)x^2+(-A+B)x-B. \end{aligned}$$

Equating the coefficients, we have

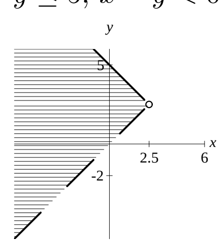
$$\begin{aligned} A+C &= 4 \\ -A+B &= 1 \\ -B &= -2. \end{aligned}$$

The solution of this system is $B=2, A=1$, and $C=3$. The answer is $\dfrac{1}{x}+\dfrac{2}{x^2}+\dfrac{3}{x-1}$.

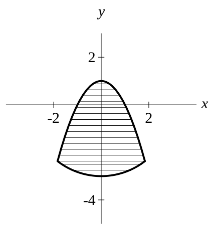
15. $2x-y<8$



16. $x+y\le 5$, $x-y<0$



17. $x^2 + y^2 \le 9$, $y \le 1 - x^2$



18. Let x and y be the number of male and female students, respectively. Then

$$\begin{aligned} \frac{1}{3}x + \frac{1}{4}y &= 15 \\ x + y &= 52. \end{aligned}$$

Multiply the first equation by 12 and multiply the second equation by -3. Then add the resulting equations.

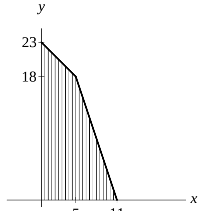
$$\begin{array}{rcr} 4x + 3y &=& 180 \\ -3x - 3y &=& -156 \\ \hline x &=& 24 \end{array}$$

The solution of this system is $x = 24$ males and $y = 28$ females.

19. Let x and y be the number of TV commercials and newspaper ads, respectively. The linear program is given below.

$$\begin{aligned} &\text{Maximize } 14,000x + 6000y \\ &\text{subject to } 9000x + 3000y \le 99,000 \\ &\qquad x + y \le 23 \\ &\qquad x, y \ge 0 \end{aligned}$$

The values of $N(x, y) = 14,000x + 6000y$ at the vertices are $N(0, 0) = 0$, $N(0, 23) = 138,000$, $N(5, 18) = 178,000$, and $N(11, 0) = 154,000$. To obtain maximum audience exposure, the hospital must have 5 TV commercials and 18 newspaper ads.



Tying It All Together

1. Multiply the equation by $24(x + 5)$.

$$\begin{aligned} 24(x-2) &= 11(x+5) \\ 24x - 48 &= 11x + 55 \\ 13x &= 103 \end{aligned}$$

The solution set is $\{103/13\}$.

2. Multiply the equation by $24x(x + 5)$.

$$\begin{aligned} 24(x+5) + 24x(x-2) &= 11x(x+5) \\ 24x + 120 + 24x^2 - 48x &= 11x^2 + 55x \\ 13x^2 - 79x + 120 &= 0 \\ (13x - 40)(x - 3) &= 0 \end{aligned}$$

The solution set is $\{40/13, 3\}$.

3.

$$\begin{aligned} 5 - 3x - 6 - 2x + 4 &= 7 \\ 3 - 5x &= 7 \\ -5x &= 4 \end{aligned}$$

The solution set is $\{-4/5\}$.

4. We will solve an equivalent statement without absolute values.

$$\begin{array}{rcl} 3 - 2x = 5 & \text{or} & 3 - 2x = -5 \\ -2x = 2 & \text{or} & -2x = -8 \\ x = -1 & \text{or} & x = 4 \end{array}$$

The solution set is $\{4, -1\}$.

5. Square both sides of the equation.

$$\begin{aligned} 3 - 2x &= 25 \\ -2x &= 22 \end{aligned}$$

The solution set is $\{-11\}$.

6. Isolate x^2 on one side and take the square roots.

$$\begin{aligned} 3x^2 &= 4 \\ x^2 &= \frac{4}{3} \\ x &= \pm\frac{2}{\sqrt{3}} \\ x &= \pm\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \end{aligned}$$

The solution set is $\left\{\pm\dfrac{2\sqrt{3}}{3}\right\}$.

7. Multiply equation by x^2.

$$\begin{aligned}(x-2)^2 &= x^2\\ x^2-4x+4 &= x^2\\ -4x &= -4\end{aligned}$$

The solution set is $\{1\}$.

8. Since $2^{x-1}=9$, we obtain $x-1=\log_2(9)$ by using the definition of a logarithm. The solution set is $\{1+\log_2(9)\}$.

9. Write left-hand side as a single logarithm.

$$\begin{aligned}\log((x+1)(x+4)) &= 1\\ x^2+5x+4 &= 10^1\\ x^2+5x-6 &= 0\\ (x+6)(x-1) &= 0\\ x &= -6, 1\end{aligned}$$

But $\log(x+1)$ is undefined when $x=-6$. The solution set is $\{1\}$.

10. Raise both sides of equation to the power $-3/2$.

$$\begin{aligned}x^{-2/3} &= \frac{1}{4}\\ x &= \pm\left(\frac{1}{4}\right)^{-3/2}\\ x &= \pm(4)^{3/2}\\ x &= \pm(4^{1/2})^3\\ x &= \pm(2)^3\end{aligned}$$

The solution set is $\{\pm 8\}$.

11. Use the quadratic formula.

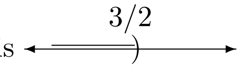
$$\begin{aligned}x^2-3x-6 &= 0\\ x &= \frac{3\pm\sqrt{33}}{2}\end{aligned}$$

The solution set is $\left\{\dfrac{3\pm\sqrt{33}}{2}\right\}$.

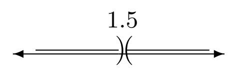
12. By using the square root property, we obtain

$$\begin{aligned}(x-3)^2 &= \frac{1}{2}\\ x-3 &= \pm\frac{\sqrt{2}}{2}\\ x &= \frac{6}{2}\pm\frac{\sqrt{2}}{2}.\end{aligned}$$

The solution set is $\left\{\dfrac{6\pm\sqrt{2}}{2}\right\}$.

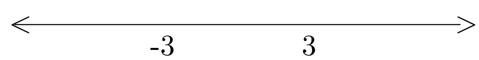
13. Since $3-2x>0$, we get $3>2x$ and $x<3/2$. The solution set is $(-\infty, 3/2)$ and its graph is 

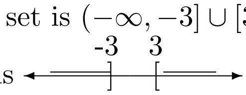
14. Since we must exclude $x=\dfrac{3}{2}$, the solution set is $(-\infty, 1.5)\cup(1.5,\infty)$ and the graph is



15. The sign graph of $(x-3)(x+3)\geq 0$ is shown below.

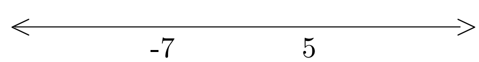
- - - - - - - - 0 + + + +
- - - - 0 + + + + + + + +

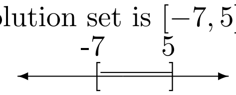


So the solution set is $(-\infty,-3]\cup[3,\infty)$ and the graph is 

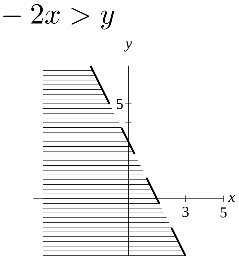
16. Note that $x^2+2x-8\leq 27$ is equivalent to $x^2+2x-35\leq 0$. The sign graph of $(x+7)(x-5)\leq 0$ is

- - - - - - - - 0 + + + +
- - - - 0 + + + + + + + +

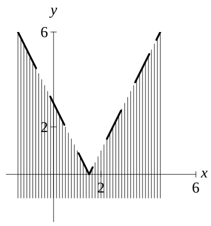


So the solution set is $[-7,5]$ and the graph is 

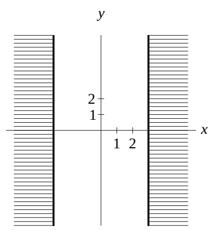
17. $3-2x>y$



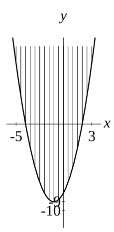
18. $|3-2x|>y$



19. $x^2 \geq 9$



20. $(x-2)(x+4) \leq y$



21. consistent

22. inconsistent

23. independent

24. dependent

25. substitution

26. addition

27. inconsistent

28. identity

29. linear, three

30. ordered triple

Concepts of Calculus

1(a) Let $f(x)=\dfrac{2x+1}{x^2+x}$.

$$\frac{f(x+h)-f(x)}{h} =$$

$$\frac{\dfrac{2(x+h)+1}{(x+h)^2+(x+h)}-\dfrac{2x+1}{x^2+x}}{h} =$$

$$\frac{[2(x+h)+1][x^2+x]}{h(x^2+x)[(x+h)^2+(x+h)]} -$$

$$\frac{(2x+1)[(x+h)^2+(x+h)]}{h(x^2+x)[(x+h)^2+(x+h)]} =$$

$$\frac{[2(x+h)+1][x^2+x]}{h(x^2+x)[(x+h)^2+(x+h)]} -$$

$$\frac{(2x+1)[(x+h)^2+(x+h)]}{h(x^2+x)[(x+h)^2+(x+h)]} =$$

$$\frac{2x^3+3x^2+2hx^2+2hx+x}{h(x^2+x)[(x+h)^2+(x+h)]} -$$

$$\frac{2x^3+3x^2+4hx^2+x+4hx+2h^2x+h^2+h}{h(x^2+x)[(x+h)^2+(x+h)]} =$$

$$\frac{-2hx^2-2hx-2h^2x-h^2-h}{h(x^2+x)[(x+h)^2+(x+h)]} =$$

$$\frac{-2x^2-2x-2hx-h-1}{(x^2+x)[(x+h)^2+(x+h)]} =$$

$$\frac{-2x^2-2x-2hx-h-1}{(x^2+x)[x^2+x+2xh+h^2+h]} =$$

1(b) We use the result from part 1(a).

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} =$$

$$\lim_{h\to 0}\frac{-2x^2-2x-2hx-h-1}{(x^2+x)[x^2+x+2xh+h^2+h]} =$$

$$\frac{-2x^2-2x-1}{(x^2+x)[x^2+x]} =$$

$$\frac{-2x^2-2x-1}{(x^2+x)^2} =$$

Thus, we have

$$f'(x)=\frac{-2x^2-2x-1}{(x^2+x)^2}.$$

2.

$$\begin{aligned}\frac{2x+1}{x(x+1)} &= \frac{A}{x}+\frac{B}{x+1}\\ 2x+1 &= A(x+1)+Bx\\ 2x+1 &= (A+B)x+A\\ A+B=2 \quad &\text{and} \quad A=1\end{aligned}$$

Substituting $A = 1$ into $A + B = 2$, we find $1 + B = 2$ or $B = 1$. The partial fraction decomposition is

$$\frac{2x+1}{x(x+1)} = \frac{1}{x}+\frac{1}{x+1}.$$

3(a) By Exercise 2, we have $f(x) = \dfrac{1}{x} + \dfrac{1}{x+1}$.

$$\begin{aligned}&\frac{f(x+h)-f(x)}{h} =\\ &\frac{\frac{1}{x+h}+\frac{1}{x+h+1}-\left(\frac{1}{x}+\frac{1}{x+1}\right)}{h} =\\ &\frac{\frac{1}{x+h}-\frac{1}{x}+\frac{1}{x+h+1}-\frac{1}{x+1}}{h} =\\ &\frac{\frac{1}{x+h}-\frac{1}{x}}{h}+\frac{\frac{1}{x+h+1}-\frac{1}{x+1}}{h} =\\ &\frac{x-x-h}{hx(x+h)}+\frac{x+1-x-h-1}{h(x+1)(x+h+1)} =\\ &\frac{-h}{hx(x+h)}+\frac{-h}{h(x+1)(x+h+1)} =\\ &\frac{-1}{x(x+h)}+\frac{-1}{(x+1)(x+h+1)} =\end{aligned}$$

3(b) Using the result from part 3(a), we obtain

$$\begin{aligned}&f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} =\\ &\lim_{h\to 0}\left(\frac{-1}{x(x+h)}+\frac{-1}{(x+1)(x+h+1)}\right) =\\ &\frac{-1}{x(x+0)}+\frac{-1}{(x+1)(x+0+1)} =\\ &\frac{-1}{x^2}+\frac{-1}{(x+1)^2} = .\end{aligned}$$

Thus, we have

$$f'(x) = \frac{-1}{x^2}+\frac{-1}{(x+1)^2}.$$

4. We rewrite the answer obtained in part 3(b).

$$\begin{aligned}f'(x) &= \frac{-1}{x^2}+\frac{-1}{(x+1)^2}\\ &= \frac{-(x+1)^2-x^2}{x^2(x+1)^2}\\ &= \frac{-(x^2+2x+1)-x^2}{(x(x+1))^2}\\ f'(x) &= \frac{-2x^2-2x-1}{(x^2+x)^2}\end{aligned}$$

This shows that the answers in parts 1(b) and 3(b) are equivalent.