For Thought

1. True **2.** False

3. False, $5°$ is coterminal with $-355°$.

4. False **5.** True, since $\frac{38\pi}{4} = \frac{19\pi}{2}$.

6. False, $210° = \frac{7\pi}{6}$.

7. False, since $25°20'40'' \neq 25.34°$; $25.34°$ is an approximation to $25°20'40''$.

8. True, since Seattle makes an angle of 2π every 24 hours, then the angular velocity is $\frac{2\pi}{24} = \frac{\pi}{12}$ radians per hour.

9. False, Seattle has a smaller linear velocity since its orbit about the axis of the earth is smaller than the orbit of Los Angeles.

10. True

5.1 Exercises

1. angle

2. central

3. standard position

4. acute

5. obtuse

6. right

7. coterminal

8. quadrantal

9. minute

10. second

11. unit

12. linear, angular

13. Substitute $k = 1, 2, -1, -2$ into $60° + k \cdot 360°$ to obtain the coterminal angles

$$420°, 780°, -300°, -660°.$$

There are other coterminal angles.

14. Substitute $k = 1, 2, -1, -2$ into $45° + k \cdot 360°$ to obtain the coterminal angles

$$405°, 765°, -315°, -675°.$$

There are other coterminal angles.

15. Substitute $k = 1, 2, -1, -2$ into $-16° + k \cdot 360°$ to find the coterminal angles

$$344°, 704°, -376°, -736°.$$

There are other coterminal angles.

16. Substitute $k = 1, 2, -1, -2$ into $-90° + k \cdot 360°$ to find the coterminal angles

$$270°, 630°, -450°, -810°.$$

There are other coterminal angles.

17. Yes, since $123.4° - (-236.6°) = 360°$ is an integral multiple of $360°$.

18. Yes, since $744° - (-336°) = 3 \cdot 360°$ is an integral multiple of $360°$.

19. No, since $1055° - (155°) = 900° = k \cdot 360°$ does not have an integral solution for any k.

20. No, since $359.9° = k \cdot 360°$ does not have an integral solution for any k.

21. Quadrant I **22.** Quadrant II

23. $-125°$ lies in Quadrant III since $-125° + 360° = 235°$ and $180° < 235° < 270°$

24. Quadrant II

25. Quadrant IV **26.** Quadrant III

27. $750°$ lies in Quadrant I since $750° - 720° = 30°$

28. $-980°$ lies in Quadrant II since $-980° + 3 \cdot 360° = 100°$

29. $45°$ **30.** $135°$

31. $60°$ **32.** $30°$

33. $120°$ **34.** $150°$

35. $400° - 360° = 40°$ **36.** $540° - 360° = 180°$

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37. $-340° + 360° = 20°$

38. $-180° + 360° = 180°$

39. $-1100° + 4 \cdot 360° = 340°$

40. $-840° + 3 \cdot 360° = 240°$

41. $13° + \dfrac{12}{60}^{\circ} = 13.2°$

42. $45° + \dfrac{6}{60}^{\circ} = 45.1°$

43. $-8° - \dfrac{30}{60}^{\circ} - \dfrac{18}{3600}^{\circ} = -8.505°$

44. $-5° - \dfrac{45}{60}^{\circ} - \dfrac{30}{3600}^{\circ} \approx -5.7583°$

45. $28° + \dfrac{5}{60}^{\circ} + \dfrac{9}{3600}^{\circ} \approx 28.0858°$

46. $44° + \dfrac{19}{60}^{\circ} + \dfrac{32}{3600}^{\circ} \approx 44.3256°$

47. $75.5° = 75°30'$ since $0.5(60) = 30$

48. $39.4° = 39°24'$ since $0.4(60) = 24$

49. $-17.33° = -17°19'48''$ since $0.33(60) = 19.8$ and $0.8(60) = 48$

50. $-9.12° = -9°7'12''$ since $0.12(60) = 7.2$ and $0.2(60) = 12$

51. $18.123° \approx 18°7'23''$ since $0.123(60) = 7.38$ and $0.38(60) \approx 23$

52. $122.786° \approx 122°47'10''$ since $0.786(60) = 47.16$ and $0.16(60) \approx 10$

53. $\dfrac{\pi}{6}$ **54.** $\dfrac{\pi}{4}$

55. $18° \cdot \dfrac{\pi}{180} = \dfrac{\pi}{10}$ **56.** $48° \cdot \dfrac{\pi}{180} = \dfrac{4\pi}{15}$

57. $-67.5° \cdot \dfrac{\pi}{180} = -\dfrac{135\pi}{360} = -\dfrac{3\pi}{8}$

58. $-105° \cdot \dfrac{\pi}{180} = \dfrac{-7\pi}{12}$

59. $630° \cdot \dfrac{\pi}{180} = \dfrac{7\pi}{2}$ **60.** $495° \cdot \dfrac{\pi}{180} = \dfrac{11\pi}{4}$

61. $37.4° \cdot \dfrac{\pi}{180} \approx 0.653$

62. $125.3° \cdot \dfrac{\pi}{180} \approx 2.187$

63. $\left(-13 - \dfrac{47}{60}\right) \cdot \dfrac{\pi}{180} \approx -0.241$

64. $\left(-99 - \dfrac{15}{60}\right) \cdot \dfrac{\pi}{180} \approx -1.732$

65. $\left(53 + \dfrac{37}{60} + \dfrac{6}{3600}\right) \cdot \dfrac{\pi}{180} \approx 0.936$

66. $\left(187 + \dfrac{49}{60} + \dfrac{36}{3600}\right) \cdot \dfrac{\pi}{180} \approx 3.278$

67. $\dfrac{5\pi}{12} \cdot \dfrac{180}{\pi} = 75°$

68. $\dfrac{17\pi}{12} \cdot \dfrac{180}{\pi} = 255°$

69. $\dfrac{7\pi}{4} \cdot \dfrac{180}{\pi} = 315°$

70. $\dfrac{13\pi}{6} \cdot \dfrac{180}{\pi} = 390°$

71. $-6\pi \cdot \dfrac{180}{\pi} = -1080°$

72. $-9\pi \cdot \dfrac{180}{\pi} = -1620°$

73. $2.39 \cdot \dfrac{180}{\pi} \approx 136.937°$

74. $0.452 \cdot \dfrac{180}{\pi} \approx 25.898°$

75. Substitute $k = 1, 2, -1, -2$ into $\dfrac{\pi}{3} + k \cdot 2\pi$ to obtain the coterminal angles

$$\frac{7\pi}{3}, \frac{13\pi}{3}, -\frac{5\pi}{3}, -\frac{11\pi}{3}.$$

There are other coterminal angles.

76. Substitute $k = 1, 2, -1, -2$ into $\dfrac{\pi}{4} + k \cdot 2\pi$ to obtain the coterminal angles

$$\frac{9\pi}{4}, \frac{17\pi}{4}, -\frac{7\pi}{4}, -\frac{15\pi}{4}.$$

There are other coterminal angles.

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77. Substitute $k = 1, 2, -1, -2$ into $-\dfrac{\pi}{6} + k \cdot 2\pi$ to find the coterminal angles

$$\frac{11\pi}{6}, \frac{23\pi}{6}, -\frac{13\pi}{6}, -\frac{25\pi}{6}.$$

There are other coterminal angles.

78. Substitute $k = 1, 2, -1, -2$ into $-\dfrac{2\pi}{3} + k \cdot 2\pi$ to find the coterminal angles

$$\frac{4\pi}{3}, \frac{10\pi}{3}, -\frac{8\pi}{3}, -\frac{14\pi}{3}.$$

There are other coterminal angles.

79. $3\pi - 2\pi = \pi$

80. $6\pi - 4\pi = 2\pi$

81. $\dfrac{9\pi}{2} - 4\pi = \dfrac{\pi}{2}$

82. $\dfrac{19\pi}{2} - 8\pi = \dfrac{3\pi}{2}$

83. $-\dfrac{5\pi}{3} + 2\pi = \dfrac{\pi}{3}$

84. $-\dfrac{7\pi}{6} + 2\pi = \dfrac{5\pi}{6}$

85. $-\dfrac{13\pi}{3} + 6\pi = \dfrac{5\pi}{3}$

86. $-\dfrac{19\pi}{4} + 6\pi = \dfrac{5\pi}{4}$

87. $8.32 - 2\pi \approx 2.04$

88. $-23.55 + 8\pi \approx 1.58$

89. No, since $\dfrac{29\pi}{4} - \dfrac{3\pi}{4} = \dfrac{26\pi}{4} = k \cdot 2\pi$ does not have an integral solution for any k.

90. Yes, since $\dfrac{5\pi}{3} - \dfrac{-\pi}{3} = \dfrac{6\pi}{3} = 2\pi$.

91. Yes, since $\dfrac{7\pi}{6} - \dfrac{-5\pi}{6} = \dfrac{12\pi}{6} = 2\pi$.

92. Yes, since $\dfrac{3\pi}{2} - \dfrac{-9\pi}{2} = 6\pi$.

93. Quadrant I **94.** Quadrant III

95. Quadrant III

96. $-\dfrac{39\pi}{20}$ lies in Quadrant I since

$$-\frac{39\pi}{20} + 2\pi = \frac{\pi}{20}$$

97. $\dfrac{13\pi}{8}$ lies in Quadrant IV since

$$\frac{3\pi}{2} = \frac{12\pi}{8} < \frac{13\pi}{8} < 2\pi$$

98. $-\dfrac{11\pi}{8}$ lies in Quadrant II since

$$-\frac{11\pi}{8} + 2\pi = \frac{5\pi}{8}$$

99. Note $2\pi \approx 6.28$ and $3\pi/2 \approx 4.71$. Since -7.3 is coterminal with $-7.3 + 2(6.28) = 5.26$ and $4.71 < 5.26 < 6.28$, it follows that -7.3 lies in Quadrant IV.

100. Note $\pi \approx 3.14$ and $3\pi/2 \approx 4.71$. Since 23.1 is coterminal with $23.1 - 3(6.28) = 4.26$ and $3.14 < 4.26 < 4.71$, we get that 23.1 lies in Quadrant III.

101. $30° = \dfrac{\pi}{6}$, $45° = \dfrac{\pi}{4}$, $60° = \dfrac{\pi}{3}$, $90° = \dfrac{\pi}{2}$,

$120° = \dfrac{2\pi}{3}$, $135° = \dfrac{3\pi}{4}$, $150° = \dfrac{5\pi}{6}$, $180° = \pi$,

$210° = \dfrac{7\pi}{6}$, $225° = \dfrac{5\pi}{4}$, $240° = \dfrac{4\pi}{3}$,

$270° = \dfrac{3\pi}{2}$, $300° = \dfrac{5\pi}{3}$, $315° = \dfrac{7\pi}{4}$,

$330° = \dfrac{11\pi}{6}$, $360° = 2\pi$

102. $-30° = -\dfrac{\pi}{6}$, $-45° = -\dfrac{\pi}{4}$, $-60° = -\dfrac{\pi}{3}$,

$-90° = -\dfrac{\pi}{2}$, $-120° = -\dfrac{2\pi}{3}$, $-135° = -\dfrac{3\pi}{4}$,

$-150° = -\dfrac{5\pi}{6}$, $-180° = -\pi$, $-210° = -\dfrac{7\pi}{6}$,

$-225° = -\dfrac{5\pi}{4}$, $-240° = -\dfrac{4\pi}{3}$, $-270° = -\dfrac{3\pi}{2}$,

$-300° = -\dfrac{5\pi}{3}$, $-315° = -\dfrac{7\pi}{4}$,

$-330° = -\dfrac{11\pi}{6}$, $-360° = -2\pi$

103. $s = 12 \cdot \dfrac{\pi}{4} = 3\pi$ ft

104. $s = 4 \cdot 1 = 4$ cm

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105. $s = 4000 \cdot \dfrac{3\pi}{180} \approx 209.4$ miles

106. $s = 2 \cdot \dfrac{\pi}{3} \approx 2.1$ m

107. radius is $r = \dfrac{s}{\alpha} = \dfrac{1}{1} = 1$ mile.

108. radius is $r = \dfrac{s}{\alpha} = \dfrac{99}{0.004} = 24,750$ km

109. radius is $r = \dfrac{s}{\alpha} = \dfrac{10}{\pi} \approx 3.18$ km

110. radius is $r = \dfrac{s}{\alpha} = \dfrac{8}{2\pi} \approx 1.27$ m

111. Distance from Peshtigo to the North Pole is

$s = r\alpha = 3950\left(45 \cdot \dfrac{\pi}{180}\right) \approx 3102$ miles.

112. Assume the helper is on an arc in a circle whose radius is the distance r between the helper and the surveyor. The angle in radians subtended is $\dfrac{37}{60} \cdot \dfrac{\pi}{180} \approx 0.01076$.

Then $r = \dfrac{s}{\alpha} \approx \left(6 + \dfrac{2}{12}\right) \div 0.01076 \approx 573$ ft.

113. central angle is $\alpha = \dfrac{2000}{3950} \approx 0.506329$ radians

$\approx 0.506329 \cdot \dfrac{180}{\pi} \approx 29.0°$

114. Fifty yards from the goal, the angle is

$\alpha = \dfrac{18.5}{50(3)} \approx 0.1233$ radians $\approx 7.06°$.

Then 50 yards from the goal, the maximum deviation from the actual trajectory is

$$\frac{7.06°}{2} = 3.53°.$$

Twenty yards from the goal, the angle is

$\alpha = \dfrac{18.5}{20(3)} \approx 0.3083$ radians $\approx 17.66°$.

Thus, 20 yards away, the maximum deviation from the actual tajectory is

$$\frac{17.66°}{2} = 8.83°.$$

115. Linear velocity is $v = \dfrac{s}{t} = \dfrac{r\alpha}{t} =$

$\dfrac{6 \cdot (10,350) \cdot 2\pi}{1} \approx 390,185.8$ cm/min.

116. Angular velocity is $w = \dfrac{\alpha}{t} =$

$\dfrac{10,350(2\pi)}{1} \approx 65,031.0$ radians/min.

117. The radius of the blade is

10 in. $= 10 \cdot \dfrac{1}{12 \cdot 5280} \approx 0.0001578$ miles.

Since the angle rotated in one hour is

$$2800 \cdot 2\pi \cdot 60 = 336,000\pi$$

the linear velocity is

$v = \dfrac{r\alpha}{t} \approx \dfrac{(0.0001578)(336,000\pi)}{1} \approx 166.6$ mph.

118. The radius of the bit is

0.5 in. $= 0.5 \cdot \dfrac{1}{12 \cdot 5280} \approx 0.0000079$ miles.

Since the angle made in one hour is $45,000 \cdot 2\pi \cdot 60 = 5,400,000\pi$, we obtain

$v = \dfrac{r\alpha}{t} \approx \dfrac{(0.0000079) \cdot 5,400,000\pi}{1}$

≈ 133.9 mph.

119. In 1 hr,the saw rotates through an angle of $3450(60) \cdot 2\pi$. After converting the radii into miles, the linear velocity is

$3450(60) \cdot 2\pi \left(\dfrac{6}{12(5280)} - \dfrac{5}{12(5280)}\right) \approx$

20.5 mph.

120. The radius of the tire in miles is

$r = \dfrac{13}{12(5280)} \approx 0.00020517677$.

So $w = \dfrac{\alpha}{t} = \dfrac{\alpha}{1} = \alpha = \dfrac{s}{r} \approx$

$\dfrac{55}{0.00020517677} \approx 268,061.5$ radians/hr.

121. The angular velocity of any point on the surface of the earth is $w = \dfrac{\pi}{12}$ rad/hr.

A point 1 mile from the North Pole is approximately 1 mile from the axis of the earth. The linear velocity of that point is $v = w \cdot r = \dfrac{\pi}{12} \cdot 1 \approx 0.26$ mph.

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122. The angular velocity is

$$w = \frac{\pi}{12} \approx 0.26 \text{ rad/hr.}$$

Let r be the distance between Peshtigo and the point on the x-axis closest to Peshtigo. Since Peshtigo is on the 45th parallel, that point on the x-axis is r miles from the center of the earth. By the Pythagorean Theorem, $r^2+r^2 = 3950^2$ or

$$r \approx 2,793.0718 \text{ miles.}$$

The linear velocity is
$v = w \cdot r \approx \frac{\pi}{12} \cdot 2,793.0718 \approx 731.2$ mph.

123. Since $7^\circ \approx 0.12217305$, the radius of the earth according to Eratosthenes is
$r = \frac{s}{\alpha} \approx \frac{800}{0.12217305} \approx 6548.089$ km.
The circumference is

$$2\pi r \approx 41,143 \text{ km.}$$

Using $r = 6378$ km, the circumference is 40,074 km.

124. Since $\alpha = 15^\circ \approx 0.261799$ radians, the linear velocity is $v = \frac{r\alpha}{t} = \frac{r\alpha}{1} = r\alpha \approx$ $6.5(3950) \cdot 0.261799 \approx 6721.7$ mph.

125. The area of a 16-inch diameter pizza is $\pi r^2 = \pi \cdot 8^2 = 64\pi$. The area of one slice is $\frac{64\pi}{6} \approx 33.5$ in^2.

126. With a central angle of α radians, the number of slices is $\frac{2\pi}{\alpha}$. The area of one slice is the area of a whole pizza divided by the number of slices i.e. $\pi r^2 \div \frac{2\pi}{\alpha} = \frac{\alpha r^2}{2}$.

127. Since the velocity at point A is 10 ft/sec, the linear velocity at B and C are both 10 ft/sec. The angular velocity at B is

$$\omega = \frac{v}{r} = \frac{10 \text{ ft/sec}}{5/12 \text{ ft}} = 24 \text{ rad/sec.}$$

While, the angular velocity at C is

$$\omega = \frac{v}{r} = \frac{10 \text{ ft/sec}}{3/12 \text{ ft}} = 40 \text{ rad/sec.}$$

128. The linear velocity at A, B, and C are all equal to $v = \omega r = \frac{1000(2\pi)}{60}\frac{5}{12}$ ft/sec
$= \frac{250\pi}{18}$ ft/sec ≈ 43.6 ft/sec.
The angular velocity at C is
$\omega = \frac{v}{r} = \frac{250\pi/18}{3/12}$ ft/sec
$= \frac{250\pi/18}{3/12} \cdot \frac{60}{2\pi}$ rev/min ≈ 1666.7 rev/min.

129. Let t be the number of minutes after 12 noon. The number of degrees spanned by the minute hand and hour hand since 12 noon are $6t$ and $t/2$, respectively. If the minute hand and the hour hand form a 90-degree angle, then

$$6t - \frac{t}{2} = 90 \text{ or } 6t - \frac{t}{2} = 270.$$

The solutions to this equation are

$$y = \frac{180}{11} \text{ min} \approx 16 \text{ min}, 22 \text{ sec.}$$

and

$$y = \frac{540}{11} \text{ min} \approx 49 \text{ min}, 5 \text{ sec.},$$

respectively. Thus, the two hands form a 90-degree angle at 12:16:22 and 12:49:05.

130. Let t be the number of minutes after 4 pm. The number of degrees spanned by the minute hand and hour hand since 4 pm are $6t$ and $t/2$, respectively. Note, at 4pm the hour hand forms a 120-degree angle with the minute hand. When the minute and hour hands coincide, we have

$$6t = \frac{t}{2} + 120.$$

The solution to this equation are

$$y = \frac{240}{11} \text{ min} \approx 21 \text{ min}, 49.1 \text{ sec.}$$

Hence, Tammy will swim for 21 minutes and 49.1 seconds.

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131. a) Given an angle α (in degrees) as in the problem, the radius r of the cone must satisfy

$$2\pi r = 8\pi - 4\alpha\frac{\pi}{180}$$

Note, 8π inches is the circumference of a circle with radius 4 inches. Then we obtain

$$r = 4 - \frac{\alpha}{90}.$$

Note, $h = \sqrt{16 - r^2}$ by the Pythagorean theorem. Since the volume $V(\alpha)$ of the cone is $\frac{\pi}{3}r^2h$,

$$V(\alpha) = \frac{\pi}{3}\left(4 - \frac{\alpha}{90}\right)^2\sqrt{16 - \left(4 - \frac{\alpha}{90}\right)^2}.$$

This reduces to

$$V(\alpha) = \frac{\pi(360-\alpha)^2\sqrt{720\alpha - \alpha^2}}{2,187,000}$$

If $\alpha = 30°$, then $V(30°) \approx 22.5$ inches^3.

b) As shown in part a), the volume of the cone obtained by an overlapping angle α is

$$V(\alpha) = \frac{\pi(360-\alpha)^2\sqrt{720\alpha - \alpha^2}}{2,187,000}$$

132. The volume $V(\alpha)$ in Exercise 131 b) is maximized when $\alpha \approx 66.06°$.

The maximum volume is

$$V(66.06°) \approx 25.8 \text{ cubic inches.}$$

133. Since $x^2 + 3x - 10 = (x+5)(x-2) = 0$, the solution set is $\{-5, 2\}$.

134. The exponential form of $\log(x^2 + 3x) = 1$ is $x^2 + 3x = 10$. From Exercise 133, $x = -5$ or $x = 2$. Note, $\log x$ is undefined if $x = -5$. Thus, the solution set is $\{2\}$.

135. Writing in exponential form, we have

$$\begin{aligned} \frac{x}{x+3} &= 2^{-3} \\ 8x &= x + 3 \\ 7x &= 3 \end{aligned}$$

The solution set is $\{3/7\}$.

136. Since $\log_4(64) = 3$, we find

$$\begin{aligned} 5^{3x-1} &= 3 \\ 3x - 1 &= \log_5 3 \\ 3x &= 1 + \log_5 3 \end{aligned}$$

The solution set is $\left\{\frac{1+\log_5 3}{3}\right\}$.

137. Set one side to zero:

$$\frac{2x-1}{x+3} - 1 = \frac{x-4}{x+3} = f(x) \ge 0$$

If $x = -2$, then $f(-2) > 0$.
If $x = 0$, then $f(0) < 0$.
If $x = 5$, then $f(5) > 0$.

+	U	−	0	+
−2	−3	0	4	5

The solution set is $(-\infty, -3) \cup [4, \infty)$.

138. Set one side to zero: $(x+2)(x+1) - 6 = x^2 + 3x - 4 = (x+4)(x-1) = f(x) > 0$

If $x = -5$, then $f(-5) > 0$.
If $x = 0$, then $f(0) < 0$.
If $x = 2$, then $f(2) > 0$.

+	0	−	0	+
−5	−4	0	1	2

The solution set is $(-\infty, -4) \cup (1, \infty)$.

Thinking Outside the Box XLV

Let r be the radius of the circle. Let p be the distance from the lower left most corner of the 1-by-1 square to the point of tangency of the left most, lower most circle at the base of the 1-by-1 square. By the Pythagorean Theorem,

$$(p + r)^2 + (r + 1 - p)^2 = 1$$

or equivalently

$$p^2 + r^2 + r - p = 0. \quad (1)$$

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Since the area of the four triangles plus the area of the small square in the middle is 1, we obtain

$$4r^2 + 4\left[\frac{1}{2}(p+r)(r+1-p)\right] = 1$$

or

$$6r^2 - 2p^2 + 2p + 2r = 1. \qquad (2)$$

Multiply (1) by two and add the result to (2). Then $8r^2 + 4r = 1$. The solution is $r = \dfrac{\sqrt{3}-1}{4}$.

5.1 Pop Quiz

1. $1267° - 1080° = 187°$

2. Quadrant III

3. $70° + (30/60)° + (36/3600)° = 70.51°$

4. $270 \cdot \dfrac{\pi}{180°} = \dfrac{3\pi}{2}$

5. $\left(\dfrac{7\pi}{4} \cdot \dfrac{180}{\pi}\right)^{\circ} = 315°$

6. Yes, since the difference

$$-\frac{3\pi}{4} - \frac{5\pi}{4} = -2\pi$$

is a multiple of 2π.

7. $s = r\alpha = 30 \cdot \dfrac{\pi}{3} = 10\pi$ ft

5.1 Linking Concepts

a) Since the circumference is 20π and the ferris wheel makes three revolutions in one minute, the linear velocity is 60π $(= 3 \cdot 20\pi)$ meters/minute.

b) Since the angle made in one revolution is 2π, the angular velocity is 6π $(= 3 \cdot 2\pi)$ radians/minute.

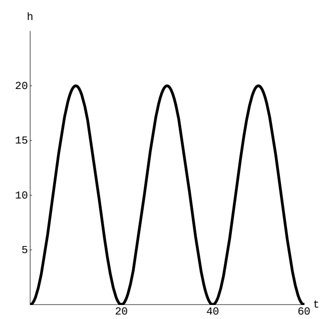
c) The length of the arc between adjacent seats (represented as points) is the circumference divided by 8, i.e., $\dfrac{20\pi}{8}$ meters or 2.5π meters.

d) Note, the ferris wheel makes one revolution in 20 seconds. For the following times, given in seconds, one finds the following heights in meters

t	0	2.5	5	7.5	10	12.5
$h(t)$	0	2.9	10	17.1	20	17.1

t	15	17.5	20
$h(t)$	10	2.9	0

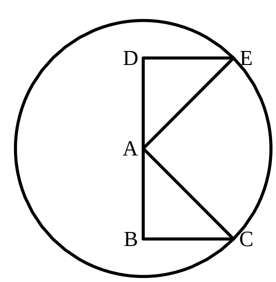
e) By connecting the points $(t, h(t))$ provided in part d) and by repeating this pattern, one obtains a sketch of a graph of $h(t)$ versus t for $0 \le t \le 60$.



f) There are six solutions to $h(t) = 18$ in the interval $[0, 60]$ since the ferris wheel makes one revolution every 20 seconds and in each revolution one attains the height of 18 meters twice (one on the way up and the other on the way down).

g) In the picture below, the ferris wheel is represented by the circle with radius 10 meters. Triangles $\triangle ABC$ and $\triangle ADE$ are isosceles triangles. By the Pythagorean Theorem, one finds

$$AD = DE = AB = BC = 5\sqrt{2}.$$



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After 2.5 seconds, one's position on the ferris wheel will be at C; and after 7.5 seconds the location will be at E.
Thus, we obtain

$$h(2.5) = 10 - 5\sqrt{2}$$

and

$$h(7.5) = 10 + 5\sqrt{2}.$$

For Thought

1. False, since $\cos 90° = 0$.

2. False, since $\cos 90 \approx -0.4$.

3. True, since $\sin(45°) = \dfrac{\sqrt{2}}{2} = \dfrac{1}{\sqrt{2}}$.

4. False, since

$$\sin\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

and

$$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}.$$

5. False, since

$$\cos\left(-\frac{\pi}{3}\right) = \frac{1}{2}$$

and

$$-\cos\left(\frac{\pi}{3}\right) = -\frac{1}{2}.$$

6. True, since the reference arc of 390° is 30° and 390° lies in Quadrant I.

7. False, since α lies in Quadrant IV.

8. False, since $\sin(\alpha) = -\dfrac{1}{2}$.

9. False, since possibly $\alpha = \dfrac{13\pi}{6}$.

10. True, since $(1 - \sin\alpha)(1 + \sin\alpha) = 1 - \sin^2\alpha = \cos^2\alpha$.

5.2 Exercises

1. $\sin\alpha$, $\cos\alpha$

2. fundamental

3. $(1,0), \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right), (0,1), \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right),$
$(-1,0), \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right), (0,-1), \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

4. $(1,0), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right), \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right), (0,1)$
$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right), \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right), (-1,0), \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right),$
$\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right), (0,-1), \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right), \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

5. 0 **6.** -1 **7.** 0 **8.** 0

9. 0 **10.** -1 **11.** 0 **12.** -1

13. $\frac{\sqrt{2}}{2}$ **14.** $-\frac{\sqrt{2}}{2}$ **15.** $-\frac{\sqrt{2}}{2}$ **16.** $\frac{\sqrt{2}}{2}$

17. $\frac{1}{2}$ **18.** $\frac{\sqrt{3}}{2}$ **19.** $\frac{1}{2}$ **20.** $-\frac{1}{2}$

21. $-\frac{\sqrt{3}}{2}$ **22.** $-\frac{1}{2}$ **23.** $\frac{\sqrt{3}}{2}$ **24.** $\frac{1}{2}$

25. $\sin(390°) = \sin(30°) = \dfrac{1}{2}$

26. $\sin(765°) = \sin(45°) = \dfrac{\sqrt{2}}{2}$

27. $\cos(-420°) = \cos(300°) = \dfrac{1}{2}$

28. $\cos(-450°) = \cos(270°) = 0$

29. $\cos\left(\dfrac{13\pi}{6}\right) = \cos\left(\dfrac{\pi}{6}\right) = \dfrac{\sqrt{3}}{2}$

30. $\cos\left(-\dfrac{7\pi}{3}\right) = \cos\left(-\dfrac{\pi}{3}\right) = \dfrac{1}{2}$

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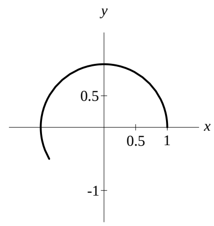
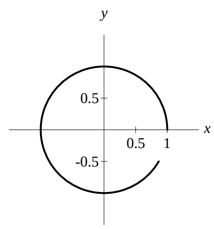
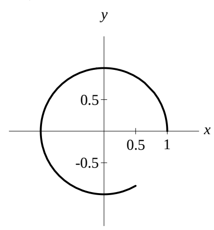
31. $30°, \pi/6$



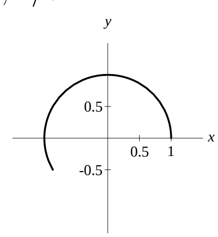
32. $30°, \pi/6$



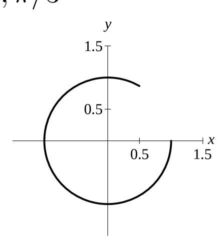
33. $60°, \pi/3$



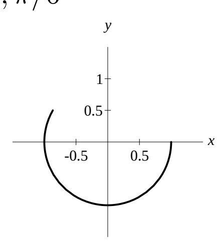
34. $30°, \pi/6$



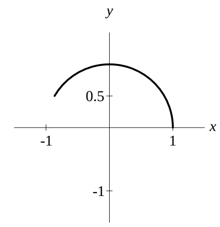
35. $60°, \pi/3$



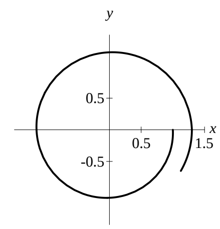
36. $30°, \pi/6$



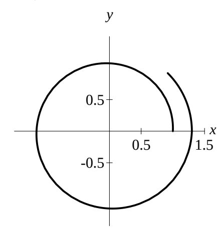
37. $30°, \pi/6$



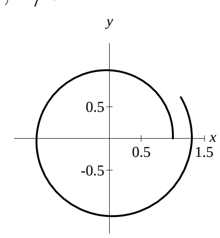
38. $30°, \pi/6$



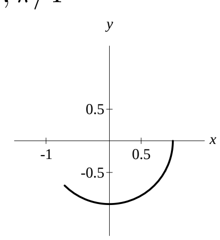
39. $45°, \pi/4$



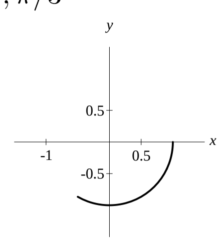
40. $30°, \pi/6$



41. $45°, \pi/4$



42. $60°, \pi/3$



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43. +, since sine is positive in the 2nd quadrant

44. −, since cosine is negative in the 2nd quadrant

45. +, since cosine is positive in the 4th quadrant

46. −, since sine is negative in the 3rd quadrant

47. −, since sine is negative in the 3rd quadrant

48. +, since cosine is positive in the 1st quadrant

49. −, since cosine is negative in the 3rd quadrant

50. +, since sine is positive in the 1st quadrant

51. $\sin(135°) = \sin(45°) = \frac{\sqrt{2}}{2}$

52. $\sin(420°) = \sin(60°) = \frac{\sqrt{3}}{2}$

53. $\cos\left(\frac{5\pi}{3}\right) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

54. $\cos\left(\frac{11\pi}{6}\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

55. $\sin\left(\frac{7\pi}{4}\right) = -\sin\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

56. $\sin\left(-\frac{13\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

57. $\cos\left(-\frac{17\pi}{6}\right) = -\cos\left(\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2}$

58. $\cos\left(-\frac{5\pi}{3}\right) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

59. $\sin(-45°) = -\sin(45°) = -\frac{\sqrt{2}}{2}$

60. $\cos(-120°) = -\cos(60°) = -\frac{1}{2}$

61. $\cos(-240°) = -\cos(60°) = -\frac{1}{2}$

62. $\sin(-225°) = \sin(45°) = \frac{\sqrt{2}}{2}$

63. $\frac{\cos(\pi/3)}{\sin(\pi/3)} = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

64. $\frac{\sin(-5\pi/6)}{\cos(-5\pi/6)} = \frac{-1/2}{-\sqrt{3}/2} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

65. $\frac{\sin(7\pi/4)}{\cos(7\pi/4)} = \frac{-\sqrt{2}/2}{\sqrt{2}/2} = -1$

66. $\frac{\sin(-3\pi/4)}{\cos(-3\pi/4)} = \frac{-\sqrt{2}/2}{-\sqrt{2}/2} = 1$

67. $\sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right) = \sin\left(\frac{\pi}{2}\right) = 1$

68. $\cos\left(\frac{\pi}{3} - \frac{\pi}{6}\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

69. $\frac{1 - \cos(5\pi/6)}{\sin(5\pi/6)} = \frac{1 - (-\sqrt{3}/2)}{1/2} \cdot \frac{2}{2} = 2 + \sqrt{3}$

70. $\frac{\sin(5\pi/6)}{1 + \cos(5\pi/6)} = \frac{1/2}{1 + (-\sqrt{3}/2)} \cdot \frac{2}{2} =$

$\frac{1}{2 - \sqrt{3}} \cdot \frac{2 + \sqrt{3}}{2 + \sqrt{3}} = \frac{2 + \sqrt{3}}{4 - 3} = 2 + \sqrt{3}$

71. $\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2}$

72. 1, since $\sin^2(\alpha) + \cos^2(\alpha) = 1$

73. 0.9999 **74.** −0.2740

75. 0.4035 **76.** 0.6532

77. −0.7438 **78.** 0.9875

79. 1.0000 **80.** −1.0000

81. −0.2588 **82.** 0.9239

83. $\sin\left(\frac{\pi}{2}\right) = 1$ **84.** $\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

85. $\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$ **86.** $\cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$

87. $\sin\left(\frac{3\pi}{4}\right) = \frac{\sqrt{2}}{2}$ **88.** $\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

89. $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$ **90.** $\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

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91. Use the Fundamental Identity.

$$\begin{aligned}\left(\frac{5}{13}\right)^2+\cos^2(\alpha) &= 1\\ \frac{25}{169}+\cos^2(\alpha) &= 1\\ \cos^2(\alpha) &= \frac{144}{169}\\ \cos(\alpha) &= \pm\frac{12}{13}\end{aligned}$$

Since α is in quadrant II, $\cos(\alpha)=-12/13$.

92. Use the Fundamental Identity.

$$\begin{aligned}\left(-\frac{4}{5}\right)^2+\sin^2(\alpha) &= 1\\ \frac{16}{25}+\sin^2(\alpha) &= 1\\ \sin^2(\alpha) &= \frac{9}{25}\\ \sin(\alpha) &= \pm\frac{3}{5}\end{aligned}$$

Since α is in quadrant III, $\sin(\alpha)=-3/5$.

93. Use the Fundamental Identity.

$$\begin{aligned}\left(\frac{3}{5}\right)^2+\sin^2(\alpha) &= 1\\ \frac{9}{25}+\sin^2(\alpha) &= 1\\ \sin^2(\alpha) &= \frac{16}{25}\\ \sin(\alpha) &= \pm\frac{4}{5}\end{aligned}$$

Since α is in quadrant IV, $\sin(\alpha)=-4/5$.

94. Use the Fundamental Identity.

$$\begin{aligned}\left(-\frac{12}{13}\right)^2+\cos^2(\alpha) &= 1\\ \frac{144}{169}+\cos^2(\alpha) &= 1\\ \cos^2(\alpha) &= \frac{25}{169}\\ \cos(\alpha) &= \pm\frac{5}{13}\end{aligned}$$

Since α is in quadrant IV, $\cos(\alpha)=5/13$.

95. Use the Fundamental Identity.

$$\begin{aligned}\left(\frac{1}{3}\right)^2+\cos^2(\alpha) &= 1\\ \frac{1}{9}+\cos^2(\alpha) &= 1\\ \cos^2(\alpha) &= \frac{8}{9}\\ \cos(\alpha) &= \pm\frac{2\sqrt{2}}{3}\end{aligned}$$

Since $\cos(\alpha)>0$, $\cos(\alpha)=\dfrac{2\sqrt{2}}{3}$.

96. Use the Fundamental Identity.

$$\begin{aligned}\left(\frac{2}{5}\right)^2+\sin^2(\alpha) &= 1\\ \frac{4}{25}+\sin^2(\alpha) &= 1\\ \sin^2(\alpha) &= \frac{21}{25}\\ \sin(\alpha) &= \pm\frac{\sqrt{21}}{5}\end{aligned}$$

Since $\sin(\alpha)<0$, $\sin(\alpha)=-\dfrac{\sqrt{21}}{5}$.

97. Since $x(t)=4\sin(t)-3\cos(t)$, the location of the weight after 3 seconds is $x(3)=4\sin(3)-3\cos(3)\approx 3.53$ cm. It is below its equilibrium position.

98. Since $x(t)=-\sqrt{3}\cdot\sin(\sqrt{3}t)+\cos(\sqrt{3}t)$, the location after 2 seconds is $x(2)=-\sqrt{3}\cdot\sin(2\sqrt{3})+\cos(2\sqrt{3})\approx -0.40$ cm. It is above the equilibrium position.

99. The angle between the tips of two adjacent teeth is $\dfrac{2\pi}{22}=\dfrac{\pi}{11}$. The actual distance is $c=6\sqrt{2-2\cos(\pi/11)}\approx 1.708$ in. The length of the arc is $s=6\cdot\dfrac{\pi}{11}\approx 1.714$ in.

100. The central angle determined by an edge of a stop sign is $\dfrac{2\pi}{8}$. If r is the radius of the circular drum, then $10=r\sqrt{2-2\cos(2\pi/8)}\approx r(0.765367)$. Thus, $r\approx 13.07$ in.

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101. Note, $\cos^2\alpha = 1 - \sin^2\alpha$ or $\cos(\alpha) = \pm\sqrt{1-\sin^2\alpha}$. Then $\cos\alpha = \sqrt{1-\sin^2\alpha}$ if the terminal side of α lies in quadrant I or IV, while $\cos\alpha = -\sqrt{1-\sin^2\alpha}$ if the terminal side of α lies in quadrant II or III.

102. Solving for v_o, one finds

$$\begin{aligned} 367 &= \frac{v_o^2}{32}\sin 86^\circ \\ \sqrt{\frac{32(367)}{\sin 86^\circ}}\text{ ft/sec} &= v_o \\ \sqrt{\frac{32(367)}{\sin 86^\circ}\frac{3600}{5280}}\text{ mph} &= v_o \\ 74\text{ mph} &\approx v_o \end{aligned}$$

103. $48^\circ 13.8' = 48^\circ 13' 48''$

104. $135^\circ \cdot \dfrac{\pi}{180^\circ} = \dfrac{3\pi}{4}$

105. The factors of 6 are $p = \pm 1 \pm 2, \pm 3, \pm 6$. The factors of 4 are $q = \pm 1, \pm 2, \pm 4$.

Then the possible rational solutions are $\dfrac{p}{q} = \pm 1, \pm 2, \pm 3, \pm 6, \pm\dfrac{1}{4}, \pm\dfrac{1}{2}, \pm\dfrac{3}{2}, \pm\dfrac{3}{4}$

106. Factor:

$$\begin{aligned} (x^{1/3}-5)(x^{1/3}+1) &= 0 \\ x^{1/3} = 5 \quad \text{or} \quad x^{1/3} &= -1 \end{aligned}$$

The solution set is $\{125, -1\}$.

107. Using $3x - 9 \geq 0$ or $x \geq 3$, it follows that the domain is $[3, \infty)$.

Note, $-\sqrt{w}$ takes all the values in $(-\infty, 0]$ as w assumes all the nonnegative values. Then the range of $y = 1 - \sqrt{3x-9}$ is $(-\infty, 0]$.

108. Since the inverse of squaring x is taking the square root of x, the inverse is $f^{-1}(x) = \sqrt{x}$. Note, $f(x) = x^2$ is one-to-one on for $x \geq 0$.

Thinking Outside the Box XLVI

a) Let t be a fraction of an hour, i.e., $0 \leq t \leq 1$. If the angle between the hour and minute hands is 120°, then

$$\begin{aligned} 360t - 30t &= 120 \\ 330t &= 120 \\ t &= \frac{12}{33}\text{hr} \\ t &\approx 21\text{ min}, 49.1\text{ sec.} \end{aligned}$$

Thus, the hour and minute hands will be 120° apart when the time is 12:21:49.1. Moreover, this is the only time between 12 noon and 1 pm that the angle is 120°.

b) We measure angles clockwise from the 12 0'clock position.

At 12:21:49.1, the hour hand is pointing to the 10.9°-angle, the minute hand is pointing to the 130.9°-angle, and the second hand is pointing to the 294.5°-angle. Thus, the three hands of the clock cannot divide the face of the clock into thirds.

c) No, as discussed in part b).

5.2 Pop Quiz

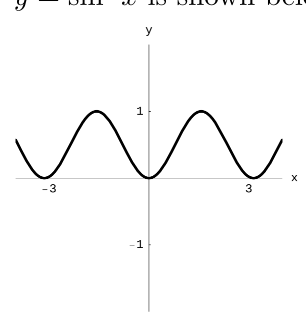
1. 60° **2.** 0 **3.** $-\dfrac{\sqrt{3}}{2}$ **4.** $\dfrac{\sqrt{2}}{2}$

5. 0 **6.** $-\dfrac{1}{2}$ **7.** $\dfrac{\sqrt{3}}{2}$

8. $\sin\alpha = -\sqrt{1 - \left(-\dfrac{3}{5}\right)^2} = -\sqrt{\dfrac{25}{25} - \dfrac{9}{25}} = -\dfrac{4}{5}$

5.2 Linking Concepts

a) The domain of $f(x) = \sin^2 x$ is $(-\infty, \infty)$ which is the same as the domain of $y = \sin x$. Since the square of a number is never negative and the range of $y = \sin x$ is $[-1, 1]$, the range of $y = \sin^2 x$ is $[0, 1]$. A sketch of the graph of $y = \sin^2 x$ is shown below.



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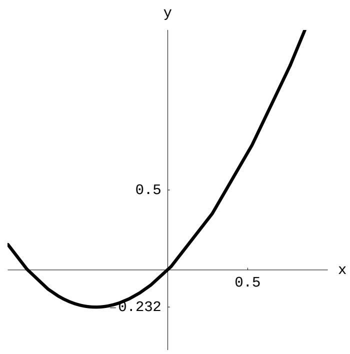
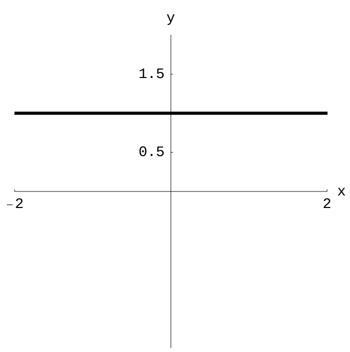
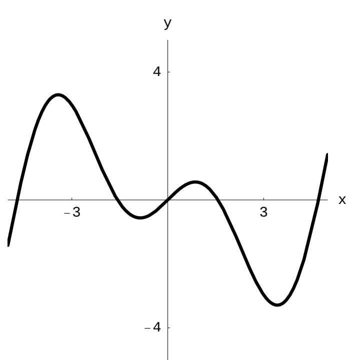
b) The domain of $f(x) = x^2 + \sin x$ is $(-\infty, \infty)$ which is the domain of $y = x^2$ and $y = \sin x$. From the graph of $y = x^2 + \sin x$, the range can be approximated to be $[-0.232, \infty)$.



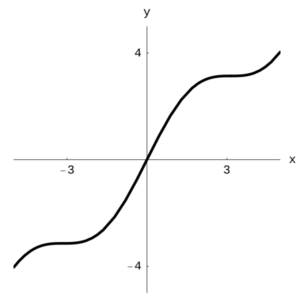
c) The domain of $f(x) = \sin^2(x) + \cos^2(x)$ is $(-\infty, \infty)$ which is the domain of $y = \sin(x)$ and $y = \cos(x)$. The graph of f is the same as the graph of $y = 1$ for $\sin^2(x) + \cos^2(x) = 1$, see Section 6.1. The range of f is $\{1\}$.



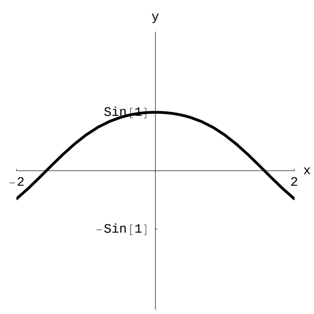
d) The domain of $f(x) = x \cdot \cos(x)$ is $(-\infty, \infty)$ which is the same as the domains of $y = x$ and $y = \cos(x)$. From the graph of f, as shown below, we can say that the range of f is $(-\infty, \infty)$.



e) The domain of $f(x) = x + \sin(x)$ is $(-\infty, \infty)$ which is the domain of $y = x$ and $y = \sin(x)$. Since the range of $y = x$ is $(-\infty, \infty)$ and by studying the graph of f, as shown below, we can say that the range of f is $(-\infty, \infty)$.



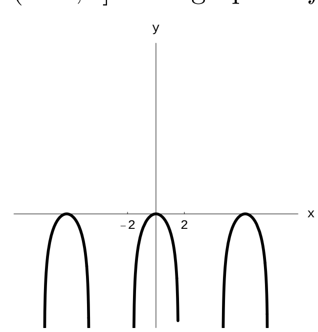
f) The domain of $f(x) = \sin(\cos(x))$ is $(-\infty, \infty)$ which is the same as the domains of $y = \cos x$ and $y = \sin(x)$. Since $-1 \leq \cos x \leq 1$ and by studying the graph of f, as shown below, we can conclude that the range of f is $[\sin(-1), \sin(1)]$.



g) The domain of $f(x) = \ln(\cos(x))$ consists of all x for which $\cos x > 0$, and which precisely consists of all intervals of the form

$$\left(\frac{(4n-1)\pi}{2}, \frac{(4n+1)\pi}{2} \right)$$

where n is an integer. The range of f is $(-\infty, 0]$. The graph of f is shown below.



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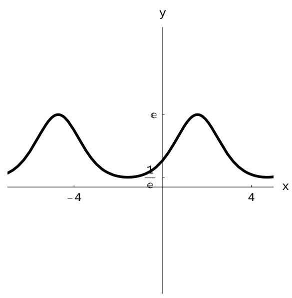
h) The domain of $f(x) = e^{\sin(x)}$ is $(-\infty, \infty)$ which is the same as the domain of $y = \sin x$. Since $-1 \leq \sin x \leq 1$ and by studying the graph of f, as shown below, we can conclude that the range of f is $[e^{-1}, e^1]$ or equivalently $[1/e, e]$.



For Thought

1. False, the period is 1.

2. False, the range is $[-1, 7]$.

3. False, the phase shift is $-\pi/12$.

4. True

5. True

6. True, $\dfrac{2\pi}{0.1\pi} = 20$.

7. False

8. True, since $\dfrac{2\pi}{b} = \dfrac{2\pi}{4} = \dfrac{\pi}{2}$.

9. True

10. True

5.3 Exercises

1. sine wave

2. fundamental cycle

3. amplitude

4. phase shift

5. period

6. frequency

7. $y = -2\sin(x)$, amplitude 2

8. $y = -2\cos(x)$, amplitude 2

9. $y = 3\cos(x)$, amplitude 3

10. $y = 3\sin(x)$, amplitude 3

11. Amplitude 2, period 2π, phase shift 0

12. Amplitude 4, period 2π, phase shift 0

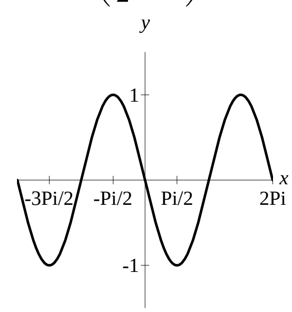
13. Amplitude 1, period 2π, phase shift $\pi/2$

14. Amplitude 1, period 2π, phase shift $-\pi/2$

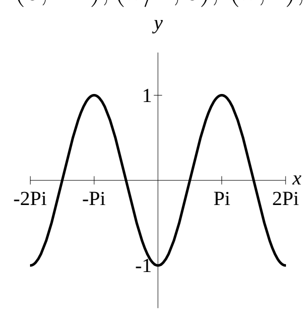
15. Amplitude 2, period 2π, phase shift $-\pi/3$

16. Amplitude 3, period 2π, phase shift $\pi/6$

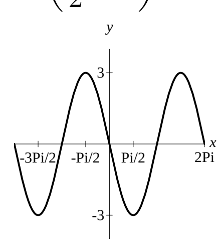
17. Amplitude 1, phase shift 0, some points are $(0,0)$, $\left(\dfrac{\pi}{2}, -1\right)$, $(\pi, 0)$, $\left(\dfrac{3\pi}{2}, 1\right)$,$(2\pi, 0)$



18. Amplitude 1, phase shift 0, some points are $(0,-1), (\pi/2, 0)$, $(\pi, 1)$, $(3\pi/2, 0)$, $(2\pi, -1)$



19. Amplitude 3, phase shift 0, some points are $(0,0)$, $\left(\dfrac{\pi}{2}, -3\right)$, $(\pi, 0)$, $\left(\dfrac{3\pi}{2}, 3\right)$, $(2\pi, 0)$



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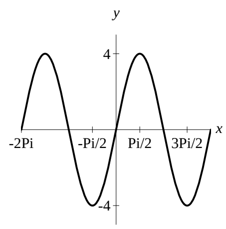
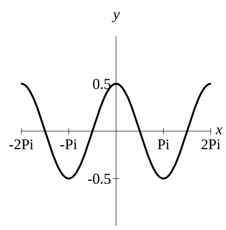
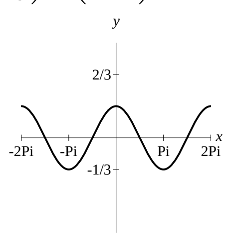
20. Amplitude 4, phase shift 0, some points are $(0,0)$, $\left(\dfrac{\pi}{2},4\right)$, $(\pi,0)$, $\left(\dfrac{3\pi}{2},-4\right)$, $(2\pi,0)$



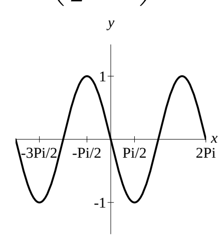
21. Amplitude 1/2, phase shift 0, some points are $(0,1/2)$, $(\pi/2,0)$, $(\pi,-1/2)$ $(3\pi/2,0)$, $(2\pi,1/2)$



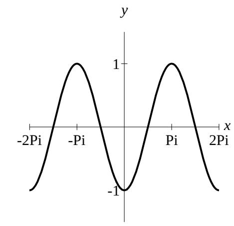
22. Amplitude 1/3, phase shift 0, some points are $\left(0,\dfrac{1}{3}\right)$, $\left(\dfrac{\pi}{2},0\right)$, $(\pi,-1/3)$, $(3\pi/2,0)$, $\left(2\pi,\dfrac{1}{3}\right)$



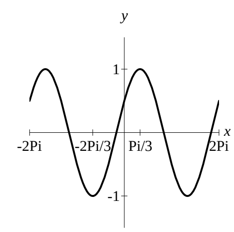
23. Amplitude 1, phase shift $-\pi$, some points are $(0,0)$, $\left(\dfrac{\pi}{2},-1\right)$, $(\pi,0)$, $\left(\dfrac{3\pi}{2},1\right)$, $(2\pi,0)$



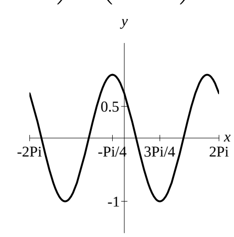
24. Amplitude 1, phase shift π, some points are $(0,-1)$, $(\pi/2,0)$, $(\pi,1)$, $(3\pi/2,0)$, $(2\pi,-1)$



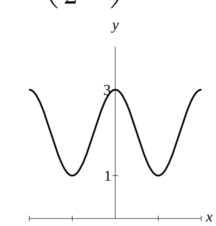
25. Amplitude 1, phase shift $\pi/3$, some points are $\left(-\dfrac{2\pi}{3},-1\right)$, $\left(-\dfrac{\pi}{6},0\right)$, $\left(\dfrac{\pi}{3},1\right)$, $\left(\dfrac{5\pi}{6},0\right)$, $\left(\dfrac{4\pi}{3},-1\right)$



26. Amplitude 1, phase shift $-\pi/4$, some points are $\left(-\dfrac{\pi}{4},1\right)$, $\left(\dfrac{\pi}{4},0\right)$, $\left(\dfrac{3\pi}{4},-1\right)$, $\left(\dfrac{5\pi}{4},0\right)$, $\left(\dfrac{7\pi}{4},1\right)$



27. Amplitude 1, phase shift 0, some points are $(0,3)$, $\left(\dfrac{\pi}{2},2\right)$, $(\pi,1)$ $\left(\dfrac{3\pi}{2},2\right)$, $(2\pi,3)$



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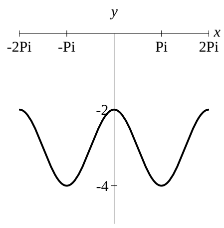
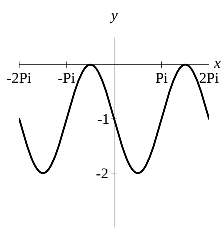
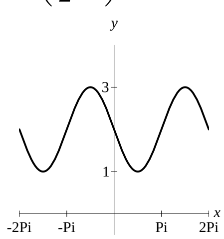
28. Amplitude 1, phase shift 0, some points are $(0,-2)$, $\left(\frac{\pi}{2},-3\right)$, $(\pi,-4)$, $\left(\frac{3\pi}{2},-3\right)$, $(2\pi,-2)$,



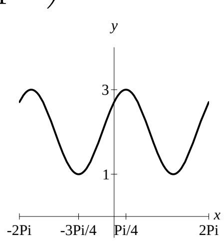
29. Amplitude 1, phase shift 0, some points are $(0,-1)$, $\left(\frac{\pi}{2},-2\right)$, $(\pi,-1)$,$\left(\frac{3\pi}{2},0\right)$, $(2\pi,-1)$



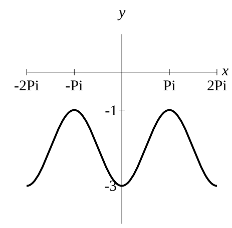
30. Amplitude 1, phase shift 0, some points are $(0,2)$,$\left(\frac{\pi}{2},1\right)$, $(\pi,2)$, $\left(\frac{3\pi}{2},3\right)$, $(2\pi,2)$



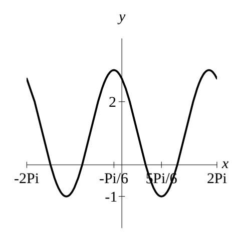
31. Amplitude 1, phase shift $-\pi/4$, some points are $\left(-\frac{\pi}{4},2\right)$, $\left(\frac{\pi}{4},3\right)$, $\left(\frac{3\pi}{4},2\right)$, $\left(\frac{5\pi}{4},1\right)$, $\left(\frac{7\pi}{4},2\right)$



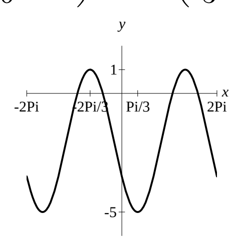
32. Amplitude 1, phase shift $\pi/2$, some points are $(0,-3)$,$\left(\frac{\pi}{2},-2\right)$, $(\pi,-1)$,$\left(\frac{3\pi}{2},-2\right)$, $(2\pi,-3)$



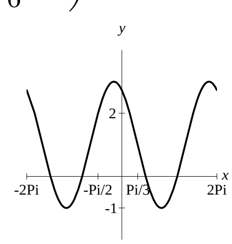
33. Amplitude 2, phase shift $-\pi/6$, some points are $\left(-\frac{\pi}{6},3\right)$, $\left(\frac{\pi}{3},1\right)$, $\left(\frac{5\pi}{6},-1\right)$, $\left(\frac{4\pi}{3},1\right)$, $\left(\frac{11\pi}{6},3\right)$



34. Amplitude 3, phase shift $-2\pi/3$, some points are $\left(-\frac{2\pi}{3},1\right)$, $\left(-\frac{\pi}{6},-2\right)$, $\left(\frac{\pi}{3},-5\right)$, $\left(\frac{5\pi}{6},-2\right)$, and $\left(\frac{4\pi}{3},1\right)$



35. Amplitude 2, phase shift $\pi/3$, some points are $\left(-\frac{\pi}{6},3\right)$, $\left(\frac{\pi}{3},1\right)$, $\left(\frac{5\pi}{6},-1\right)$, $\left(\frac{4\pi}{3},1\right)$, $\left(\frac{11\pi}{6},3\right)$



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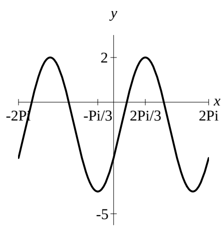
36. Amplitude 3, phase shift $-\pi/3$, some points are $\left(-\frac{\pi}{3}, -4\right)$, $\left(\frac{\pi}{6}, -1\right)$, $\left(\frac{2\pi}{3}, 2\right)$, $\left(\frac{7\pi}{6}, -1\right)$, $\left(\frac{5\pi}{3}, -4\right)$



37. Amplitude 3, period $\pi/2$, phase shift 0

38. Amplitude 2, period $2\pi/3$, phase shift 0

39. Amplitude 1, period $\frac{2\pi}{1/2}$ or 4π, phase shift 0

40. Amplitude 1, period $\frac{2\pi}{1/3}$ or 6π, phase shift 0

41. Amplitude 2, period 2π, phase shift π

42. Amplitude 5, period 2π, phase shift -4

43. Amplitude 2; since $y = -2\cos(2(x+\pi/4))$, the period is π and the phase shift is $-\pi/4$

44. Amplitude 4; since $y = 4\cos(3(x-\pi/12))$, the period is $2\pi/3$ and the phase shift is $\pi/12$

45. Amplitude 2; since $y = -2\cos\left(\frac{\pi}{2}(x+2)\right)$, the period is $\frac{2\pi}{\pi/2}$ or 4, and the phase shift is -2

46. Amplitude 8; since $y = 8\sin\left(\frac{\pi}{3}\left(x-\frac{3}{2}\right)\right)$, the period is $\frac{2\pi}{\pi/3}$ or 6, and the phase shift is $\frac{3}{2}$

47. Note, $A = (7-3)/2 = 2$. Since $A + D = 2 + D = 7$, we find that $D = 5$. Since $C = -\pi/2$ and $2\pi/B = \pi$, we obtain $B = 2$. Thus,

$$y = 2\sin\left(2\left(x+\frac{\pi}{2}\right)\right) + 5.$$

48. Note, $A = (4-(-2))/2 = 3$. Since $A + D = 3 + D = 4$, we find that $D = 1$. Since $C = \pi$ and $2\pi/B = \pi/2$, we obtain $B = 4$. Thus,

$$y = 3\sin\left(4\left(x-\pi\right)\right) + 1.$$

49. Note, $A = (9-(-1))/2 = 5$. Since $A + D = 5 + D = 9$, we obtain that $D = 4$. Since $C = 2$ and $2\pi/B = 2$, we find $B = \pi$. Thus,

$$y = 5\sin\left(\pi\left(x-2\right)\right) + 4.$$

50. Note, $A = (25-5)/2 = 10$. Since $A + D = 10 + D = 25$, we find that $D = 15$. Since $C = 7$ and $2\pi/B = 4$, we find $B = \pi/2$. Thus,

$$y = 10\sin\left(\frac{\pi}{2}(x-7)\right) + 15.$$

51. Note, $A = (3-(-9))/2 = 6$. Since $A + D = 6 + D = 3$, we find that $D = -3$. Since $C = -\pi$ and $2\pi/B = 1/2$, we find $B = 4\pi$. Hence,

$$y = 6\sin\left(4\pi\left(x+\pi\right)\right) - 3.$$

52. Note, $A = (2-(-6))/2 = 4$. Since $A + D = 4 + D = 2$, we find that $D = -2$. Since $C = 2\pi$ and $2\pi/B = 1/3$, we find $B = 6\pi$. Hence,

$$y = 4\sin\left(6\pi\left(x-2\pi\right)\right) - 2.$$

53. $y = -\sin\left(x - \frac{\pi}{4}\right) + 1$

54. $y = -\cos\left(x + \frac{\pi}{6}\right) - 2$

55. $y = -\left[3\cos\left(x-\pi\right) - 2\right]$ or equivalently $y = -3\cos\left(x-\pi\right) + 2$

56. $y = -4\left[\sin\left(x + \frac{\pi}{2}\right) + 1\right]$ or equivalently $y = -4\sin\left(x + \frac{\pi}{2}\right) - 4$

57. $F(x) = \sin\left(3x - \frac{\pi}{4}\right)$

58. $F(x) = 3\sin\left(x - \frac{\pi}{4}\right)$

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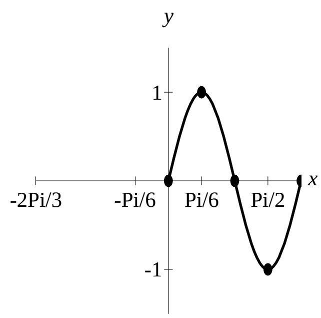
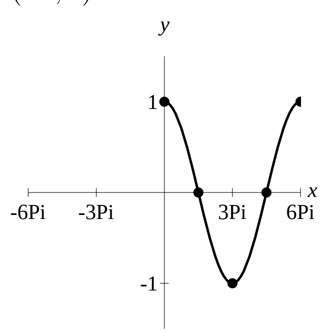
59. $F(x) = \sin\left(3x - \frac{3\pi}{4}\right)$

60. $F(x) = \sin(3x) - \frac{\pi}{4}$

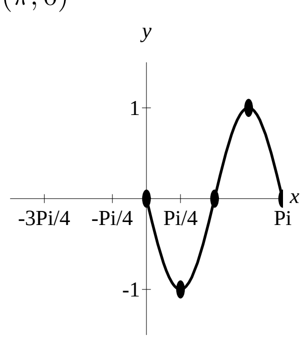
61. Period $2\pi/3$, phase shift 0, range $[-1, 1]$, labeled points are $(0, 0)$, $\left(\frac{\pi}{6}, 1\right)$, $\left(\frac{\pi}{3}, 0\right)$, $\left(\frac{\pi}{2}, -1\right)$, $\left(\frac{2\pi}{3}, 0\right)$



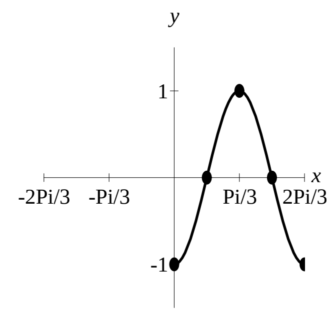
62. Period 6π, phase shift 0, range $[-1, 1]$, labeled points are $(0, 1)$, $\left(\frac{3\pi}{2}, 0\right)$, $(3\pi, -1)$, $\left(\frac{9\pi}{2}, 0\right)$, $(6\pi, 1)$



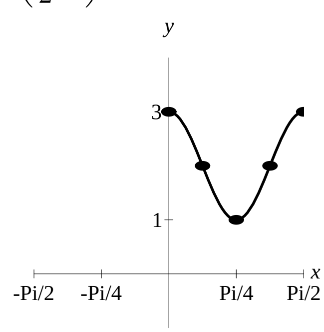
63. Period π, phase shift 0, range $[-1, 1]$, labeled points are $(0, 0)$, $\left(\frac{\pi}{4}, -1\right)$, $\left(\frac{\pi}{2}, 0\right)$, $\left(\frac{3\pi}{4}, 1\right)$, $(\pi, 0)$



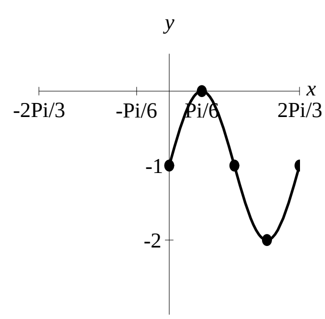
64. Period $2\pi/3$, phase shift 0, range $[-1, 1]$, labeled points are $(0, -1)$, $\left(\frac{\pi}{6}, 0\right)$, $\left(\frac{\pi}{3}, 1\right)$, $\left(\frac{\pi}{2}, 0\right)$, $\left(\frac{2\pi}{3}, -1\right)$



65. Period $\pi/2$, phase shift 0, range $[1, 3]$, labeled points are $(0, 3)$, $\left(\frac{\pi}{8}, 2\right)$, $\left(\frac{\pi}{4}, 1\right)$, $\left(\frac{3\pi}{8}, 2\right)$, $\left(\frac{\pi}{2}, 3\right)$



66. Period $2\pi/3$, phase shift 0, range $[-2, 0]$, labeled points are $(0, -1)$, $\left(\frac{\pi}{6}, 0\right)$, $\left(\frac{\pi}{3}, -1\right)$, $\left(\frac{\pi}{2}, -2\right)$, $\left(\frac{2\pi}{3}, -1\right)$



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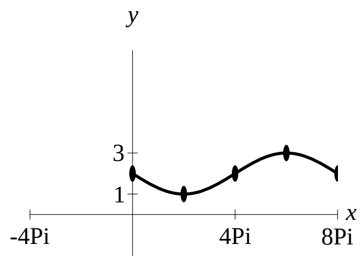
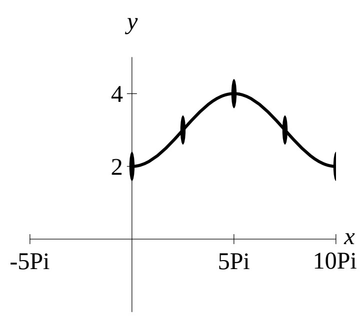
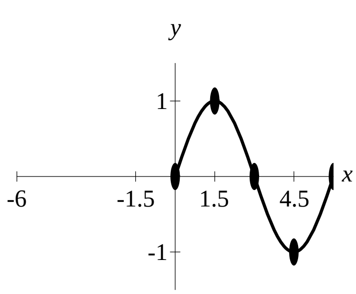
67. Period 8π, phase shift 0, range $[1, 3]$, labeled points are $(0, 2)$, $(2\pi, 1)$, $(4\pi, 2)$, $(6\pi, 3)$, $(8\pi, 2)$



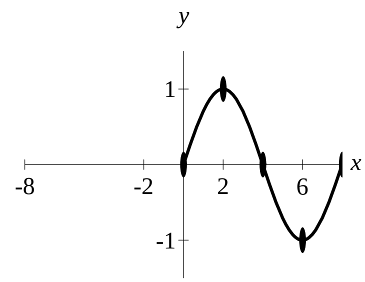
68. Period 10π, phase shift 0, range $[2, 4]$, labeled points are $(0, 2)$, $\left(\frac{5\pi}{2}, 3\right)$, $(5\pi, 4)$, $\left(\frac{15\pi}{2}, 3\right)$, $(10\pi, 2)$



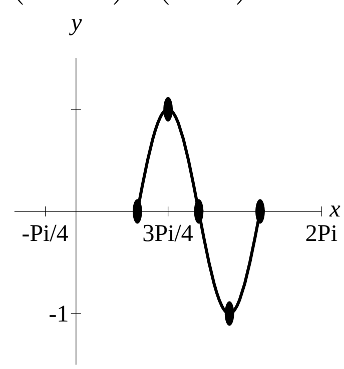
69. Period 6, phase shift 0, range $[-1, 1]$, labeled points are $(0, 0)$, $(1.5, 1)$, $(3, 0)$, $(4.5, -1)$, $(6, 0)$



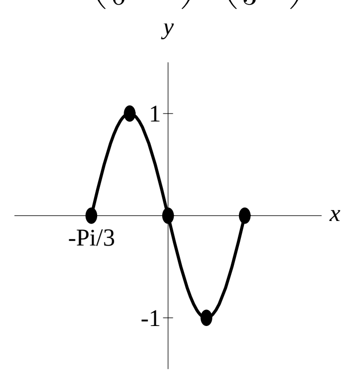
70. Period 8, phase shift 0, range $[-1, 1]$, labeled points are $(0, 0)$, $(2, 1)$, $(4, 0)$, $(6, -1)$, $(8, 0)$



71. Period π, phase shift $\pi/2$, range $[-1, 1]$, labeled points are $\left(\frac{\pi}{2}, 0\right)$, $\left(\frac{3\pi}{4}, 1\right)$, $(\pi, 0)$, $\left(\frac{5\pi}{4}, -1\right)$, $\left(\frac{3\pi}{2}, 0\right)$



72. Period $2\pi/3$, phase shift $-\pi/3$, range $[-1, 1]$, labeled points are $\left(-\frac{\pi}{3}, 0\right)$, $\left(-\frac{\pi}{6}, 1\right)$, $(0, 0)$, $\left(\frac{\pi}{6}, -1\right)$, $\left(\frac{\pi}{3}, 0\right)$



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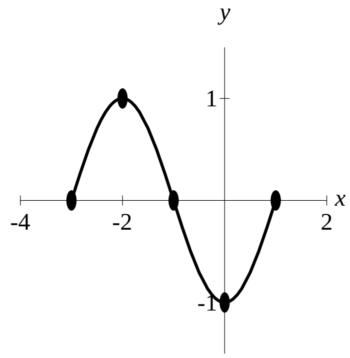
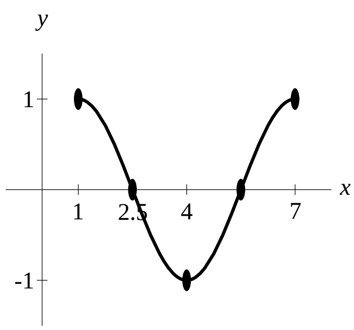
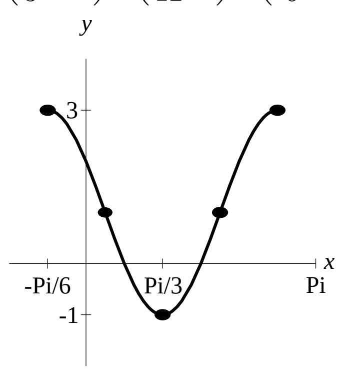
73. Period 4, phase shift -3, range $[-1, 1]$, labeled points are $(-3, 0)$, $(-2, 1)$, $(-1, 0)$, $(0, -1)$, $(1, 0)$



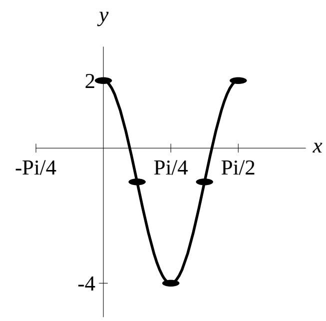
74. Period 6, phase shift 1, range $[-1, 1]$, labeled points are $(1, 1)$, $(2.5, 0)$, $(4, -1)$, $(5.5, 0)$, $(7, 1)$



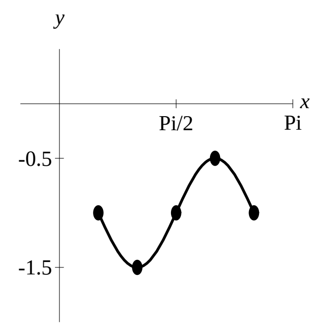
75. Period π, phase shift $-\pi/6$, range $[-1, 3]$, labeled points are $\left(-\frac{\pi}{6}, 3\right)$, $\left(\frac{\pi}{12}, 1\right)$, $\left(\frac{\pi}{3}, -1\right)$, $\left(\frac{7\pi}{12}, 1\right)$, $\left(\frac{5\pi}{6}, 3\right)$



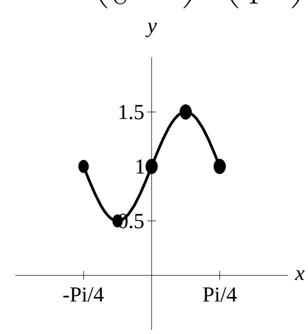
76. Period $\pi/2$, phase shift $\pi/2$, range $[-4, 2]$, labeled points are $(0, 2)$, $\left(\frac{\pi}{8}, -1\right)$, $\left(\frac{\pi}{4}, -4\right)$, $\left(\frac{3\pi}{8}, -1\right)$, $\left(\frac{\pi}{2}, 2\right)$



77. Period $\frac{2\pi}{3}$, phase shift $\frac{\pi}{6}$, range $\left[-\frac{3}{2}, -\frac{1}{2}\right]$, labeled points are $\left(\frac{\pi}{6}, -1\right)$, $\left(\frac{\pi}{3}, -\frac{3}{2}\right)$, $\left(\frac{\pi}{2}, -1\right)$, $\left(\frac{2\pi}{3}, -\frac{1}{2}\right)$, $\left(\frac{5\pi}{6}, -1\right)$



78. Period $\pi/2$, phase shift $-\pi/4$, range $[1/2, 3/2]$, labeled points are $\left(-\frac{\pi}{4}, 1\right)$, $\left(-\frac{\pi}{8}, 0.5\right)$, $(0, 1)$, $\left(\frac{\pi}{8}, 1.5\right)$, $\left(\frac{\pi}{4}, 1\right)$



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79. $y = 2\sin\left(2\left(x - \frac{\pi}{4}\right)\right)$

80. $y = -\sin\left(\frac{x}{2}\right)$

81. $y = 3\sin\left(\frac{3}{2}\left(x + \frac{\pi}{3}\right)\right) + 3$

82. $y = 2\sin\left(2\left(x + \frac{\pi}{4}\right)\right) - 1$

83. 100 cycles per second

84. 1/2000 cycle per second

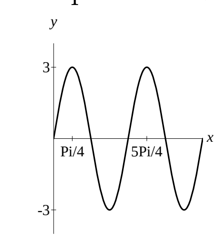
85. Frequency is $\frac{1}{0.025} = 40$ cycles per hour

86. Period is $\frac{1}{40,000} = 0.000025$ second.

87. Substitute $v_o = 6$, $\omega = 2$, and $x_o = 0$ into $x(t) = \frac{v_o}{\omega} \cdot \sin(\omega t) + x_o \cdot \cos(\omega t)$. Then

$$x(t) = 3\sin(2t).$$

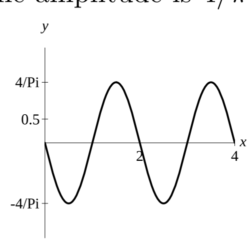
The amplitude is 3 and period is π.



88. Substitute $v_o = -4$, $\omega = \pi$, and $x_o = 0$ into $x(t) = \frac{v_o}{\omega} \cdot \sin(\omega t) + x_o \cdot \cos(\omega t)$. Then

$$x(t) = -\frac{4}{\pi} \cdot \sin(\pi t).$$

The amplitude is $4/\pi$ and period is 2.



89. 11 years

90. Approximately 1.4 seconds

91. Note that the maximum and minimum values of $\sin(t)$ are 1 and -1.
(a) Maximum volume is 1300 cc and miminum volume is 500 cc
(b) The runner takes a breath every 1/30 (which is the period) of a minute.Thus, a runner makes 30 breaths/minute.

92. (a) Maximum velocity is 8 cm/sec and mininum velocity is 0.
(b) The rodent's heart makes a beat every 1/3 (which is the period) of a second or it makes 180 beats in a minute.

93. Period is 12, amplitude is 15,000, phase-shift is -3, vertical translation is 25,000, a formula for the curve is

$$y = 15,000\sin\left(\frac{\pi}{6}x + \frac{\pi}{2}\right) + 25,000.$$

For April (when $x = 4$), the revenue is $15,000\sin\left(\frac{\pi}{6}x + \frac{\pi}{2}\right) + 25,000 \approx \$17,500$.

94. Period is 12, amplitude is 150, phase-shift is -2, vertical translation is 350, a formula for the curve is

$$y = 150\sin\left(\frac{\pi x}{6} + \frac{\pi}{3}\right) + 350.$$

For November (when $x = 11$), the utility bill is $150\sin\left(\frac{\pi x}{6} + \frac{\pi}{3}\right) + 350 \approx \$425$

95.

a) period is 40, amplitude is 65, an equation for the sine wave is $y = 65\sin\left(\frac{\pi}{20}x\right)$

b) 40 days

c) $65\sin\left(\frac{\pi}{20}(36)\right) \approx -38.2$ meters/second

d) The planet is between the Earth and Rho.

96. **a)** Ganymede's period is 7.155 days, or 7 days and 8 hours; Callisto's period is 16.689 days, or 16 days and 17 hours; Io's period is 1.769 days, or 1 day and 18 hours; Europa's period is 3.551 days, or 3 days and 13 hours.

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To the nearest hour, it is easiest to find Io's period since it is the satellite with the smallest period.

b) Io's amplitude is 262,000 miles, Europa's amplitude is 417,000 miles, Ganymede's amplitude is 666,000 miles, Callisto's amplitude is 1,170,000 miles

97. Since the period is 20, the amplitude is 1, and the vertical translation is 1, an equation for the swell is $y = \sin\left(\frac{\pi}{10}x\right) + 1$.

98. Since the period is 200, the amplitude is 15, and the vertical translation is 15, an equation for the tsunami is $y = 15\sin\left(\frac{\pi}{100}x\right) + 15$.

99. With a calculator we obtain the sine regression equation is $y = a\sin(bx + c) + d$ where $a = 51.62635869$, $b = 0.1985111953$, $c = 1.685588984$, $d = 50.42963472$, or about

$$y = 51.6\sin(0.20x + 1.69) + 50.43.$$

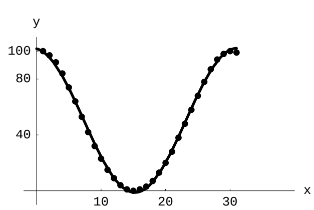
The period is

$$\frac{2\pi}{b} = \frac{2\pi}{0.1985111953} \approx 31.7 \text{ days}.$$

When $x = 35$, we find

$$a\sin(b(35) + c) + d \approx 87\%$$

of the moon is illuminated on February 4, 2010. Shown below is a graph of the regression equation and the data points.



100. Using a calculator, the sine regression equation is $y = a\sin(bx + c) + d$ where $a = 95.05446443$, $b = 0.5138444646$, $c = -1.847302606$, $d = 727.4320744$, or about

$$y = 95.1\sin(0.51x - 1.85) + 727.4.$$

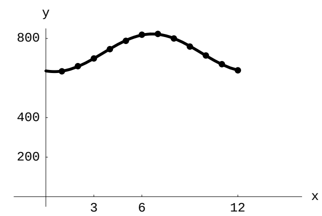
The period is

$$\frac{2\pi}{b} = \frac{2\pi}{0.5138444646} \approx 12.2 \text{ months}.$$

When $x = 14$, we obtain

$$a\sin(b(14) + c) + d \approx 651 \text{ minutes}$$

between sunrise and sunset on February 1, 2011.



102. $y = \sin x$ is an odd function since its graph is symmetric about the origin, and $y = \cos x$ is an even function since its graph is symmetric about the y-axis.

103. $225^\circ \cdot \frac{\pi}{180^\circ} = \frac{5\pi}{4}$

104. $\frac{7\pi}{6} \cdot \frac{180^\circ}{\pi} = 210^\circ$

105. Since the radius is $r = 15$ inches, the linear velocity of the tip of the blade that is rotating at 2000 revolutions/minute is given by

$$v = 15(2000)2\pi \cdot \frac{60}{12(5280)} = 178.5 \text{ mph}$$

106. At 200 revolutions/sec, the angular velocity is given by

$$\omega = 200(2\pi)(6) \approx 75,398.2 \frac{\text{radians}}{\text{minute}}$$

107. a) $\sin 45^\circ = \frac{\sqrt{2}}{2}$, **b)** $\cos\left(\frac{-\pi}{4}\right) = \frac{\sqrt{2}}{2}$

108. a) $\sin 60^\circ = \frac{\sqrt{3}}{2}$, **b)** $\cos\left(\frac{-5\pi}{6}\right) = \frac{-\sqrt{3}}{2}$

Thinking Outside the Box XLVII

a) The contestants that leave the table are # 1, # 3, # 5, # 7, # 9, # 11, # 13, # 4, # 8, # 12, # 6, #2 (in this order). Then contestant # 10 is the unlucky contestant.

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b) Let $n = 8$. The contestants that leave the table are # 1, # 3, # 5, # 7, # 2, # 6, # 4 (in this order). Thus, contestant # 8 is the unlucky contestant.

Let $n = 16$. The contestants that leave the table are # 1, # 3, # 5, # 7, # 9, # 11, # 13, # 15, # 2, # 6, # 10, # 14, # 4, # 12, # 8 (in this order). Thus, contestant # 16 is the unlucky contestant.

Let $n = 41$. The contestants that leave the table are # 1, # 3, # 5, # 7, # 9, # 11, # 13, # 15, # 17, # 19, # 21, # 23, # 25, # 27, # 29, # 31, # 33, # 35, # 37, # 39, # 4, # 8 # 12, # 16, # 20, # 24, # 28, # 32, # 36, # 40, # 6, # 14, # 22, # 30, # 38, # 10, # 26, # 2, # 34 (in this order). Thus, contestant # 18 is the unlucky contestant.

c) Let $m \geq 1$ and let n satisfy

$$2^m < n \leq 2^{m+1} \tag{3}$$

where

$$n = 2^m + k. \tag{4}$$

We claim the unlucky number is $2k$. One can check the claim is true for all n when $m = 1$. Suppose the claim is true for $m - 1$ and all such n.

Consider the case when k is an even integer satisfying (3) and (4). After selecting the survivors in round 1, the remaining contestants are

$$2, 4, 6, ..., 2^m + k.$$

Renumber, the above contestant by the rule

$$f(x) = x/2$$

so that the remaining contestant are renumbered as

$$1, 2, 3, ..., 2^{m-1} + k/2.$$

Note,

$$2^{m-1} < 2^{m-1} + k/2 \leq 2^m$$

Since the claim is true for $m - 1$, the unlucky contestant in the renumbering is

$$2\left(\frac{k}{2}\right) = k.$$

But by the renumbering, we find

$$f(x) = \frac{x}{2} = k$$

or the unlucky contestant is $2k$.

Finally, let k be an odd integer satisfying (3) and (4). After selecting the survivors in round 1, the remaining contestants are

$$2, 4, 6, ..., 2^m + (k - 1).$$

Note, the next survivor is 4 since the last survivor chosen is $2^m + k$.

Renumber, the above contestants by the rule

$$g(x) = \frac{x}{2} - 1$$

so the remaining contestant are renumbered as follows:

$$0, 1, 2, 3, ..., [2^{m-1} + (k - 1)/2 - 1].$$

Since the claim is true for $m - 1$, the unlucky contestant using the previous renumbering is

$$2\left(\frac{k-1}{2}\right) \quad \text{or} \quad k - 1$$

Using the original numbering, the unlucky contestant is

$$g(x) = \frac{x}{2} - 1 = k - 1 \quad \text{or} \quad x = 2k.$$

Equivalently, the unlucky number is

$$2k = 2(n - 2^m).$$

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5.3 Pop Quiz

1. Amplitude 5; since

$$y = -5\sin\left(2\left(x + \frac{\pi}{3}\right)\right)$$

the period is $\frac{2\pi}{2}$ or π, and the phase shift is $-\frac{\pi}{3}$

2. $(0,0)$, $\left(\frac{\pi}{4}, 3\right)$, $\left(\frac{\pi}{2}, 0\right)$, $\left(\frac{3\pi}{4}, -3\right)$, $(\pi, 0)$

3. $y = -\cos\left(x - \frac{\pi}{2}\right) + 3$

4. The range is $[-4, 4] + 2$, or $[-2, 6]$.

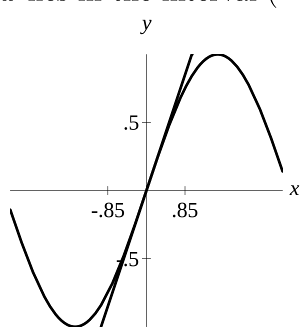
5. Since the period is

$$\frac{2\pi}{500\pi} = \frac{1}{250}$$

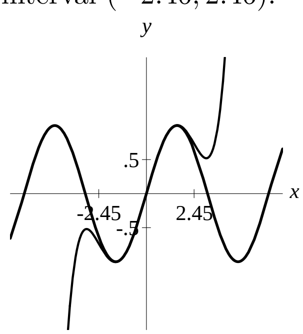
the frequency is 250 cycles per minute.

5.3 Linking Concepts

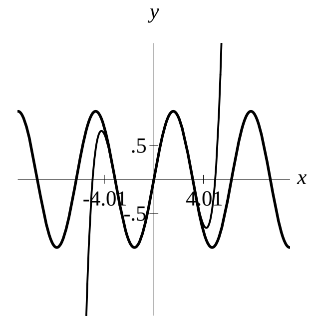
a) From the graphs of $y_1 = x$ and $y_2 = \sin x$, we obtain that y_1 and y_2 differ by less than 0.1 if x lies in the interval $(-0.85, 0.85)$.



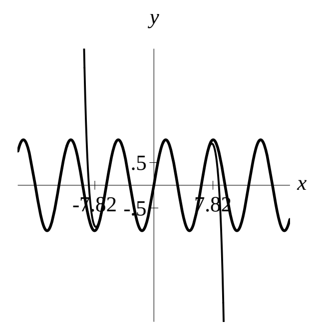
b) From the graphs of $y = x - \frac{x^3}{3!} + \frac{x^5}{5!}$ and $y_2 = \sin x$, it follows that y and y_2 differ by less than 0.1 if x lies in the interval $(-2.46, 2.46)$.



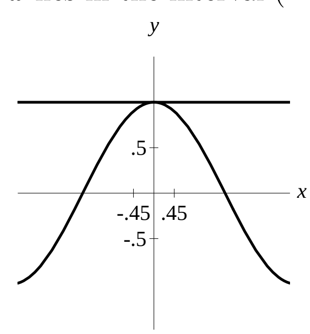
From the graphs of $y = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!}$ and $y_2 = \sin x$, one obtains that y and y_2 differ by less than 0.1 if x lies in the interval $(-4.01, 4.01)$.



From the graphs of $y = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!} + \frac{x^{13}}{13!} - \frac{x^{15}}{15!} + \frac{x^{17}}{17!} - \frac{x^{19}}{19!}$ and $y_2 = \sin(x)$, one finds that y and y_2 differ by less than 0.1 if x lies in the interval $(-7.82, 7.82)$.



c) From the graphs of $y_1 = 1$ and $y = \cos x$, one obtains that y and y_1 differ by less than 0.1 if x lies in the interval $(-0.45, 0.45)$.



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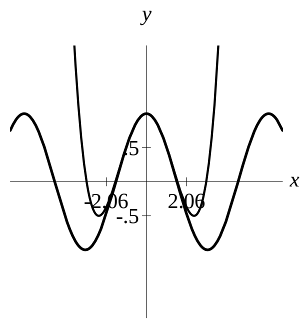
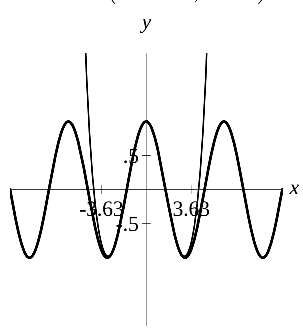
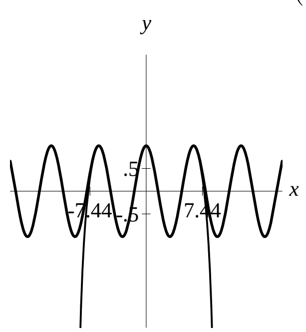
From the graphs of $y = 1 - \frac{x^2}{2!} + \frac{x^4}{4!}$ and $y_2 = \cos x$, one finds y and y_2 differ by less than 0.1 if x lies in the interval $(-2.06, 2.06)$.



From the graphs of $y = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!}$ and $y_2 = \cos x$, one derives that y and y_2 differ by less than 0.1 if x lies in the interval $(-3.63, 3.63)$.



From the graphs of $y = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!} + \frac{x^{12}}{12!} - \frac{x^{14}}{14!} + \frac{x^{16}}{16!} - \frac{x^{18}}{18!}$ and $y_2 = \cos(x)$, one finds that y and y_2 differ by less than 0.1 if x lies in the interval $(-7.44, 7.44)$.



d) To find $\sin x$, first, let y be the reference angle of x. Note, $0 \le y \le \frac{\pi}{2}$. Using the 3rd degree Taylor polynomial for $\sin y$, we get

$$\sin x = \pm \sin y \approx \pm\left(y - \frac{y^3}{3!}\right)$$

where the sign on the right side depends on the quadrant in which angle x lies. Note, the error in the approximation is less than 0.1

For Thought

1. True, since $\sin(\pi/4) = \cos(\pi/4)$.

2. False, since $\cot(\pi/2) = 0$ and $\frac{1}{\tan(\pi/2)}$ is undefined for $\tan(\pi/2)$ is undefined.

3. True, $\csc(60^\circ) = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$.

4. False, since $\tan(5\pi/2)$ is undefined.

5. False, $\sec(95^\circ) < 0$.

6. True, since $\sin 120^\circ = \frac{\sqrt{3}}{2}$ and $\csc 120^\circ = 1/\sin 120^\circ$.

7. False, since the ranges are $(-\infty, -2] \cup [2, \infty)$ and $(-\infty, -0.5] \cup [0.5, \infty)$, respectively.

8. True, since $|\csc x| \ge 1$ and $|0.5 \csc x| \ge 0.5$.

9. True, since $\tan\left(3 \cdot \frac{\pm\pi}{6}\right) = \tan\left(\pm\frac{\pi}{2}\right)$ is undefined.

10. True, since $\cot\left(4 \cdot \frac{\pm\pi}{4}\right) = \cot(\pm\pi)$ is undefined.

5.4 Exercises

1. $\tan\alpha$, $\sec\alpha$

2. vertical asymptote

3. $\tan(0) = 0$, $\tan(\pi/4) = 1$, $\tan(\pi/2)$ undefined, $\tan(3\pi/4) = -1$, $\tan(\pi) = 0$, $\tan(5\pi/4) = 1$, $\tan(3\pi/2)$ undefined, $\tan(7\pi/4) = -1$

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4. $\tan(0) = 0$, $\tan(\pi/6) = \sqrt{3}/3$, $\tan(\pi/3) = \sqrt{3}$, $\tan(\pi/2)$ undefined, $\tan(2\pi/3) = -\sqrt{3}$, $\tan(5\pi/6) = -\sqrt{3}/3$, $\tan(\pi) = 0$, $\tan(7\pi/6) = \sqrt{3}/3$, $\tan(4\pi/3) = \sqrt{3}$, $\tan(3\pi/2)$ undefined, $\tan(5\pi/3) = -\sqrt{3}$, $\tan(11\pi/6) = -\sqrt{3}/3$

5. $\sqrt{3}$ **6.** 1 **7.** -1 **8.** $\sqrt{3}/3$

9. 0 **10.** $-\sqrt{3}/3$

11. $-\sqrt{3}/3$ **12.** undefined

13. $\dfrac{2\sqrt{3}}{3}$ **14.** 2

15. Undefined **16.** -1

17. Undefined **18.** 2

19. $\sqrt{2}$ **20.** $-2\sqrt{3}/3$

21. -1 **22.** undefined

23. $\sqrt{3}$ **24.** $-\sqrt{3}/3$

25. -2 **26.** undefined

27. $-\sqrt{2}$ **28.** $-2\sqrt{3}/3$

29. 0 **30.** 0

31. 48.0785 **32.** -34.2325 **33.** -2.8413

34. 2.3080 **35.** 500.0003 **36.** 1.0005

37. 1.0353 **38.** 1.0824 **39.** 636.6192

40. -95.4895 **41.** -1.4318 **42.** -1.2134

43. 71.6221 **44.** 1.0000 **45.** -0.9861

46. -0.9004

47. $\sec^2\left(2\left(\frac{\pi}{6}\right)\right) = \sec^2\left(\frac{\pi}{3}\right) = 2^2 = 4$

48. $\csc^2\left(2\left(\frac{\pi}{8}\right)\right) = \csc^2\left(\frac{\pi}{4}\right) = \left(\sqrt{2}\right)^2 = 2$

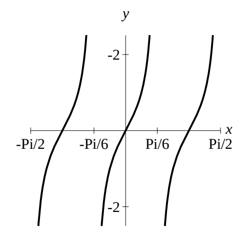
49. $\tan\left(\frac{\frac{\pi}{3}}{2}\right) = \tan\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{3}$

50. $\csc\left(\frac{\frac{\pi}{2}}{2}\right) = \csc\left(\frac{\pi}{4}\right) = \sqrt{2}$

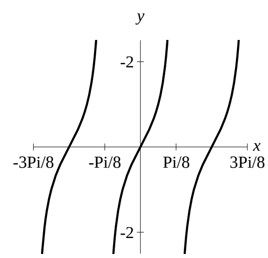
51. $\sec\left(\frac{\frac{3\pi}{2}}{2}\right) = \sec\left(\frac{3\pi}{4}\right) = -\sqrt{2}$

52. $\cot\left(\frac{\frac{2\pi}{3}}{2}\right) = \cot\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{3}$

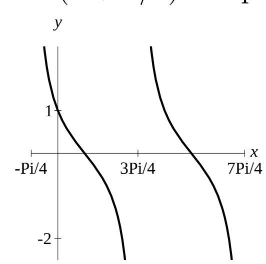
53. $y = \tan(3x)$ has period $\pi/3$



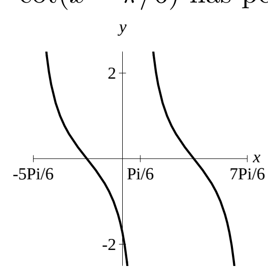
54. $y = \tan(4x)$ has period $\pi/4$



55. $y = \cot(x + \pi/4)$ has period π



56. $y = \cot(x - \pi/6)$ has period π



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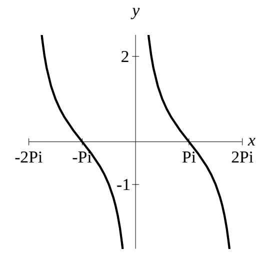
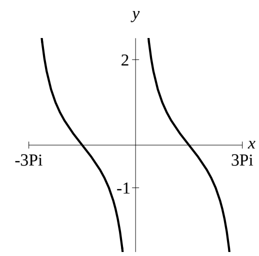
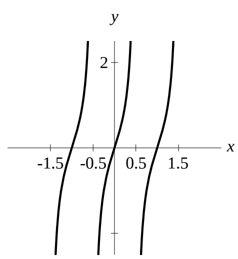
57. $y = \cot(x/2)$ has period 2π



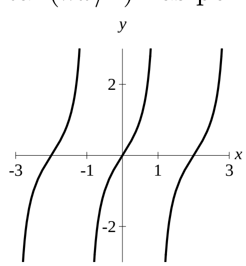
58. $y = \cot(x/3)$ has period 3π



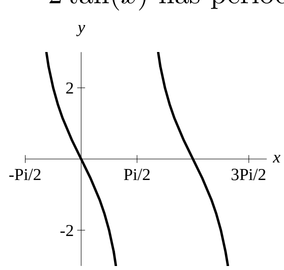
59. $y = \tan(\pi x)$ has period 1



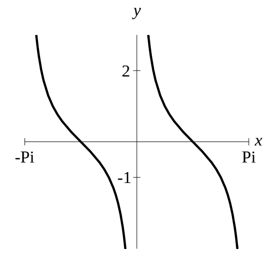
60. $y = \tan(\pi x/2)$ has period 2



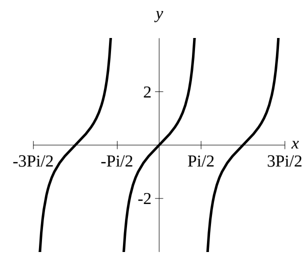
61. $y = -2\tan(x)$ has period π



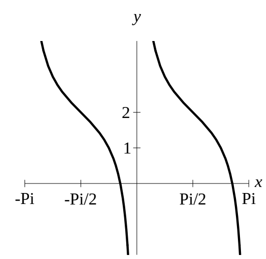
62. $y = -\tan(x - \pi/2)$ has period π



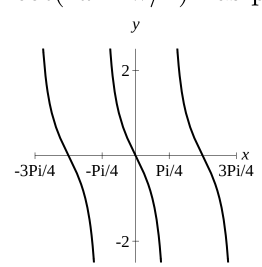
63. $y = -\cot(x + \pi/2)$ has period π



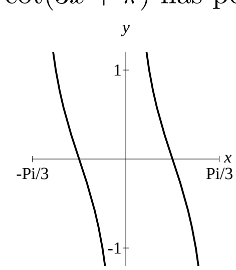
64. $y = 2 + \cot(x)$ has period π



65. $y = \cot(2x - \pi/2)$ has period $\pi/2$



66. $y = \cot(3x + \pi)$ has period $\pi/3$



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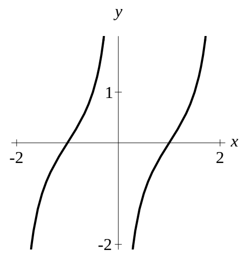
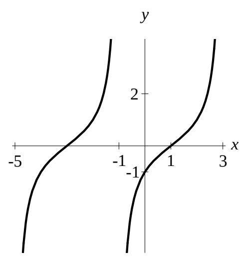
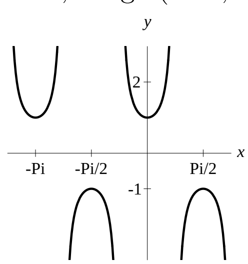
67. $y = \tan\left(\dfrac{\pi}{2} \cdot x - \dfrac{\pi}{2}\right)$ has period 2



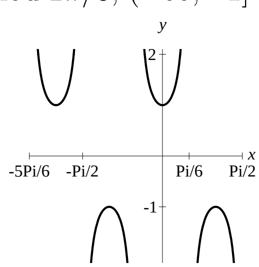
68. $y = \tan\left(\dfrac{\pi}{4} \cdot x + \dfrac{3\pi}{4}\right)$ has period 4



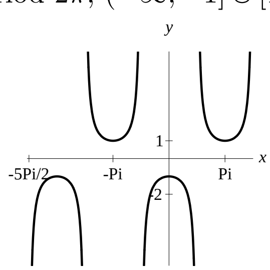
69. period π, range $(-\infty, -1] \cup [1, \infty)$



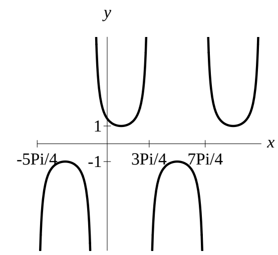
70. period $2\pi/3$, $(-\infty, -1] \cup [1, \infty)$



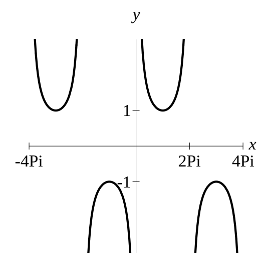
71. period 2π, $(-\infty, -1] \cup [1, \infty)$



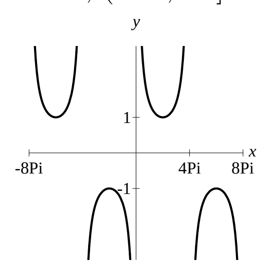
72. period 2π, $(-\infty, -1] \cup [1, \infty)$



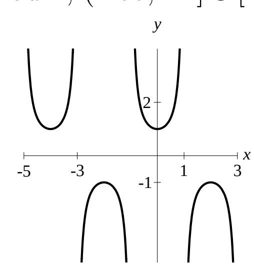
73. period 4π, $(-\infty, -1] \cup [1, \infty)$



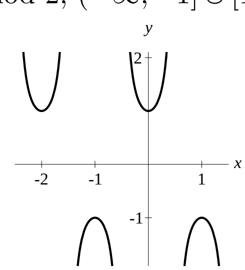
74. period 8π, $(-\infty, -1] \cup [1, \infty)$



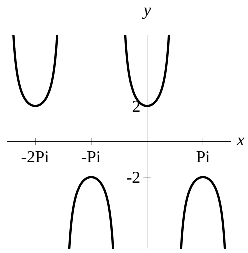
75. period 4, $(-\infty, -1] \cup [1, \infty)$



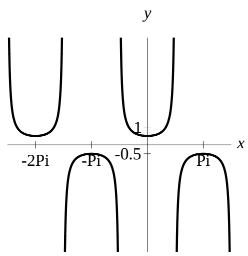
76. period 2, $(-\infty, -1] \cup [1, \infty)$



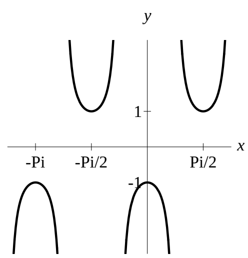
77. period 2π, $(-\infty, -2] \cup [2, \infty)$



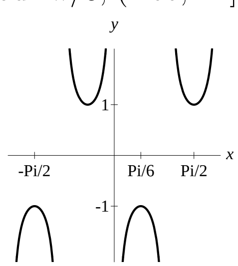
78. period 2π, $(-\infty, -0.5] \cup [0.5, \infty)$



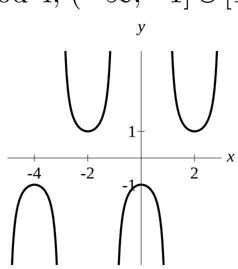
79. period π, $(-\infty, -1] \cup [1, \infty)$



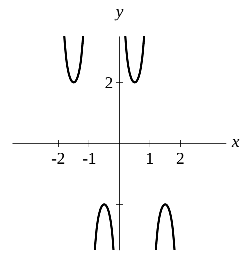
80. period $2\pi/3$, $(-\infty, -1] \cup [1, \infty)$



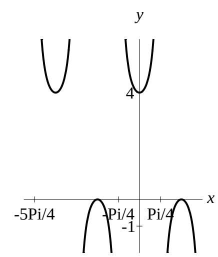
81. period 4, $(-\infty, -1] \cup [1, \infty)$



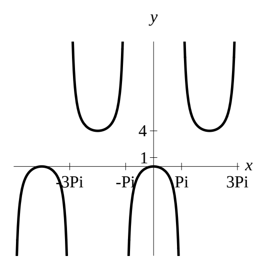
82. period 2, $(-\infty, -2] \cup [2, \infty)$



83. period π, $(-\infty, 0] \cup [4, \infty)$



84. period 4π, $(-\infty, 0] \cup [4, \infty)$



85. Period $\pi/B = \pi/2$, range $(-\infty, \infty)$

86. Period $\pi/B = \pi/3$, range $(-\infty, \infty)$

87. Period $2\pi/B = 2\pi/(1/2) = 4\pi$, range $(-\infty, -2-1] \cup [2-1, \infty)$ or $(-\infty, -3] \cup [1, \infty)$

88. Period $2\pi/B = 2\pi/(1/3) = 6\pi$, range $(-\infty, -2+3] \cup [2+3, \infty)$ or $(-\infty, 1] \cup [5, \infty)$

89. Period $2\pi/B = 2\pi/2 = \pi$, range $(-\infty, -3-4] \cup [3-4, \infty)$ or $(-\infty, -7] \cup [-1, \infty)$

90. Period $2\pi/B = 2\pi/3$, range $(-\infty, -4+5] \cup [4+5, \infty)$ or $(-\infty, 1] \cup [9, \infty)$

91. $y = 3\tan\left(x - \frac{\pi}{4}\right) + 2$

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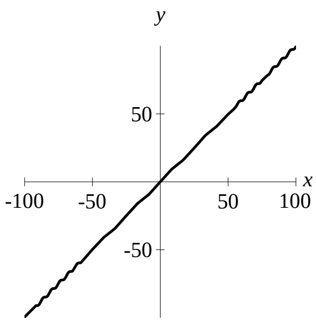
92. $y = -\cot\left(x + \frac{\pi}{2}\right) + 1$

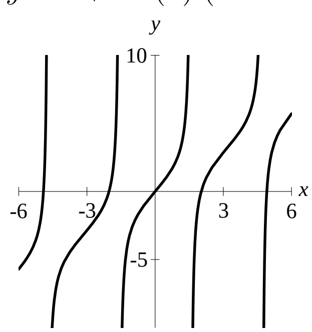
93. $y = -\sec(x + \pi) + 2$

94. $y = -[\csc(x-2) - 3]$ or equivalently $y = -\csc(x-2) + 3$

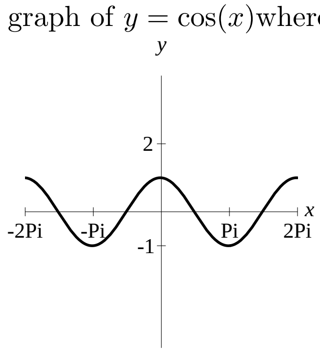
95. By adding the ordinates of $y = x$ and $y = \sin(x)$, we obtain the graph of $y = x + \sin(x)$ (which is given below).



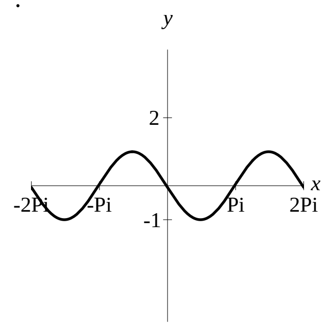
96. By adding the ordinates of $y = x$ and $y = \tan(x)$, we obtain the graph of $y = x + \tan(x)$ (which is given below).



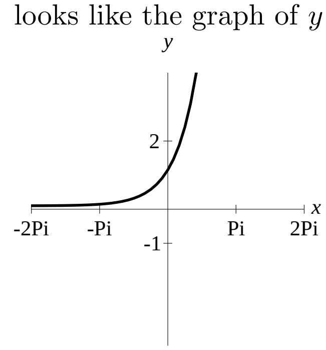
97. a) The graph of y_2 (as shown) looks like the graph of $y = \cos(x)$where $y_1 = \sin(x)$.



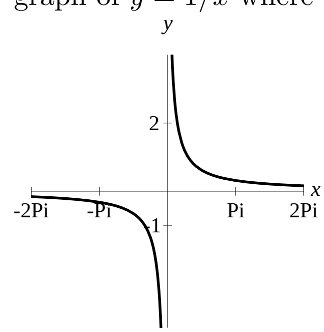
b) The graph of y_2 (as shown) looks like the graph of $y = -\sin(x)$ where $y_1 = \cos(x)$.



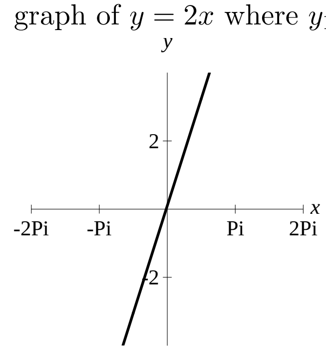
The graph of y_2 (as shown) where $y_1 = e^x$ looks like the graph of $y = e^x$.



The graph of y_2 (as shown) looks like the graph of $y = 1/x$ where $y_1 = \ln(x)$.



The graph of y_2 (as shown) looks like the graph of $y = 2x$ where $y_1 = x^2$.



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98.

a) period is about 2.3 years

b) It looks like the graph of a tangent function.

99. $-210°$

100. Quadrant III, since $\pi < \dfrac{17\pi}{12} < \dfrac{3\pi}{2}$

101. Using $s = r\alpha$, the linear velocity is

$$3950(2\pi) \cdot \frac{5280}{(24)60^2} \approx 1517 \text{ ft/sec}$$

102. Amplitude $\dfrac{1}{2}$, period 2π,

phase shift $\dfrac{\pi}{2}$, range $[2.5, 3.5]$.

103. $y = -\cos(x + \pi) + 2$

104. $\sin\alpha = -\sqrt{1 - \cos^2\alpha} = -\sqrt{1 - 1/9} = -\dfrac{2\sqrt{2}}{3}$

Thinking Outside the Box XLVIII

Let P stand for Porsche, N for Nissan, and C for Chrysler. The fifteen preferences could be the following:

Six preferences: P, N, C (1st, 2nd, 3rd, respectively)

Five preferences: C, N, P

Three preferences: N, C, P

One preference: N, P, C

5.4 Pop Quiz

1. Period is $\pi/B = \pi/3$

2. Since $y = \cot(2x)$, let $2x = k\pi$. The asymptotes are $x = \dfrac{k\pi}{2}$ where k is an integer.

3. Since $y = \sec(2x)$, let $2x = \dfrac{\pi}{2} + k\pi$. The asymptotes are $x = \dfrac{\pi}{4} + \dfrac{k\pi}{2}$ where k is an integer.

4. The range is $(-\infty, -3] \cup [3, \infty)$

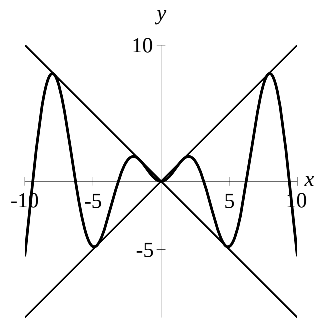
5. 1 **6.** $\dfrac{\sqrt{3}/2}{-1/2} = -\sqrt{3}$

7. $\dfrac{1/2}{-\sqrt{3}/2} = -\dfrac{\sqrt{3}}{3}$ **8.** $\dfrac{1}{1/2} = 2$

9. $\dfrac{1}{-1/\sqrt{2}} = -\sqrt{2}$

5.4 Linking Concepts

a) Shown below are the graphs of $y_1 = x\sin(x)$, $y_2 = x$, and $y_3 = -x$.



Note, if $x \neq 0$, then $x\sin(x) = x$ has the same solution set as $\sin(x) = 1$. Thus, the exact values of x satisfying $x\sin(x) = x$ are

$$x = 0, \frac{\pi}{2} + 2\pi k$$

where k is an integer.

Similarly, if $x \neq 0$, then $x\sin(x) = -x$ has the same solution set as $\sin(x) = -1$. Thus, the exact values of x satisfying $x\sin(x) = -x$ are

$$x = 0, \frac{3\pi}{2} + 2\pi k.$$

b) The exact values of x satisfying $x^2\sin(x) = x^2$ are

$$x = 0, \frac{\pi}{2} + 2\pi k$$

where k is an integer.

The exact values of x satisfying $x^2\sin(x) = -x^2$ are

$$x = 0, \frac{3\pi}{2} + 2\pi k.$$

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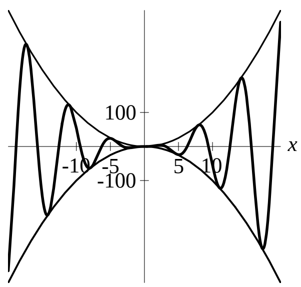
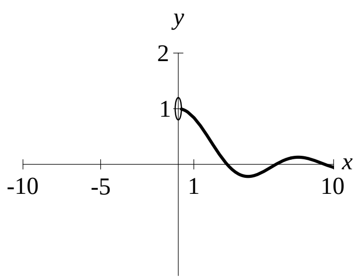
Shown below are the graphs of $y = x^2\sin(x)$, $y = x^2$, and $y = -x^2$. The points of intersection between $y = x^2\sin(x)$ and $y = x^2$ ($y = x^2\sin(x)$ and $y = -x^2$) give the exact solutions to $x^2\sin(x) = x^2$ ($x^2\sin(x) = -x^2$, respectively).



c) Given is a graph of $y_1 = \dfrac{1}{x}\sin(x)$ where $0 \le x \le 10$.



Note, the inequality $-\dfrac{1}{x} < \dfrac{1}{x}\sin(x) < \dfrac{1}{x}$

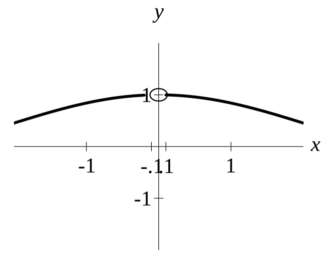
does not hold true if $x = \dfrac{\pi}{2}$.

In fact, $\dfrac{1}{x}\sin(x) = \dfrac{1}{x}$ if $x = \dfrac{\pi}{2}$.

d) From the graph of $f(x) = \dfrac{1}{x}\sin(x)$, we note that $f(0)$ is undefined and we conclude that for each x in $[-0.1, 0.1]$, except when $x = 0$, we have

$$1 > f(x) \ge f(0.1) = f(-0.1) \approx 0.9983.$$

Yes, $0.99 < f(x) < 1$ if x lies in $[-0.1, 0.1]$ and $x \ne 0$.



e) $y = \dfrac{8\sin(3x)}{x}$

For Thought

1. True, $\sin^{-1}(0) = 0 = \sin(0)$.

2. True, since $\sin(3\pi/4) = \dfrac{\sqrt{2}}{2} = \dfrac{1}{\sqrt{2}}$.

3. False, $\cos^{-1}(0) = \pi/2$.

4. False, $\sin^{-1}(\sqrt{2}/2) = 45°$.

5. False, since it equals $\tan^{-1}(1/5)$.

6. True, since $1/5 = 0.2$.

7. True, $\sin(\cos^{-1}(\sqrt{2}/2)) = \sin(\pi/4) = 1/\sqrt{2}$.

8. True by definition of $y = \sec^{-1}(x)$.

9. False, since $f^{-1}(x) = \sin(x)$ where $-\pi/2 \le x \le \pi/2$.

10. False, the secant and cosecant functions are not one-to-one functions and therefore do not have inverse functions.

5.5 Exercises

1. domain

2. range

3. domain

4. range

5. $-\pi/6$ **6.** 0 **7.** $\pi/6$ **8.** $\pi/3$

9. $\pi/4$ **10.** $\pi/2$ **11.** $-45°$ **12.** $60°$

13. $30°$ **14.** $-90°$ **15.** $0°$ **16.** $45°$

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17. $-19.5°$ **18.** $24.4°$

19. $34.6°$ **20.** $-19.9°$

21. $3\pi/4$ **22.** 0 **23.** $\pi/3$ **24.** $5\pi/6$

25. π **26.** $\pi/2$ **27.** $135°$ **28.** $30°$

29. $180°$ **30.** $90°$ **31.** $120°$ **32.** $0°$

33. $173.2°$ **34.** $42.3°$

35. $89.9°$ **36.** $119.9°$

37. $-\pi/4$ **38.** $\pi/3$ **39.** $\pi/3$ **40.** $\pi/3$

41. $\pi/4$ **42.** $-\pi/6$ **43.** $-\pi/6$ **44.** $5\pi/6$

45. 0 **46.** 0 **47.** $\pi/2$ **48.** $-\pi/2$

49. $3\pi/4$ **50.** $\pi/2$ **51.** $2\pi/3$ **52.** $\pi/4$

53. 0.60 **54.** −0.43 **55.** 3.02

56. 0.74 **57.** −0.14 **58.** 0.23

59. 1.87 **60.** 0.15 **61.** 1.15

62. −1.38 **63.** −0.36 **64.** 2.72

65. 3.06 **66.** 1.57 **67.** 0.06 **68.** 2.36

69. $\tan(\pi/3) = \sqrt{3}$ **70.** $\sec(\pi/4) = \sqrt{2}$

71. $\sin^{-1}(-1/2) = -\pi/6$ **72.** $\tan^{-1}(1) = \pi/4$

73. $\cot^{-1}(\sqrt{3}) = \pi/6$ **74.** $\sec^{-1}(2) = \pi/3$

75. $\arcsin(\sqrt{2}/2) = \pi/4$

76. $\arccos(1/2) = \pi/3$

77. $\tan(\pi/4) = 1$ **78.** $\cot(0) = 0$

79. $\cos^{-1}(0) = \pi/2$ **80.** $\sin(-\pi/6) = -1/2$

81. $\cos(2 \cdot \pi/4) = \cos(\pi/2) = 0$

82. $\tan(2 \cdot \pi/3) = -\sqrt{3}$

83. $\sin^{-1}(2 \cdot 1/2) = \sin^{-1}(1) = \pi/2$

84. $\cos^{-1}(0.5 \cdot 1) = \cos^{-1}(1/2) = \pi/3$

85. 0.8930 **86.** 0.0226

87. undefined **88.** 1.1781 **89.** −0.9802

90. undefined **91.** −0.4082 **92.** undefined

93. 3.4583 **94.** 1.1000 **95.** 1.0183

96. 3.2573

97. Solving for y, one finds

$$\begin{aligned} x &= \sin(2y) \\ \sin^{-1}(x) &= 2y \\ y &= \frac{\sin^{-1}(x)}{2}. \end{aligned}$$

Then

$$f^{-1}(x) = 0.5\sin^{-1}(x).$$

As x takes the values in $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$, $2x$ takes all the values in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, and $f(x) = \sin(2x)$ takes all the values in $[-1, 1]$. Thus, the range of f is $[-1, 1]$ which is the domain of f^{-1}.

98. Solving for y, one finds

$$\begin{aligned} x &= \cos(3y) \\ \cos^{-1}(x) &= 3y \\ y &= \frac{\cos^{-1}(x)}{3}. \end{aligned}$$

Thus, $f^{-1}(x) = \dfrac{\cos^{-1}(x)}{3}$ and the domain is $[-1, 1]$ (which is the range of f).

99. Solving for y, one obtains

$$\begin{aligned} x &= 3 + \tan(\pi y) \\ x - 3 &= \tan(\pi y) \\ \tan^{-1}(x-3) &= \pi y \\ \frac{\tan^{-1}(x-3)}{\pi} &= y. \end{aligned}$$

Thus, we find

$$f^{-1}(x) = \frac{\tan^{-1}(x-3)}{\pi}.$$

As x takes the values in $\left(-\frac{1}{2}, \frac{1}{2}\right)$, πx takes all the values in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, $\tan(\pi x)$ takes all the

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values in $(-\infty, \infty)$, and $f(x) = 3 + \tan(\pi x)$ will take all the values in $(-\infty, \infty)$. Thus, the range of f is $(-\infty, \infty)$, which is the domain of f^{-1}.

100. Solving for y, one obtains

$$\begin{aligned} x &= 2 - \sin(\pi y - \pi) \\ \sin(\pi y - \pi) &= 2 - x \\ \pi y - \pi &= \sin^{-1}(2 - x) \\ y &= \frac{\sin^{-1}(2-x) + \pi}{\pi}. \end{aligned}$$

Thus, $f^{-1}(x) = \dfrac{1}{\pi}\sin^{-1}(2-x) + 1$ and the domain is $[1, 3]$ (which is the range of f).

101. Solving for y, one obtains

$$\begin{aligned} x &= \sin^{-1}\left(\frac{y}{2}\right) + 3 \\ x - 3 &= \sin^{-1}\left(\frac{y}{2}\right) \\ \sin(x-3) &= \frac{y}{2} \\ y &= 2\sin(x-3). \end{aligned}$$

Thus, we obtain

$$f^{-1}(x) = 2\sin(x-3).$$

As x takes the values in $[-2, 2]$, $\dfrac{x}{2}$ takes all the values in $[-1, 1]$, $\sin^{-1}\left(\dfrac{x}{2}\right)$ takes all the values in $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$, and $f(x) = \sin^{-1}\left(\dfrac{x}{2}\right) + 3$ will take all the values in $\left[-\dfrac{\pi}{2} + 3, \dfrac{\pi}{2} + 3\right]$. Thus, the range of f is $\left[3 - \dfrac{\pi}{2}, 3 + \dfrac{\pi}{2}\right]$, which is the domain of f^{-1}.

102. Solving for y, one obtains

$$\begin{aligned} x &= 2\cos^{-1}(5y) + 3 \\ \frac{x-3}{2} &= \cos^{-1}(5y) \\ \cos\left(\frac{x-3}{2}\right) &= 5y \\ y &= \frac{1}{5}\cos\left(\frac{x-3}{2}\right). \end{aligned}$$

Thus, $f^{-1}(x) = \dfrac{1}{5}\cos\left(\dfrac{x-3}{2}\right)$ and the domain is $[3, 2\pi + 3]$ (which is the range of f).

103. Consider the right triangle with hypotenuse 2400, altitude 2000, and the angle between the hypotenuse and the altitude is $\dfrac{\theta}{2}$. Since

$$\cos\left(\frac{\theta}{2}\right) = \frac{2000}{2400},$$

we obtain

$$\begin{aligned} \theta &= 2\cos^{-1}\left(\frac{2000}{2400}\right) \\ &\approx 67.1^\circ. \end{aligned}$$

Thus, the airplane is within the range of the gun for $\theta \approx 67.1^\circ$.

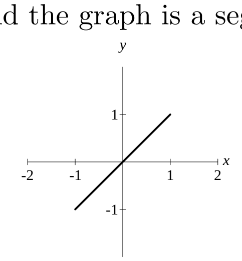
104. First, the smallest angle is opposite the shortest side (which is 1.3 meters). Since one can realize $c = 1.3$ as the length of a chord of a circle with radius 5.2 meters, then the smallest angle is given by

$$\begin{aligned} \theta &= \cos^{-1}\left(1 - \frac{c^2}{2r^2}\right) \\ &= \cos^{-1}\left(1 - \frac{1.3^2}{2(5.2)^2}\right) \\ \theta &\approx 14.4^\circ. \end{aligned}$$

105. Note that

$$y = \sin\left(\sin^{-1} x\right) = x$$

and the graph is a segment of the line $y = x$.



106. $y = \sin^{-1}(\sin x) =$

$$\begin{cases} x + 2\pi & \text{if } -2\pi \le x \le -3\pi/2 \\ -x - \pi & \text{if } -3\pi/2 \le x \le -\pi/2 \\ x & \text{if } -\pi/2 \le x \le \pi/2 \\ \pi - x & \text{if } \pi/2 \le x \le 3\pi/2 \\ x - 2\pi & \text{if } 3\pi/2 \le x \le 2\pi \end{cases}$$

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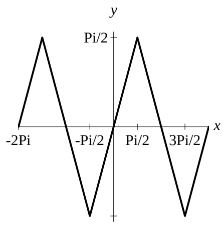
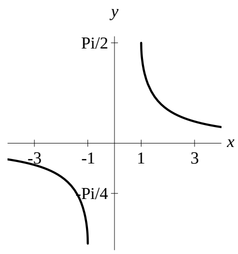
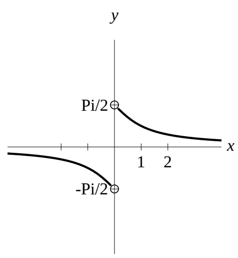


107. Note, $\sin^{-1}(1/x) = \csc^{-1}(x)$ for $-1 \leq x \leq 1$ and $x \neq 0$.



108. The range of $y = \tan^{-1}(1/x)$ and $y = \cot^{-1}(x)$ are $(-\pi/2, \pi/2)$ and $(0, \pi)$, respectively, which are not identical.



109. **a)** -1 **b)** $\frac{\sqrt{2}}{2}$ **c)** $\sqrt{3}$

d) undefined **e)** -2

f) undefined **g)** $-\sqrt{3}$ **h)** $-\frac{\sqrt{2}}{2}$

110. Using the arc length formula $s = r\alpha$, the linear velocity is

$$\frac{93 \cdot 10^6 \cdot 2\pi}{365(24)} \approx 67,000 \text{ mph}$$

111. $\frac{\pi}{6}$

112. $\frac{7\pi}{4} \cdot \frac{180^\circ}{\pi} = 315^\circ$

113. Period π

Solve for x:

$$\begin{aligned} 2x - \pi &= \frac{\pi}{2} + k\pi \\ 2x &= \frac{\pi}{2} + (k+1)\pi \\ x &= \frac{\pi}{4} + \frac{(k+1)\pi}{2} \end{aligned}$$

Since $k+1$ is an integer, the vertical asymptotes are

$$x = \frac{\pi}{4} + \frac{k\pi}{2}$$

where k is an integer

The range is $(-\infty, -3] \cup [3, \infty)$.

114. $\cos\beta = -\sqrt{1 - \sin^2\beta} = -\sqrt{1 - \frac{1}{16}} = -\frac{\sqrt{15}}{4}$

Thinking Outside the Box XLIX

a) Draw a line segment along a diameter of the porthole with one endpoint at the point where the wiper blade is attached. Then form a right triangle such that this line segment-diameter is a hypotenuse of length 2. Let θ be the angle of the sector that is formed by the wiper blades. Then the angle of the right triangle at the point of attachment is $\theta/2$. Using right triangle trigonometry, we obtain

$$\theta = 2\cos^{-1}\left(\frac{x}{2}\right).$$

Since the area of the sector is $A = \frac{\theta x^2}{2}$, we find

$$A = x^2 \cos^{-1}\left(\frac{x}{2}\right).$$

b) The area of a unit circle, i.e., radius 1, is π. If the area of the sector in part a) is one-half of the area of a unit circle, then

$$x^2 \cos^{-1}\left(\frac{x}{2}\right) = \frac{\pi}{2}.$$

Note, the graph of

$$y = x^2 \cos^{-1}\left(\frac{x}{2}\right) - \frac{\pi}{2}, \quad 0 < x < 2$$

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has only two x-intercepts. By inspection, these are $x = \sqrt{2}$ ft or $x = \sqrt{3}$ ft.

c) Using a graphing calculator, we find that the area

$$A = x^2 \cos^{-1}\left(\frac{x}{2}\right)$$

is maximized when $x \approx 1.5882$ ft.

5.5 Pop Quiz

1. Solving for y, we obtain

$$\begin{aligned} x &= \cos^{-1}(2y) \\ \cos^{-1} x &= 2y \\ \frac{\cos^{-1} x}{2} &= y. \end{aligned}$$

Thus, the inverse is

$$f^{-1}(x) = \frac{1}{2}\cos^{-1} x.$$

2. $-\frac{\pi}{2}$ **3.** $\frac{\pi}{6}$ **4.** π

5. $-\frac{\pi}{4}$ **6.** $\tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$

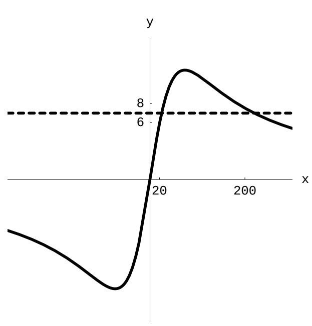
7. $\sin^{-1}\frac{\sqrt{2}}{2} = \frac{\pi}{4}$

5.5 Linking Concepts

a) Shown below are the graphs of

$$y = \alpha = \tan^{-1}(90/x) - \tan^{-1}(60/x)$$

with α in degrees, and the graph of $y = 7$.



b) From the graph, α is greater than 7° if x lies in the interval

$$(24.6 \text{ ft}, 219.8 \text{ ft}).$$

c) The distance a motorist will be traveling with a viewing angle greater than 7° is $\frac{219.8 - 24.6}{5280}$ of a mile. The time duration is

$$\frac{\frac{219.8-24.6}{5280}}{35} \approx 0.001056 \text{ of an hour}$$

or 3.8 seconds.

For Thought

1. False, since $\sin\alpha = -10/\sqrt{125}$.

2. True, since

$$r = \sqrt{(-1)^2 + 2^2} = \sqrt{5}$$

we get

$$\sec\alpha = \frac{r}{x} = \frac{\sqrt{5}}{-1} = -\sqrt{5}.$$

3. False, α may not lie in $[-\pi/2, \pi/2]$.

4. False, α may not lie in $[0, \pi]$.

5. True, since the side opposite α is the side adjacent to β.

6. False, $c = \sqrt{20}$.

7. False, since $\tan\beta = 1/3$.

8. True, since $\tan(55°) = 8/b$.

9. True, the smallest angle is $\cos^{-1}(4/5)$.

10. False, $\sin(90°) = 1 \neq$ hyp/adj.

5.6 Exercises

1. opposite side, hypotenuse

2. adjacent side, hypotenuse

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3. opposite side, adjacent side

4. sum, 180°

5. $\sin(\alpha) = 4/5, \cos(\alpha) = 3/5, \tan(\alpha) = 4/3,$
$\csc(\alpha) = 5/4, \sec(\alpha) = 5/3, \cot(\alpha) = 3/4$

6. $\sin(\alpha) = \sqrt{2}/2, \cos(\alpha) = \sqrt{2}/2, \tan(\alpha) = 1,$
$\csc(\alpha) = \sqrt{2}, \sec(\alpha) = \sqrt{2}, \cot(\alpha) = 1$

7. $\sin(\alpha) = 3\sqrt{10}/10, \cos(\alpha) = -\sqrt{10}/10,$
$\tan(\alpha) = -3, \csc(\alpha) = \sqrt{10}/3,$
$\sec(\alpha) = -\sqrt{10}, \cot(\alpha) = -1/3$

8. $\sin(\alpha) = 2\sqrt{5}/5, \cos(\alpha) = -\sqrt{5}/5,$
$\tan(\alpha) = -2, \csc(\alpha) = \sqrt{5}/2, \sec(\alpha) = -\sqrt{5},$
$\cot(\alpha) = -1/2$

9. $\sin(\alpha) = -\sqrt{3}/3, \cos(\alpha) = -\sqrt{6}/3,$
$\tan(\alpha) = \sqrt{2}/2, \csc(\alpha) = -\sqrt{3},$
$\sec(\alpha) = -\sqrt{6}/2, \cot(\alpha) = \sqrt{2}$

10. $\sin(\alpha) = -\sqrt{3}/2, \cos(\alpha) = -1/2,$
$\tan(\alpha) = \sqrt{3}, \csc(\alpha) = -2\sqrt{3}/3,$
$\sec(\alpha) = -2, \cot(\alpha) = \sqrt{3}/3$

11. $\sin(\alpha) = -1/2, \cos(\alpha) = \sqrt{3}/2,$
$\tan(\alpha) = -\sqrt{3}/3, \csc(\alpha) = -2,$
$\sec(\alpha) = 2\sqrt{3}/3, \cot(\alpha) = -\sqrt{3}$

12. $\sin(\alpha) = -1/2, \cos(\alpha) = \sqrt{3}/2,$
$\tan(\alpha) = -\sqrt{3}/3, \csc(\alpha) = -2,$
$\sec(\alpha) = 2\sqrt{3}/3, \cot(\alpha) = -\sqrt{3}$

13. $\sin(\alpha) = \sqrt{5}/5, \cos(\alpha) = 2\sqrt{5}/5, \tan(\alpha) = 1/2,$
$\sin(\beta) = 2\sqrt{5}/5, \cos(\beta) = \sqrt{5}/5, \tan(\beta) = 2$

14. $\sin(\alpha) = 7\sqrt{58}/58, \cos(\alpha) = 3\sqrt{58}/58,$
$\tan(\alpha) = 7/3, \sin(\beta) = 3\sqrt{58}/58,$
$\cos(\beta) = 7\sqrt{58}/58, \tan(\beta) = 3/7$

15. $\sin(\alpha) = 3\sqrt{34}/34, \cos(\alpha) = 5\sqrt{34}/34,$
$\tan(\alpha) = 3/5, \sin(\beta) = 5\sqrt{34}/34,$
$\cos(\beta) = 3\sqrt{34}/34, \tan(\beta) = 5/3$

16. $\sin(\alpha) = 2\sqrt{13}/13, \cos(\alpha) = 3\sqrt{13}/13,$
$\tan(\alpha) = 2/3, \sin(\beta) = 3\sqrt{13}/13,$
$\cos(\beta) = 2\sqrt{13}/13, \tan(\beta) = 3/2$

17. $\sin(\alpha) = 4/5, \cos(\alpha) = 3/5, \tan(\alpha) = 4/3,$
$\sin(\beta) = 3/5, \cos(\beta) = 4/5, \tan(\beta) = 3/4$

18. $\sin(\alpha) = 20/\sqrt{481}, \cos(\alpha) = 9/\sqrt{481},$
$\tan(\alpha) = 20/9, \sin(\beta) = 9/\sqrt{481},$
$\cos(\beta) = 20/\sqrt{481}, \tan(\beta) = 9/20$

19. $\tan^{-1}(9/1.5) \approx 80.5°$

20. $\tan^{-1}(5/4) \approx 51.3°$

21. $\tan^{-1}(\sqrt{3}) = 60°$

22. $\tan^{-1}(6.9/4.3) \approx 58.1°$

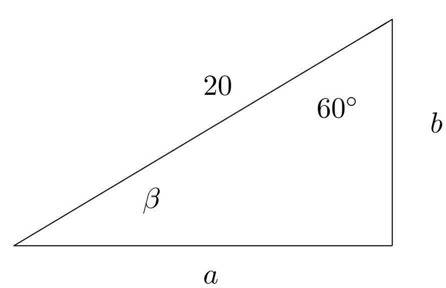
23. $\tan^{-1}(6.3/4) \approx 1.0$

24. $\tan^{-1}(3/2) \approx 1.0$

25. $\tan^{-1}(1/\sqrt{5}) \approx 0.4$

26. $\tan^{-1}(\sqrt{3/7}) \approx 0.6$

27. Form the right triangle with $\alpha = 60°$, $c = 20$.



Since $\sin 60° = \dfrac{a}{20}$, we get

$$a = 20 \cdot \frac{\sqrt{3}}{2} = 10\sqrt{3}.$$

Since $\cos 60° = \dfrac{b}{20}$, we find

$$b = 20 \cdot \frac{1}{2} = 10.$$

Also, $\beta = 90° - 60° = 30°$.

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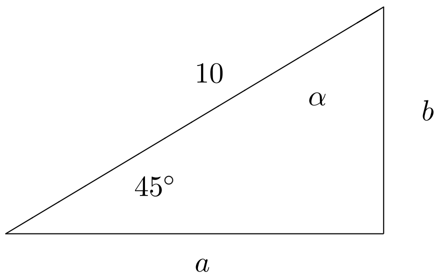
28. Form the right triangle with $\beta = 45°$, $c = 10$.



Since $\sin 45° = \dfrac{b}{10}$, we get

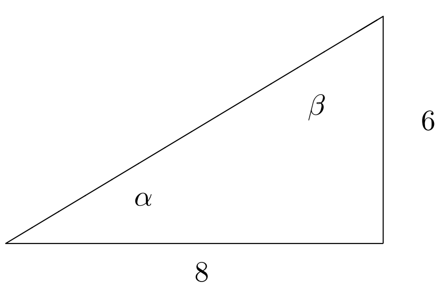
$$b = 10 \cdot \frac{\sqrt{2}}{2} = 5\sqrt{2}.$$

Since $\cos 45° = \dfrac{a}{10}$, we find

$$a = 10 \cdot \frac{\sqrt{2}}{2} = 5\sqrt{2}.$$

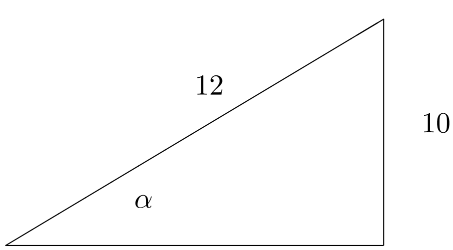
Also, $\alpha = 90° - 45° = 45°$.

29. Form the right triangle with $a = 6$, $b = 8$.



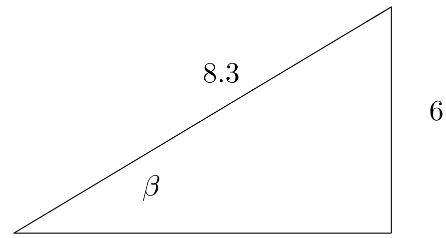
Note: $c = \sqrt{6^2 + 8^2} = 10$, $\tan(\alpha) = 6/8$, so $\alpha = \tan^{-1}(6/8) \approx 36.9°$ and $\beta \approx 53.1°$.

30. Form the right triangle with $a = 10$, $c = 12$.



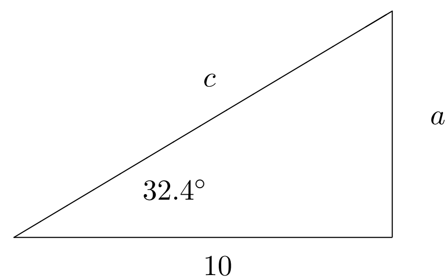
Note: $b = \sqrt{12^2 - 10^2} = \sqrt{44}$, $\sin(\alpha) = 10/12$, and so $\alpha = \sin^{-1}(5/6) \approx 56.4°$ and $\beta \approx 33.6°$.

31. Form the right triangle with $b = 6$, $c = 8.3$.



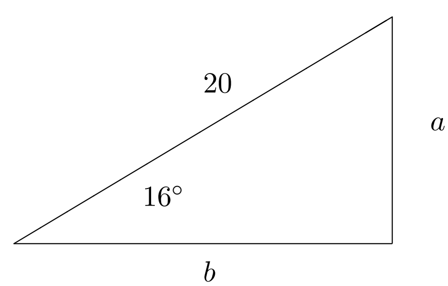
Note: $a = \sqrt{8.3^2 - 6^2} \approx 5.7$, $\sin(\beta) = 6/8.3$, so $\beta = \sin^{-1}(6/8.3) \approx 46.3°$ and $\alpha \approx 43.7°$.

32. Form the right triangle with $\alpha = 32.4°$, $b = 10$.



Since $\tan(32.4°) = a/10$ and $\cos(32.4) = 10/c$, $a = 10\tan(32.4°) \approx 6.3$ and $c = 10/\cos(32.4°) \approx 11.8$. Also $\beta = 57.6°$.

33. Form the right triangle with $\alpha = 16°$, $c = 20$.



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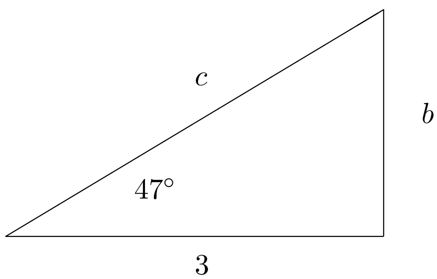
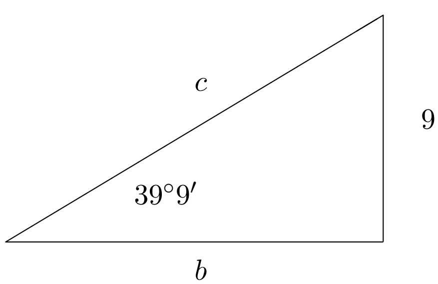
Since $\sin(16°) = a/20$ and $\cos(16°) = b/20$, $a = 20\sin(16°) \approx 5.5$ and $b = 20\cos(16°) \approx 19.2$. Also $\beta = 74°$.

34. Form the right triangle with $\beta = 47°$, $a = 3$.



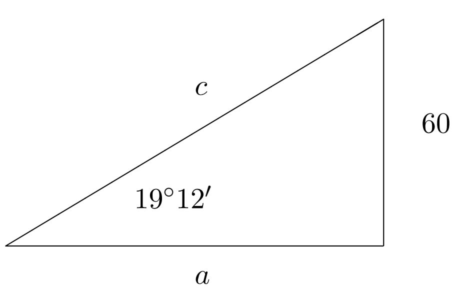
Since $\tan(47°) = b/3$ and $\cos(47°) = 3/c$ then $b = 3 \cdot \tan(47°) \approx 3.2$ and $c = 3/\cos(47°) \approx 4.4$. Also $\alpha = 43°$.

35. Form the right triangle with $\alpha = 39°9'$, $a = 9$.



Since $\sin(39°9') = 9/c$ and $\tan(39°9') = 9/b$, then $c = 9/\sin(39°9') \approx 14.3$ and $b = 9/\tan(39°9') \approx 11.1$. Also $\beta = 50°51'$.

36. Form the right triangle with $\beta = 19°12'$, $b = 60$.



Since $\sin(19°12') = 60/c$ and $\tan(19°12') = 60/a$, then $c = 60/\sin(19°12') \approx 182.4$ and $a = 60/\tan(19°12') \approx 172.3$. Also $\alpha = 70°48'$.

37. 25, the least number of significant digits is 2.

38. 90.6, the least number of significant digits is 3.

39. 0.831, the least number of significant digits is 3.

40. 1×10^1, since the least number of significant digits is 1.

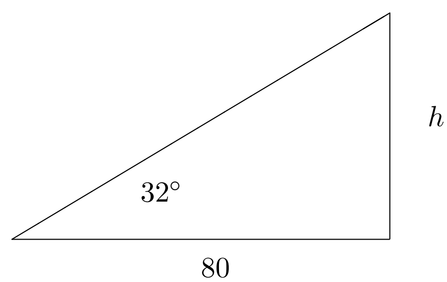
41. 18.8, the least number of significant digits is 3.

42. 0.006, least number of significant digits is 1.

43. −289, least number of significant digits is 3.

44. −38.3, least number of significant digits is 3.

45. Let h be the height of the buliding.



Since $\tan(32°) = h/80$, we obtain

$$h = 80 \cdot \tan(32°) \approx 50 \text{ ft.}$$

46. Let h be the height of the tree.

$$\begin{aligned} \tan 75° &= \frac{h}{80} \\ 80\tan 75° &= h \\ h &\approx 299 \text{ ft} \end{aligned}$$

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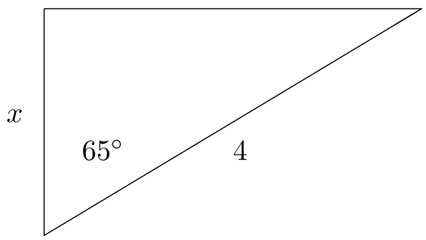
47. Let x be the distance between Muriel and the road at the time she encountered the swamp.



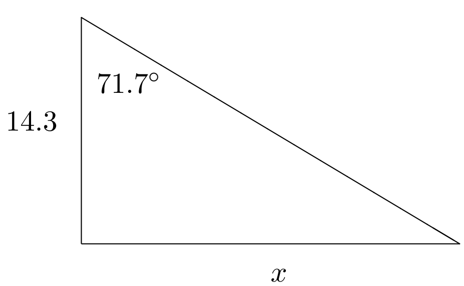
Since $\cos(65°) = x/4$, we find

$$x = 4 \cdot \cos(65°) \approx 1.7 \text{ miles.}$$

48. Let h be the height of the antenna.

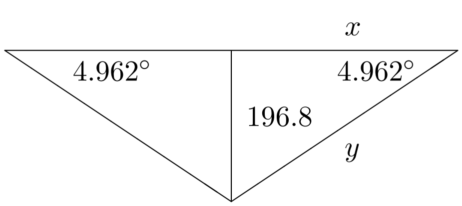
$$\begin{aligned} \sin 62° &= \frac{h}{100} \\ 100 \sin 62° &= h \\ h &\approx 88 \text{ ft} \end{aligned}$$

49. Let x be the distance between the car and a point on the highway directly below the observer.



Since $\tan(71.7°) = x/14.3$, we obtain $x = 14.3 \cdot \tan(71.7°) \approx 43.2$ meters.

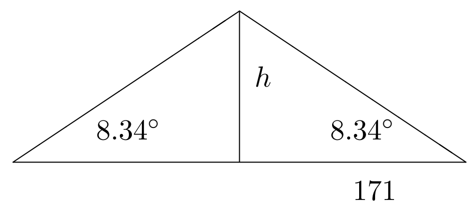
50. Let x and y be as in the picture below.



Since $\tan(4.962°) = 196.8/x$, the distance on the surface between the entrances is $2x = 2 \cdot \dfrac{196.8}{\tan(4.962°)} \approx 4533$ ft.

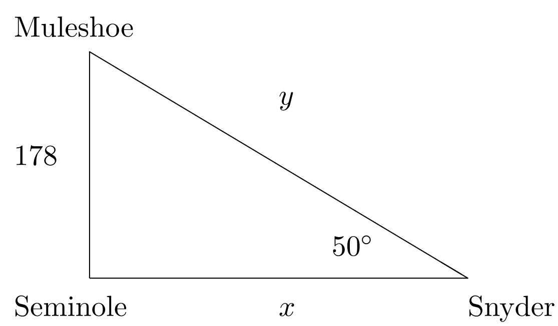
Similarly, since $\sin(4.962°) = 196.8/y$, we get that the length of the tunnel is $2y = 2 \cdot \dfrac{196.8}{\sin(4.962°)} \approx 4551$ ft.

51. Let h be the height as in the picture below.



Since $\tan(8.34°) = h/171$, we obtain $h = 171 \cdot \tan(8.34°) \approx 25.1$ ft.

52.



Note that $x = \dfrac{178}{\tan 50°} \approx 149.36$, $y = \dfrac{178}{\sin 50°} \approx 232.36$, and $178 + 149.36 - 232.36 = 95$. Harry drove 95 more miles than Harriet.

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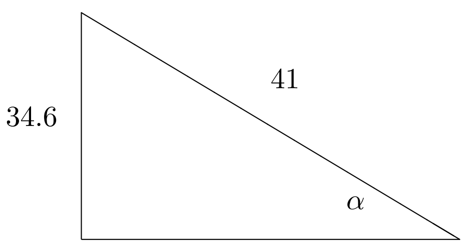
53. Let α be the angle the guy wire makes with the ground.

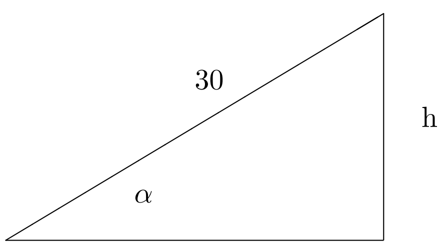


From the Pythagorean Theorem, the distance of the point to the base of the antenna is

$$\sqrt{41^2 - 34.6^2} \approx 22 \text{ meters.}$$

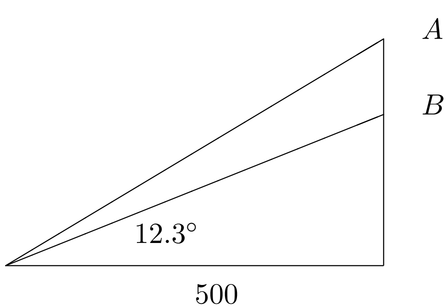
Also, $\alpha = \sin^{-1}(34.6/41) \approx 57.6^\circ$.

54. Let α be an angle of elevation of the ladder.



Since $\sin(\alpha) = h/30$, the minimum and maximum heights are $30\sin(55^\circ) \approx 24.6$ ft and $30\sin(70^\circ) \approx 28.2$ ft, respectively.

55. Note, 1.75 sec. = 1.75/3600 hour.



The distance in miles between A and B is

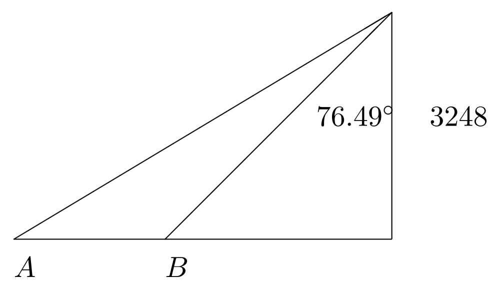
$$\frac{500\,(\tan(15.4^\circ) - \tan(12.3^\circ))}{5280} \approx 0.0054366$$

The speed is

$$\frac{0.0054366}{(1.75/3600)} \approx 11.2 \text{ mph}$$

and the car is not speeding.

56. Note, 5 min. = 1/12 hour.

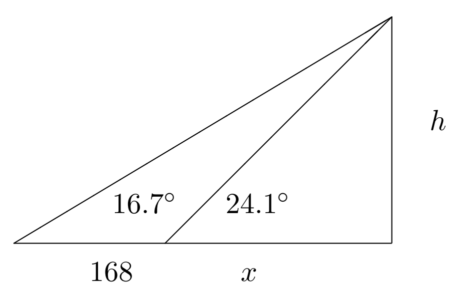


The distance in miles between A and B is

$$\frac{3248\,(\tan(78.66^\circ) - \tan(76.49^\circ))}{5280} \approx 0.50706$$

The speed is $\dfrac{0.50706}{1/12} \approx 6.1$ mph.

57. Let h be the height.



Note, $\tan 24.1^\circ = \dfrac{h}{x}$ and $\tan 16.7^\circ = \dfrac{h}{168 + x}$. Solve for h in the second equation and substitute $x = \dfrac{h}{\tan 24.1^\circ}$.

$$h = \tan(16.7^\circ) \cdot \left(168 + \frac{h}{\tan 24.1^\circ}\right)$$

$$h - \frac{h\tan(16.7^\circ)}{\tan 24.1^\circ} = \tan(16.7^\circ) \cdot 168$$

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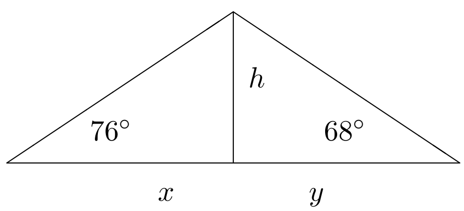
$$h = \frac{168 \cdot \tan(16.7^\circ)}{1 - \tan(16.7^\circ)/\tan(24.1^\circ)}$$

$$h \approx 153.1\text{meters}$$

The height is 153.1 meters.

58. Let h be the height of the balloon.

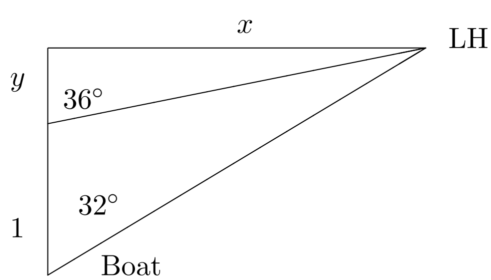


Note that $1.2 = x + y = \frac{h}{\tan(76^\circ)} + \frac{h}{\tan(68^\circ)}$.

If we factor h, then we obtain

$$h = \frac{1.2}{1/\tan(76^\circ) + 1/\tan(68^\circ)} \approx 1.8 \text{ miles.}$$

59. Let x be the closest distance the boat can come to the lighthouse LH.



Since $\tan(36^\circ) = x/y$ and $\tan(32^\circ) = x/(1+y)$, we obtain

$$\tan(32^\circ) = \frac{x}{1 + x/\tan(36^\circ)}$$

$$\tan(32^\circ) + \frac{\tan(32^\circ)x}{\tan(36^\circ)} = x$$

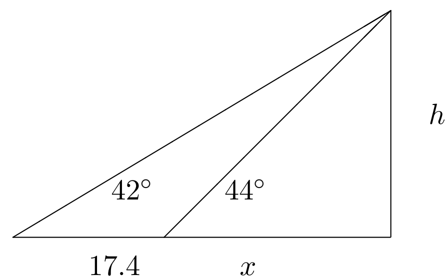
$$\tan(32^\circ) = x\left(1 - \frac{\tan(32^\circ)}{\tan(36^\circ)}\right)$$

$$x = \frac{\tan(32^\circ)}{1 - \tan(32^\circ)/\tan(36^\circ)}$$

$$x \approx 4.5 \text{ km.}$$

The closest the boat will come to the lighthouse is 4.5 km.

60. Let h be the height of the Woolworth skyscraper.



Since $\tan(44^\circ) = h/x$ and $\tan(42^\circ) = \frac{h}{17.4 + x}$, we find

$$\tan(42^\circ) = \frac{h}{17.4 + h/\tan(44^\circ)}$$

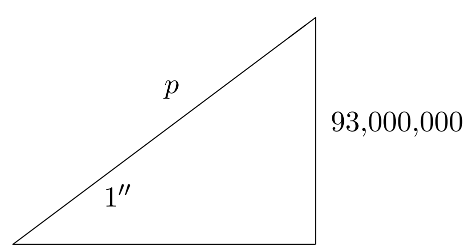
$$17.4 \cdot \tan(42^\circ) + \frac{\tan(42^\circ)h}{\tan(44^\circ)} = h$$

$$17.4 \cdot \tan(42^\circ) = h\left(1 - \tan(42^\circ)/\tan(44^\circ)\right)$$

$$h = \frac{17.4 \cdot \tan(42^\circ)}{1 - \tan(42^\circ)/\tan(44^\circ)}$$

$$h \approx 232 \text{ meters.}$$

61. Let p be the number of miles in one parsec.



Since $\sin(1'') = \frac{93,000,000}{p}$, we obtain

$$p = \frac{93,000,000}{\sin(1/3600^\circ)} \approx 1.9 \times 10^{13} \text{ miles.}$$

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Light travels one parsec in 3.3 years for
$$\frac{p}{186,000 \text{ miles/sec.}} \cdot \frac{1}{365} \cdot \frac{1}{24} \cdot \frac{1}{3600} \approx 3.3$$

62.

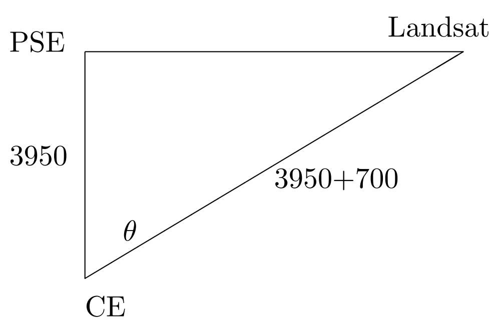
a) $\alpha = \cos^{-1}\left(\frac{4000(5280)}{4000(5280)+2}\right) \approx 0.0249°$

b) $\alpha = \cos^{-1}\left(\frac{4000(5280)}{4000(5280)+6}\right) \approx 0.0432°$

c) If the earth rotates every 24 hours, then the number of seconds it takes the earth to rotate 1° is
$$p = \frac{360}{24(60)(60)} = \frac{1}{240} \text{ sec}$$

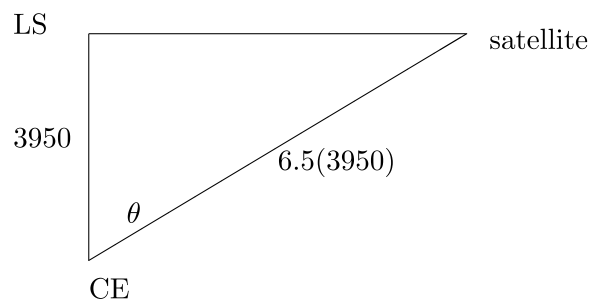
Then the number of seconds between the times when Diane and Ed sees the green flash is
$$\frac{0.0432 - 0.0249}{p} \approx 4.4 \text{ sec}$$

63. In the triangle below CE stands for the center of the earth and PSE is a point on the surface of the earth on the horizon of the cameras of Landsat.



Since $\cos(\theta) = \frac{3950}{3950+700}$, we have
$\theta = \cos^{-1}\left(\frac{3950}{3950+700}\right) \approx 0.5558$ radians.
But 2θ is the central angle, with vertex at CE, intercepted by the path on the surface of the earth as can be seen by Landsat. The width of this path is the arclength subtended by 2θ, i.e., $s = r \cdot 2\theta = 3950 \cdot 2 \cdot 0.5558 \approx 4391$ miles

64. In the triangle below CE stands for the center of the earth, and LS is a point on the surface of the earth lying in the line of sight of the satellite.



Since $\cos(\theta) = \frac{3950}{6.5(3950)} = 1/6.5$, we get
$\theta = \cos^{-1}(1/6.5) \approx 1.41634$ radians. But 2θ is the widest angle formed by a sender and receiver of a signal with vertex CE. The maximum distance is the arclength subtended by 2θ, i.e., $s = r \cdot 2\theta = 3950 \cdot 2 \cdot 1.41634 \approx 11,189$ miles.

65. Let h be the height of the building, and let x be the distance between C and the building. Using right triangle trigonometry, we obtain
$$\frac{1}{\sqrt{3}} = \tan 30° = \frac{h}{40+x}$$
and
$$1 = \tan 45° = \frac{h}{20+x}.$$

Solving simultaneously, we find $x = 10(\sqrt{3}-1)$ and $h = 10(\sqrt{3}+1)$. However,
$$\tan C = \frac{h}{x} = \frac{10(\sqrt{3}+1)}{10(\sqrt{3}-1)} = 2 + \sqrt{3}$$

Since $\tan 75° = 2+\sqrt{3}$ by the addition formula for tangent, we obtain
$$C = 75°$$

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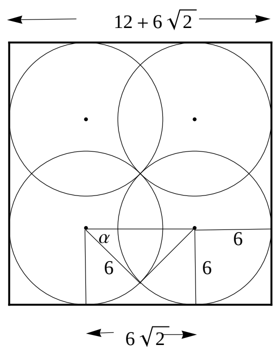
66. In two minutes, the height increases by $250(\tan 58^\circ - \tan 24^\circ)$. Then the rate of ascent is

$$\frac{250(\tan 58^\circ - \tan 24^\circ)}{2(60)} \approx 2.4 \ \frac{\text{m}}{\text{sec}}$$

67. a) The distance from a center to the nearest vertex of the square is $6\sqrt{2}$ by the Pythagorean theorem. Then the diagonal of the square is $12 + 12\sqrt{2}$. From which, the side of the square is $12 + 6\sqrt{2}$ by the Pythagorean theorem as shown below.



The area A_c of the region in one corner of the box that is *not* watered is obtained by subtracting one-fourth of the area of a circle of radius 6 from the area of a 6-by-6 square:

$$A_c = 36 - 9\pi.$$

Note, the distance between two horizontal centers is $6\sqrt{2}$ as shown above. Then the angle α between the line joining the centers and the line to the intersection of the circles is $\alpha = \pi/4$.

Thus, the area A_b of the region between two adjacent circles that is not watered is the area of a 6-by-$6\sqrt{2}$ square minus the area, 18, of the isosceles triangle with base angle $\alpha = \pi/4$, and minus the combined area 9π of two sectors with central angle $\pi/4$:

$$A_b = 36\sqrt{2} - 18 - 9\pi$$

Hence, the total area not watered is

$$\begin{aligned} 4(A_c + A_b) &= 4\left(36 - 9\pi + 36\sqrt{2} - 18 - 9\pi\right) \\ &= 72\left(1 + 2\sqrt{2} - \pi\right) \end{aligned}$$

b) Since the side of the square is $12 + 6\sqrt{2}$, the area that is watered by at least one sprinkler is

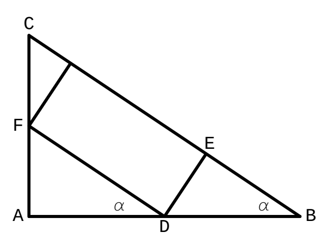
$$100\% - \frac{4(A_c + A_b)}{(12 + 6\sqrt{2})^2} \cdot 100\% \approx 88.2\%$$

68. The triangle that encloses the three circles is an equilateral triangle. From this, it follows that the central angle subtended by the metal band along the circumference of a circle is 120°. Then the length of the metal band along the circumference of a circle of radius 1 ft is

$$s = r\alpha = 1 \cdot \frac{2\pi}{3} = \frac{2\pi}{3}$$

Thus, the length of the band around the three circles is 2π. Note, the length of the remaining metal band (i.e., part of triangle) is 12 ft. Thus, the length of the metal band around the circles is $2\pi + 12$ ft.

69. First, consider the figure below.



Suppose $AC = s$, $AB = \dfrac{3s}{2}$, $BD = h$, $DE = w$, and $DF = L$. Note,

$$\tan\alpha = \frac{AC}{AB} = \frac{2}{3}.$$

Since $\sin\alpha = \dfrac{2}{\sqrt{13}}$ and $\sin\alpha = \dfrac{DE}{BD}$, we obtain $w = \dfrac{2h}{\sqrt{13}}$. Since $\cos\alpha = \dfrac{AD}{DF}$, we get

$$\cos\alpha = \frac{\frac{3s}{2} - h}{L}.$$

Solving for L. we find $L = \dfrac{\frac{3s}{2} - h}{\cos\alpha}$

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and since $\cos\alpha = \dfrac{3}{\sqrt{13}}$, we get

$L = \dfrac{\sqrt{13}}{3}\left(\dfrac{3s}{2} - \dfrac{w\sqrt{13}}{2}\right)$.

So, the area of the parking lot is

$$wL = \frac{w\sqrt{13}}{3}\left(\frac{3s}{2} - \frac{w\sqrt{13}}{2}\right).$$

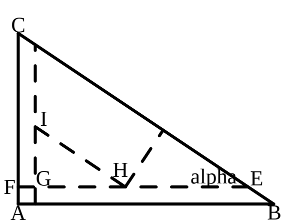
Since this area represents a quadratic function of w, one can find the vertex of its graph and conclude that the maximum area of the rectangle is obtained if one chooses $w = \dfrac{3s}{2\sqrt{13}}$.

Correspondingly, we obtain $L = \dfrac{s\sqrt{13}}{4}$.

Finally, given $s = 100$ feet, the dimensions of the house with maximum area are

$$w = \frac{3(100)}{2\sqrt{13}} \approx 41.60 \text{ ft and } L = \frac{100\sqrt{13}}{4} \approx 90.14 \text{ ft.}$$

70. Consider the right triangle below.



The triangle with vertices at G, H, and I is the part of the lot where the parking lot must be built so that the parking lot is 10 feet from each side of the property and 40 feet from the street.

Note, $AC = 100$, $AB = 150$, and $AF = FG = 10$. Since the ratios of corresponding sides of similar triangles are equal, we have

$$\begin{aligned} \frac{100}{150} &= \frac{90}{EF} \\ EF &= 90\left(\frac{3}{2}\right) \\ EF &= 135. \end{aligned}$$

Since H is 40 feet from BC, we get

$$\begin{aligned} \sin\alpha &= \frac{40}{EH} \\ \frac{2}{\sqrt{13}} &= \frac{40}{EH} \\ EH &= 20\sqrt{13}. \end{aligned}$$

Then

$$\begin{aligned} GH &= EF - EH - 10 \\ &= 125 - 20\sqrt{13} \end{aligned}$$

and by using the ratios of similar triangles one finds

$$\begin{aligned} \frac{100}{150} &= \frac{GI}{GH} \\ GI &= \frac{2}{3}(125 - 20\sqrt{13}). \end{aligned}$$

From the discussion in Exercise 61, the dimensions of the parking lot with maximum area that can be built inside the triangle with vertices G, H, and I are (recall $s = GI$)

$$\begin{aligned} w &= \frac{3s}{2\sqrt{13}} \\ &= \frac{3(GI)}{2\sqrt{13}} \\ &= \frac{3\left(\frac{2}{3}(125 - 20\sqrt{13})\right)}{2\sqrt{13}} \\ &= \frac{125 - 20\sqrt{13}}{\sqrt{13}} \\ w &\approx 14.7 \text{ feet} \end{aligned}$$

and

$$\begin{aligned} L &= \frac{s\sqrt{13}}{4} \\ &= \frac{\left(\frac{2}{3}(125 - 20\sqrt{13})\right)\sqrt{13}}{4} \\ &= \frac{\sqrt{13}}{6}(125 - 20\sqrt{13}) \\ L &\approx 31.8 \text{ feet.} \end{aligned}$$

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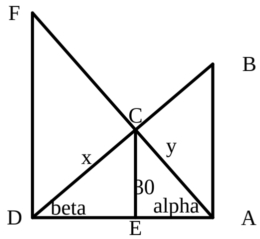
71. In the figure below,



let $AF = 120$, $AC = y$, $BD = 90$, and $CD = x$. Also, let α be the angle formed by AE and AC, and let β be the angle formed by CD and DE.

Choose the point Q in DF so that DF is perpendicular to CQ. In $\triangle CQF$, we have $\sin(\pi/2 - \alpha) = \dfrac{QC}{120 - y}$. In $\triangle CQD$, we get $\sin(\pi/2 - \beta) = \dfrac{QC}{x}$. Therefore,

$$\begin{aligned} \sin(\pi/2 - \alpha)(120 - y) &= x \sin(\pi/2 - \beta) \\ \frac{x}{\sin(\pi/2 - \alpha)} &= \frac{120 - y}{\sin(\pi/2 - \beta)} \\ \frac{x}{\cos\alpha} &= \frac{120 - y}{\cos\beta}. \end{aligned}$$

Similarly, $\dfrac{90 - x}{\cos\alpha} = \dfrac{y}{\cos\beta}$. Thus, $\dfrac{x}{120 - y} = \dfrac{\cos\alpha}{\cos\beta} = \dfrac{90 - x}{y}$. Consequently, $4x + 3y = 360$ or equivalently

$$y = \frac{360 - 4x}{3}.$$

From the right triangles $\triangle$ ACE and $\triangle$ CDE, one finds $\sin\alpha = \dfrac{30}{y}$ and $\sin\beta = \dfrac{30}{x}$.

Consequently,

$$\cos\alpha = \frac{\sqrt{y^2 - 30^2}}{y} \quad \text{and} \quad \cos\beta = \frac{\sqrt{x^2 - 30^2}}{x}.$$

Moreover, using triangles $\triangle$ ABD and $\triangle$ ADF, one obtains $\cos\beta = \dfrac{AD}{90}$ and $\cos\alpha = \dfrac{AD}{120}$. Combining all of these, one derives

$$\begin{aligned} 90\cos\beta &= 120\cos\alpha \\ \frac{3}{4} &= \frac{\cos\alpha}{\cos\beta} \\ \frac{3}{4} &= \frac{\frac{\sqrt{y^2-30^2}}{y}}{\frac{\sqrt{x^2-30^2}}{x}} \\ \frac{3}{4} &= \frac{\sqrt{y^2 - 30^2}}{\sqrt{x^2 - 30^2}} \cdot \frac{x}{y} \\ \frac{3}{4} &= \frac{\sqrt{\left(\frac{360-4x}{3}\right)^2 - 30^2}}{\sqrt{x^2 - 30^2}} \cdot \frac{x}{\frac{360-4x}{3}} \\ \frac{3}{4} &= \frac{3x}{360 - 4x} \cdot \frac{\sqrt{\left(\frac{360-4x}{3}\right)^2 - 30^2}}{\sqrt{x^2 - 30^2}} \\ \frac{1}{16} &= \frac{x^2}{(360 - 4x)^2} \cdot \frac{\left(\frac{360-4x}{3}\right)^2 - 30^2}{x^2 - 30^2} \end{aligned}$$

$$(360 - 4x)^2(x^2 - 30^2) = \\ = 16x^2\left[\left(\frac{360 - 4x}{3}\right)^2 - 30^2\right]$$

Using a graphing calculator, for $0 < x < 90$, one finds $x \approx 60.4$. Working backwards, one derives $\sin\beta = \dfrac{30}{60.4}$ or $\beta \approx 29.8°$, and $\cos(29.8) = \dfrac{AD}{90}$. Hence, the width of the property is $AD = 78.1$ feet.

72. Let r be the radius of the earth. Let α_1 and α_2 be the angles in degrees associated with Diane and Ed in the Figure for Exercise 62.

$$\alpha_1 = \cos^{-1}\left(\frac{r}{r + \frac{2}{5280}}\right)$$

$$\alpha_2 = \cos^{-1}\left(\frac{r}{r + \frac{6}{5280}}\right)$$

In part c) of Exercise 62, $p = 1/240$ is the number of seconds it takes the earth to rotate through an angle of 1°. If there is a 4 sec difference between the moments when Diane and Ed sees the green flash, then

$$\alpha_2 - \alpha_1 = 4p.$$

Using a calculator, we find $r = 4798$ miles.

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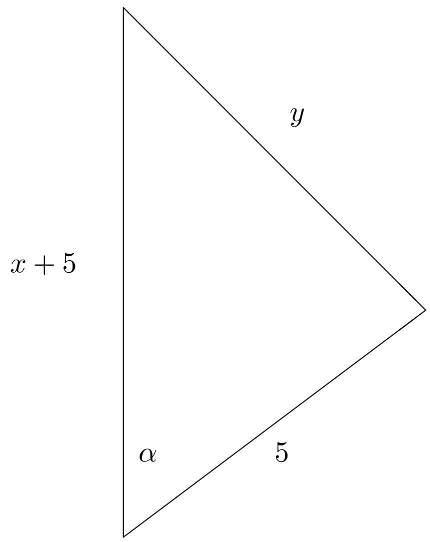
73. Consider the right triangle formed by the hook, the center of the circle, and a point on the circle where the chain is tangent to the circle.



Then $\tan\alpha = \frac{y}{5}$ or $y = 5\tan\alpha$. Since the chain is 40 ft long and the angle $2\pi - 2\alpha$ intercepts an arc around the pipe where the chain wraps around the circle, we obtain

$$2y + 5(2\pi - 2\alpha) = 40.$$

By substitution, we get

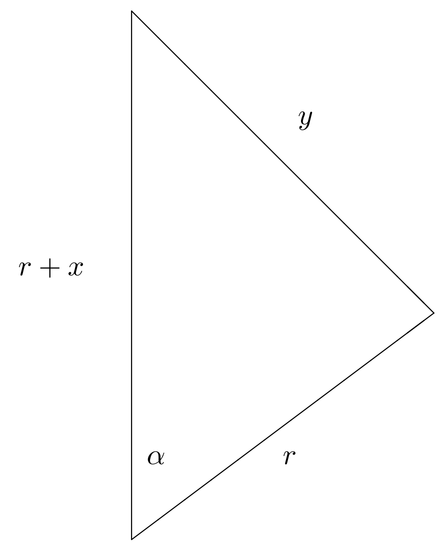
$$10\tan\alpha + 10\pi - 10\alpha = 40.$$

With a graphing calculator, we obtain $\alpha \approx 1.09835$ radians. From the figure above, we get $\cos\alpha = \frac{5}{5+x}$. Solving for x, we obtain

$$x = \frac{5 - 5\cos\alpha}{\cos\alpha} \approx 5.987 \text{ ft.}$$

74. Consider the right triangle where

$$r = 6,400,000 \text{ meters}$$



Then $\cos\alpha = \frac{r}{r+x}$ or

$$\alpha = \cos^{-1}\left(\frac{r}{r+x}\right)$$

By the Pythagorean Theorem, we find $r^2 + y^2 = (r+x)^2$ from which we derive

$$y = \sqrt{x^2 + 2rx}.$$

Since the length of the rope is 1 meter longer than the earth's diameter, we obtain

$$2y + (2\pi - 2\alpha)r \quad = \quad 2\pi r + 1$$

or by substitution

$$2\sqrt{x^2 + 2rx} + 2r\left(\pi - \cos^{-1}\left(\frac{r}{r+x}\right)\right) = 2\pi r + 1$$

Using a calculator, we obtain $r = 121.6$ meters

75. Assume the circle is given by

$$x^2 + (y - r)^2 = r^2$$

where r is the radius, see figure below.

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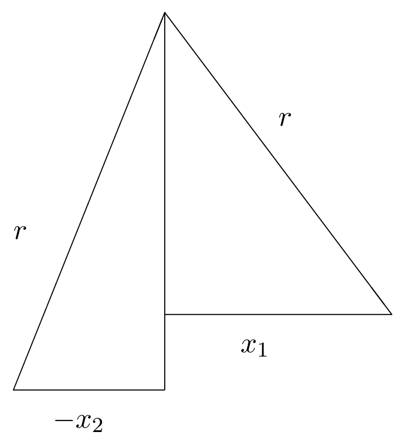


Using the Pythagorean Theorem, we find

$$x_1^2 + (r-2)^2 = r^2 \text{ and } x_2^2 + (r-1)^2 = r^2.$$

From which we obtain

$$x_1^2 + 4 - 4r = 0 \text{ and } x_2^2 + 1 - 2r = 0.$$

Then

$$x_1 = \sqrt{4r-4} \text{ and } x_2 = \sqrt{2r-1}.$$

Note, the central angle of the arc joining the points $(x_1, 2)$ and $(-x_2, 1)$ on the blocks is

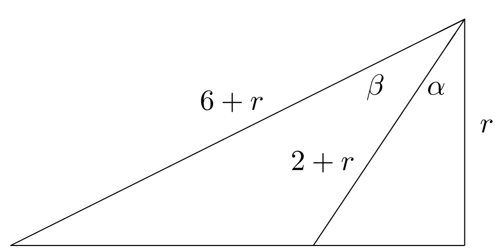
$$\alpha = \arcsin \frac{x_1}{r} + \arcsin \frac{x_2}{r}$$

Since the length $r\alpha$ of the arc is 6 ft, we obtain

$$r\left(\arcsin \frac{\sqrt{4r-4}}{r} + \arcsin \frac{\sqrt{2r-1}}{r}\right) = 6$$

Using a calculator, we find $r \approx 2.768$ ft.

76. In the figure, r is the radius of the circle.



Note,

$$\alpha = \cos^{-1}\left(\frac{r}{2+r}\right) \text{ and } \beta = \cos^{-1}\left(\frac{r}{6+r}\right)$$

which we assume are in degree measure. With the aid of a graphing calculator, we find that the solutions to

$$\cos^{-1}\left(\frac{r}{6+r}\right) - \cos^{-1}\left(\frac{r}{2+r}\right) - 18 = 0$$

are $r \approx 3.626$ ft and $r \approx 9.126$ ft.

When we use 19°, with a graphing calculator we see that

$$\cos^{-1}\left(\frac{r}{6+r}\right) - \cos^{-1}\left(\frac{r}{2+r}\right) - 19 = 0$$

has no solution.

79. **a)** $-30°$ **b)** $120°$ **c)** $-45°$

80. **a)** $-\frac{1}{2}$ **b)** -1 **c)** -1

d) undefined **e)** $\frac{2\sqrt{3}}{3}$ **f)** -1

81. Amplitude 3; period is $\frac{2\pi}{B} = \frac{2\pi}{\pi/2} = 4$

Since $\frac{\pi x}{2} - \frac{\pi}{2} = \frac{\pi}{2}(x-1)$, the phase shift is 1

If we add 7 to the interval $[-3, 3]$, we find that the range is $[4, 10]$

82. The period satisfies

$$\frac{2\pi}{B} = \frac{5\pi}{4} - \frac{\pi}{4} = \pi$$

Then $B = 2$.

We want an equation of the form

$$y = A\cos(B(x-C)) + D$$

Since the second key point is $(\pi/2, 5)$ which is the beginning point for a cosine curve, the phase shift is $C = \pi/2$.

Since the third key point is $(3\pi/4, 2)$, the vertical translation is $D = 2$.

Since the maximum y-value is 5, the amplitude satisfies $D + A = 5$ or $2 + A = 5$. Thus, $A = 3$ and

$$y = 3\cos\left(2\left(x - \frac{\pi}{2}\right)\right) + 2$$

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83. period is $\frac{2\pi}{B} = \frac{2\pi}{\pi} = 2$
range $(-\infty, -5] \cup [5, \infty)$

84. Since $y = \csc(2x)$ is well-defined whenever $2x \neq k\pi$ for all integers k, the domain is

$$\{x|x \neq k\pi/2 \text{ integer } k\}$$

Thinking Outside the Box L

We begin by writing the length of a diagonal of a rectangular box. If the dimensions of a box are a-by-b-by-c, then the length of a diagonal is $\sqrt{a^2+b^2+c^2}$.

Suppose the ball is placed at the center of the field and 60 feet from the goal line. Then the distance between the ball and the right upright of the goal is

$$A = \sqrt{90^2 + 10^2 + 9.25^2} \approx 91.025.$$

Consider the triangle formed by the ball, and the left and right uprights of the goal. Opposite the angle θ_1 is the 10-ft horizontal bar. Using the cosine law in Chapter 7, we find

$$18.5^2 = 2A^2 - 2A^2 \cos\theta$$

and $\theta_1 \approx 11.66497°$.

Now, place the ball on the right hash mark which is 9.25 ft from the centerline. The ball is also 60 feet from the goal line. Then the distance between the ball and the right upright of the goal is

$$B = \sqrt{90^2 + 10^2} \approx 90.554.$$

And, the distance between the ball and the left upright of the goal is

$$C = \sqrt{90^2 + 10^2 + 18.5^2} \approx 92.424.$$

Similarly, consider the triangle formed by the ball, and the left and right uprights of the goal. Opposite the angle θ is the 10-ft horizontal bar. Using the cosine law in Chapter 7, we find

$$18.5^2 = B^2 + C^2 - 2BC\cos\theta_2$$

and $\theta_2 \approx 11.54654°$.

Thus, the difference between the values of θ is

$$\theta_1 - \theta_2 \approx 11.66497° - 11.54654° \approx 0.118°$$

5.6 Pop Quiz

1. Since $5 = \sqrt{(-3)^2 + 4^2}$, we find $\sin\alpha = 4/5$, $\cos\alpha = -3/5$, and $\tan\alpha = -4/3$.

2. Since $\sqrt{3^2+6^2} = 3\sqrt{5}$, we find

$$\sin\alpha = \frac{3}{3\sqrt{5}} = \frac{\sqrt{5}}{5},$$

$$\cos\alpha = \frac{6}{3\sqrt{5}} = \frac{2\sqrt{5}}{5},$$

and $\tan\alpha = 3/6 = 1/2$.

3. The height of the building is

$$1000\tan 36° \approx 727 \text{ ft.}$$

Review Exercises

1. $388° - 360° = 28°$

2. $-840° + 3 \cdot 360° = 240°$

3. $-153°14'27'' + 359°59'60'' = 206°45'33''$

4. $455°39'24'' - 360° = 95°39'24''$

5. $180°$ **6.** $-35\pi/6 + 6\pi = \pi/6 = 30°$

7. $13\pi/5 - 2\pi = 3\pi/5 = 3 \cdot 36° = 108°$

8. $29\pi/12 - 2\pi = 5\pi/12 = 5 \cdot 15° = 75°$

9. $5\pi/3 = 5 \cdot 60° = 300°$ **10.** $-135°$

11. $270°$ **12.** $150°$ **13.** $11\pi/6$

14. $9\pi/4$ **15.** $-5\pi/3$ **16.** $-7\pi/6$

17., 18.

θ deg	0	30	45	60	90	120	135	150	180
θ rad	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
$\tan\theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	NA	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0

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19. $-\sqrt{2}/2$ **20.** $-1/2$ **21.** $\sqrt{3}$

22. $2\sqrt{3}/3$ **23.** $-2\sqrt{3}/3$ **24.** -1

25. 0 **26.** 0 **27.** 0 **28.** 2

29. -1 **30.** $-\sqrt{3}/3$ **31.** $\cot(60°) = \sqrt{3}/3$

32. $\sin(30°) = 1/2$ **33.** $-\sqrt{2}/2$ **34.** 1

35. -2 **36.** $-\sqrt{2}$ **37.** $-\sqrt{3}/3$ **38.** $-1/2$

39. $\sin(\alpha) = 5/13, \cos(\alpha) = 12/13, \tan(\alpha) = 5/12,$ $\csc(\alpha) = 13/5, \sec(\alpha) = 13/12, \cot(\alpha) = 12/5$

40. $\sin(\alpha) = 3\sqrt{13}/13, \cos(\alpha) = 2\sqrt{13}/13,$ $\tan(\alpha) = 3/2, \csc(\alpha) = \sqrt{13}/3,$ $\sec(\alpha) = \sqrt{13}/2, \cot(\alpha) = 2/3$

41. 0.6947 **42.** -0.9063 **43.** -0.0923

44. 0.0016 **45.** 0.1869 **46.** -1.4528

47. 1.0356 **48.** 0.6988 **49.** $-\pi/6$

50. $2\pi/3$ **51.** $-\pi/4$

52. $\pi/3$ **53.** $\pi/4$

54. $\pi/6$ **55.** $\pi/6$

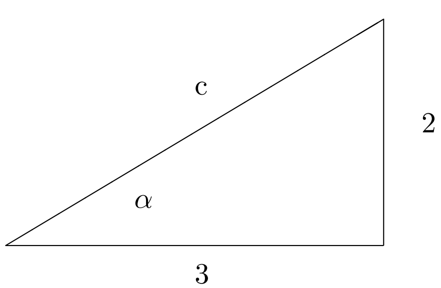
56. $-\sqrt{3}/2$ **57.** 90°

58. 45° **59.** 135°

60. 60° **61.** 30°

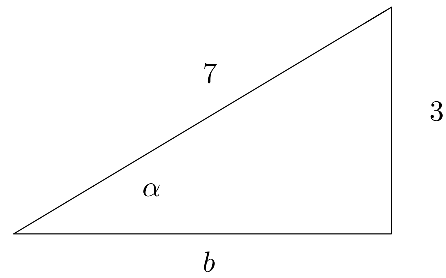
62. 150° **63.** 90° **64.** 120°

65. Form the right triangle with $a = 2$, $b = 3$.



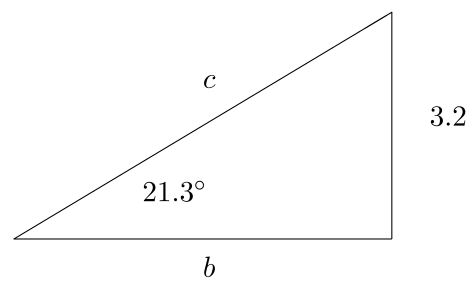
Note that $c = \sqrt{2^2 + 3^2} = \sqrt{13}$, $\tan(\alpha) = 2/3$, so $\alpha = \tan^{-1}(2/3) \approx 33.7°$ and $\beta \approx 56.3°$.

66. Form the right triangle with $a = 3$, $c = 7$.



Note that $b = \sqrt{7^2 - 3^2} = 2\sqrt{10}$, $\sin(\alpha) = 3/7$, so $\alpha = \sin^{-1}(3/7) \approx 25.4°$ and $\beta \approx 64.6°$.

67. Form the right triangle with $a = 3.2$, $\alpha = 21.3°$.



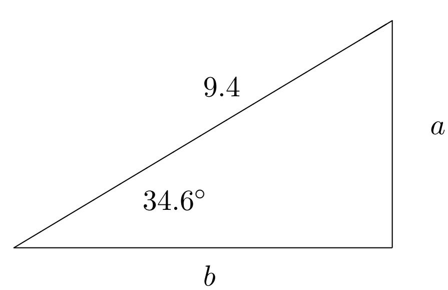
Since $\sin 21.3° = \dfrac{3.2}{c}$ and

$\tan 21.3° = \dfrac{3.2}{b}$, $c = \dfrac{3.2}{\sin 21.3°} \approx 8.8$

and $b = \dfrac{3.2}{\tan 21.3°} \approx 8.2$

Also, $\beta = 90° - 21.3° = 68.7°$

68. Form the right triangle with $c = 9.4$, $\alpha = 34.6°$.



Since $\sin 34.6° = \dfrac{a}{9.4}$ and $\cos 34.6° = \dfrac{b}{9.4}$,

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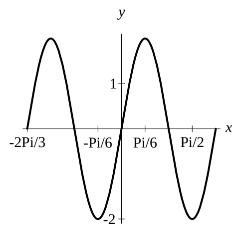
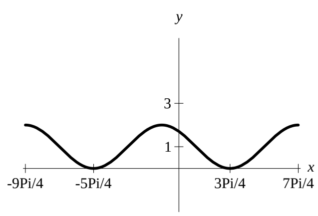
we get $a = 9.4 \cdot \sin 34.6° \approx 5.3$ and
$b = 9.4 \cdot \cos 34.6° \approx 7.7$
Also, $\beta = 90° - 34.6° = 55.4°$

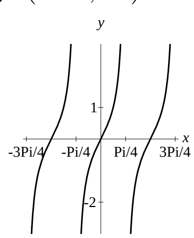
69. $f(x) = 2\sin(3x)$ has period $2\pi/3$, range $[-2, 2]$



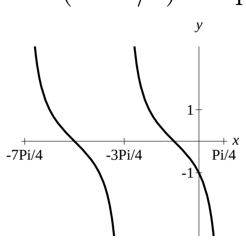
70. $f(x) = 1 + \cos(x + \pi/4)$ has period 2π, range $[0, 2]$



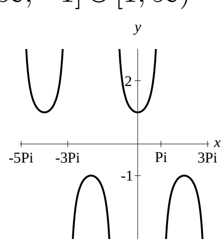
71. $f(x) = \tan(2x + \pi)$ has period $\pi/2$, range $(-\infty, \infty)$



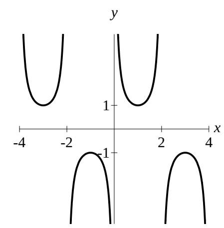
72. $y = \cot(x - \pi/4)$ has period π, range $(-\infty, \infty)$



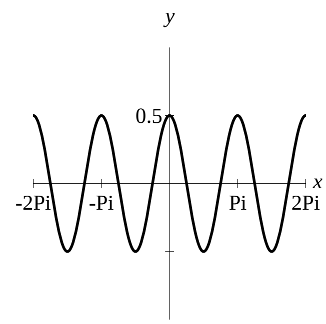
73. $f(x) = \sec(x/2)$ has period 4π, range $(-\infty, -1] \cup [1, \infty)$



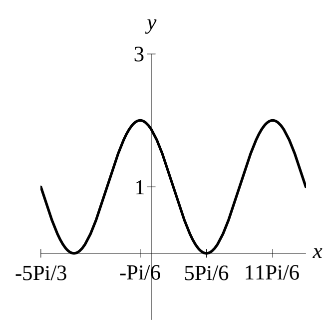
74. $f(x) = \csc\left(\dfrac{\pi}{2} \cdot x\right)$ has period 4, range $(-\infty, -1] \cup [1, \infty)$



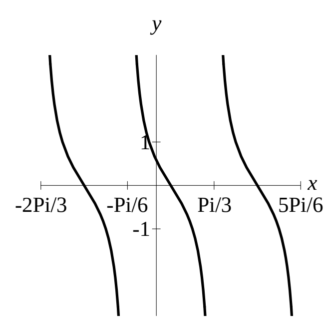
75. $f(x) = \dfrac{1}{2} \cdot \cos(2x)$ has period π, range $[-1/2, 1/2]$



76. $f(x) = 1 - \sin\left(x - \dfrac{\pi}{3}\right)$ has period 2π, range $[0, 2]$



77. $f(x) = \cot\left(2x + \dfrac{\pi}{3}\right)$ has period $\dfrac{\pi}{2}$, range $(-\infty, \infty)$



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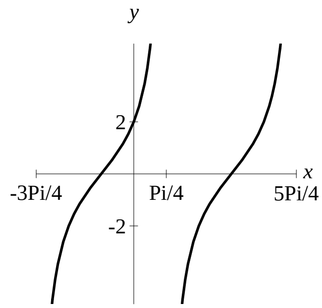
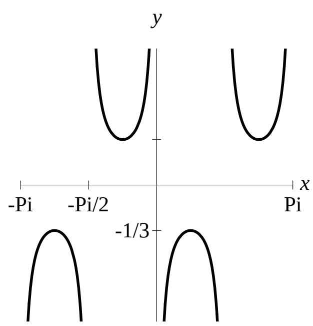
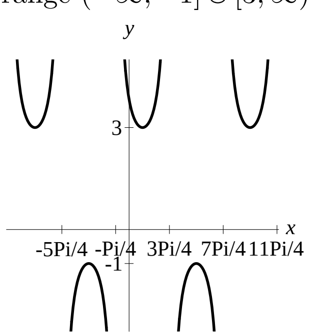
78. $f(x) = 2\tan\left(x + \frac{\pi}{4}\right)$ has period π, range $(-\infty, \infty)$



79. $f(x) = \frac{1}{3}\csc(2x + \pi)$ has period π, range $(-\infty, -1/3] \cup [1/3, \infty)$



80. $f(x) = 1 + 2\sec\left(x - \frac{\pi}{4}\right)$ has period 2π, range $(-\infty, -1] \cup [3, \infty)$



81. One full period is between $2 \le x \le 6$. So $\frac{2\pi}{b} = 4$ or $b = \frac{\pi}{2}$ and there is a right shift of 2 units. An equation is $y = 2\sin\left(\frac{\pi}{2}(x-2)\right)$.

82. One full period is between $0 \le x \le \pi$ and there is a vertical upward shift of one unit. From the period, we have $\frac{2\pi}{b} = \pi$ or $b = 2$. An equation is $y = \sin(2x) + 1$.

83. Note, one full period is between $8\pi \le x \le 10\pi$, there is a vertical upward shift of 40 units, and $a = 20$. An equation is $y = 20\sin(x) + 40$.

84. One full period is between $80 \le x \le 100$. So $\frac{2\pi}{b} = 20$ or $b = \frac{\pi}{10}$. We can realize a right shift of 90 units, an upward shift of 75, and $a = 5$. An equation is $y = 5\sin\left(\frac{\pi}{10}(x-90)\right) + 75$.

85. $\alpha = 150°$ **86.** $\beta = 210°$

87. $\sin(\alpha) = -\sqrt{1-(1/5)^2} = -\sqrt{24/25} = -2\sqrt{6}/5$

88. Since $\cos(\alpha) = -\sqrt{1-\left(\frac{1}{3}\right)^2} = -\sqrt{\frac{8}{9}} = -\frac{2\sqrt{2}}{3}$, $\tan(\alpha) = \frac{\sin\alpha}{\cos\alpha} = \frac{1/3}{-2\sqrt{2}/3} = -\frac{1}{2\sqrt{2}} = -\sqrt{2}/4$

89. In the given right triangle, the hypotenuse is 24 feet and the side adjacent to the 16° angle is 18 feet. Since $\sin(16°) = \frac{x}{24}$, the shortest side is $x = 24\sin(16°) \approx 6.6$ feet.

90. In this right triangle, 12 meters is the shortest side and is opposite the 22° angle. Since $\sin(22°) = \frac{12}{x}$, the length of the hypotenuse is $x = \frac{12}{\sin(22°)} \approx 32.0$ meters.

91. Since the largest angle α must be opposite the 8 cm leg (which is the longest leg) and the leg adjacent to α is 6 cm long, we obtain $\alpha = \tan^{-1}\left(\frac{8}{6}\right) \approx 53.1°$.

92. If γ is the angle formed by the hypotenuse (which is 19 yards) and the leg with length 8 yards, then $\gamma = \cos^{-1}\left(\frac{8}{19}\right) \approx 65.1°$. Since the other acute angle is $90° - 65.1° = 24.9°$, then the largest acute angle is $\beta = \gamma \approx 65.1°$

93. Period is $\frac{1}{92.3 \times 10^6} \approx 1.08 \times 10^{-8}$ sec

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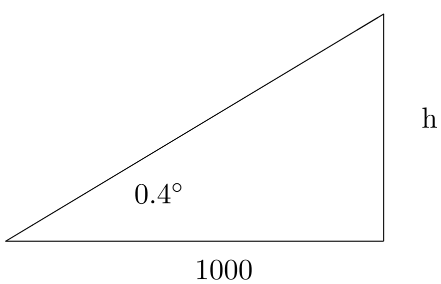
94. Period is $\dfrac{1}{890 \times 10^3} \approx 1.1 \times 10^{-6}$ sec

95. In one hour the nozzle revolves through an angle of $2\pi/8$. The linear velocity, $v = r \cdot \alpha = 120 \cdot 2\pi/8 \approx 94.2$ ft/hr.

96. Note that 1 mile $= 5280 \cdot 12$ inches. Then $\omega = s/r = \dfrac{16 \cdot 5280 \cdot 12}{13} \approx 77{,}981.5$ rad/hr.

97. The height of the man is $s = r \cdot \alpha = 1000(0.4) \cdot \dfrac{\pi}{180} \approx 6.9813$ ft.

98. Let h be the height of the man.

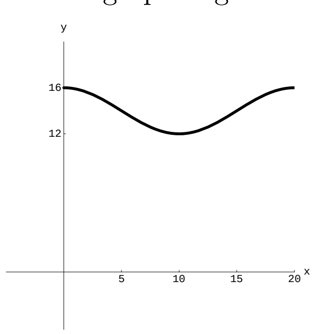


Since $\tan(0.4^\circ) = h/1000$, we obtain $h = 1000 \cdot \tan(0.4^\circ) \approx 6.9814$ ft.

99. Since the period is 20 minutes, $\dfrac{2\pi}{b} = 20$ or $b = \dfrac{\pi}{10}$. Since the depth is between 12 ft and 16 ft, the vertical upward shift is 14 and $a = 2$. Since the depth is 16 ft at time $t = 0$, one can assume there is a left shift of 5 minutes. An equation is

$$y = 2\sin\left(\frac{\pi}{10}(x+5)\right) + 14$$

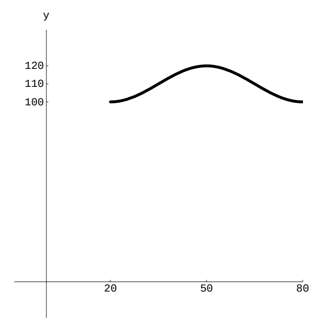
and its graph is given.



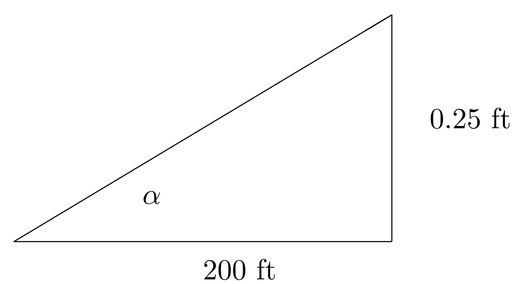
100. Since temperature oscillates between 100°F and 120°F, there is a vertical upward shift of 110 and $a = 10$. Note, the period is 60 minutes. Thus, $\dfrac{2\pi}{b} = 60$ or $b = \dfrac{\pi}{30}$.
Since the temperature is 100°F when $t = 20$, we conclude the temperature is 110°F at $t = 35$. Hence, an equation is

$$y = 10\sin\left(\frac{\pi}{30}(x-35)\right) + 110$$

The graph is given below.

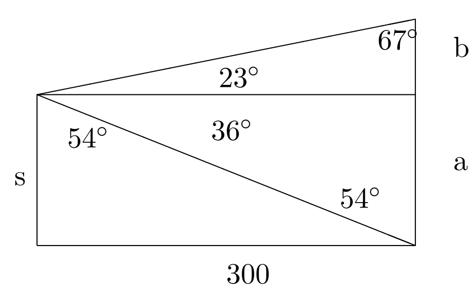


101. Form the right triangle below.



Since $\tan(\alpha) = 0.25/200$, $\alpha = \tan^{-1}(0.25/200) \approx 0.0716^\circ$. She will not hit the target if she deviates by 0.1° from the center of the circle.

102. Let s be the height of the shorter building and let $a + b$ the height of the taller building.



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Since $\tan 54° = \dfrac{300}{s}$, $\tan 36° = \dfrac{a}{300}$, and $\tan 23° = \dfrac{b}{300}$, we obtain

$$s = \frac{300}{\tan 54°} \approx 218 \text{ ft.}$$

The height of the taller building is

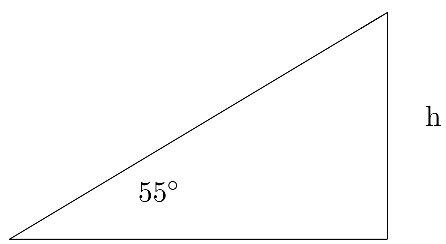
$$a + b = 300 \tan 36° + 300 \tan 23° \approx 345 \text{ ft.}$$

103. If $r = 90$ meters, then the time for one revolution is given by

$$\begin{aligned} T^2(9.8) &= 4\pi^2(90) \\ T &\approx 19.04 \text{ sec.} \end{aligned}$$

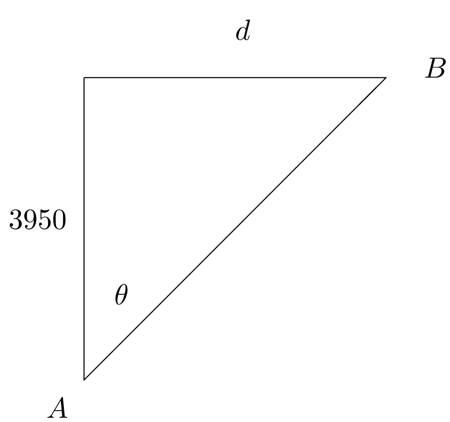
Angular velocity when the period $T = 19.04$ sec is $\omega = 2\pi/19.04 \approx 0.33$ radians/sec.

104. Let h be the height of the cloud cover.



Since $\tan 55° = \dfrac{h}{500}$, $h = 500 \tan 55° \approx 714$ ft.

105. Consider the right triangle below where vertex A represents the center of the earth, the cargo ship is located at vertex B, and d is the distance to the horizon.



From the triangle, we find $\tan\theta = \dfrac{d}{3950}$ and since $AB = 3950 + \dfrac{90}{5280}$ we obtain

$$\sin\theta = \frac{d}{3950 + \frac{90}{5280}}.$$

Also, we have

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{d}{\sqrt{d^2 + 3950^2}}.$$

Then the distance to the horizon is obtained as follows:

$$\begin{aligned} \frac{d}{\sqrt{d^2+3950^2}} &= \frac{d}{3950 + \frac{90}{5280}} \\ \frac{1}{\sqrt{d^2+3950^2}} &= \frac{1}{3950 + \frac{90}{5280}} \\ 3950 + \frac{90}{5280} &= \sqrt{d^2+3950^2} \\ \left(3950 + \frac{90}{5280}\right)^2 &= d^2 + 3950^2 \\ \left(3950 + \frac{90}{5280}\right)^2 - 3950^2 &= d^2 \\ \sqrt{\left(3950 + \frac{90}{5280}\right)^2 - 3950^2} &= d \\ 11.6 \text{ miles} &= d \end{aligned}$$

106. Note, using the equation given in the second to the last line of the solution given for Exercise 105, one can conclude that the distance D from the mast (mounted 25 feet above the water) to the horizon is

$$\begin{aligned} D &= \sqrt{\left(3950 + \frac{25}{5280}\right)^2 - 3950^2} \\ &\approx 6.1 \text{ miles.} \end{aligned}$$

Since the distance of the cargo ship from its horizon is 11.6 miles (see Exercise 105), the sailboat will be on the radar image of the cargo ship when it is 17.7 $(= 6.1 + 11.6)$ miles from the cargo ship.

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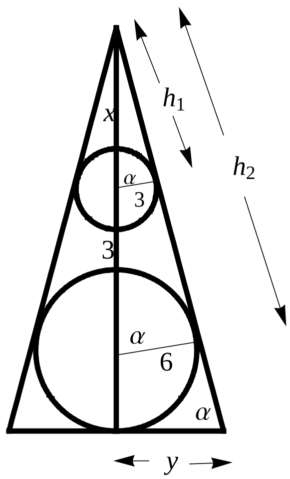
107. Draw an isosceles triangle containing the two circles as shown below. The indicated radii are perpendicular to the sides of the triangle.



Using similar triangles, we obtain

$$\frac{x+3}{3} = \frac{x+15}{6}$$

from which we solve $x = 9$. Applying the Pythagorean theorem, we find

$$h_1 = 3\sqrt{15}, \quad h_2 = 6\sqrt{15}$$

Note, the height of the triangle is 30 ft. By similar triangles,

$$\frac{3\sqrt{15}}{3} = \frac{30}{y}$$

or $y = 2\sqrt{15}$. Thus, the base angle α of the isosceles triangle is $\alpha = \arctan\sqrt{15}$.

Then the arc lengths of the belt that wrap around the two pulleys are (using $s = r\alpha$)

$$s_1 = 6 \cdot (2\pi - 2\alpha)$$

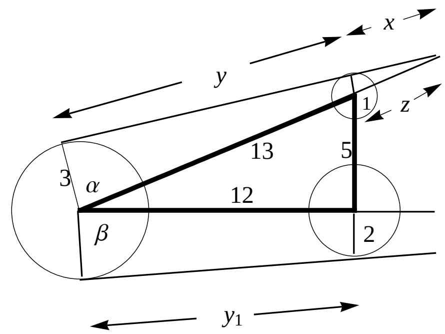
and

$$s_2 = 3 \cdot 2\alpha = 6\alpha.$$

Hence, the length of the belt is

$$\begin{aligned} s_1 + s_2 + 2(h_2 - h_1) &= \\ 12\pi - 6\arctan(\sqrt{15}) + 6\sqrt{15} &\approx \\ 53.0 \text{ in.} \end{aligned}$$

108. In the figure below, the radii are perpendicular to the belts.



By the Pythagorean theorem and similar triangles, we obtain

a) $1 + x^2 = z^2$

b) $3^2 + (x+y)^2 = (13+z)^2$

c) $\dfrac{x}{1} = \dfrac{x+y}{3}$

Solving, we find $x = \sqrt{165}/2$, $y = \sqrt{165}$, and $z = 13/2$. Then $\alpha = \arctan(\sqrt{165}/2)$.

Repeating the above process, we find $y_1 = \sqrt{143}$ and $\beta = \arctan\sqrt{143}$.

Then the length of the belt that wraps around the circle of radius 3 inches is

$$s_3 = 3\left(2\pi - \alpha - \beta - \arctan(5/12)\right)$$

We repeat the calculations.

Then the length y_2 of the belt between the points of tangency for the circles with radii 1 and 2 is $y_2 = 2\sqrt{6}$. Likewise, the length of the belt that wraps around the circle of radius 2 is

$$s_2 = 2\left(\beta + \frac{\pi}{2} - \arctan(2\sqrt{6})\right).$$

Also, the length of the belt that wraps around the circle of radius 1 is

$$s_1 = \alpha + \arctan(2\sqrt{6}) - \arctan\frac{12}{5}.$$

Finally, the total length of the belt is

$$s_3 + s_2 + s_1 + y + y_1 + y_2 \approx 43.6 \text{ in.}$$

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Thinking Outside the Box LI

We assume that the center of the bridge moves straight upward to the center of the arc where the bridge expands. The arc has length $100 + \frac{1}{12}$ feet. To find the central angle α, we use $s = r\alpha$ and $\sin(\alpha/2) = 50/r$. Then $\alpha = 2\sin^{-1}(50/r)$. Solving the equation

$$100 + \frac{1}{12} = 2r\sin^{-1}\left(\frac{50}{r}\right)$$

with a graphing calculator, we find

$$r \approx 707.90241 \text{ ft.}$$

The distance from the chord to the center of the circle can be found by using the Pythagorean theorem, and it is

$$d = \sqrt{r^2 - 50^2} \approx 706.1344221 \text{ ft.}$$

Then the distance from the center of the arc to the cord is

$$r - d \approx 1.767989 \text{ ft} \approx 21.216 \text{ inches.}$$

Chapter 5 Test

1. $\cos(60°) = 1/2$ **2.** $\sin(-30°) = -1/2$

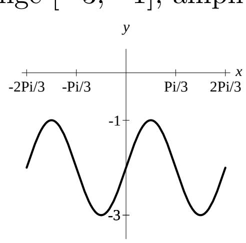
3. -1 **4.** 2 **5.** -2 **6.** $\sqrt{3}/3$

7. $-\pi/6$ **8.** $2\pi/3$ **9.** $-\pi/4$

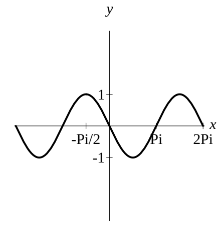
10. Undefined since $0 < \dfrac{\sqrt{3}}{2} < 1$

11. Undefined since $-1 < -\dfrac{\sqrt{2}}{2} < 0$ **12.** $2\sqrt{2}/3$

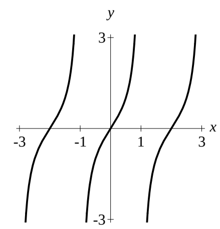
13. $y = \sin(3x) - 2$ has period $2\pi/3$, range $[-3, -1]$, amplitude 1



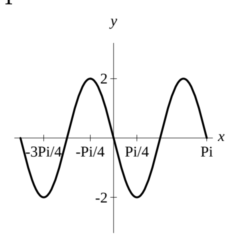
14. $y = \cos(x + \pi/2)$ has period 2π, range $[-1, 1]$, amplitude 1



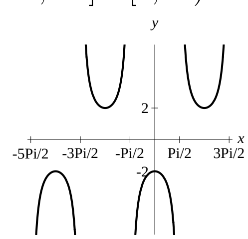
15. $y = \tan(\pi x/2)$ has period 2, range $(-\infty, \infty)$



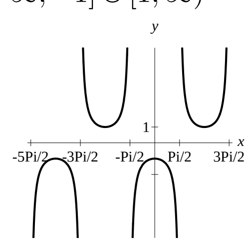
16. $y = 2\sin(2x + \pi)$ has period π, range $[-2, 2]$, amplitude 2



17. $y = 2\sec(x - \pi)$ has period 2π, range $(-\infty, -2] \cup [2, \infty)$



18. $y = \csc(x - \pi/2)$ has period 2π, range $(-\infty, -1] \cup [1, \infty)$



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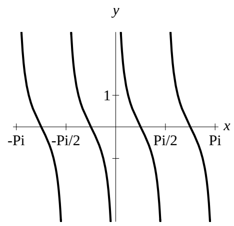
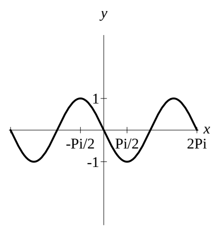
19. $y = \cot(2x)$ has period $\pi/2$, range $(-\infty, \infty)$



20. $y = -\cos(x - \pi/2)$ has period 2π, range $[-1, 1]$, amplitude 1



21. Since $46°24'6'' \approx 0.8098619$, the arclength is $s = r\alpha = 35.62(0.8098619) \approx 28.85$ meters.

22. $2.34 \cdot \dfrac{180°}{\pi} \approx 134.1°$

23. $\cos(\alpha) = -\sqrt{1 - (1/4)^2} = -\sqrt{15}/4$

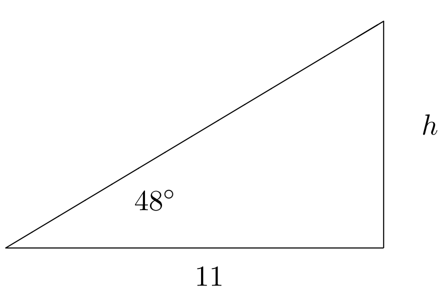
24. $r = \sqrt{5^2 + (-2)^2} = \sqrt{29}$
$\sin(\alpha) = -2/\sqrt{29}$, $\cos(\alpha) = 5/\sqrt{29}$,
$\tan(\alpha) = -2/5$, $\csc(\alpha) = -\sqrt{29}/2$,
$\sec(\alpha) = \sqrt{29}/5$, $\cot(\alpha) = -5/2$

25. $\omega = 103 \cdot 2\pi \approx 647.2$ radians/minute

26. In one minute, the wheel turns through an arclength of $13(103 \cdot 2\pi)$ inches.

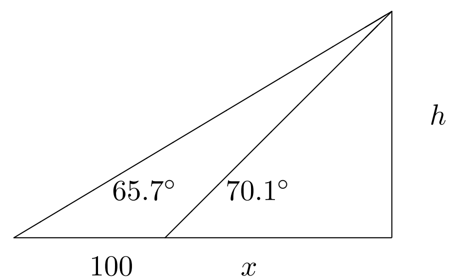
Multiplying this by $\dfrac{60}{12 \cdot 5280}$ results in the speed in mph which is 7.97 mph.

27. Let h be the height of the head.



Since $\tan(48°) = h/11$, we obtain $h = 11 \tan 48° \approx 12.2$ m.

28. Let h be the height of the building.



Since $\tan 70.1° = \dfrac{h}{x}$ and $\tan 65.7° = \dfrac{h}{100 + x}$, we obtain

$$\tan(65.7°) = \frac{h}{100 + h/\tan(70.1°)}$$

$$100 \cdot \tan(65.7°) + h \cdot \frac{\tan(65.7°)}{\tan(70.1°)} = h$$

$$100 \cdot \tan(65.7°) = h\left(1 - \frac{\tan(65.7°)}{\tan(70.1°)}\right)$$

$$\begin{aligned} h &= \frac{100 \cdot \tan(65.7°)}{1 - \tan(65.7°)/\tan(70.1°)} \\ h &\approx 1117 \text{ ft.} \end{aligned}$$

29. Since the pH oscillates between 7.2 and 7.8, there is a vertical upward shift of 7.5 and $a = 0.3$. Note, the period is 4 days. Thus, $\dfrac{2\pi}{b} = 4$ or $b = \dfrac{\pi}{2}$. Since the pH is 7.2 on day 13, the pH is 7.5 on day 14. Using the period, the pH is also 7.5 on day 2. We can assume a right shift of 2 days. Hence, an equation is

$$y = 0.3 \sin\left(\frac{\pi}{2}(x - 2)\right) + 7.5.$$

A graph of one cycle is given.

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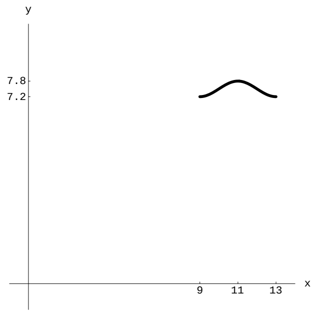
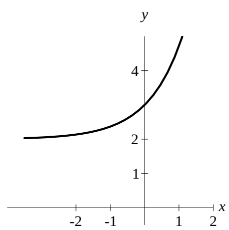
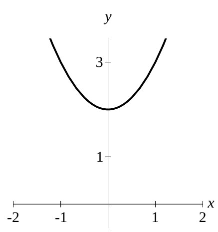


Tying It All Together

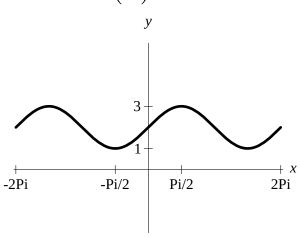
1. $y = 2 + e^x$ has domain $(-\infty, \infty)$, range $(2, \infty)$



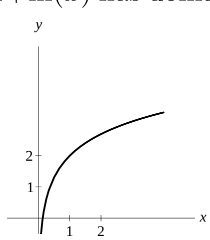
2. $y = 2 + x^2$ has domain $(-\infty, \infty)$, range $[2, \infty)$



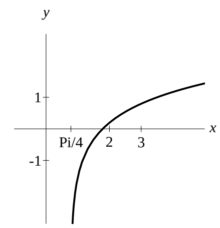
3. $y = 2 + \sin(x)$ has domain $(-\infty, \infty)$, range $[1, 3]$



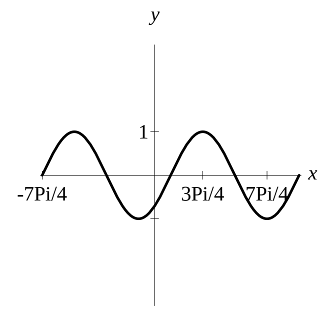
4. $y = 2 + \ln(x)$ has domain $(0, \infty)$, range $(-\infty, \infty)$



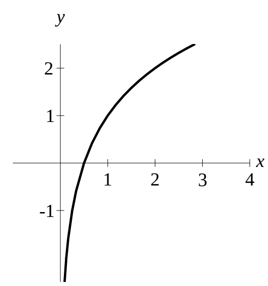
5. $y = \ln(x - \pi/4)$ has domain $(\pi/4, \infty)$, range $(-\infty, \infty)$



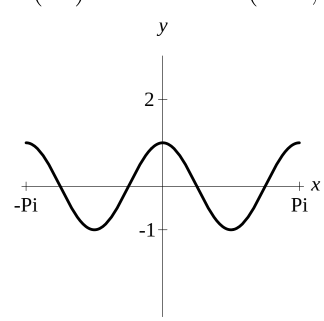
6. $y = \sin(x - \pi/4)$ has domain $(-\infty, \infty)$, range $[-1, 1]$



7. $y = \log_2(2x)$ has domain $(0, \infty)$, range $(-\infty, \infty)$



8. $y = \cos(2x)$ has domain $(-\infty, \infty)$, range $[-1, 1]$



9. Neither

10. Odd

11. Even

12. Even

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13. Odd

14. Even

15. Neither

16. Odd

17. Increasing

18. Increasing

19. Increasing

20. Decreasing

21. Decreasing

22. Increasing

23. perfect square

24. exponential

25. logarithmic

26. common

27. natural

28. angle

29. central

30. acute

31. quadrantal

32. coterminal

Concepts of Calculus

1. Letting $x = 0.1$, we obtain

$$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} \approx 1.105170833.$$

With a calculator, we find

$$e^{0.1} \approx 1.105170918.$$

2. Letting $x = 1$, and by using the first 65 terms of the series we find

$$\begin{aligned} \ln(2) &= \ln(1+1) \\ &\approx x - \frac{x^2}{2} + \frac{x^3}{3} - ... + \frac{x^{65}}{65} \\ \ln(2) &\approx 0.7007803237. \end{aligned}$$

With a calculator, we get

$$\ln(2) \approx 0.6931471806.$$

3. When $x = 4$, we cannot use the series for $\ln(x)$ since the series is valid only for $-1 < x \le 1$.

If $x = 0.25$, then we obtain

$$\begin{aligned} \ln(1+0.25) &\approx x - \frac{x^2}{2} + \frac{x^3}{3} - ... + \frac{x^{65}}{65} \\ &\approx 0.2231435513. \end{aligned}$$

Note,

$$\ln(5) = 2\ln(2) + \ln(1 + 0.25).$$

Thus, we have

$$\begin{aligned} \ln(5) &\approx 2(0.6931471806) + 0.2231435513 \\ &\approx 1.624704199. \end{aligned}$$

4. If $x = 0.125$, then we get

$$\begin{aligned} \ln(1+0.125) &\approx x - \frac{x^2}{2} + \frac{x^3}{3} - ... + \frac{x^{65}}{65} \\ \ln(1+0.125) &\approx 0.1177830357. \end{aligned}$$

Note,

$$\ln(9) = 3\ln(2) + \ln(1 + 0.125).$$

Thus, we obtain

$$\begin{aligned} \ln(9) &\approx 3(0.6931471806) + 0.1177830357 \\ &\approx 2.220124007. \end{aligned}$$

5. Let $x = \pi/6$. Then

$$x - \frac{x^3}{3!} \approx 0.4996741794$$

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$$x - \frac{x^3}{3!} + \frac{x^5}{5!} \approx 0.5000021326$$
$$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \approx 0.4999999919$$
$$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} \approx 0.5000000000$$

The series with five terms is the first expression to give 0.5, assuming an accuracy of ten decimal places.

6. Let $x = 13\pi/6$. Then

$$x - \frac{x^3}{3!} \approx -45.75555378$$
$$x - \frac{x^3}{3!} + \frac{x^5}{5!} \approx 76.01117164$$
$$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \approx -58.31581919$$
$$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} \approx 28.12417692$$
$$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ... - \frac{x^{11}}{11!} \approx -8.284592690$$
$$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ... + \frac{x^{13}}{13!} \approx 2.528884130$$
$$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ... - \frac{x^{15}}{15!} \approx 0.1431062477$$
$$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ... + \frac{x^{17}}{17!} \approx 0.5494981571$$
$$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ... - \frac{x^{19}}{19!} \approx 0.4944423801$$
$$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ... + \frac{x^{21}}{21!} \approx 0.5005158594$$
$$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ... - \frac{x^{23}}{23!} \approx 0.4999597362$$
$$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ... + \frac{x^{25}}{25!} \approx 0.5000026803$$
$$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ... - \frac{x^{27}}{27!} \approx 0.4999998460$$
$$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ... + \frac{x^{29}}{29!} \approx 0.5000000077$$
$$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ... - \frac{x^{31}}{31!} \approx 0.4999999997$$
$$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ... + \frac{x^{33}}{33!} \approx 0.5000000000$$

The series with 17 terms is the first expression to give 0.5, assuming an accuracy of ten decimal places.

7. Let $x = \pi/4$. Then

$$1 - \frac{x^2}{2!} \approx 0.6915748625$$
$$1 - \frac{x^2}{2!} + \frac{x^4}{4!} \approx 0.7074292067$$
$$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} \approx 0.7071032148$$
$$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - ... + \frac{x^8}{8!} \approx 0.7071068057$$
$$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - ... - \frac{x^{10}}{10!} \approx 0.7071067811$$
$$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - ... + \frac{x^{12}}{12!} \approx 0.7071067812$$

Since

$$\cos(\pi/4) \approx 0.7071067812,$$

the series with 7 terms is the first expression to give $\cos(\pi/4)$, assuming an accuracy of ten decimal places.

8. Let $x = 9\pi/4$. Then

$$1 - \frac{x^2}{2!} \approx -23.98243614$$
$$1 - \frac{x^2}{2!} + \frac{x^4}{4!} \approx 80.03791644$$
$$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} \approx -93.20753794$$
$$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - ... + \frac{x^8}{8!} \approx 61.36722994$$
$$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - ... - \frac{x^{10}}{10!} \approx -24.44730933$$
$$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - ... + \frac{x^{12}}{12!} \approx 8.035361081$$
$$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - ... - \frac{x^{14}}{14!} \approx -0.8821800100$$
$$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - ... + \frac{x^{16}}{16!} \approx 0.9743358303$$
$$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - ... - \frac{x^{18}}{18!} \approx 0.6711966903$$

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$$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - ... + \frac{x^{20}}{20!} \approx 0.7110553966$$

$$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - ... - \frac{x^{22}}{22!} \approx 0.7067447145$$

$$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - ... + \frac{x^{24}}{24!} \approx 0.7071349005$$

$$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - ... - \frac{x^{26}}{26!} \approx 0.7071049073$$

$$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - ... + \frac{x^{28}}{28!} \approx 0.7071068895$$

$$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - ... - \frac{x^{30}}{30!} \approx 0.7071067757$$

$$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - ... + \frac{x^{32}}{32!} \approx 0.7071067814$$

$$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - ... - \frac{x^{34}}{34!} \approx 0.7071067812$$

Since

$$\cos(9\pi/4) \approx 0.7071067812,$$

the series with 18 terms is the first expression to give $\cos(9\pi/4)$, assuming an accuracy of ten decimal places.

9. As accuracy increases, the number of terms needed for the accuracy increases.

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