For Thought

1. False, the base of an exponential function is positive. **2.** True

3. True, since $2^{-3} = \frac{1}{8}$.

4. True **5.** True **6.** True **7.** True

8. False, since it is decreasing.

9. True, since $0.25 = 4^{-1}$.

10. True, since $\sqrt[100]{2^{173}} = \left(2^{173}\right)^{1/100}$.

4.1 Exercises

1. algebraic

2. transcendental

3. exponential

4. all real numbers

5. increasing, decreasing

6. horizontal asymptote

7. range

8. exponential family

9. 27 **10.** 32

11. $-(2^0) = -1$ **12.** $-(4^0) = -1$

13. $\frac{1}{2^3} = \frac{1}{8}$ **14.** $\frac{1}{3^2} = \frac{1}{9}$

15. $\left(\frac{2}{1}\right)^4 = 2^4 = 16$ **16.** $\left(\frac{3}{1}\right)^2 = 3^2 = 9$

17. $\left(8^{1/3}\right)^2 = 2^2 = 4$ **18.** $\left(9^{1/2}\right)^3 = (3)^3 = 27$

19. $-\left(9^{1/2}\right)^{-3} = -(3)^{-3} = -\frac{1}{3^3} = -\frac{1}{27}$

20. $-\left(4^{1/2}\right)^{-3} = -(2)^{-3} = -\frac{1}{2^3} = -\frac{1}{8}$

21. $3^2 = 9$ **22.** $3^4 = 81$ **23.** $3^{-2} = 1/9$

24. $3^{-3} = 1/27$ **25.** $2^{-1} = 1/2$ **26.** $2^0 = 1$

27. $2^3 = 8$

28. $2^4 = 16$

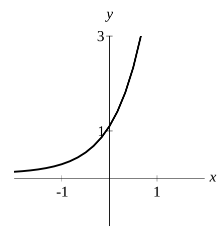
29. $(1/4)^{-1} = 4$

30. $(1/4)^{-2} = 16$

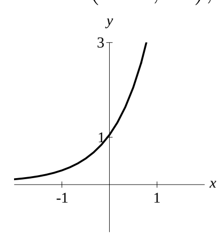
31. $4^{1/2} = 2$

32. $\left((1/4)^{1/2}\right)^3 = (1/2)^3 = 1/8$

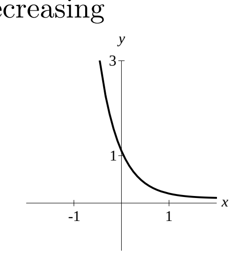
33. $f(x) = 5^x$ goes through $(-1, 1/5), (0, 1), (1, 5)$, domain is $(-\infty, \infty)$, range is $(0, \infty)$, increasing



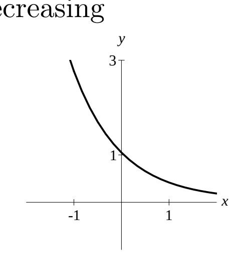
34. $f(x) = 4^x$ goes through $(-1, 1/4), (0, 1), (1, 4)$, domain is $(-\infty, \infty)$, range is $(0, \infty)$, increasing



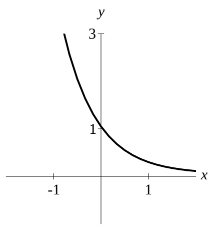
35. $f(x) = 10^{-x}$ goes through $(-1, 10), (0, 1), (1, 1/10)$, domain is $(-\infty, \infty)$, range is $(0, \infty)$, decreasing



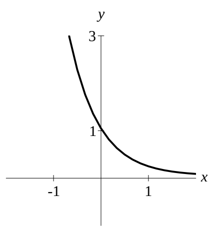
36. $f(x) = e^{-x}$ goes through $(-1, e)$, $(0, 1)$, $(1, 1/e)$, domain is $(-\infty, \infty)$, range is $(0, \infty)$, decreasing



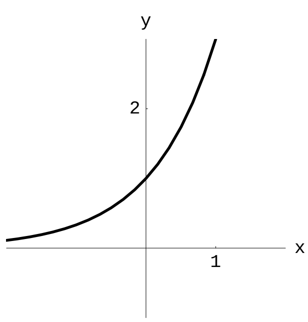
37. $f(x) = (1/4)^x$ goes through $(-1, 4), (0, 1)$, $(1, 1/4)$, domain is $(-\infty, \infty)$, range is $(0, \infty)$, decreasing



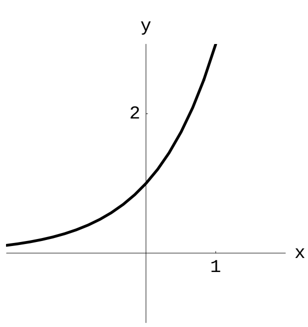
38. $f(x) = (0.2)^x$ goes through $(-1, 5), (0, 1)$, $(1, 0.2)$, domain is $(-\infty, \infty)$, range is $(0, \infty)$, decreasing



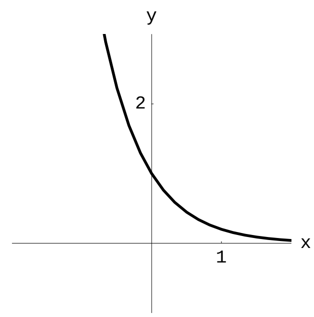
39. From the graph, we find $\lim_{x\to\infty} 3^x = \infty$.



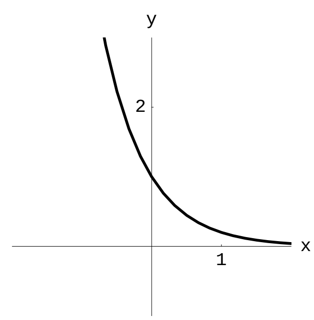
40. From the graph, we find $\lim_{x\to-\infty} 3^x = 0$.



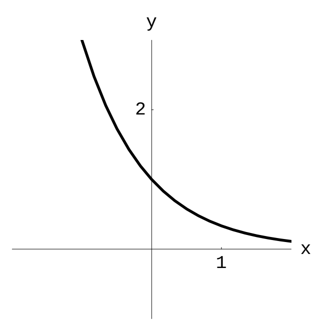
41. Using the graph, we obtain $\lim_{x\to\infty} 5^{-x} = 0$.



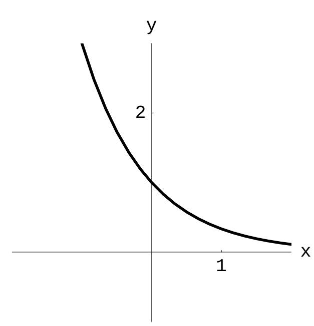
42. Using the graph, we obtain $\lim_{x\to-\infty} 5^{-x} = \infty$.



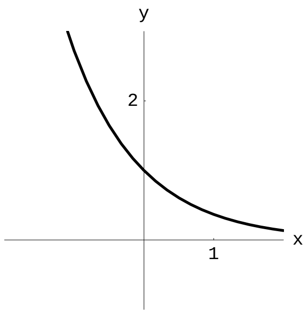
43. We see from the graph that $\lim_{x\to\infty} \left(\frac{1}{3}\right)^x = 0$.



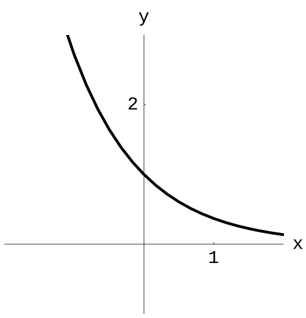
44. We see from the graph that $\lim_{x\to-\infty} \left(\frac{1}{3}\right)^x = \infty$.



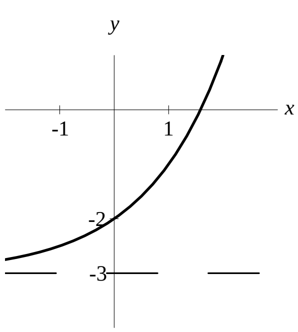
45. Using the graph, we get $\lim_{x \to -\infty} e^{-x} = \infty$.



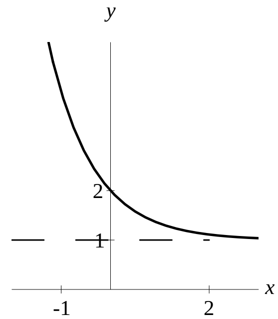
46. Using the graph, we get $\lim_{x \to \infty} e^{-x} = 0$.



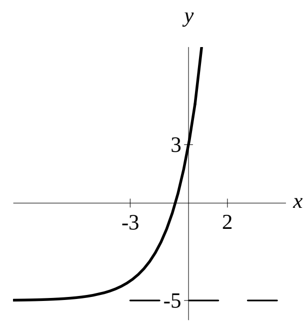
47. Shift $y = 2^x$ down by 3 units; $f(x) = 2^x - 3$ goes through $(-1, -2.5), (0, -2), (2, 1)$, domain $(-\infty, \infty)$, range $(-3, \infty)$, asymptote $y = -3$, increasing



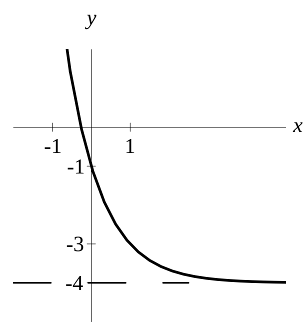
48. Shift $y = 3^{-x}$ up by 1 unit; $f(x) = 3^{-x} + 1$ goes through $(-1, 4)$, $(0, 2)$, $(1, 4/3)$, domain $(-\infty, \infty)$, range $(1, \infty)$, asymptote $y = 1$, decreasing



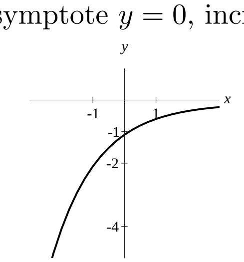
49. Shift $y = 2^x$ to left by 3 units and down by 5 units; $f(x) = 2^{x+3} - 5$ goes through $(-4, -4.5)$,$(-3, -4)$,$(0, 3)$, domain $(-\infty, \infty)$, range $(-5, \infty)$, asymptote $y = -5$, increasing



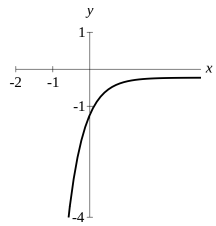
50. Shift $y = 3^{-x}$ to right by 1 unit and down by 4 units; $f(x) = 3^{1-x} - 4$ goes through $(-1, 5)$,$(0, -1)$,$(1, -3)$, domain $(-\infty, \infty)$, range $(-4, \infty)$, asymptote $y = -4$, decreasing



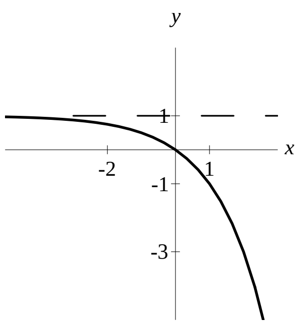
51. Reflect $y = 2^{-x}$ about x-axis, $f(x) = -2^{-x}$ goes through $(-1, -2)$,$(0, -1)$,$(1, -1/2)$, domain $(-\infty, \infty)$, range $(-\infty, 0)$, asymptote $y = 0$, increasing



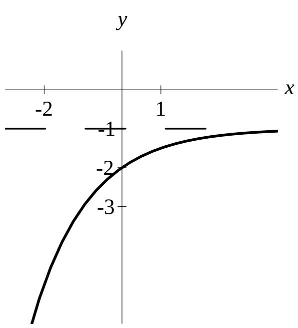
52. Reflect $y = 10^{-x}$ about x-axis, $f(x) = -10^{-x}$ goes through $(-1, -10)$,$(0, -1)$,$(1, -0.1)$, domain $(-\infty, \infty)$, range $(-\infty, 0)$, asymptote $y = 0$, increasing



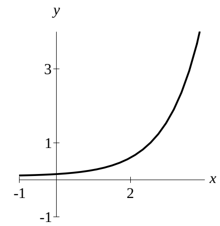
53. Reflect $y = 2^x$ about x-axis and shift up by 1 unit, $f(x) = 1 - 2^x$ goes through $(-1, 0.5)$,$(0, 0)$,$(1, -1)$, domain $(-\infty, \infty)$, range $(-\infty, 1)$, asymptote $y = 1$, decreasing



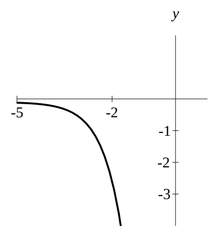
54. Reflect $y = 2^{-x}$ about x-axis and shift down by 1 unit, $f(x) = -1 - 2^{-x}$ goes through $(-1, -3)$,$(0, -2)$,$(1, -1.5)$, domain $(-\infty, \infty)$, range $(-\infty, -1)$, asymptote $y = -1$, increasing



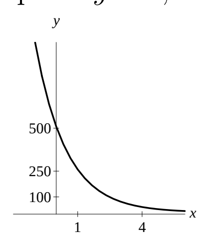
55. Shift $y = 3^x$ to right by 2 and shrink by a factor of 0.5, $f(x) = 0.5 \cdot 3^{x-2}$ goes through $(0, 1/18)$,$(2, 0.5)$,$(3, 1.5)$, domain $(-\infty, \infty)$, range $(0, \infty)$, asymptote $y = 0$, increasing



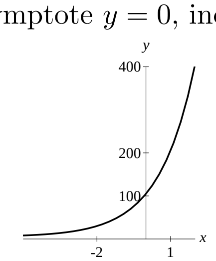
56. Shift $y = 5^x$ to left by 4, shrink by a factor of 0.1, reflect about x-axis, $f(x) = -0.1 \cdot 5^{x+4}$ goes through $(-5, -.02)$,$(-3, -0.5)$,$(-2, -2.5)$, domain $(-\infty, \infty)$, range $(-\infty, 0)$, asymptote $y = 0$, decreasing



57. Stretch $y = (0.5)^x$ by a factor of 500, $f(x) = 500 \cdot (0.5)^x$ goes through $(0, 500)$, $(1, 250)$, domain $(-\infty, \infty)$, range $(0, \infty)$, asymptote $y = 0$, decreasing



58. Stretch $y = 2^x$ by a factor of 100, $f(x) = 100 \cdot 2^x$ goes through $(0, 100)$,$(1, 200)$, domain $(-\infty, \infty)$, range $(0, \infty)$, asymptote $y = 0$, increasing



59. $y = 2^{x-5} - 2$

60. $y = e^{x+3} + 1$

61. $y = -\left(\frac{1}{4}\right)^{x-1} - 2$

62. $y = -\left(10^{x+2} + 3\right)$ or $y = -10^{x+2} - 3$

63. Since $2^x = 2^6$, solution set is $\{6\}$.

64. $\{0\}$ **65.** $\{-1\}$

66. Since $10^{2x} = 10^3$, $2x = 3$ and the solution set is $\left\{\dfrac{3}{2}\right\}$.

67. Multiplying the equation by -1, $3^x = 3^3$ and the solution set is $\{3\}$.

68. Multiplying the equation by -1, $2^x = \dfrac{1}{2}$ and the solution set is $\{-1\}$.

69. $\{-2\}$ **70.** $\{-3\}$

71. Since $(2^3)^x = 2^{3x} = 2$, $3x = 1$. The solution set is $\left\{\dfrac{1}{3}\right\}$.

72. Since $(3^2)^x = 3^{2x} = 3$, $2x = 1$. The solution set is $\left\{\dfrac{1}{2}\right\}$.

73. $\{-2\}$ **74.** $\{1\}$

75. Since $(2^{-1})^x = 2^{-x} = 2^3$, $-x = 3$. The solution set is $\{-3\}$.

76. Since $\left(\left(\dfrac{3}{2}\right)^{-1}\right)^x = \left(\dfrac{3}{2}\right)^{-x} = \left(\dfrac{3}{2}\right)^2$, $-x = 2$ and the solution set is $\{-2\}$.

77. Since $10^{x-1} = 10^{-2}$, $x - 1 = -2$. The solution set is $\{-1\}$.

78. Since $10^{|x|} = 10^3$, $|x| = 3$. The solution set is $\{\pm 3\}$.

79. Since $2^{2x} = 4^x$, the original equation may be written as $4^x = 64$. Then the solution set is $\{3\}$.

80. Since $\left(\frac{2}{3}\right)^{2x}\left(\frac{2}{3}\right)^{3-3x} = \left(\frac{2}{3}\right)^{3-x} = \frac{2}{3}$, we obtain $3 - x = 1$. Then the solution set is $\{2\}$.

81. Since $2^x = 4$, we find $x = 2$.

82. Since $2^x = 32$, we obtain $x = 5$.

83. Since $2^x = \dfrac{1}{2}$, we obtain $x = -1$.

84. Since $2^x = 1$, we get $x = 0$.

85. Since $\left(\dfrac{1}{3}\right)^x = 1$, we get $x = 0$.

86. Since $\left(\dfrac{1}{3}\right)^x = 3^2$, we find $x = -2$.

87. Since $\left(\dfrac{1}{3}\right)^x = 3^3$, we find $x = -3$.

88. Since $\left(\dfrac{1}{3}\right)^x = \dfrac{1}{9}$, we obtain $x = 2$.

89. Since $10^x = 1000$, we obtain $x = 3$.

90. Since $10^x = 10^5$, we get $x = 5$.

91. Since $10^x = 0.1 = 10^{-1}$, we find $x = -1$.

92. Since $10^x = 0.0001 = 10^{-4}$, we find $x = -4$.

93. 1 **94.** 3 **95.** -1 **96.** 0

97. $(2, 9), (1, 3), (-1, 1/3), (-2, 1/9)$

98. $(3, 1000), (0, 1), (-1, 0.1), (-2, 0.01)$

99. $(0, 1), (-2, 25), (-1, 5), (1, 1/5)$

100. $(1, 1/e), (-1, e), (0, 1), (-2, e^2)$

101. $(4, -16), (-2, -1/4), (-1, -1/2), (5, -32)$

102. $(3, -1/16), (0, -4), (-1, -16), (3, -1/16)$

103. When interest is compounded n times a year, the amount at the end of 6 years is

$$A(n) = 5000\left(1 + \frac{0.08}{n}\right)^{6n}$$

and the interest earned after 6 years is

$$I(n) = A(n) - 5000.$$

a) If $n = 1$, then $A(1) = \$7934.37$ and $I(1) = \$2934.37$.

b) If $n = 4$, then $A(4) = \$8042.19$ and $I(4) = \$3042.19$.

c) If $n = 12$, then $A(12) = \$8067.51$ and $I(12) = \$3067.51$.

d) If $n = 365$, then $A(365) = \$8079.95$ and $I(365) = \$3079.95$.

104. When interest is compounded n times a year, the amount at the end of 20 years is

$$A(n) = 80,000\left(1 + \frac{0.09}{n}\right)^{20n}$$

and the interest earned after 20 years is

$$I(n) = A(n) - 80,000.$$

a) If $n = 1$, then $A(1) = \$448,352.86$ and $I(1) = \$368,352.86$.

b) If $n = 4$, then $A(4) = \$474,411.62$ and $I(4) = \$394,411.62$.

c) If $n = 12$, then $A(12) = \$480,732.12$ and $I(12) = \$400,732.12$.

d) If $n = 365$, then $A(365) = \$483,864.42$ and $I(365) = \$403,864.42$.

105. After t years, a deposit of \$5000 will amount to

$$A(t) = 5000e^{0.08t}.$$

We use 30 days per month and 365 days per year.

a) After 6 years, the amount is $A(6) = \$8080.37$.

b) After 8 years and 3 months, the amount is $A\left(8 + \frac{90}{365}\right) = \$9671.31$.

c) After 5 years, 4 months, and 22 days, the amount is

$$A\left(8 + \frac{4(30) + 22}{365}\right) = \$7694.93.$$

d) After 20 years and 321 days, the amount is

$$A\left(20 + \frac{321}{365}\right) = \$26,570.30.$$

106. After t years, a deposit of \$9000 will amount to

$$A(t) = 9000e^{0.0675t}.$$

We use 30 days per month and 365 days per year.

a) After 13 years, the amount is $A(13) = \$21,643.92$.

b) After 12 years and 8 months, the amount is $A\left(12 + \frac{240}{365}\right) = \$21,149.33$.

c) After 10 years, 6 months, and 14 days, the amount is

$$A\left(10 + \frac{6(30) + 14}{365}\right) = \$18,321.98.$$

d) After 40 years and 66 days, the amount is

$$A\left(40 + \frac{66}{365}\right) = \$135,562.13.$$

107. Assume there are 365 days in a year and 30 days in a month. The present value is

$$3000\left(1 + \frac{0.065}{365}\right)^{-(365)(5+120/365)} = \$2121.82.$$

108. Assume there are 365 days in a year and 30 days in a month. The present value is

$$40,000\left(1 + \frac{0.0675}{365}\right)^{-(365)(3+60/365)} = \$32,307.63.$$

109. $20,000e^{-0.0542(30)} = \$3934.30$

110. $100,000e^{-0.03(40)} = \$30,119.42$

111. a) The interest for first hour is

$$10^6 \cdot e^{0.06\left(\frac{1}{(24)(365)}\right)} - 10^6 = \$6.85.$$

b) The interest for 500th hour is the difference between the amounts in the account at the end of the 500th and 499th hours i.e.

$$10^6 e^{0.06\left(\frac{500}{(24)(365)}\right)} - 10^6 e^{0.06\left(\frac{499}{(24)(365)}\right)} = \$6.87$$

112. a) At 5.5% compounded continuously annually, the interest for one day is

$$I = 10^{13}e^{0.055/365} - 10^{13} = \$1,506,962,851.$$

b) If the annual interest rate is 5.5% compounded daily, then the amount that could be saved in the first day is

$$\left[10^{13}\left(1 + \frac{0.055}{365}\right) - 10^{13}\right] - I = -\$113,536.$$

Thus, the amount saved is \$113,536

113. If $t = 0$, then $A = 200e^0 = 200$ g.
If $t = 500$, then $A = 200e^{-0.001(500)} \approx 121.3$ g.

114. a) In 2000 when $t = 0$, the population is $2.4e^{0.03(0)} \approx 2.4$ million.

b) In 2020 when $t = 20$, the population is $2.4e^{0.03(20)} \approx 4.4$ million.

115. a) In 2006 when $t = 1$, the number of Facebook users was $0.1e^{1.45}$ million or about 400,000 users.

b) In 2011 when $t = 6$, the number of Facebook users was about $0.1e^{1.45(6)} \approx 600$ million.

116. a) In 2006 when $t = 0$, the revenue was was $52e^0 = \$52$ million .

b) In 2010 when $t = 4$, the revenue was was $52e^{0.91(4)} = \$1981$ million or $1.981 billion

117. a) Using a calculator, we find that the exponential regression curve is

$$y = 8.44(1.23)^x$$

b) Yes

c) In 2010 when $t = 20$, the number of subscribers is

$$8.44(1.23)^{20} \approx 530 \text{ million.}$$

118. a) The exponential regression curve is $y = 62.19(0.989)^x$.

b) Yes

c) In 2012 when $t = 22$, the circulation is

$$62.19(0.989)^{22} \approx 48.8 \text{ million.}$$

119. $P = 10\left(\frac{1}{2}\right)^n$ **120.** $C = 15 \cdot 2^{n-1}$

121. When $t = 31$, the number of damaged O-rings is $n = 644e^{-0.15(31)} \approx 6$.

122. The cost of 500 units is $C(500) = 500e^{0.5} = \$824.36$.
As $x \to \infty$, $AC(x) = e^{0.001x} \to \infty$.

125. $y = 2$, $x = -7$

126. Rewrite $|2x - 5| = 7x$ as follows:

$$\begin{aligned} 2x - 5 = 7x &\quad \text{or} \quad 2x - 5 = -7x \\ -5 = 5x &\quad \text{or} \quad 9x = 5 \\ x = -1 &\quad \text{or} \quad x = 5/9 \end{aligned}$$

Since $x = -1$ is an extraneous root, the solution set is $\{5/9\}$.

127. Rewrite $|2x - 5| > 7$ as follows:

$$\begin{aligned} 2x - 5 > 7 &\quad \text{or} \quad 2x - 5 < -7 \\ 2x > 12 &\quad \text{or} \quad 2x < -2 \\ x > 6 &\quad \text{or} \quad x < -1 \end{aligned}$$

The solution set is $(-\infty, -1) \cup (6, \infty)$.

128. Since $2x - 5 = 7x$, we have $-5 = 5x$. Then the solution set is $\{-1\}$.

129. Solve for x:

$$\begin{aligned} 2x - 5 &= y \\ 2x &= y + 5 \\ x &= \frac{y + 5}{2} \end{aligned}$$

130. The slope of the parallel line is $m = 2$. Using $y = mx + b$ and the point $(4, 3)$, we find b as follows:

$$\begin{aligned} 3 &= 2(4) + b \\ 3 &= 8 + b \\ -5 &= b \end{aligned}$$

Then the line is $y = 2x - 5$.

Thinking Outside the Box

XXXIX. Let x be the rate of the swimmer who after 40 feet passes the other swimmer. Let y be the rate of the other swimmer. If x is the length of the pool, then

$$\frac{40}{x} = \frac{d - 40}{y}$$

and
$$\frac{d+45}{x}=\frac{2d-45}{y}.$$
Since we may solve for the ratio y/x from both equations, we find
$$\frac{y}{x}=\frac{d-40}{40}=\frac{2d-45}{d+45}.$$
Solving for d, we obtain $d=75$ or $d=0$. Thus, the length of the pool is 75 feet.

XL. Move the second "I" in "VII", and form a square root symbol to obtain the fraction
$$\frac{1}{\sqrt{1}}$$
which reads as "one divided by the square root of one".

4.1 Pop Quiz

1. $f(4)=-2^4=-16$

2. Since the base of $f(x)=\left(\frac{1}{3}\right)^x$ is less than 1, the function is decreasing.

3. Domain $(-\infty,\infty)$, range $(2,\infty)$

4. $y=-1$

5. Since $\left(\frac{1}{4}\right)^x=4^{-x}=64=4^3$, we obtain $-x=3$ or $x=-3$.

6. Since $f(a)=2^{-a}=8=2^3$, we find $-a=3$ or $a=-3$.

7. $1000\left(1+\frac{0.04}{4}\right)^{4(20)}=\$2216.72$

8. $1000e^{0.04(20)}=\$2225.54$

4.1 Linking Concepts

a)

$[a,b]$	$f(a)$	$A[a,b]$	$\frac{A[a,b]}{f(a)}$
$[3.00,3.05]$	$1,344.5$	680.7	0.51
$[7.50,7.51]$	$12,756.3$	$6,394.1$	0.50
$[8.623,8.624]$	$22,365.7$	$11,185.6$	0.50

b) number of bacteria per hour

c) 0.5

d) The shorter the interval $[a,b]$, the closer is the ratio $\frac{A[a,b]}{f(a)}$ to 0.5

e) If b is sufficiently close but not equal to a, then the average rate of change $A[a,b]$ is directly proportional to $f(a)$. The proportion constant is 0.5.

f)

$[a,b]$	$f(a)$	$A[a,b]$	$\frac{A[a,b]}{f(a)}$
$[3.00,3.05]$	$2,700$	800	0.30
$[7.50,7.51]$	$6,300$	800	0.13
$[8.623,8.624]$	$7,198.4$	800	0.11

The ratio $\frac{A[a,b]}{f(a)}$ approaches 0 as $f(a)$ gets larger, because the average rate of change is constant.

g) In a linear model, the average rate of change is constant. But in an exponential model, the average rate of change is proportional to the population size. Thus, an exponential model is better than a linear model in describing the growth of a bacteria population.

For Thought

1. True **2.** False, since $\log_{100}(10)=1/2$.

3. True **4.** True

5. False, the domain is $(0,\infty)$. **6.** True

7. True

8. False, since $\log_a(0)$ is undefined.

9. True **10.** True

4.2 Exercises

1. logarithmic

2. common

3. natural

4. increasing, decreasing

5. vertical asymptote

6. domain

7. logarithmic family

8. one-to-one

9. 6 **10.** 4 **11.** -4 **12.** 0

13. $\frac{1}{4}$ **14.** 1 **15.** -3 **16.** 3

17. Since $2^6 = 64$, $\log_2(64) = 6$.

18. Since $2^4 = 16$, $\log_2(16) = 4$.

19. Since $3^{-4} = \frac{1}{81}$, $\log_3\left(\frac{1}{81}\right) = -4$.

20. Since $3^0 = 1$, $\log_3(1) = 0$.

21. Since $16^{1/4} = 2$, $\log_{16}(2) = \frac{1}{4}$.

22. Since $16^1 = 16$, $\log_{16}(16) = 1$.

23. Since $\left(\frac{1}{5}\right)^{-3} = 125$, $\log_{1/5}(125) = -3$.

24. Since $\left(\frac{1}{5}\right)^{3} = \frac{1}{125}$, $\log_{1/5}\left(\frac{1}{125}\right) = 3$.

25. Since $10^{-1} = 0.1$, $\log(0.1) = -1$

26. $\log\left(10^6\right) = 6$

27. Since $10^0 = 1$, $\log(1) = 0$.

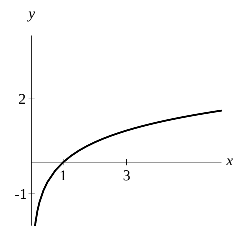
28. Since $10^1 = 10$, $\log(10) = 1$.

29. Since $e^1 = e$, $\ln(e) = 1$.

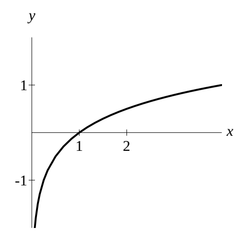
30. Undefined since $e^x = 0$ has no solution.

31. -5 **32.** 9

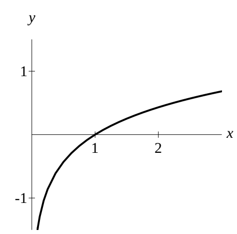
33. $y = \log_3(x)$ goes through $(1/3, -1)$, $(1, 0)$, $(3, 1)$, domain $(0, \infty)$, range $(-\infty, \infty)$



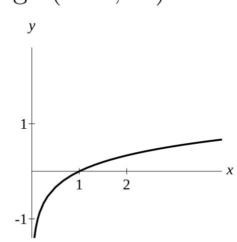
34. $y = \log_4(x)$ goes through $(1/4, -1)$, $(1, 0)$, $(4, 1)$, domain $(0, \infty)$, range $(-\infty, \infty)$



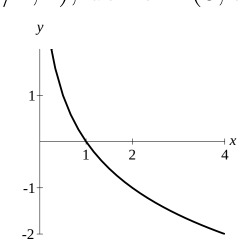
35. $f(x) = \log_5(x)$ goes through $(1/5, -1)$,$(1, 0)$, and $(5, 1)$, domain $(0, \infty)$, range $(-\infty, \infty)$



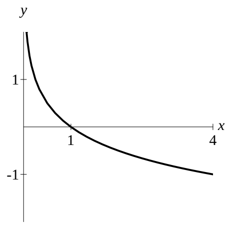
36. $g(x) = \log_8(x)$ goes through $(1/8, -1)$, and $(1, 0)$,$(8, 1)$, domain $(0, \infty)$, range $(-\infty, \infty)$



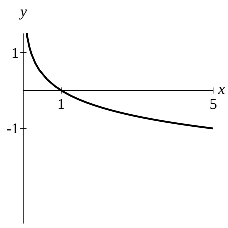
37. $y = \log_{1/2}(x)$ goes through $(2, -1)$,$(1, 0)$, $(1/2, 1)$, domain $(0, \infty)$, range $(-\infty, \infty)$



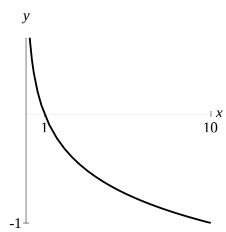
38. $y = \log_{1/4}(x)$ goes through $(4, -1)$, $(1, 0)$, and $(1/4, 1)$, domain $(0, \infty)$, range $(-\infty, \infty)$



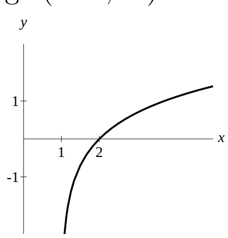
39. $h(x) = \log_{1/5}(x)$ goes through $(5, -1)$, $(1, 0)$,$(1/5, 1)$, domain $(0, \infty)$, range $(-\infty, \infty)$



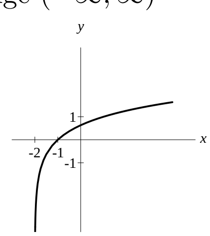
40. $s(x) = \log_{1/10}(x)$ goes through $(10, -1)$, $(1, 0)$, $(1/10, 1)$, domain $(0, \infty)$, range $(-\infty, \infty)$



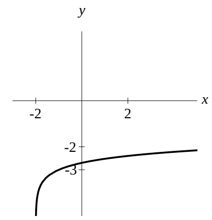
41. $f(x) = \ln(x - 1)$ goes through $\left(1 + \frac{1}{e}, -1\right)$, $(2, 0)$, $(1 + e, 1)$, domain $(1, \infty)$, range $(-\infty, \infty)$



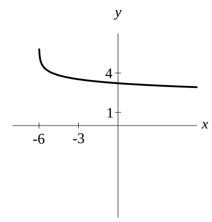
42. $f(x) = \log_3(x + 2)$ goes through $\left(-\frac{5}{3}, -1\right)$, $(-1, 0)$, $(1, 1)$, domain $(-2, \infty)$, range $(-\infty, \infty)$



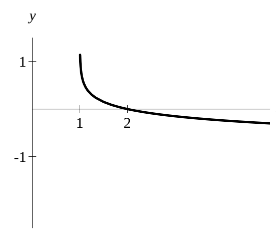
43. $f(x) = -3 + \log(x + 2)$ goes through $(-1.9, -4)$, $(-1, -3)$, $(8, -2)$, domain $(-2, \infty)$, range $(-\infty, \infty)$



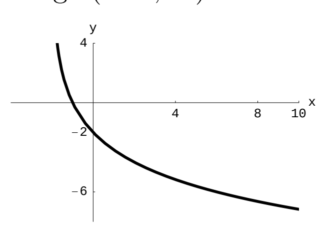
44. $y = 4 - \log(x + 6)$ goes through $(-5.9, 5)$, $(-5, 4)$, $(4, 3)$, domain $(-6, \infty)$, range $(-\infty, \infty)$



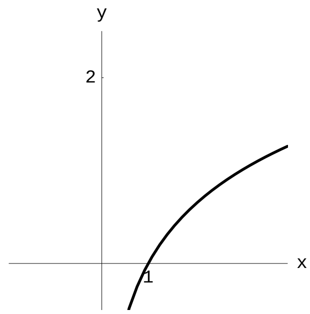
45. $f(x) = -\dfrac{1}{2}\log(x - 1)$ goes through $(1.1, 0.5)$, $(2, 0)$, $(11, -0.5)$, domain $(1, \infty)$, range $(-\infty, \infty)$



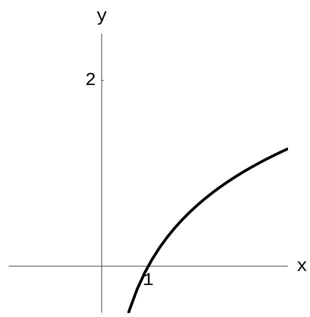
46. $f(x) = -2\log_2(x + 2)$ goes through $(-1, 0)$, $(2, -4)$, $(6, -6)$, domain $(-2, \infty)$, range $(-\infty, \infty)$



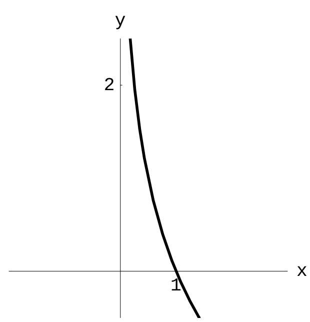
47. From the graph, we find $\lim_{x\to\infty} \log_3 x = \infty$.



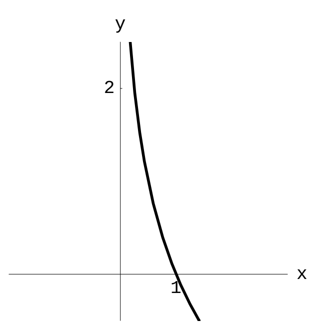
48. From the graph, we find $\lim_{x\to 0^+} \log_3 x = -\infty$.



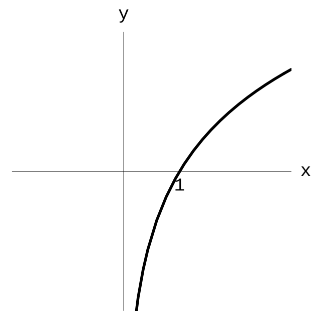
49. Using the graph, we obtain $\lim_{x\to 0^+} \log_{1/2} x = \infty$.



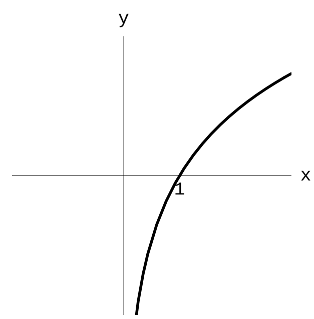
50. Using the graph, we obtain $\lim_{x\to\infty} \log_{1/2} x = -\infty$.



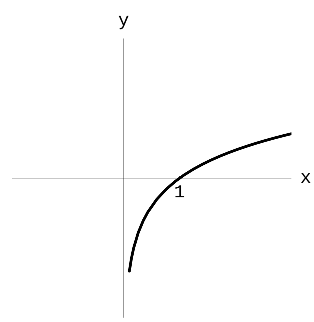
51. From the graph, we get $\lim_{x\to 0^+} \ln x = -\infty$.



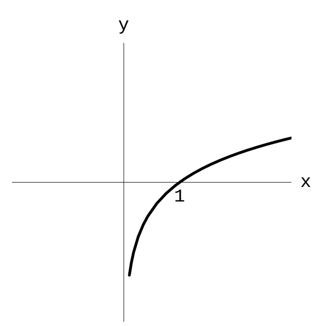
52. From the graph, we get $\lim_{x\to\infty} \ln x = \infty$.



53. From the graph, we find $\lim_{x\to\infty} \log x = \infty$.



54. From the graph, we find $\lim_{x\to 0^+} \log x = -\infty$.



55. $y = \ln(x-3) - 4$

56. $y = \log(x+5) + 7$

57. $y = -\log_2(x-5) - 1$

58. $y = -(\log_3(x+6)+4)$ or $y = -\log_3(x+6) - 4$

59. $2^5 = 32$ **60.** $3^4 = 81$ **61.** $5^y = x$

62. $4^b = a$ **63.** $10^z = 1000$ **64.** $e^3 = y$

65. $e^x = 5$ **66.** $10^2 = y$

67. $a^m = x$ **68.** $b^t = q$

69. $\log_5(125) = 3$ **70.** $\log_2(128) = 7$

71. $\ln(y) = 3$ **72.** $\log(w) = 5$

73. $\log(y) = m$ **74.** $\ln p = x$

75. $\log_a(y) = z$ **76.** $\log_b(w) = k$

77. $\log_a(n) = x - 1$ **78.** $\log_w(r) = x + 2$

79. $f^{-1}(x) = \log_2(x)$ **80.** $f^{-1}(x) = \log_5(x)$

81. $f^{-1}(x) = 7^x$ **82.** $f^{-1}(x) = 10^x$

83. Replace $f(x)$ by y, interchange x and y, solve for y, and replace y by $f^{-1}(x)$.

$$\begin{aligned} y &= \ln(x-1) \\ x &= \ln(y-1) \\ e^x &= y-1 \\ y = f^{-1}(x) &= e^x + 1 \end{aligned}$$

84. Replace $f(x)$ by y, interchange x and y, solve for y, and replace y by $f^{-1}(x)$.

$$\begin{aligned} y &= \log(x+4) \\ x &= \log(y+4) \\ 10^x &= y+4 \\ y = f^{-1}(x) &= 10^x - 4 \end{aligned}$$

85. Replace $f(x)$ by y, interchange x and y, solve for y, and replace y by $f^{-1}(x)$.

$$\begin{aligned} y &= 3^{x+2} \\ x &= 3^{y+2} \\ y+2 &= \log_3(x) \\ y = f^{-1}(x) &= \log_3(x) - 2 \end{aligned}$$

86. Replace $f(x)$ by y, interchange x and y, solve for y, and replace y by $f^{-1}(x)$.

$$\begin{aligned} y &= 6^{x-1} \\ x &= 6^{y-1} \\ y-1 &= \log_6(x) \\ y = f^{-1}(x) &= \log_6(x) + 1 \end{aligned}$$

87. Replace $f(x)$ by y, interchange x and y, solve for y, and replace y by $f^{-1}(x)$.

$$\begin{aligned} x &= \frac{1}{2}10^{y-1} + 5 \\ x-5 &= \frac{1}{2}10^{y-1} \\ 2x-10 &= 10^{y-1} \\ \log(2x-10) &= y-1 \\ \log(2x-10)+1 &= y \\ f^{-1}(x) &= \log(2x-10)+1 \end{aligned}$$

88. Replace $f(x)$ by y, interchange x and y, solve for y, and replace y by $f^{-1}(x)$.

$$\begin{aligned} x &= 2^{3y+1} - 6 \\ x+6 &= 2^{3y+1} \\ \log_2(x+6) &= 3y+1 \\ \log_2(x+6)-1 &= 3y \\ \frac{1}{3}(\log_2(x+6)-1) &= y \\ f^{-1}(x) &= \frac{1}{3}(\log_2(x+6)-1) \end{aligned}$$

89. Since $2^8 = x$, the solution is $x = 256$.

90. Since $5^3 = x$, the solution is $x = 125$.

91. Since $3^{1/2} = x$, the solution is $x = \sqrt{3}$.

92. Since $4^{1/3} = x$, the solution is $x = \sqrt[3]{4}$.

93. Since $x^2 = 16$ and the base of a logarithm is positive, $x = 4$.

94. Since $x^4 = 16$ and the base of a logarithm is positive, $x = 2$.

95. By using the definition of the logarithm, we get $x = \log_3(77)$.

96. Since $2^x = \frac{1}{5}$, the solution is $x = \log_2\left(\frac{1}{5}\right)$.

97. Since $y = \ln(x)$ is one-to-one, $x - 3 = 2x - 9$. The solution is $x = 6$.

98. Since $y = \log_2(x)$ is one-to-one, $4x = x + 6$. The solution is $x = 2$.

99. Since $x^2 = 18$, we obtain $x = \sqrt{18} = 3\sqrt{2}$. The base of a logarithm is positive.

100. Since $x^{1/2} = 9$, $x = 81$.

101. Since $x + 1 = \log_3(7)$, $x = \log_3(7) - 1$.

102. Since $3 - x = \log_5(12)$, $x = 3 - \log_5(12)$.

103. Since $y = \log(x)$ is one-to-one, we get

$$\begin{aligned} x &= 6 - x^2 \\ x^2 + x - 6 &= 0 \\ (x+3)(x-2) &= 0 \\ x &= -3, 2. \end{aligned}$$

But $\log(-3)$ is undefined, so the solution is $x = 2$.

104. Since $y = \log_3(x)$ is a one-to-one function, we obtain

$$\begin{aligned} 2x &= 24 - x^2 \\ x^2 + 2x - 24 &= 0 \\ (x+6)(x-4) &= 0 \\ x &= -6, 4. \end{aligned}$$

Checking $x = -6$ we get $\log_3(-12)$ which is undefined. The solution set is $x = 4$.

105. Since $x^{-2/3} = \frac{1}{9}$, we find

$$x = \pm\left(\frac{1}{9}\right)^{-3/2} = \pm\left(\frac{1}{3}\right)^{-3} = \pm 27.$$

The solution is $x = 27$ since a negative number cannot be a base.

106. Since $x^{4/3} = \frac{1}{16}$, we have

$$x = \pm\left(\frac{1}{16}\right)^{3/4} = \pm\left(\frac{1}{2}\right)^{3} = \pm\frac{1}{8}.$$

But the base must be positive, so the solution is $x = \frac{1}{8}$.

107. Since $(2^2)^{2x-1} = 2^{4x-2} = 2^{-1}$, we get $4x - 2 = -1$. The solution is $x = \frac{1}{4}$.

108. Since $e^{3x-4} = e^0$, we obtain $3x - 4 = 0$. The solution is $x = \frac{4}{3}$.

109. Since $32^x = 64$, we get $2^{5x} = 2^6$. Thus, the solution is $x = \frac{6}{5}$.

110. Since $x = \ln\left(\frac{1}{\sqrt{e}}\right) = \ln\left(e^{-1/2}\right) = -\frac{1}{2}$, the solution is $x = -\frac{1}{2}$.

111. Since $\log_3(\log_4(x)) = 1$, we obtain $\log_4 x = 3$. Then $x = 4^3 = 64$.

112. Since $\log_2\left(2^{4x}\right) = 4x$, the given equation may be written as $\log_2(\log_2(4^x)) = 3$. Then $\log_2(4^x) = 8$ and thus, $4^x = 2^8 = 4^4$. Hence, $x = 4$ by the one-to-one property.

113. $x = \log 25 \approx 1.3979$

114. $x = \ln(2) \approx 0.6931$

115. Solving for x, we obtain

$$\begin{aligned} e^{2x} &= 3 \\ 2x &= \ln(3) \\ x &= \frac{1}{2}\ln(3) \\ x &\approx 0.5493. \end{aligned}$$

116. Solving for x, we obtain

$$\begin{aligned} 10^{3x} &= 5 \\ 3x &= \log 5 \\ x &= \frac{1}{3}\log 5 \\ x &\approx 0.2330. \end{aligned}$$

117. Solving for x, we get

$$\begin{aligned} e^{x} &= \frac{4}{5} \\ x &= \ln\left(\frac{4}{5}\right) \\ x &\approx -0.2231. \end{aligned}$$

118. Solving for x, we get

$$\begin{aligned} 10^x &= 8 \\ x &= \log(8) \\ x &\approx 0.9031. \end{aligned}$$

119. Solving for x, we find

$$\begin{aligned} \frac{1}{10^x} &= 2 \\ \frac{1}{2} &= 10^x \\ x &= \log\left(\frac{1}{2}\right) \\ x &\approx -0.3010. \end{aligned}$$

120. Solving for x, we find

$$\begin{aligned} \frac{1}{e^{x-1}} &= 5 \\ \frac{1}{5} &= e^{x-1} \\ \ln\left(\frac{1}{5}\right) &= x-1 \\ x &= \ln\left(\frac{1}{5}\right)+1 \\ x &\approx -0.6094. \end{aligned}$$

121. Solving for the year t when \$10 grows to \$20, we find

$$\begin{aligned} 20 &= 10e^{rt} \\ 2 &= e^{rt} \\ \ln 2 &= rt \\ \frac{\ln 2}{r} &= t. \end{aligned}$$

a) If $r = 2\%$, then $t = \dfrac{\ln 2}{0.02} \approx 34.7$ years.

b) If $r = 4\%$, then $t = \dfrac{\ln 2}{0.04} \approx 17.3$ years.

c) If $r = 8\%$, then $t = \dfrac{\ln 2}{0.08} \approx 8.7$ years.

d) If $r = 16\%$, then $t = \dfrac{\ln 2}{0.16} \approx 4.3$ years.

122. Solving for the year t when \$10 becomes \$40, we find

$$\begin{aligned} 40 &= 10e^{rt} \\ 4 &= e^{rt} \\ \ln 4 &= rt \\ \frac{\ln 4}{r} &= t. \end{aligned}$$

a) If $r = 1\%$, then $t = \dfrac{\ln 4}{0.01} \approx 138.6$ years.

b) If $r = 2\%$, then $t = \dfrac{\ln 4}{0.02} \approx 69.3$ years.

c) If $r = 8.327\%$, then $t = \dfrac{\ln 4}{0.08327} \approx 16.6$ years.

d) If $r = \dfrac{23}{3}\%$, then $t = \dfrac{\ln 4}{(23/3)100} \approx 18.1$ years.

123. Solving for the annual percentage rate r when \$10 becomes \$30, we find

$$\begin{aligned} 30 &= 10e^{rt} \\ 3 &= e^{rt} \\ \ln 3 &= rt \\ \frac{\ln 3}{t} &= r. \end{aligned}$$

a) If $t = 5$ years, then $r = \dfrac{\ln 3}{5} \approx 0.2197 \approx 22\%$.

b) If $t = 10$ years, then $r = \dfrac{\ln 3}{10} \approx 0.10986 \approx 11\%$.

c) If $t = 20$ years, then $r = \dfrac{\ln 3}{20} \approx 0.0549 \approx 5.5\%$.

d) If $t = 40$ years, then $r = \dfrac{\ln 3}{40} \approx 0.027465 \approx 2.7\%$.

124. Solving for the annual percentage rate r when \$10 grows to \$50, we find

$$\begin{aligned} 50 &= 10e^{rt} \\ 5 &= e^{rt} \\ \ln 5 &= rt \\ \frac{\ln 5}{t} &= r. \end{aligned}$$

a) If $t = 4$ years, then $r = \frac{\ln 5}{4} \approx 0.40236 \approx 40.2\%$.

b) If $t = 8$ years, then $r = \frac{\ln 5}{8} \approx 0.20118 \approx 20.1\%$.

c) If $t = 16$ years, then $r = \frac{\ln 5}{16} \approx 0.10059 \approx 10.1\%$.

d) If $t = 32$ years, then $r = \frac{\ln 5}{32} \approx 0.050295 \approx 5.0\%$.

125. Let t be the number of years.

$$\begin{aligned} 1000 \cdot e^{0.14t} &= 10^6 \\ e^{0.14t} &= 1000 \\ 0.14t &= \ln(1000) \\ t &\approx 49.341 \text{ years} \end{aligned}$$

Note, $0.341(365) \approx 125$.
It will take 49 years and 125 days.

126. Let t be the number of years. Then

$$\begin{aligned} 1000 \cdot e^{0.08t} &= 2000 \\ e^{0.08t} &= 2 \\ 0.08t &= \ln(2) \\ t &\approx 8.664 \text{ years} \end{aligned}$$

Note, $0.664(365) \approx 242.4$.
It will take 8 years and 242 days.

127. Since $e^{rt} = A/P$, $rt = \ln(A/P)$ and

$$r = \frac{\ln(A/P)}{t}.$$

Thus, \$1000 will double in 3 years if the rate is $r = \frac{\ln(2000/1000)}{3} \approx 0.231$ or 23.1%.

128. From Exercise 127, the rate is $r = \frac{\ln(A/P)}{t}$.

Thus, $r = \dfrac{\ln\left(\dfrac{2,540,689}{30,000}\right)}{40} \approx 0.111$ or 11.1%.

129.

(a) Let t be the number of years.

$$\begin{aligned} P \cdot e^{0.1t} &= 2P \\ e^{0.1t} &= 2 \\ 0.1t &= \ln(2) \\ t = \frac{\ln(2)}{0.1} &\approx 6.9 \end{aligned}$$

An investment at 10% doubles every 6.9 years.

(b) If t is the number of years it takes before an investment doubles, then

$$\begin{aligned} P \cdot e^{rt} &= 2P \\ e^{rt} &= 2 \\ rt &= \ln(2) \\ t &= \frac{\ln(2)}{r} \\ t &\approx \frac{0.70}{r}. \end{aligned}$$

In particular, if $r = 0.07$ then

$$t \approx \frac{0.70}{0.07} = 10 \text{ years}.$$

That is, at 10%, an investment will double in about 10 years.

130.

(a) At a 2% annual rate, according to the Rule of 70, Connie's \$1000 will double in approximately $\frac{70}{2} = 35$ years.

(b) At 10% yearly, according to the Rule of 70, \$1000 will double in about $\frac{70}{10}$ or 7 years.

(c) After 35 years, Celeste's money (which would have doubled five times) will be worth $2^5(1000) = \$32,000$ and Connie's money will be worth \$2000. Since $\frac{32,000}{2000} = 16$, the ratio of Celeste's money to Connie's is 16 to 1.

131. Let r be the interest rate.

$$\begin{aligned} 4,000 \cdot e^{200r} &= 4,500,000 \\ e^{200r} &= 1,125 \\ 200r &= \ln(1,125) \\ r = \frac{\ln(1,125)}{200} &\approx 0.035 \end{aligned}$$

The rate is 3.5% .

132. Let r be the interest rate.

$$\begin{aligned} 4,000 \cdot e^{200r} &= 2,000,000 \\ e^{200r} &= 500 \\ 200r &= \ln(500) \\ r = \frac{\ln(500)}{200} &\approx 0.031 \end{aligned}$$

The rate is 3.1% .

133. Let t be the number of years.

$$\begin{aligned} F_o \cdot e^{-0.052t} &= 0.6F_o \\ e^{-0.052t} &= 0.6 \\ -0.052t &= \ln(0.6) \\ t = \frac{\ln(0.6)}{-0.052} &\approx 9.8 \end{aligned}$$

Only 60% of the present forest will remain after 9.8 years.

134. Let r be the annual rate.

$$\begin{aligned} F_o \cdot e^{-20r} &= 0.53F_o \\ e^{-20r} &= 0.53 \\ -20r &= \ln(0.53) \\ r = \frac{\ln(0.53)}{-20} &\approx 0.032 \end{aligned}$$

The annual rate is 3.2%

135. Let r be the annual rate from 1950 to 1987.

$$\begin{aligned} 2.5 \cdot e^{37r} &= 5 \\ e^{37r} &= 2 \\ 37r &= \ln(2) \\ r = \frac{\ln(2)}{37} &\approx 0.0187 \end{aligned}$$

The annual rate is 1.87%.

If the annual rate is 1.63% and the initial population is 5 billion in 1987, the world population in year 2010 will be

$$5 \cdot e^{0.0163(23)} \approx 7.3 \text{ billion.}$$

136. Let r be the annual growth for the period 1400 to 2000. Then we obtain the following:

$$\begin{aligned} 0.37 \times 10^9 \cdot e^{600r} &= 6.07 \times 10^9 \\ e^{600r} &= \frac{6.07}{0.37} \\ 600r &= \ln\left(\frac{6.07}{0.37}\right) \\ r &\approx 0.00466. \end{aligned}$$

Thus, the annual growth rate from 1400 to 2000 is about

$$0.466\%.$$

After 600 years, if the 100 million people had not been lost, the world population in 2000 would have been bigger by an additional

$$100e^{0.00466(600)} \approx 1637.9 \text{ million}$$

or 1.638 billion.

137.

(a) $30e^{1.2(7)} \approx 133,412$ acres

(b) Solve for t:

$$\begin{aligned} 30e^{1.2t} &= 53,480,960 \\ 1.2t &= \ln\left(\frac{53,480,960}{30}\right) \\ t &\approx 11.9 \end{aligned}$$

It will take about 12 days.

138.

(a) $0.1e^{0.02(110)} - 0.1e^{0.02(50)} \approx 0.6°$ Celsius

(b) Solving for t, we find

$$\begin{aligned} 0.1e^{0.02t} - 0.1e^{0.02(100)} &= 4 \\ e^{0.02t} - e^{0.02(100)} &= 40 \\ e^{0.02t} &= 40 + e^2 \\ 0.02t &= \ln(40 + e^2) \\ t &= \frac{\ln(40 + e^2)}{0.02} \\ t &\approx 193. \end{aligned}$$

The year when the global temperature is predicted to be 4° Celsius greater than the global temperature in 2000 is the year 2093 ($\approx 2000+193$).

139.

a) The function is given by

$$\begin{aligned} p-100 &= \frac{100-10}{2-4}(x-2) \\ p-100 &= -45(x-2) \\ p &= -45x+90+100 \\ p &= -45x+190 \\ p &= -45\log(I)+190. \end{aligned}$$

b) $p=-45\log(100,000)+190=-35\%$ or 0% of the population is expected to be without safe drinking water, i.e., everyone is expected to have safe water.

140.

(a) The function is given by

$$\begin{aligned} w-400 &= \frac{400-100}{5-2}(x-5) \\ w-400 &= 100(x-5) \\ w &= 100x-100 \\ w &= 100\log(I)-100. \end{aligned}$$

(b) $w=100\log(200,000)-100 \approx 430$ kg.

141. $pH=-\log\left(10^{-4.1}\right)=4.1$

142. $pH=-\log\left(10^{-1}\right)=1$

143. $pH=-\log\left(10^{-3.7}\right)=3.7$

144. $pH=-\log\left(10^{-7.4}\right)=7.4$

145. By substituting $x=1$, we find a formula for c.

$$\begin{aligned} a\cdot b^x &= a\cdot e^{cx} \\ b^x &= e^{cx} \\ b &= e^c \\ c &= \ln b. \end{aligned}$$

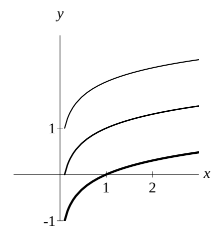
By using this formula, we find

$$y=500(1.036)^x=500e^{\ln(1.036)x}$$

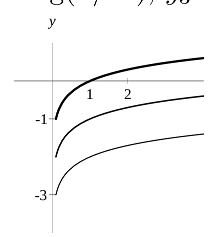
and the continuous growth rate is

$$\ln(1.036)\cdot 100 \approx 3.54\%.$$

146. The graphs of $y_1=\log(x)$, $y_2=\log(10x)$, and $y_3=\log(100x)$ are drawn simultaneously



as well as the graphs of $y_1=\log(x)$, $y_2=\log(x/10)$, $y_3=\log(x/100)$

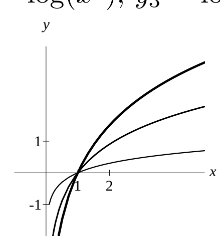


The graph of

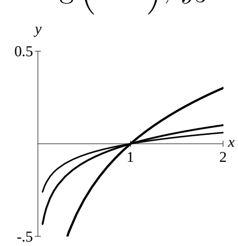
$$y=\log(10^n x)=n+\log(x)$$

is obtained by translating the graph of $y=\log(x)$ vertically $|n|$ units upward if $n>0$ and downward if $n<0$.

A sketch of the graphs of $y_1=\log(x)$, $y_2=\log(x^3)$, $y_3=\log(x^5)$ are given



and also the graphs of $y_1=\log(x)$, $y_2=\log\left(x^{1/3}\right)$, $y_3=\log\left(x^{1/5}\right)$



Assuming $n > 1$, the graph of

$$y = \log(x^n) = n \cdot \log(x)$$

is obtained by stretching the graph of $y = \log(x)$ by n units and the graph of $y = \log(x^{1/n})$ is obtained by shrinking the graph of $y = \log(x)$ by $1/n$ units.

149. Domain $(-\infty, \infty)$, range $(-\infty, 7]$

150. Since $2^{x-3} = 2^{10x-2}$, we have $x-3 = 10x-2$. Then $-1 = 9x$. The solution set is $\{-1/9\}$.

151. $(8 \times 10^{-27})(25 \times 10^6) = 200 \times 10^{-21} = 2 \times 10^{-19}$

152. If x is the number of bass, then

$$\begin{aligned} 200 + x &= 0.2(2000 + x) \\ 200 + x &= 400 + 0.2x \\ 0.8x &= 200 \\ x &= 250 \text{ bass} \end{aligned}$$

153. Rewrite the equation:

$$\begin{aligned} x(x^2 - 4x + 13) &= 0 \\ x((x-2)^2 + 9) &= 0 \end{aligned}$$

Then $x = 0$ or $x - 2 = \pm 3i$.

The solution set is $\{0, 2 \pm 3i\}$.

154. Let $C = kLW$. Solve for the proportion constant k:

$$\begin{aligned} 875.60 &= k(8)(11) \\ 875.60 &= 88k \\ 9.95 &= k. \end{aligned}$$

The cost for a 10 ft by 14 ft room is

$$C = 9.95(10)(14) = \$1393$$

Thinking Outside the Box XLI

First, \$59 is not a possible total, i.e., for all whole numbers x and y we have

$$7x + 11y \neq 59.$$

But for each $60 \leq n \leq 69$, there are whole numbers x and y satisfying

$$n = 7x + 11y.$$

Suppose

$$m = 7a + 11b \geq 69$$

and a, b are whole numbers. Since

$$m + 1 = 7(a-3) + 11(b+2)$$

we see that $m+1$ is a possible total if $a - 3 \geq 0$.

However, if $a - 3 < 0$ then a is either 0, 1, or 2 for $a \geq 0$. Since $m - 7a = 11b$ and $m \geq 69$, we find

$$\begin{aligned} b - 5 &= \frac{m - 7a - 55}{11} \\ &\geq \frac{69 - 7a - 55}{11} \\ &= \frac{14 - 7a}{11} \\ &\geq 0 \end{aligned}$$

for $a = 0, 1, 2$. In any case, $m+1$ is a possible total since

$$m + 1 = 7(a+8) + 11(b-5).$$

Since $m \geq 69$, all whole numbers greater than 59 is a possible total. Hence, 59 is the largest integer that is not a possible total.

4.2 Pop Quiz

1. 5

2. 5

3. Since the base $b = 3$ is greater than 1, the function is increasing.

4. $x = 1$

5. $\log_3(b) = a$

6. Replace $f(x)$ by y, interchange x and y, solve for y, and replace y by $f^{-1}(x)$.

$$\begin{aligned} y &= \log(x+3) \\ x &= \log(y+3) \\ 10^x &= y+3 \\ y &= 10^x - 3 \\ f^{-1}(x) &= 10^x - 3 \end{aligned}$$

7. $x = 5^3 = 125$

8. Since $x^2 = 36$, we have $x = \pm 6$. Since the base of a logarithm is positive, we obtain $x = 6$.

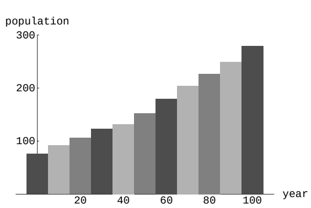
9. Let t be the number of years.

$$\begin{aligned} 2e^{0.05t} &= 4 \\ e^{0.05t} &= 2 \\ 0.05t &= \ln 2 \\ t = \frac{\ln 2}{0.05} &\approx 13.863 \end{aligned}$$

Since 0.863 yr $\approx$ 315 days, we find $t = 13$ years, and 315 days.

4.2 Linking Concepts

a) The population data is plotted as shown in

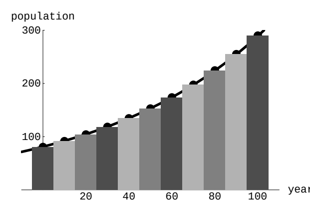


b) If x is the number of years since 1900 and y is the population in millions, then an exponential model is approximately $y = 81.19(1.0126)^x$.

c) A table that shows the predicted population (in millions) based on the exponential model in part (b) is

year	predicted population
1900	81
1910	92
1920	104
1930	118
1940	134
1950	152
1960	172
1970	195
1980	221
1990	241
2000	284
2010	321

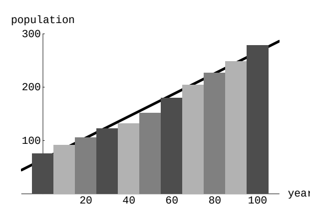
d) A sketch of the exponential curve and the data points from part c) is shown below.



e) A linear regression model is approximately

$$y = 2.102x + 61.8.$$

where x is the number of years since 1900. Its graph is sketched together with the given population data points.



f) According to the exponential model, the population in the year 2020 is

$$y = 81.19(1.0126)^{120} \approx 365 \text{ million.}$$

In the linear model, the population in the same year is

$$2.102(120) + 61.8 \approx 314 \text{ million.}$$

g) Judging from the graphs, one could have the most confidence by using the prediction of the exponential model in part (f).

h) Based on the data for $x = 0$ and $x = 110$, we obtain

$$\begin{aligned} P(t) &= P_0 \cdot e^{rt} \\ 309 &= 76e^{r(110)} \\ \frac{\ln(309/76)}{110} &= r \\ 0.013 &\approx r. \end{aligned}$$

Then $P(t) = 76e^{0.013t}$.

i) Using the formula in part (h), the predicted population for the year 2020 is $P(120) = 76 \cdot e^{0.013(120)} \approx 362$ million.

For Thought

1. False, since $\log(8) - \log(3) = \log(8/3) \neq \dfrac{\log(8)}{\log(3)}$.

2. True, since $\ln(3^{1/2}) = \dfrac{1}{2} \cdot \ln(3) = \dfrac{\ln(3)}{2}$.

3. True, since $\dfrac{\log_{19}(8)}{\log_{19}(2)} = \log_2(8) = 3 = \log_3(27)$.

4. True, because of the base-change formula.

5. False

6. False, since $\log(x) - \log(2) = \log(x/2)$.

7. False, since the solution of the first equation is $x = -2$ and the second equation is not defined when $x = -2$.

8. True **9.** False, since x can be negative.

10. False, since a can be negative and so $\ln(a)$ will not be a real number.

4.3 Exercises

1. sum

2. quotient

3. power

4. base-change

5. $\sqrt{y}$ **6.** $3x + 1$ **7.** $y + 1$

8. $2k$ **9.** 999

10. $\log_4((2^2)^{150}) = \log_4(4^{150}) = 150$

11. $\log(15)$ **12.** $\ln(12)$

13. $\log_2((x-1)x) = \log_2(x^2 - x)$

14. $\log_3((x+2)(x-1)) = \log_3(x^2 + x - 2)$

15. $\log_4(6)$ **16.** $\log_2(5)$ **17.** $\ln\left(\dfrac{x^8}{x^3}\right) = \ln(x^5)$

18. $\log\left(\dfrac{(x-2)(x+2)}{x-2}\right) = \log(x+2)$

19. $\log_2(3x) = \log_2(3) + \log_2(x)$

20. $\log_3(xy) = \log_3(x) + \log_3(y)$

21. $\log\left(\dfrac{x}{2}\right) = \log(x) - \log(2)$

22. $\log\left(\dfrac{a}{b}\right) = \log(a) - \log(b)$

23. $\log((x-1)(x+1)) = \log(x-1) + \log(x+1)$

24. $\log((a-3)(a+3)) = \log(a-3) + \log(a+3)$

25. $\ln\left(\dfrac{x-1}{x}\right) = \ln(x-1) - \ln(x)$

26. $\ln\left(\dfrac{a+b}{b}\right) = \ln(a+b) - \ln(b)$

27. $\log_a(5^3) = 3\log_a(5)$ **28.** $\log_a(5^2) = 2\log_a(5)$

29. $\log_a(5^{1/2}) = \dfrac{1}{2} \cdot \log_a(5)$

30. $\log_a(5^{1/3}) = \dfrac{1}{3} \cdot \log_a(5)$

31. $\log_a(5^{-1}) = -\log_a(5)$

32. $\log_a(5^{-3}) = -3\log_a(5)$

33. $\log_a(2) + \log_a(5)$

34. $\log_a(2/5) = \log_a(2) - \log_a(5)$

35. $\log_a(5/2) = \log_a(5) - \log_a(2)$

36. $\log_a(2 \cdot 5^3) = \log_a(2) + 3\log_a(5)$

37. $\log_a(\sqrt{2^2 \cdot 5}) = \frac{1}{2}(2\log_a(2) + \log_a(5)) =$
$\log_a(2) + \frac{1}{2}\log_a(5)$

38. $\log_a(5(10)^{-4}) = \log_a(5(2^{-4}5^{-4})) =$
$\log_a(2^{-4}5^{-3}) = \log_a(5^{-3}) + \log_a(2^{-4}) =$
$-3\log_a(5) - 4\log_a(2)$

39. $\log_a(4) - \log_a(25) = \log_a(2^2) - \log_a(5^2) =$
$2\log_a(2) - 2\log_a(5)$

40. $\log_a(10^{-1}) = -\log_a(2 \cdot 5) =$
$-(\log_a(2) + \log_a(5)) = -\log_a(2) - \log_a(5)$

41. $\log_3(5) + \log_3(x)$

42. $\log_2(x) + \log_2(y) + \log_2(z)$

43. $\log_2(5) - \log_2(2y) = \log_2(5) - \log_2(2) - \log_2(y)$

44. $\log(4a) - \log(3b) =$
$\log(4) + \log(a) - \log(3) - \log(b)$

45. $\log(3) + \frac{1}{2}\log(x)$

46. $\frac{1}{2}\ln(x/4) = \frac{1}{2}(\ln(x) - 2\ln(2)) = \frac{1}{2}\ln(x) - \ln(2)$

47. $\log(3) + (x-1)\log(2)$

48. $\ln(5^{-x}) - \ln(2) = -x \cdot \ln(5) - \ln(2)$

49. $\frac{1}{3}\cdot\ln(xy) - \frac{4}{3}\ln(t) = \frac{1}{3}\cdot\ln(x) + \frac{1}{3}\cdot\ln(y) - \frac{4}{3}\ln(t)$

50. $\log(3x^2) - \log((ab)^{2/3}) =$
$\log(3) + 2\log(x) - \frac{2}{3}\cdot\log(a) - \frac{2}{3}\cdot\log(b)$

51. $\ln(6\sqrt{x-1}) - \ln(5x^3) =$
$\ln(6) + \frac{1}{2}\cdot\ln(x-1) - \ln(5) - 3\cdot\ln(x)$

52. $\log_4(3x\sqrt{y}) - \frac{1}{3}\log_4(x-1) =$
$\log_4(3) + \log_4(x) + \frac{1}{2}\log_4(y) - \frac{1}{3}\log_4(x-1)$

53. $\log_2(5x^3)$ **54.** $6 \cdot \log(x) = \log(x^6)$

55. $\log_7(x^5) - \log_7(x^8) = \log(x^5/x^8) = \log_7(x^{-3})$

56. $\frac{1}{3}(\ln(6) - \ln(2)) = \frac{1}{3}\cdot\ln(3) = \ln(\sqrt[3]{3})$

57. $\log(2xy/z)$ **58.** $\ln(2 \cdot 3 \cdot 5 \div 7) = \ln(30/7)$

59. $\log\left(\frac{\sqrt{x}}{y}\right) + \log\left(\frac{z}{\sqrt[3]{w}}\right) = \log\left(\frac{z\sqrt{x}}{y\sqrt[3]{w}}\right)$

60. $\log_2\left(\frac{x^{5/6}\cdot y^{2/3}}{x^{1/2}\cdot y}\right) = \log_2\left(\frac{x^{1/3}}{y^{1/3}}\right) =$
$\log_2\left(\sqrt[3]{\frac{x}{y}}\right)$

61. $\log_4(x^6) + \log_4(x^{12}) + \log_4(x^2) = \log_4(x^{20})$

62. $\log(\sqrt{xy}) - \log(z) = \log\left(\frac{\sqrt{xy}}{z}\right)$

63. Since $2^x = 9$, we get $x = \frac{\log 9}{\log 2} \approx 3.1699$.

64. Since $3^x = 12$, we get $x = \frac{\log 12}{\log 3} \approx 2.2619$.

65. Since $0.56^x = 8$, we get
$x = \frac{\log 8}{\log 0.56} \approx -3.5864$.

66. Since $0.23^x = 18.4$, we get
$x = \frac{\log 18.4}{\log 0.23} \approx -1.9816$.

67. Since $1.06^x = 2$, we get
$x = \frac{\log 2}{\log 1.06} \approx 11.8957$.

68. Since $1.09^x = 3$, we get
$x = \frac{\log 3}{\log 1.09} \approx 12.7482$.

69. Since $0.73^x = 0.5$, we get
$x = \frac{\log 0.5}{\log 0.73} \approx 2.2025$.

70. Since $0.62^x = 0.25$, we get
$x = \frac{\log 0.25}{\log 0.62} \approx 2.9000$.

71. $\frac{\ln(9)}{\ln(4)} \approx \frac{2.1972246}{1.3862944} \approx 1.5850$

72. $\frac{\ln(4.78)}{\ln(3)} \approx \frac{1.5644405}{1.0986123} \approx 1.4240$

73. $\dfrac{\ln(2.3)}{\ln(9.1)} \approx \dfrac{0.8329091}{2.2082744} \approx 0.3772$

74. $\dfrac{\ln(13.7)}{\ln(1.2)} \approx \dfrac{2.6173958}{0.1823216} \approx 14.3559$

75. $\dfrac{\ln(12)}{\ln(1/2)} \approx -3.5850$

76. $\dfrac{\ln(3.66)}{\ln(1.05)} \approx 26.5927$

77. Since $4t = \log_{1.02}(3) = \dfrac{\ln(3)}{\ln(1.02)}$,

we find $t = \dfrac{\ln(3)}{4 \cdot \ln(1.02)} \approx 13.8695$.

78. Since $12t = \log_{1.025}(3) = \dfrac{\ln(3)}{\ln(1.025)}$,

we get $t = \dfrac{\ln(3)}{12 \cdot \ln(1.025)} \approx 3.7076$.

79. Since $365t = \log_{1.0001}(3.5) = \dfrac{\ln(3.5)}{\ln(1.0001)}$,

we get $t = \dfrac{\ln(3.5)}{365 \cdot \ln(1.0001)} \approx 34.3240$.

80. Since $365t = \log_{1.00012}(2.4) = \dfrac{\ln(2.4)}{\ln(1.00012)}$,

we obtain $t = \dfrac{\ln(2.4)}{365 \cdot \ln(1.00012)} \approx 19.9891$.

81. $1 + r = \sqrt[3]{2.3}$, so $r = \sqrt[3]{2.3} - 1 \approx 0.3200$

82.

$$\begin{aligned} 1 + \frac{r}{4} &= \pm \sqrt[20]{3} \\ \frac{r}{4} &= \pm \sqrt[20]{3} - 1 \\ r &= 4\left(\pm \sqrt[20]{3} - 1\right) \\ r &\approx 0.2259, -8.2259 \end{aligned}$$

83.

$$\begin{aligned} \left(1 + \frac{r}{12}\right)^{360} &= 4.2 \\ 1 + \frac{r}{12} &= \pm \sqrt[360]{4.2} \\ r &= 12\left(\pm \sqrt[360]{4.2} - 1\right) \\ r &\approx 0.0479, -24.0479 \end{aligned}$$

84.

$$\begin{aligned} \left(1 + \frac{r}{360}\right)^{720} &= 2.4 \\ 1 + \frac{r}{360} &= \pm \sqrt[720]{2.4} \\ r &= 360\left(\pm \sqrt[720]{2.4} - 1\right) \\ r &\approx 0.4380, -720.4380 \end{aligned}$$

85. Since $x^5 = 33.4$, we get $x = \sqrt[5]{33.4} \approx 2.0172$.

86. Since $x^{2.3} = 12.33$, we get $x = 12.33^{1/2.3} \approx 2.9808$.

87. Since $x^{-1.3} = 0.546$, we have $x = 0.546^{1/(-1.3)}$ or $x \approx 1.5928$.

88. Since $x^{-3.2} = 0.915$, we obtain $x = 0.915^{1/(-3.2)}$ or $x \approx 1.0281$.

89. Let t be the number of years.

$$\begin{aligned} 800\left(1 + \frac{0.08}{365}\right)^{365t} &= 2000 \\ \left(1 + \frac{0.08}{365}\right)^{365t} &= 2.5 \\ (1.0002192)^{365t} &\approx 2.5 \\ 365t &\approx \log_{1.0002192}(2.5) \\ t &\approx \frac{1}{365} \cdot \frac{\ln(2.5)}{\ln(1.0002192)} \\ t &\approx 11.454889 \\ t &\approx 11 \text{ years}, 166 \text{ days} \end{aligned}$$

90. Let t be the number of years.

$$\begin{aligned} 10^4 (1 + 0.0775)^t &= 10^6 \\ (1.0775)^t &= 100 \\ t &= \log_{1.0775}(100) \\ t &= \frac{\ln(100)}{\ln(1.0775)} \\ t &\approx 62 \text{ years} \end{aligned}$$

91. Let t be the number of years.

$$\begin{aligned} W\left(1 + \frac{0.1}{4}\right)^{4t} &= 3W \\ (1.025)^{4t} &= 3 \end{aligned}$$

$$\begin{aligned} 4t &\approx \log_{1.025}(3) \\ t &\approx \frac{1}{4}\cdot\frac{\ln(3)}{\ln(1.025)} \\ t &\approx 11.123 \text{ years} \\ t &\approx 11.123(4) \approx 44 \text{ quarters} \end{aligned}$$

92. Let t be the number of years.

$$\begin{aligned} W\left(1+\frac{0.12}{12}\right)^{12t} &= 2W \\ (1.01)^{12t} &= 2 \\ 12t &\approx \log_{1.01}(2) \\ t &\approx \frac{1}{12}\cdot\frac{\ln(2)}{\ln(1.01)} \\ t &\approx 5.805\text{years} \\ t &\approx 5.805(12) \approx 70 \text{ months} \end{aligned}$$

93. Let t be the number of years.

$$\begin{aligned} 500\,(1+r)^{25} &= 2000 \\ (1+r)^{25} &= 4 \\ 1+r &\approx \sqrt[25]{4} \\ r &= \sqrt[25]{4}-1 \\ r &\approx 0.057 \text{ or } 5.7\% \end{aligned}$$

94. Let t be the number of years.

$$\begin{aligned} 1000\left(1+\frac{r}{12}\right)^{12(8)} &= 2500 \\ \left(1+\frac{r}{12}\right)^{96} &= 2.5 \\ r &= 12\left(\sqrt[96]{2.5}-1\right) \\ r &\approx 0.115 \text{ or } 11.5\% \end{aligned}$$

95. Let t be the number of years.

$$\begin{aligned} 4000\,(1+r)^{200} &= 4.5\times 10^6 \\ (1+r)^{200} &= 1125 \\ r &= \sqrt[200]{1125}-1 \\ r &\approx 0.035752 \text{ or } 3.58\% \end{aligned}$$

96. Let t be the number of years.

$$\begin{aligned} 4000\left(1+\frac{r}{12}\right)^{12(200)} &= 2\times 10^6 \\ \left(1+\frac{r}{12}\right)^{2400} &= 500 \\ r &= 12\left(\sqrt[2400]{500}-1\right) \\ r &\approx 0.0311 \text{ or } 3.11\% \end{aligned}$$

97. Let r be the annual growth rate.

$$\begin{aligned} 1995\,(1+r)^{39} &= 10,890 \\ r &= \left(\frac{10,890}{1995}\right)^{1/39}-1 \\ r &\approx 0.0444 \\ r &\approx 4.4\% \end{aligned}$$

98. Let r be the annual growth rate.

$$\begin{aligned} 0.49\,(1+r)^{49} &= 4.59 \\ r &= \left(\frac{4.59}{0.49}\right)^{1/49}-1 \\ r &\approx 0.0467 \\ r &\approx 4.7\% \end{aligned}$$

99. The Richter scale rating is

$$\log(I)-\log(I_o)=\log\left(\frac{I}{I_o}\right).$$

When $I=1000\cdot I_o$, the Richter scale rating is

$$\log\left(\frac{1000\cdot I_o}{I_o}\right)=\log(1000)=3.$$

100. Since $8.6=\log\left(\frac{I}{I_o}\right)$, we get $10^{8.6}=\frac{I}{I_o}$ and consequently $I=10^{8.6}\cdot I_o$.

101. $t=\frac{1}{r}\ln(P/P_o)=\frac{1}{r}\ln(P)-\frac{1}{r}\ln(P_o)$

102. $pH=\log((H^+)^{-1})=-\log(H^+)$

103.

(a) p decreases as n increases

(b) Solving for n, one can take the logarithm of both sides and to note that $y = \log(x)$ is an increasing function.

$$\begin{aligned} \left(\frac{7,059,051}{7,059,052}\right)^n &> \frac{1}{2} \\ \log\left(\left(\frac{7,059,051}{7,059,052}\right)^n\right) &> \log\left(\frac{1}{2}\right) \\ n\log\left(\frac{7,059,051}{7,059,052}\right) &> \log\left(\frac{1}{2}\right) \end{aligned}$$

$$\begin{aligned} n &< \frac{\log(1/2)}{\log(7,059,051/7,059,052)} \\ n &< 4,892,962 \end{aligned}$$

If at most 4,892,961 tickets are purchsed, then the probability of a rollover is greater than 50%.

104. Solving for n, one can take the logarithm of both sides and to note that $y = \log(x)$ is an increasing function.

$$\begin{aligned} 1.3(0.75)^n &< 0.02(1.3) \\ (0.75)^n &< 0.02 \\ \log((0.75)^n) &< \log(0.02) \\ n\log(0.75) &< \log(0.02) \\ n &> \frac{\log(0.02)}{\log(0.75)} \\ n &> 13.598 \end{aligned}$$

Hence, the money will have no impact after 13.598 respendings or after $13.598(4) \approx$ 54 days.

105. We note that $MR(x) = R(x+1) - R(x) = 500 \cdot \log(x+2) - 500 \cdot \log(x+1) =$

$500 \cdot \log\left(\frac{x+2}{x+1}\right) = \log\left(\left(\frac{x+2}{x+1}\right)^{500}\right)$.

Hence, as $x \to \infty$, then $\frac{x+2}{x+1} \to 1$

and $MR(x) \to 0$.

106. We note that $m = \ln(e^{80}) - \ln((t+1)^7) = \ln(e^{80}) - 7 \cdot \ln(t+1) = 80 - 7 \cdot \ln(t+1)$.

The mean scores for $t = 0, 5, 12$ are
$80 - 7 \cdot \ln(1) = 80$,
$80 - 7 \cdot \ln(6) \approx 67.5$, and
$80 - 7 \cdot \ln(13) \approx 62$, respectively.

107.

a) Let x be the number of years since 1990 and let y be the number of computers per 1000 people. With the aid of a calculator, we find that an exponential regression curve is

$$y = 217.9(1.084)^x$$

b) $y = 217.9(e^{\ln 1.084})^x \approx 217.9e^{0.0807x}$

c) From part b), the continuous growth is 8.07%

d) Substitute $y = 1500$ into the equation in part a):

$$\begin{aligned} 1500 &= 217.9(1.084)^x \\ \frac{1500}{217.9} &= 1.084^x \\ \ln\left(\frac{1500}{217.9}\right) &= x\ln 1.084 \\ 23.9 &\approx x \end{aligned}$$

In 2014 ($= 1990 + 24$), there will be 1500 computers per 1000 people.

e) No, the data does not look exponential, rather it looks linear.

108.

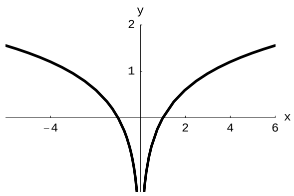
a) Let x be the number of years since 2000 and let y be the average cost in dollars of a barrel of oil. With the aid of a calculator, we find that an exponential regression curve is

$$y = 18.923(1.217)^x$$

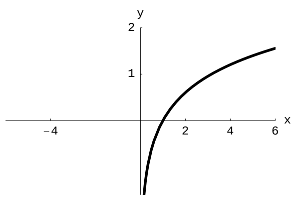
b) In 2015, i.e., $x = 15$, the cost of oil is

$$y = 18.923(1.217)^{15} \approx \$360/\text{barrel}$$

109. Note, $x^2 \geq 0$ for all real numbers x. Then the domain of $y = \log(x^2)$ is $(-\infty, 0) \cup (0, \infty)$ and the domain of $y = 2\log(x)$ is $(0, \infty)$. Thus, these two functions are not the same. The graph of $y = \log(x^2)$ is shown below



and the graph of $y = 2\log(x)$ is given next.



Note, the domains of $y = \log(x(x-1))$ and $y = \log(x) + \log(x-1)$ are not the same.

110. Let $a > 0$, $a \neq 1$. Using the definition of a logarithm, we have

$$\frac{M}{N} = \frac{a^{\log_a(M)}}{a^{\log_a(N)}} = a^{\log_a(M) - \log_a(N)}$$

and

$$\frac{M}{N} = a^{\log_a(M/N)}$$

Thus, $a^{\log_a(M/N)} = a^{\log_a(M) - \log_a(N)}$ and $\log_a(M) - \log_a(N) = \log_a(M/N)$.

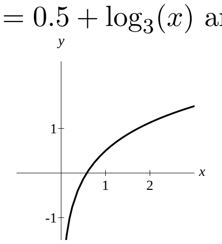
111. Let $a > 0$, $a \neq 1$. Applying the definition of a logarithm, we have

$$a^{\log_a(M^p)} = M^p = \left(a^{\log_a(M)}\right)^p = a^{p \cdot \log_a(M)}.$$

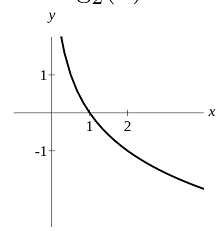
Then $a^{\log_a(M^p)} = a^{p \cdot \log_a(M)}$. Since $a^s = a^t$ implies $s = t$, we have $\log_a(M^p) = p \cdot \log_a(M)$.

112.

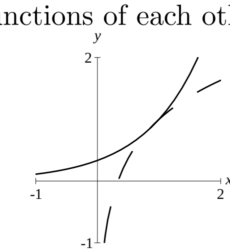
a) The graphs of $y_1 = \log_3(\sqrt{3}x)$ and $y_2 = 0.5 + \log_3(x)$ are identical.



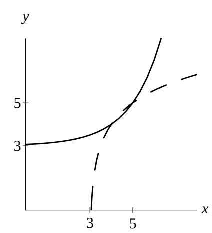
b) The graphs of $y_1 = \log_2(1/x)$ and $y_2 = -\log_2(x)$ are identical.



c) $y_1 = 3^{x-1}$ and $y_2 = \log_3(x) + 1$ are inverse functions of each other



d) $y_1 = 3 + 2^{x-4}$ and $y_2 = \log_2(x-3) + 4$ are inverse functions of each other



113. Domain $(1, \infty)$, range $(-\infty, \infty)$

114. Since $x - 3 = 2^8 = 256$, we find $x = 259$. The solution set is $\{259\}$.

115. $10,000e^{0.043(5.25)} = \$12,532.6$

116. In a non-leap year, the first 3 months has 90 days. Then

$$10,000\left(1 + \frac{0.043}{365}\right)^{5(365)+90} = \$12,530.61$$

117. Solve for x:

$$\begin{aligned} \frac{3}{8}x - \frac{15}{8} &= \frac{3}{2}x + \frac{5}{6} \\ -\frac{15}{8} - \frac{5}{6} &= \frac{3}{2}x - \frac{3}{8}x \\ -\frac{65}{24} &= \frac{9}{8}x \end{aligned}$$

$$-\frac{65}{24} \cdot \frac{8}{9} = x$$
$$-\frac{65}{27} = x$$

The solution set is $\{-\frac{65}{27}\}$.

118. $-(4^{1/2})^3 = -(2)^3 = -8$ and $(4^{1/2})^{-3} = (2)^{-3} = \frac{1}{8}$

Thinking Outside the Box XLII

a) $\frac{6}{23} = \frac{1}{4} + \frac{1}{92}$

b) $\frac{14}{15} = \frac{1}{2} + \frac{1}{3} + \frac{1}{10}$

c) $\frac{7}{11} = \frac{1}{2} + \frac{1}{11} + \frac{1}{22}$

4.3 Pop Quiz

1. $\log(9 \cdot 3) = \log(27)$

2. $\log\left(\frac{9}{3}\right) = \log(3)$

3. $\ln(x^3) + \ln y = \ln(x^3 y)$

4. $\ln(5) + \ln x$

5. $\ln\left((3^2 \cdot 2)^{1/2}\right) = \ln\left(3 \cdot 2^{1/2}\right) = \ln 3 + \frac{1}{2}\ln 2$

6. $\log_3(11) = \frac{\ln 11}{\ln 3} \approx 2.1827$

7. Since $x^3 = 22.5$, we find $x = \sqrt[3]{22.5} \approx 2.8231$.

4.3 Linking Concepts

a) Since $n = 1$ and $p_1 = 1$, we get $d = -\log_2(1) = 0$.

b)

$$\begin{aligned} d &= -[p_1 \cdot \log_2(p_1) + p_2 \cdot \log_2(p_2) + p_3 \cdot \log_2(p_3)] \\ &= -[\log_2(p_1^{p_1}) + \log_2(p_2^{p_2}) + \log_2(p_3^{p_3})] \\ &= -[\log_2(p_1^{p_1} p_2^{p_2} p_3^{p_3})] \\ &= \log_2\left(\frac{1}{p_1^{p_1} p_2^{p_2} p_3^{p_3}}\right) \\ d &= -\log_2\left(p_1^{p_1} p_2^{p_2} p_3^{p_3}\right) \end{aligned}$$

c)

$$\begin{aligned} d &= -[0.2 \cdot \log_2(0.2) + 0.3 \cdot \log_2(0.3) + \\ &\quad 0.5 \cdot \log_2(0.5)] \\ &\approx 1.485 \end{aligned}$$

d) The diversity index (where there are 100 terms inside the bracket) is

$$\begin{aligned} d &= -\left[\frac{1}{100}\log_2\left(\frac{1}{100}\right) + ... + \frac{1}{100}\log_2\left(\frac{1}{100}\right)\right] \\ &= -\log_2\left(\frac{1}{100}\right) \\ &= \log_2(100) \\ &= \frac{\log(100)}{\log(2)} \\ d &\approx 6.644 \end{aligned}$$

e) The diversity index (where there are 99 terms inside the bracket) is given by

$$d = -.99\log_2(.99) - \left[\frac{.01}{99}\log_2\left(\frac{.01}{99}\right) + ... + \frac{.01}{99}\log_2\left(\frac{.01}{99}\right)\right] \approx 0.147$$

f) The answer to this question may vary depending on how the dogs are classified. Suppose the breeds of the dogs in the movie '101 Dalmatians' are the ones listed below.

breed	number
Dalmatians	101
Afghan	1
Bulldog	2
Poodle	1
Cocker Spaniel	1
Mixed breed	10
German Sheperd	1
Great Dane	1
Schnauzer	1
Scottie	1
Lhaso Apso	5
Bloodhound	1
Giant Schnauzer	1
Collie	1
Labrador	1

The diversity index is

$$d = -\left[\frac{101}{129}\log_2\left(\frac{101}{129}\right) + \frac{2}{129}\log_2\left(\frac{2}{129}\right) + \frac{5}{129}\log_2\left(\frac{5}{129}\right) + \frac{10}{129}\log_2\left(\frac{10}{129}\right) + 11\cdot\frac{1}{129}\log_2\left(\frac{1}{129}\right)\right] \approx 1.4$$

g) Listed below are the travel times, in minutes, from home to work, of workers 16 years old and over. The data was taken from the 1990 U.S. Census.

travel time t	Number
$0 \le t < 30$	77,675,032
$30 \le t < 59$	27,244,564
$60 \le t < 89$	4,980,662
$90 \le t$	1,763,991

The diversity index is

$$d = -\left[\frac{77,675,032}{111,664,249}\log_2\left(\frac{77,675,032}{111,664,249}\right) + \frac{27,244,564}{111,664,249}\log_2\left(\frac{27,244,564}{111,664,249}\right) + \frac{4,980,662}{111,664,249}\log_2\left(\frac{4,980,662}{111,664,249}\right) + \frac{1,763,991}{111,664,249}\log_2\left(\frac{1,763,991}{111,664,249}\right)\right]$$

$d \approx 1.2$.

For Thought

1. True, since $(1.02)^x = 7$ is equivalent to $x = \log_{1.02}(7)$.

2. True, for $x(1-\ln(3)) = 8$ implies $x = \dfrac{8}{1-\ln(3)}$.

3. False, $\ln(1-\sqrt{6})$ is undefined.

4. True, by the definition of a logarithm.

5. False, the exact solution is $x = \log_3(17)$ which is not the same as 2.5789.

6. False, since $x = -2$ is not a solution of the first equation but is a solution of the second one.

7. True, since $4^x = 2^{2x} = 2^{x-1}$ is equivalent to $2x = x - 1$.

8. True, since we may take the ln of both sides of $1.09^x = 2.3$.

9. True, since $\dfrac{\ln(2)}{\ln(7)} = \dfrac{\log(2)}{\log(7)}$.

10. True, since $\log(e)\cdot\ln(10) = \ln\left(10^{\log(e)}\right) = \ln(e) = 1$.

4.4 Exercises

1. Since $x = 2^3$, we get $x = 8$.

2. Since $x = 3^0$, we get $x = 1$.

3. Since $10^2 = x + 20$, $x = 80$.

4. Since $10^1 = x - 6$, $x = 16$.

5. Since $10^1 = x^2 - 15$, we get $x^2 = 25$. The solutions are $x = \pm 5$.

6. Since $10^1 = x^2 - 5x + 16$, we get $x^2 - 5x + 6 = 0$. And, since $(x-3)(x-2) = 0$, we find $x = 2, 3$.

7. Note, $x^2 = 9$ and $x = \pm 3$. Since the base of a logarithm is positive, the solution is $x = 3$.

8. Note, $x^4 = 16$ and $x = \pm 2$. Since the base of a logarithm is positive, the solution is $x = 2$.

9. Note, $x^{-2} = 4$ or $\dfrac{1}{4} = x^2$. Since the base of a logarithm is positive, the solution is $x = \dfrac{1}{2}$.

10. Note, $x^{-1/2} = 9$ or $\dfrac{1}{9} = x^{1/2}$. The solution is $x = \dfrac{1}{81}$.

11. Since $x^3 = 10$, the solution is $x = \sqrt[3]{10}$.

12. Since $x^2 = 5$ and $x > 0$, the solution is $x = \sqrt{5}$.

13. Since $x = \left(8^{-1/3}\right)^2$, the solution is $x = \dfrac{1}{4}$.

14. Since $x = \left(4^{-1/2}\right)^5$, the solution is $x = \dfrac{1}{32}$.

15.

$$\begin{aligned} \log_2(x^2-4) &= 5 \\ x^2 - 4 &= 2^5 \\ x^2 &= 36 \\ x &= \pm 6 \end{aligned}$$

Checking $x = -6$, one gets $\log_2(-6+2)$ which is undefined. The solution is $x = 6$.

16.

$$\begin{aligned} \log_6(w^2 - 3w + 2) &= 1 \\ w^2 - 3w + 2 &= 6^1 \\ w^2 - 3w - 4 &= 0 \\ (w-4)(w+1) &= 0 \end{aligned}$$

Note, if $w = -1$, then $\log_6(w-1)$ is undefined. Thus, $w = 4$.

17.

$$\begin{aligned} \log_6\left(\frac{x^2 - x - 6}{14}\right) &= 0 \\ \frac{x^2 - x - 6}{14} &= 6^0 = 1 \\ x^2 - x - 6 &= 14 \\ x^2 - x - 20 &= 0 \\ (x-5)(x+4) &= 0 \end{aligned}$$

Note, if $x = -4$, then $\log \frac{x-3}{2}$ is undefined. Thus, $x = 5$.

18.

$$\begin{aligned} \log_2\left(\frac{a^2 + a - 6}{50}\right) &= 0 \\ \frac{a^2 + a - 6}{50} &= 2^0 = 1 \\ a^2 + a - 6 &= 50 \\ a^2 + a - 56 &= 0 \\ (a+8)(a-7) &= 0 \end{aligned}$$

Note, if $a = -8$, then $\log_2\left(\frac{a-2}{5}\right)$ is undefined. Thus, $a = 7$.

19.

$$\begin{aligned} \log\left(\frac{x+1}{x}\right) &= 3 \\ \frac{x+1}{x} &= 10^3 \\ x + 1 &= 1000x \\ 1 &= 999x \\ x &= \frac{1}{999} \end{aligned}$$

20.

$$\begin{aligned} \log_5\left(\frac{x}{x-2}\right) &= 3 \\ \frac{x}{x-2} &= 5^3 \\ x &= 125x - 250 \\ 250 &= 124x \\ x &= \frac{125}{62} \end{aligned}$$

21.

$$\begin{aligned} \log_4\left(\frac{x}{x+2}\right) &= 2 \\ \frac{x}{x+2} &= 16 \\ x &= 16x + 32 \\ -\frac{32}{15} &= x \end{aligned}$$

Note, if $x = -32/15$, then $\log_4 x$ is undefined. The solution set is $\emptyset$.

22.

$$\begin{aligned} \log_3\left(\frac{x-6}{2x}\right) &= 4 \\ \frac{x-6}{2x} &= 81 \\ x - 6 &= 162x \\ -\frac{6}{161} &= x \end{aligned}$$

Note, if $x = -6/161$, then $\log_3(x-6)$ is undefined. The solution set is $\emptyset$.

23.

$$\begin{aligned} \log(5) + \log(x) &= 2 \\ \log(5x) &= 2 \\ 5x &= 10^2 \\ x &= 20 \end{aligned}$$

24.

$$\begin{aligned}\log(4)-\log(x-1) &= 1\\ \log\left(\frac{4}{x-1}\right) &= 1\\ \frac{4}{x-1} &= 10^1\\ 4 &= 10x-10\\ 14 &= 10x\\ x &= 1.4\end{aligned}$$

25. Since $\ln(x(x+2)) = \ln(8)$ and $y = \ln(x)$ is one-to-one, we get

$$\begin{aligned}x^2+2x &= 8\\ x^2+2x-8 &= 0\\ (x+4)(x-2) &= 0\\ x &= -4, 2\end{aligned}$$

Since $\ln(-4)$ is undefined, the solution is $x = 2$.

26.

$$\begin{aligned}\log_3(x)+\log_3(x-2) &= \log_3(2)\\ \log_3(x(x-2)) &= \log_3(2)\\ x^2-2x &= 2\\ x^2-2x+1 &= 2+1\\ (x-1)^2 &= 3\\ x &= 1\pm\sqrt{3}.\end{aligned}$$

But $\log_3(1-\sqrt{3})$ is undefined, so $x = 1+\sqrt{3}$.

27. Since $\log(4x) = \log\left(\frac{5}{x}\right)$ and $y = \log(x)$ is one-to-one, we obtain

$$\begin{aligned}4x &= \frac{5}{x}\\ 4x^2 &= 5\\ x^2 &= \frac{5}{4}\\ x &= \pm\frac{\sqrt{5}}{2}.\end{aligned}$$

But $\log\left(-\frac{\sqrt{5}}{2}\right)$ is undefined, so $x = \frac{\sqrt{5}}{2}$.

28. Since $\ln\left(\frac{x}{x+1}\right) = \ln\left(\frac{x+3}{x+5}\right)$ and $y = \ln(x)$ is one-to-one, we have

$$\begin{aligned}\frac{x}{x+1} &= \frac{x+3}{x+5}\\ x(x+5) &= (x+3)(x+1)\\ x^2+5x &= x^2+4x+3\\ x &= 3.\end{aligned}$$

29. Since $\log_2\left(\frac{x}{3x-1}\right) = 0$, we get

$$\begin{aligned}\frac{x}{3x-1} &= 1\\ x &= 3x-1\\ 1 &= 2x\\ x &= \frac{1}{2}.\end{aligned}$$

30. Since $\log_3(x)+\log_3(1/x) = \log_3\left(x\cdot\frac{1}{x}\right) = \log_3(1) = 0$, the solution set is $(0,\infty)$.

31.

$$\begin{aligned}x\cdot\ln(3)+x\cdot\ln(2) &= 2\\ x(\ln(3)+\ln(2)) &= 2\\ &= \frac{2}{\ln(3)+\ln(2)}\\ x &= \frac{2}{\ln(6)}\end{aligned}$$

32. Note, $x(\log(5)+\log(7)) = x\cdot\log(35) = \log(9)$. Thus, $x = \frac{\log(9)}{\log(35)}$.

33. Since $x-1 = \log_2(7)$, $x = \frac{\ln(7)}{\ln(2)}+1 \approx 3.8074$.

34. Since $3x = \log_5(29)$, $x = \frac{1}{3}\cdot\frac{\ln(29)}{\ln(5)} \approx 0.6974$.

35. Since $4x = \log_{1.09}(3.4)$, we find

$$x = \frac{1}{4}\cdot\frac{\ln(3.4)}{\ln(1.09)} \approx 3.5502.$$

36. Since $2x = \log_{1.04}(2.5)$, we obtain

$$x = \frac{1}{2} \cdot \frac{\ln(2.5)}{\ln(1.04)} \approx 11.6812.$$

37. Since $-x = \log_3(30)$, we obtain

$$x = -\frac{\ln(30)}{\ln(3)} \approx -3.0959.$$

38. Since $-x + 3 = \log(102)$, we find

$$x = 3 - \log(102) \approx 0.9914.$$

39. Note, $-3x^2 = \ln(9)$. There is no solution since the left-hand side is non-negative and the right-hand side is positive.

40. Since $-2x = \log(25)$, we obtain

$$x = -\frac{\log(25)}{2} \approx -0.6990.$$

41.

$$\begin{aligned} \ln(6^x) &= \ln(3^{x+1}) \\ x \cdot \ln(6) &= (x+1) \cdot \ln(3) \\ x \cdot \ln(6) &= x \cdot \ln(3) + \ln(3) \\ x(\ln(6) - \ln(3)) &= \ln(3) \\ x &= \frac{\ln(3)}{\ln(6) - \ln(3)} \\ x &\approx 1.5850 \end{aligned}$$

42.

$$\begin{aligned} \ln(2^x) &= \ln(7^{x-1}) \\ x \cdot \ln(2) &= (x-1) \cdot \ln(7) \\ x \cdot \ln(2) &= x \cdot \ln(7) - \ln(7) \\ x(\ln(2) - \ln(7)) &= -\ln(7) \\ x &= \frac{-\ln(7)}{\ln(2) - \ln(7)} \\ x &\approx 1.5533 \end{aligned}$$

43.

$$\begin{aligned} \ln(e^{x+1}) &= \ln(10^x) \\ (x+1) \cdot \ln(e) &= x \cdot \ln(10) \\ x + 1 &= x \cdot \ln(10) \\ 1 &= x(\ln(10) - 1) \\ x &= \frac{1}{\ln(10) - 1} \\ x &\approx 0.7677 \end{aligned}$$

44.

$$\begin{aligned} \ln(e^x) &= \ln(2^{x+1}) \\ x \cdot \ln(e) &= (x+1) \cdot \ln(2) \\ x &= x \cdot \ln(2) + \ln(2) \\ x(1 - \ln(2)) &= \ln(2) \\ x &= \frac{\ln(2)}{1 - \ln(2)} \\ x &\approx 2.2589 \end{aligned}$$

45.

$$\begin{aligned} 2^{x-1} &= (2^2)^{3x} \\ 2^{x-1} &= 2^{6x} \\ x - 1 &= 6x \\ -1 &= 5x \\ x &= -0.2 \end{aligned}$$

46.

$$\begin{aligned} 3^{3x-4} &= (3^2)^x \\ 3^{3x-4} &= 3^{2x} \\ 3x - 4 &= 2x \\ x &= 4 \end{aligned}$$

47.

$$\begin{aligned} \ln(6^{x+1}) &= \ln(12^x) \\ (x+1) \cdot \ln(6) &= x \cdot \ln(12) \\ x \cdot \ln(6) + \ln(6) &= x \cdot \ln(12) \\ \ln(6) &= x(\ln(12) - \ln(6)) \\ x &= \frac{\ln(6)}{\ln(12) - \ln(6)} \\ x &\approx 2.5850 \end{aligned}$$

48.

$$\begin{aligned} 2^{2x+1} &= (2^2)^{x^2+x} \\ 2^{2x+1} &= 2^{2x^2+2x} \\ 2x+1 &= 2x^2+2x \\ 1 &= 2x^2 \\ x &= \pm\frac{1}{\sqrt{2}} \\ x &\approx \pm 0.7071 \end{aligned}$$

49. Since $3 = e^{-\ln(w)} = e^{\ln(1/w)} = 1/w$, we have $\frac{1}{w} = 3$ and $w = \frac{1}{3}$.

50. We observe that

$$10^{2\cdot\log(y)} = 10^{\log(y^2)} = y^2.$$

Since $y^2 = 4$, we get $y = \pm 2$.
Since $\log(-2)$ is undefined, we find $y = 2$.

51.

$$\begin{aligned} (\log(z))^2 &= 2\cdot\log(z) \\ (\log(z))^2 - 2\cdot\log(z) &= 0 \\ \log(z)\cdot(\log(z)-2) &= 0 \\ \log(z)=0 \quad &\text{or} \quad \log(z)=2 \\ z=10^0 \quad &\text{or} \quad z=10^2 \\ z=1 \quad &\text{or} \quad z=100 \end{aligned}$$

The solutions are $z = 1, 100$.

52. Since $\ln\left(\frac{e^x}{e^6}\right) = \ln\left(e^{x-6}\right) = \ln(e^2)$, we get $x - 6 = 2$. Thus, $x = 8$.

53. Divide the equation by $4(1.03)^x$.

$$\begin{aligned} \left(\frac{1.02}{1.03}\right)^x &= \frac{3}{4} \\ \ln\left(\left(\frac{1.02}{1.03}\right)^x\right) &= \ln\left(\frac{3}{4}\right) \\ x\cdot\ln\left(\frac{1.02}{1.03}\right) &= \ln\left(\frac{3}{4}\right) \\ x &= \frac{\ln\left(\frac{3}{4}\right)}{\ln\left(\frac{1.02}{1.03}\right)} \\ x &\approx 29.4872 \end{aligned}$$

54. Take the natural logarithm of both sides.

$$\begin{aligned} \ln(500) + \ln((1.06)^x) &= \ln(400) + \ln((1.02)^{4x}) \\ \ln(500) + x\cdot\ln(1.06) &= \ln(400) + 4x\cdot\ln(1.02) \\ \ln(500) - \ln(400) &= x(4\cdot\ln(1.02) - \ln(1.06)) \\ x &= \frac{\ln(500)-\ln(400)}{4\cdot\ln(1.02)-\ln(1.06)} \\ x &\approx 10.6555 \end{aligned}$$

55. Note that $e^{\ln((x^2)^3)-\ln(x^2)} = e^{\ln(x^6)-\ln(x^2)} = e^{\ln(x^6/x^2)} = e^{\ln(x^4)} = x^4$.
Thus, $x^4 = 16$ and $x = \pm 2$.
But $\ln(-2)$ is undefined, so $x = 2$.

56. Square both sides of the equation.

$$\begin{aligned} \log(x) - 3 = (\log(x))^2 &- 6\log(x) + 9 \\ (\log(x))^2 - 7\log(x) + 12 &= 0 \\ (\log(x)-4)(\log(x)-3) &= 0 \\ \log(x)=4 \quad &\text{or} \quad \log(x)=3 \\ x=10^4 \quad &\text{or} \quad x=10^3 \\ x=10,000 \quad &\text{or} \quad x=1000 \end{aligned}$$

The solutions are $x = 1000,\ 10,000$.

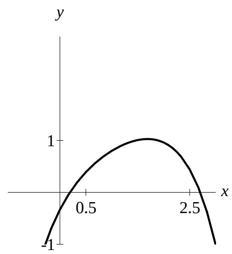
57. Since $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$, we find

$$\begin{aligned} \left(\frac{1}{2}\right)^{2x-1} &= \left(\frac{1}{2}\right)^{6x+4} \\ 2x-1 &= 6x+4 \\ -5 &= 4x \\ x &= -\frac{5}{4}. \end{aligned}$$

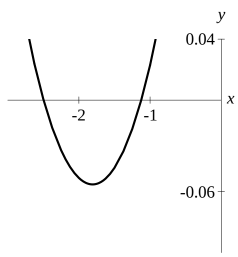
58. Since $\left(\frac{2}{3}\right)^{-2} = \frac{9}{4}$, we obtain

$$\begin{aligned} \left(\frac{2}{3}\right)^{x+1} &= \left(\frac{2}{3}\right)^{-2x-4} \\ x+1 &= -2x-4 \\ 3x &= -5 \\ x &= -\frac{5}{3}. \end{aligned}$$

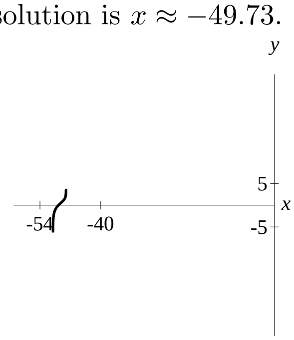
59. By approximating the x-intercepts of the graph $y = 2^x - 3^{x-1} - 5^{-x}$, we find that the solutions are $x \approx 0.194, 2.70$.



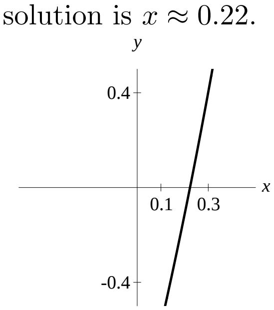
60. By approximating the x-intercepts of the graph $y = 2^x - \log(x + 4)$, we find that the solutions are $x \approx -2.50, -1.12$.



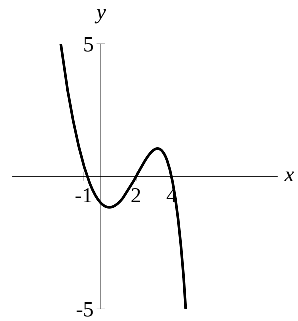
61. By approximating the x-intercept of the graph $y = \ln(x + 51) - \log(-48 - x)$, we obtain that the solution is $x \approx -49.73$.



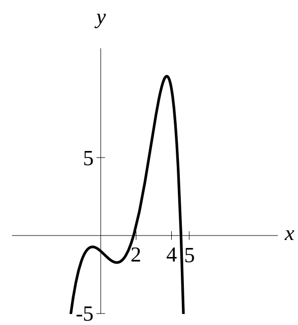
62. By approximating the x-intercept of the graph $y = 2^x - 5 + 3^{x+1}$, we obtain that the solution is $x \approx 0.22$.



63. By approximating the x-intercepts of the graph $y = x^2 - 2^x$, we find that the solutions are $x \approx -0.767, 2, 4$.



64. By approximating the x-intercepts of the graph $y = x^3 - e^x$, we get that the solutions are $x \approx 4.536, 1.857$.



65. Solving for the rate of decay r, we find

$$\begin{aligned} A_0/2 &= A_0 e^{10{,}000r} \\ 1/2 &= e^{10{,}000r} \\ \ln(1/2) &= 10{,}000r \\ \frac{\ln(1/2)}{10{,}000} &= r \end{aligned}$$

Approximately, $r \approx -6.93 \times 10^{-5}$.

66. If the rate is r, then

$$\begin{aligned} A_0/2 &= A_0 e^{12r} \\ 1/2 &= e^{12r} \\ \ln(1/2) &= 12r \\ \frac{\ln(1/2)}{12} &= r. \end{aligned}$$

That is, $r \approx -0.0578$.

67. Using $A = A_o e^{rt}$ with $A_o = 1$ and the half-life, we obtain $\frac{1}{2} = e^{5730r}$. Thus, $5730r = \ln\left(\frac{1}{2}\right)$ and $r \approx -0.000120968$. When $A = 0.1$, we get

$$\begin{aligned} 0.1 &= e^{-0.000120968t} \\ \ln(0.1) &= -0.000120968t \\ t = \frac{\ln(0.1)}{-0.000120968} &\approx 19,035 \text{ years} \end{aligned}$$

68. From Number 67, $r \approx -0.000120968$. When $A = 0.85$, we get

$$\begin{aligned} 0.85 &= e^{-0.000120968t} \\ \ln(0.85) &= -0.000120968t \\ t = \frac{\ln(0.85)}{-0.000120968} &\approx 1343 \text{ years.} \end{aligned}$$

69. From Number 67, $r \approx -0.000120968$. When $A = 10$ and $A_o = 12$, we obtain

$$\begin{aligned} 10 &= 12 \cdot e^{-0.000120968t} \\ \ln(10/12) &= -0.000120968t \\ t = \frac{\ln(5/6)}{-0.000120968} &\approx 1507 \text{ years.} \end{aligned}$$

70. From Number 67, $r \approx -0.000120968$. When $A_0 = 2.4$ and $A = 1.3$, we find

$$\begin{aligned} 1.3 &= 2.4 \cdot e^{-0.000120968t} \\ \ln\left(\frac{1.3}{2.4}\right) &= -0.000120968t \\ t &\approx 5068 \text{ years.} \end{aligned}$$

71. Let $A = A_o e^{rt}$ where $A_o = 25$, $A = 20$, and $t = 8000$. Then

$$\begin{aligned} 20 &= 25 \cdot e^{8000r} \\ \ln(0.8) &= 8000r \\ r &\approx -0.000027893. \end{aligned}$$

To find the half-life, let $A = 12.5$. Thus, we have

$$\begin{aligned} 12.5 &= 25 \cdot e^{-0.000027893t} \\ \ln(0.5) &= -0.000027893t \\ t &\approx 24,850 \text{ years.} \end{aligned}$$

72. Let $A = A_o e^{rt}$ with $A = 0.8$, $A_o = 1$, and $t = 2.5 \times 10^8$.

$$\begin{aligned} 0.8 &= e^{(2.5\times 10^8)r} \\ \ln(0.8) &= (2.5 \times 10^8)r \\ r &\approx -8.9257 \times 10^{-10} \end{aligned}$$

The percentage that remains when $t = 6 \times 10^8$ is $A = e^{(-8.9257\times 10^{-10})\cdot(6\times 10^8)} \approx 0.59$ or 59%. For the half-life, solve for t when $A = 0.5$.

$$\begin{aligned} 0.5 &= e^{(-8.9257\times 10^{-10})\cdot t} \\ \ln(0.5) &= (-8.9257 \times 10^{-10}) \cdot t \\ t &\approx 777 \text{ million years} \end{aligned}$$

The half-life is 777 million years.

73. $\frac{2.5(0.5)^{24/14}}{2.5} \times 100 \approx 30.5\%$, the percentage of the last dosage that remains before the next dosage is taken

74.

a) $L = \frac{2.5}{1 - 0.5^{(24/14)}} \approx 3.6$ mg.

b) The number of hours n is given by

$$\begin{aligned} 5.58 &= \frac{2.5}{1 - .5^{n/14}} \\ 1 - .5^{n/14} &= \frac{2.5}{5.58} \\ 1 - \frac{2.5}{5.58} &= .5^{n/14} \\ \ln\left(1 - \frac{2.5}{5.58}\right) &= \frac{n}{14}\ln(.5) \\ \frac{14\ln\left(1 - \frac{2.5}{5.58}\right)}{\ln(.5)} &= n \\ 12.003 &\approx n. \end{aligned}$$

The dosage must be taken every 12 hours.

c) 5 mg every 24 hours builds to $\frac{5}{1 - 0.5^{(24/14)}} \approx 7.19$ mg, while 2.5 mg every 12 hours builds to 5.58 mg.

75. Since the half-life is $t = 5730$ years and $A_o = 1$ and $A = 0.5$, we get

$$\begin{aligned} 0.5 &= e^{(5730)\cdot r} \\ \ln(0.5) &= 5730 \cdot r \\ r &\approx -0.000121. \end{aligned}$$

If 79.3% of the carbon is still present, then

$$\begin{aligned} 0.793 &= e^{(-0.000121)\cdot t} \\ \ln(0.793) &= -0.000121 \cdot t \\ t &\approx 1917. \end{aligned}$$

The scrolls were made in the year $1951 - 1917 = 34$ AD.

76. Since the half-life is $t = 1.31 \times 10^9$ years and $A_o = 1$ and $A = 0.5$, we have

$$\begin{aligned} 0.5 &= e^{(1.31\times 10^9)\cdot r} \\ \ln(0.5) &= 1.31 \times 10^9 \cdot r \\ r &\approx -5.2912 \times 10^{-10}. \end{aligned}$$

If 91.2% of the potassium is still present, then

$$\begin{aligned} 0.912 &= e^{(-5.2912\times 10^{-10})\cdot t} \\ \ln(0.912) &= -5.2912 \times 10^{-10} \cdot t \\ t &\approx 174 \text{ million years.} \end{aligned}$$

77.

a) Solve for r:

$$\begin{aligned} 11,981 &= 21,075e^{3r} \\ \ln\left(\frac{11,981}{21,075}\right) &= 3r \\ -0.188255 &= r \\ -18.8\% &\approx r \end{aligned}$$

b) If $t = 5$ years, then

$$P = 21,075e^{5(-0.188)} = \$8,200$$

78.

a) Solve for r:

$$\begin{aligned} 49,900 &= 107,500e^{4r} \\ \ln\left(\frac{49,900}{107,500}\right) &= 4r \\ -0.191867 &= r \\ -19.2\% &\approx r \end{aligned}$$

b) If $t = 7$ years, then

$$P = 107,500e^{7(-0.192)} = \$28,000$$

79. Let $y = 0.1e^{kt}$, and $y = 4.8$ when $t = 2$. Then

$$\begin{aligned} 4.8 &= 0.1e^{2k} \\ e^k &= \sqrt{48} \end{aligned}$$

Thus, $y = 0.1(\sqrt{48})^t$. In 2014 or $t = 5$, we find $y = 0.1(48^{2.5}) \approx 1596$ million bloggers.

80. Let $y = 1.7e^{kt}$, and $y = 28.8$ when $t = 13$. Then

$$\begin{aligned} 28.8 &= 1.7e^{13k} \\ e^k &= \sqrt[13]{\frac{28.8}{1.7}} \end{aligned}$$

Thus, $y = 1.7\left(\sqrt[13]{\frac{28.8}{1.7}}\right)^t$.

If $y = 100$, we find

$$\begin{aligned} 100 &= 1.7\left(\sqrt[13]{\frac{28.8}{1.7}}\right)^t \\ \frac{100}{1.7} &= \left(\sqrt[13]{\frac{28.8}{1.7}}\right)^t \\ \frac{\ln(100/1.7)}{\ln(\sqrt[13]{28.8/1.7})} &= t \\ t &\approx 19 \end{aligned}$$

In year 2016 $(= 1997 + 19)$, the percentage will reach 100%.

81. The initial difference in temperature is $325-35=290$, and after $t=3$ hours the difference is $325-140=185$. Then

$$\begin{aligned} 185 &= 290\cdot e^{3k} \\ \ln\left(\frac{185}{290}\right) &= 3k \\ k &\approx -0.1498417. \end{aligned}$$

The difference in temperature when the roast is well-done is $325-170=155$. Thus,

$$\begin{aligned} 155 &= 290\cdot e^{(-0.1498417)\cdot t} \\ \ln\left(\frac{155}{290}\right) &= (-0.1498417)\cdot t \\ t &\approx 4.18 \text{ hr} \\ t &\approx 4 \text{ hr and } 11 \text{ min.} \end{aligned}$$

James must wait 1 hour and 11 minutes longer.

If the oven temperature is set at 170^o, then the initial and final differences are 135 and 0, respectively. Since $0=135\cdot e^{(-0.1498417)\cdot t}$ has no solution, James has to wait forever.

82. The initial difference in temperature is $74-40=34$, and $t=2$ hours after the difference in temperature is $74-58=16$.

$$\begin{aligned} 16 &= 34\cdot e^{2k} \\ \ln\left(\frac{16}{34}\right) &= 2k \\ k &\approx -0.3768859 \end{aligned}$$

For best results, the difference in temperature must be $74-68=6$.

$$\begin{aligned} 6 &= 34\cdot e^{(-0.3768859)\cdot t} \\ \ln\left(\frac{6}{34}\right) &= (-0.3768859)\cdot t \\ t &\approx 4.6 \text{ hr} \\ t &\approx 4 \text{ hr and } 36 \text{ min} \end{aligned}$$

Marlene must wait 2 hr, 36 min longer.

83. At 7:00 a.m., the difference in temperature is $80-40=40$, and $t=1$ hour later the difference in temperature is $72-40=32$. Then

$$\begin{aligned} 32 &= 40\cdot e^{1\cdot k} \\ \ln\left(\frac{32}{40}\right) &= k \\ k &\approx -0.2231436. \end{aligned}$$

Let n be the number of hours before 7:00 a.m. when death occured. At the time of death, the difference in temperature is $98-40=58$. Then

$$\begin{aligned} 40 &= 58\cdot e^{-0.2231436\cdot n} \\ \ln\left(\frac{40}{58}\right) &= -0.2231436\cdot n \\ n &\approx 1.665 \text{ hr} \\ n &\approx 1 \text{ hr and } 40 \text{ min.} \end{aligned}$$

The death occured at 5:20 a.m.

84. Since the initial difference in temperature is $600-65=535$, and after $t=1$ min the difference in temperature is $200-65=135$, we have

$$\begin{aligned} 135 &= 535\cdot e^{1\cdot k} \\ \ln\left(\frac{135}{535}\right) &= k \\ k &\approx -1.376992. \end{aligned}$$

When the difference is $100-65=35$, one has

$$\begin{aligned} 35 &= 535\cdot e^{(-1.376992)\cdot t} \\ \ln\left(\frac{35}{535}\right) &= (-1.376992)\cdot t \\ t &\approx 1.98 \text{ min} \\ t &\approx 1 \text{ min } 59 \text{ sec.} \end{aligned}$$

The blacksmith must wait an additional 59 sec.

85. Since $R=P\dfrac{i}{1-(1+i)^{-nt}}$, we obtain

$$\begin{aligned} 1-(1+i)^{-nt} &= Pi/R \\ 1-Pi/R &= (1+i)^{-nt} \\ \ln(1-Pi/R) &= -nt\ln(1+i) \\ \frac{-\ln(1-Pi/R)}{n\ln(1+i)} &= t. \end{aligned}$$

Let $i=0.09/12=0.0075$, $P=100,000$, and $R=1250$. Then

$$t=\frac{-\ln(1-Pi/R)}{n\ln(1+i)}\approx 10.219.$$

It will take 10 yr, 3 mo to pay off the loan.

86. From Exercise 85, we obtained

$$t = \frac{-\ln(1 - Pi/R)}{n\ln(1+i)}.$$

Let $i = 0.0875/12$, $P = 48,265$, and $R = 700$. Then

$$t = \frac{-\ln(1 - Pi/R)}{n\ln(1+i)} \approx 8.01403.$$

It will take 8 yr, 0 mo to pay off the loan.

87. The future values of the \$1000 and \$1100 investments are equal. Then

$$\begin{aligned} 1,000 \cdot e^{0.06t} &= 1100\left(1 + \frac{0.06}{365}\right)^{365t} \\ e^{0.06t} &= 1.1\left(1 + \frac{0.06}{365}\right)^{365t} \\ .06t &= \ln(1.1) + 365t\ln\left(1 + \frac{0.06}{365}\right) \end{aligned}$$

$$.06t - 365t\ln\left(1 + \frac{0.06}{365}\right) = \ln(1.1)$$

$$\begin{aligned} t &= \frac{\ln(1.1)}{.06 - 365\ln\left(1 + \frac{0.06}{365}\right)} \\ t &\approx 19,328.84173 \text{ years} \end{aligned}$$

They will be equal after 19,328 yr, 307 days.

88. The value of the luxury car is equal to the cost of the compact car.

$$\begin{aligned} 35,000 \cdot (0.92)^t &= 10,000(1 + 0.05)^t \\ 3.5 &= \left(\frac{1.05}{0.92}\right)^t \\ 3.5 &\approx (1.1413043)^t \\ t &\approx \frac{\ln(3.5)}{\ln(1.1413043)} \\ t &\approx 9.5 \text{ years.} \end{aligned}$$

He will make a trade after 9.5 yr.

89. **a)** Let $t = 0$. The present number of rabbits is

$$P = 12,300 + 1000 \cdot \ln(1) = 12,300 + 0 = 12,300.$$

b) In about 15 years.

c) The number of years before there will be 15,000 rabbits is given by

$$\begin{aligned} 12,300 + 1000 \cdot \ln(t+1) &= 15,000 \\ 1000 \cdot \ln(t+1) &= 2,700 \\ \ln(t+1) &= 2.7 \\ t+1 &= e^{2.7} \\ t &\approx 13.9 \text{ yr.} \end{aligned}$$

90. An expression for 5% of the rabbit population is

$$0.05\,[12,300 + 1000 \cdot \ln(t+1)]$$

or

$$615 + 50 \cdot \ln(t+1).$$

Since this is the same as the fox population, we have

$$\begin{aligned} 400 + 50 \cdot \ln(90t+1) &= 615 + 50 \cdot \ln(t+1) \\ 50\,[\ln(90t+1) - \ln(t+1)] &= 215 \\ \ln\left(\frac{90t+1}{t+1}\right) &= 4.3 \\ \frac{90t+1}{t+1} &= e^{4.3} \\ 90t+1 &= e^{4.3} \cdot t + e^{4.3} \\ t(90 - e^{4.3}) &= e^{4.3} - 1 \\ t &= \frac{e^{4.3} - 1}{90 - e^{4.3}} \\ t &\approx 4.46 \text{ years} \end{aligned}$$

The system is out of balance after 4.46 yr.

91. **a)** Let $n = 2500$ and $A = 400$. Since $n = k\log(A)$, $2500 = k\log(400)$. Then $k = \frac{2500}{\log(400)}$. When $A = 200$, the number of species left is

$$n = \frac{2500}{\log(400)}\log(200) \approx 2211 \text{ species.}$$

b) Let $n = 3500$ and $A = 1200$.
Since $n = k\log(A)$, $3500 = k\log(1200)$.
Then $k = \dfrac{3500}{\log(1200)}$. When $n = 1000$, the remaining forest area is given by

$$\begin{aligned} 1000 &= \frac{3500}{\log(1200)}\log(A) \\ \frac{1000}{3500}\log(1200) &= \log(A) \\ \frac{2}{7}\log(1200) &= \log(A) \\ \log\left(1200^{2/7}\right) &= \log(A) \\ 1200^{2/7} &= A \end{aligned}$$

Thus, the percentage of forest that has been destroyed is $100 - \dfrac{A}{1200}(100) \approx 99\%$.

92. Let A' be the amount of rain forest in year 2000. Since $n = k\log(A)$ where A is the amount of rain forest in 1980 and n is the number of species in the same year, we have $n = k\log(A)$ or $k = \dfrac{n}{\log(A)}$. Thus,

$$\begin{aligned} \frac{n}{2} &= k\log(A') \\ \frac{n}{2} &= \frac{n}{\log(A)}\log(A') \\ \frac{\log(A)}{2} &= \log(A') \\ \sqrt{A} &= A'. \end{aligned}$$

The area in year 2000 is $\sqrt{A}$ where A is the area in 1980.

93. Since $m = 0$ and $M_v = 4.39$, the distance to α Centauri is given by

$$\begin{aligned} 4.39 - 5 + 5\cdot\log(d) &= 0 \\ 5\cdot\log(d) &= 0.61 \\ \log(d) &= 0.122 \\ d = 10^{0.122} &\approx 1.32 \text{ parsecs.} \end{aligned}$$

94. For the star Deneb, $d = 490$ and $m = 1.26$. Its absolute visual magnitude M_v is given by

$$\begin{aligned} M_v - 5 + 5\cdot\log(490) &= 1.26 \\ M_v &= 6.26 - 5\cdot\log(490) \\ M_v &\approx -7.19 \end{aligned}$$

Deneb is a very bright star and will appear brighter than α Centauri if they were both 490 parsecs away.

95.

a) Let $y = \log(P)$. A formula for P is

$$\begin{aligned} y - 4.5 &= \frac{4.5 - 0.3}{0 - 23}(x - 0) \\ y &= -\frac{21}{115}x + 4.5 \\ \log(P) &= -\frac{21}{115}x + 4.5 \\ P &= 10^{(-21x/115 + 4.5)} \\ P &\approx 10^{-0.1826x + 4.5} \end{aligned}$$

b) In 1991, $x = 9$ and

$$P \approx 10^{-0.1826(9)+4.5} = \$718.79.$$

c) Solving for x, we derive

$$\begin{aligned} 0.25 &= 10^{(-21x/115 + 4.5)} \\ \log(0.25) &= -\frac{21}{115}x + 4.5 \\ \frac{\log(0.25) - 4.5}{-21/115} &= x \\ x &\approx 28 \end{aligned}$$

In the year $2010 = (1982 + 28)$, according to this model a 1-gigabit hard drive will cost \$0.25.

96.

a) Let $y = \log(C)$. A formula for C is

$$\begin{aligned} y + 2.2 &= \frac{-2.2 - 2.2}{0 - 23}(x - 0) \\ y &= \frac{4.4}{23}x - 2.2 \\ \log(C) &= \frac{4.4}{23}x - 2.2 \\ C &= 10^{4.4x/23 - 2.2} \\ C &\approx 10^{0.1913x - 2.2} \end{aligned}$$

b) In 1994, $x = 12$ and

$$C = 10^{4.4(12)/23 - 2.2)} = 1.25 \text{ gigabytes.}$$

c) Solving for x, we derive

$$\begin{aligned} 10^3 &= 10^{(4.4x/23 - 2.2)} \\ 3 &= \frac{4.4x}{23} - 2.2 \\ \frac{5.2(23)}{4.4} &= x \\ 27 &\approx x \end{aligned}$$

In the year $2009 = (1982 + 27)$, the capacity will be 1000 gigabytes.

97. If the sound level is 90 db, then the intensity of the sound is given by

$$\begin{aligned} 10 \cdot \log(I \times 10^{12}) &= 90 \\ \log(I) + \log(10^{12}) &= 9 \\ \log(I) + 12 &= 9 \\ \log(I) &= -3 \\ I &= 10^{-3} \text{ watts per/m}^2. \end{aligned}$$

98. If the sound level is 50 db, then the intensity is given by

$$\begin{aligned} 10 \cdot \log(I \times 10^{12}) &= 50 \\ \log(I) + \log(10^{12}) &= 5 \\ \log(I) + 12 &= 5 \\ \log(I) &= -7 \\ I &= 10^{-7} \text{ watts per}/m^2. \end{aligned}$$

If the intensity is doubled to $2I$, then

$$\begin{aligned} 10 \cdot \log(2I \times 10^{12}) &= \\ 10\left[\log(2) + \log(I \times 10^{12})\right] &= \\ 10 \cdot \log(2) + 10 \cdot \log(I \times 10^{12}) &\approx \\ 3 + 10 \cdot \log(I \times 10^{12}). \end{aligned}$$

The sound level is increased by 3 db to 53 db.

The intensity at the sound level of 100 db is given by

$$\begin{aligned} 10 \cdot \log(I \times 10^{12}) &= 100 \\ \log(I) + \log(10^{12}) &= 10 \\ \log(I) + 12 &= 10 \\ \log(I) &= -2 \\ I &= 10^{-2} \text{ watts per } m^2. \end{aligned}$$

99. Since $P \cdot e^{(0.06)18} = 20,000$, the investment will grow to $P = \dfrac{20,000}{e^{(0.06)18}} \approx \$6791.91$.

100. The present value of the bond is given by

$$\begin{aligned} P \cdot \left(1 + \frac{0.0622}{12}\right)^{12\left(11\frac{1}{6}\right)} &= 50 \\ P \cdot (1.005183333)^{134} &\approx 50 \\ P &\approx \frac{50}{(1.005183333)^{134}} \\ P &\approx \$25.01. \end{aligned}$$

101.

a) The logarithmic regression line is

$$y = 114.7865206 - 12.75457939 \ln(x)$$

or approximately is

$$y = 114.8 - 12.8 \ln(x)$$

where the year is $1960 + x$.

b) Let $x = 50$. Since

$$114.7865206 - 12.75457939 \ln(50) \approx 65,$$

the percentage of households in 2010 with two parents is 65%.

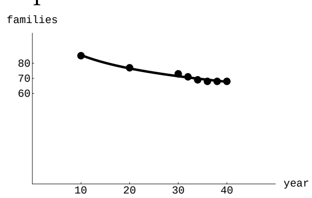
c) Let $y = 50$. Then

$$50 = 114.7865206 - 12.75457939\ln(x)$$

$$\begin{aligned} \ln(x) &= \frac{114.7865206 - 50}{12.75457939} \\ x &\approx 161. \end{aligned}$$

Since $1960 + 161 = 2121$, the percentage of two-parent families will reach 50% in the year 2121.

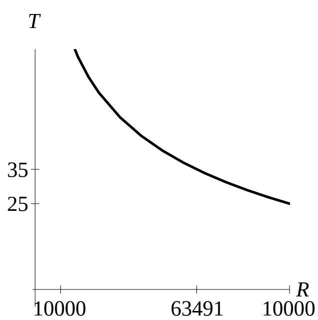
d) The data looks like a quadratic or exponential model.



102. If $R = 1 \times 10^5$, then $T \approx 24.999^o$ C or

$$T \approx 25^o\text{C}.$$

One finds $R \approx 63,491$ Ohms if $T = 35^o$ C as shown in the graph below.



103. By using the first five terms of the formula,

$$\begin{aligned} e^{0.1} &\approx 1 + 0.1 + \frac{(0.1)^2}{2} + \frac{(0.1)^3}{6} + \frac{(0.1)^4}{24} \\ &\approx 1.105170833 \end{aligned}$$

From a calculator, $e^{0.1} \approx 1.105170918$.

104. If $x = 0.4$, then $\ln(1.4) \approx$

$$0.4 - \frac{0.4^2}{2} + \frac{0.4^3}{3} - \frac{0.4^4}{4} + \frac{0.4^5}{5} \approx 0.33698.$$

With a calculator, $\ln(1.4) \approx 0.33647$.

105. $\log_6(4 \cdot 9) = \log_6(36) = 2$

106. Take the natural logarithm of both sides:

$$\begin{aligned} \ln\left(1.56^{x-1}\right) &= \ln 9.8 \\ x - 1 &= \frac{\ln 9.8}{\ln 1.56} \\ x &\approx 6.1326 \end{aligned}$$

The solution set is $\{6.1326\}$.

107. Convert to exponential form:

$$\begin{aligned} x^{2.4} &= 9.8 \\ x &= 9.8^{1/2.4} \\ x &\approx 2.5883 \end{aligned}$$

The solution set is $\{2.5883\}$.

108. $y = -2^{x-5} + 9$

109. By the identity $\log_3(3^w) = w$, we obtain

$$\begin{aligned} (g \circ f)(x) &= \log_3\left(3^{(x-5)}\right) + 5 \\ &= (x - 5) + 5 \\ &= x \end{aligned}$$

110. Factor by grouping;

$$\begin{aligned} 3x - 9 + wx - 3w &= \\ 3(x-3) + w(x-3) &= \\ (3+w)(x-3) \end{aligned}$$

Thinking Outside the Box XLIII

Consider the point of intersection between the two left-most circles on the bottom row. The (vertical) distance between this point of intersection and the center of the left-most circle in the middle row is $\sqrt{5}$.

Let x be the radius of the circle at the top row. We draw another right triangle but this time its hypotenuse is the line segment that joins the center of the left-most circle in the middle row and the center of the circle in the top row. The length of the hypotenuse of this triangle is $x + 1$.

From the center of the left-most circle in the middle row, draw a horizontal side that is 2-units long in such a way that the endpoint of this side should be directly below the center of the circle on the top row. Then a right triangle is formed when this endpoint is joined to the center of the top-most circle by a line segment.

Applying the Pythagorean Theorem, we find

$$2^2 + (x + 2 - \sqrt{5})^2 = (x + 1)^2.$$

Solving for x, we find

$$x = \sqrt{5} - 1.$$

4.4 Pop Quiz

1. $x = 4^5 = 625$

2. Since $x + 1 = 10^3$, we find $x = 999$.

3.

$$\begin{aligned} \ln(x(x-1)) &= \ln 12 \\ x(x-1) &= 12 \\ x^2 - x - 12 &= 0 \\ (x-4)(x+3) &= 0 \\ x &= 4, -3. \end{aligned}$$

Since $\ln(-3)$ is not a real number, the solution is $x = 4$.

4. Since $\log_3(10) = x - 5$, we find

$$x = 5 + \frac{\log 10}{\log 3} = 5 + \frac{1}{\log 3} \approx 7.0959.$$

5. Take the ln of both sides of the equation. Then we find

$$\begin{aligned} x \ln 8 &= (x+5)\ln 3 \\ x(\ln 8 - \ln 3) &= 5 \ln 3 \\ x &= \frac{\ln 243}{\ln(8/3)} \\ x &\approx 5.6004. \end{aligned}$$

4.4 Linking Concepts

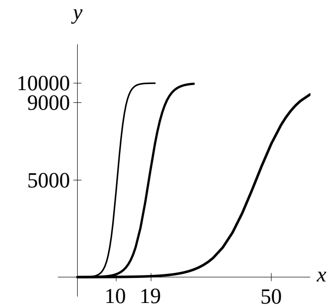
a) $\dfrac{P}{1 + (P-1)e^{-c(0)}} = \dfrac{P}{1 + (P-1)} = \dfrac{P}{P} = 1,$

i.e., one person caught the virus at time $t = 0$

b) Given are sketches of the graphs of

$$y_1 = \frac{10,000}{1 + 9,999e^{-cx}}$$

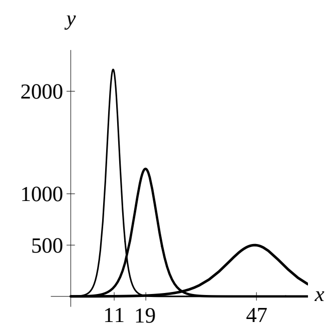
for $c = 0.2, 0.5, 0.9$. (heaviest drawn curve is for $c = 0.2$, lightest one is for $c = 0.9$)



and the graphs for

$$y_2 = y_1(x) - y_1(x-1)$$

(for the same values of c) are given below.



For each value of c, we have tabulated the value of x that maximizes the value of $y_2(x)$.

c	x
0.1	93
0.2	47
0.3	31
0.4	24
0.5	19
0.6	16
0.7	14
0.8	12
0.9	11

c) If the greatest number of new cases occurs on the 19th day, then choose $c = 0.5$. On this day, the number of new cases is

$y_2(19) = y_1(19) - y_1(18) \approx 1243$.

d) The number of days, x, when 9000 students are infected is given by

$$\begin{aligned} 9000 &= \frac{10,000}{1 + 9,999e^{-0.5x}} \\ 9000 + 9000(9999)e^{-0.5x} &= 10,000 \\ e^{-0.5x} &= \frac{1}{9(9999)} \\ x &= \frac{\ln\left(\frac{1}{9(9999)}\right)}{-0.5} \\ x &\approx 23 \text{ days.} \end{aligned}$$

e) On the 30th day, the number of students infected is $y_1 = \dfrac{10,000}{1 + 9,999e^{-0.5(30)}} \approx 9970$.

The doctors came when almost all the students have been infected. The doctors came late to stop the spread of the virus.

Chapter 4 Review Exercises

1. 64 **2.** 2 **3.** 6 **4.** $3 + 2 \cdot 1 = 5$

5. 0 **6.** 99 **7.** 17 **8.** 2

9. $4^{1/\log_3 4 + \log_9 3} = 4^{\log_4 3 + 1/2} =$
$4^{\log_4 3}4^{1/2} = 3 \cdot 2 = 6$

10. $\sqrt[3]{5^{\log_5 2} + 3^{\log_3 6}} = \sqrt[3]{2 + 6} =$
$\sqrt[3]{8} = 2$

11. $2^5 = 32$ **12.** $10^{-1} = 0.1$

13. $\log(10^3) = 3$ **14.** $10^{\log(5)} = 5$

15. $\log_2(2^9) = 9$ **16.** $2^{\log_2(7)} = 7$

17. $\log(1000) = 3$ **18.** $\log(1) = 0$

19. $\log_2(1) - \log_2(8) = 0 - 3 = -3$

20. $\sqrt{2}$ **21.** $\log_2(8) = 3$

22. 13, since $(f^{-1} \circ f)(x) = x$

23. $\log((x - 3)x) = \log(x^2 - 3x)$

24. $\ln(\sqrt{x}) - \ln(y^2) = \ln\left(\dfrac{\sqrt{x}}{y^2}\right)$

25. $\ln(x^2) + \ln(3y) = \ln(3x^2y)$

26. $\log_2(x^3) - \log_2(y^2) + \log_2(z) = \log_2\left(\dfrac{x^3z}{y^2}\right)$

27. $\log(3) + \log(x^4) = \log(3) + 4 \cdot \log(x)$

28. $\ln(x^5) - \ln(y^3) = 5 \cdot \ln(x) - 3 \cdot \ln(y)$

29. $\log_3(5) + \log_3(x^{1/2}) - \log_3(y^4) =$
$\log_3(5) + \dfrac{1}{2} \cdot \log_3(x) - 4 \cdot \log_3(y)$

30. $\log_2((xy^3)^{1/2}) = \dfrac{1}{2} \cdot (\log_2(x) + 3 \cdot \log_2(y)) =$
$\dfrac{1}{2} \cdot \log_2(x) + \dfrac{3}{2} \cdot \log_2(y)$

31. $\ln(2 \cdot 5) = \ln(2) + \ln(5)$

32. $\ln\left(\dfrac{2}{5}\right) = \ln(2) - \ln(5)$

33. $\ln(5^2 \cdot 2) = \ln(5^2) + \ln(2) = 2 \cdot \ln(5) + \ln(2)$

34. $\ln(2\sqrt{5}) = \ln(2) + \ln(5^{1/2}) = \ln(2) + \dfrac{1}{2} \cdot \ln(5)$

35. Since $\log_{10}(x) = 10$, we get $x = 10^{10}$.

36. Since $3^{-1} = x + 1$, we obtain $x = \dfrac{1}{3} - 1 = -\dfrac{2}{3}$.

37. Since $x^4 = 81$ and $x > 0$, $x = 3$.

38. Since $x^0 = 1$, $x > 0$, and $x \neq 1$, the solution set is $(0, 1) \cup (1, \infty)$.

39. Since $\log_{1/3}(27) = -3 = x + 2$, we get $x = -5$.

40. Since $\log_{1/2}(4) = -2 = x - 1$, we find $x = -1$.

41. Since $3^{x+2} = 3^{-2}$, $x + 2 = -2$. So $x = -4$.

42. Since $2^{x-1} = 2^{-2}$, $x - 1 = -2$. So $x = -1$.

43. Since $x - 2 = \ln(9)$, we obtain $x = 2 + \ln(9)$.

44. Note, $2^{x-1} = 3$ is equivalent to $x - 1 = \log_2(3)$. Then $x = 1 + \log_2(3)$.

45. Since $(2^2)^{x+3} = 2^{2x+6} = 2^{-x}$, we get $2x + 6 = -x$. Then $6 = -3x$ and so $x = -2$.

46.

$$\begin{aligned} 3^{2x-1}3^{2x} &= 3^0 \\ 3^{4x-1} &= 3^0 \\ 4x - 1 &= 0 \\ 4x &= 1 \\ x &= \frac{1}{4} \end{aligned}$$

47.

$$\begin{aligned} \log(2x^2) &= 5 \\ 2x^2 &= 10^5 \\ x^2 &= 50,000 \\ x &= \pm 100\sqrt{5} \end{aligned}$$

Since $\log(-100\sqrt{5})$ is undefined, $x = 100\sqrt{5}$.

48.

$$\begin{aligned} \log\left(\frac{x+90}{x}\right) &= 1 \\ \frac{x+90}{x} &= 10^1 \\ x + 90 &= 10x \\ 90 &= 9x \\ x &= 10 \end{aligned}$$

49.

$$\begin{aligned} \log_2\left(x^2 - 4x\right) &= \log_2(x + 24) \\ x^2 - 4x &= x + 24 \\ x^2 - 5x - 24 &= 0 \\ (x - 8)(x + 3) &= 0 \\ x &= 8, -3 \end{aligned}$$

Since $\log(-3)$ is undefined, $x = 8$.

50.

$$\begin{aligned} \log_5((x + 18)(x - 6)) &= \log_5(x^2) \\ x^2 + 12x - 108 &= x^2 \\ 12x &= 108 \\ x &= 9 \end{aligned}$$

51. Since $\ln((x + 2)^2) = \ln(4^3)$ and $y = \ln(x)$ is a one-to-one function, we obtain

$$\begin{aligned} (x + 2)^2 &= 64 \\ x + 2 &= \pm 8 \\ x &= -2 \pm 8 \\ x &= 6, -10. \end{aligned}$$

Checking $x = -10$ one gets $2\ln(-8)$ which is undefined. So $x = 6$.

52.

$$\begin{aligned} x \cdot \log_2(12) - x \cdot \log_2(3) &= 1 \\ x(\log_2(12) - \log_2(3)) &= 1 \\ x \cdot \log_2(4) &= 1 \\ x \cdot 2 &= 1 \\ x &= \frac{1}{2} \end{aligned}$$

53.

$$\begin{aligned} x \cdot \log(4) + x \cdot \log(25) &= 6 \\ x(\log(4) + \log(25)) &= 6 \\ x \cdot \log(100) &= 6 \\ x \cdot 2 &= 6 \\ x &= 3 \end{aligned}$$

54. Since $\log(\log(x)) = 1$ and by using the definition of a logarithm, we get $\log(x) = 10$. So $x = 10^{10}$.

55. The missing coordinates are

(i) 3 since $\left(\frac{1}{3}\right)^{-1} = 3$,

(ii) -3 since $\left(\frac{1}{3}\right)^{-3} = 27$,

(iii) $\sqrt{3}$ since $\left(\frac{1}{3}\right)^{-1/2} = \sqrt{3}$, and

(iv) 0 since $\left(\frac{1}{3}\right)^{0} = 1$.

56. The missing coordinates are

(i) 0 since $\log_9(2 - 1) = \log_9(1) = 0$,

(ii) $\dfrac{1}{2}$ since $\log_9(4-1) = \log_9(3) = \dfrac{1}{2}$,

(iii) 28 since $\log_9(x-1) = \dfrac{3}{2}$,

$9^{3/2} = 27 = x - 1$, so $x = 28$,

(iv) $\dfrac{10}{9}$ since $\log_9(x-1) = -1$,

$\dfrac{1}{9} = x - 1$, so $x = \dfrac{10}{9}$.

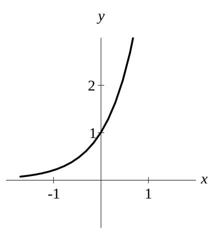
57. c **58.** h

59. b **60.** g

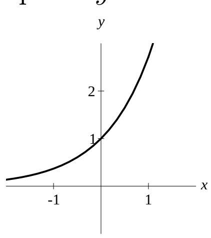
61. d **62.** a

63. e **64.** f

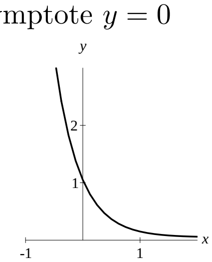
65. Domain $(-\infty, \infty)$, range $(0, \infty)$, increasing, asymptote $y = 0$



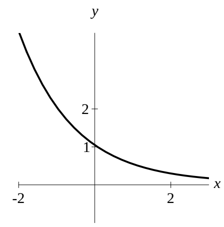
66. Domain $(-\infty, \infty)$, range $(0, \infty)$, increasing, asymptote $y = 0$



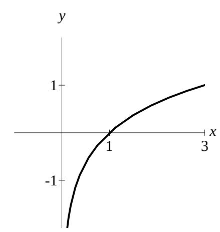
67. Domain $(-\infty, \infty)$, range $(0, \infty)$, decreasing, asymptote $y = 0$



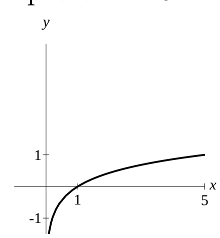
68. Domain $(-\infty, \infty)$, range $(0, \infty)$, decreasing, asymptote $y = 0$



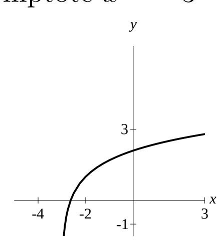
69. Domain $(0, \infty)$, range $(-\infty, \infty)$, increasing, asymptote $x = 0$



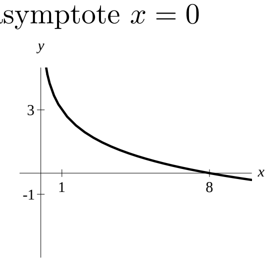
70. Domain $(0, \infty)$, range $(-\infty, \infty)$, increasing, asymptote $x = 0$



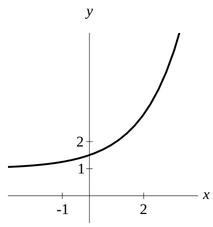
71. Domain $(-3, \infty)$, range $(-\infty, \infty)$, increasing, asymptote $x = -3$



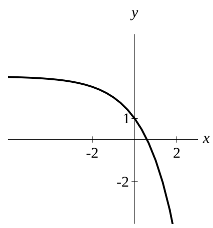
72. Domain $(0, \infty)$, range $(-\infty, \infty)$, decreasing, asymptote $x = 0$



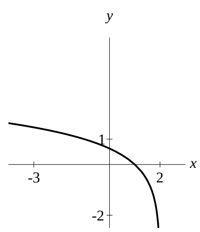
73. Domain $(-\infty, \infty)$, range $(1, \infty)$, increasing, asymptote $y = 1$



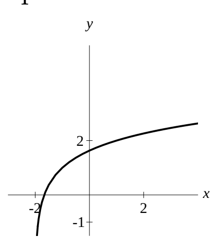
74. Domain $(-\infty, \infty)$, range $(-\infty, 3)$, decreasing, asymptote $y = 3$



75. Domain $(-\infty, 2)$, range $(-\infty, \infty)$, decreasing, asymptote $x = 2$



76. Domain $(-2, \infty)$, range $(-\infty, \infty)$, increasing, asymptote $x = -2$



77. $f^{-1}(x) = \log_7(x)$ **78.** $f^{-1}(x) = \log_3(x)$

79. $f^{-1}(x) = 5^x$ **80.** $f^{-1}(x) = 8^x$

81. Replace $f(x)$ by y, interchange x and y, solve for y, and replace y by $f^{-1}(x)$.

$$\begin{aligned} y &= 3\log(x-1) \\ x &= 3\log(y-1) \\ \frac{x}{3} &= \log(y-1) \\ 10^{x/3} &= y-1 \\ 10^{x/3}+1 &= y \\ f^{-1}(x) &= 10^{x/3}+1 \end{aligned}$$

82. Replace $f(x)$ by y, interchange x and y, solve for y, and replace y by $f^{-1}(x)$.

$$\begin{aligned} y &= \log_2(x+3)-5 \\ x &= \log_2(y+3)-5 \\ x+5 &= \log_2(y+3) \\ 2^{x+5} &= y+3 \\ 2^{x+5}-3 &= y \\ f^{-1}(x) &= 2^{x+5}-3 \end{aligned}$$

83. Replace $f(x)$ by y, interchange x and y, solve for y, and replace y by $f^{-1}(x)$.

$$\begin{aligned} y &= e^{x+2}-3 \\ x &= e^{y+2}-3 \\ x+3 &= e^{y+2} \\ \ln(x+3) &= y+2 \\ \ln(x+3)-2 &= y \\ f^{-1}(x) &= \ln(x+3)-2 \end{aligned}$$

84. Replace $f(x)$ by y, interchange x and y, solve for y, and replace y by $f^{-1}(x)$.

$$\begin{aligned} y &= 2\cdot 3^x+1 \\ x &= 2\cdot 3^y+1 \\ \frac{x-1}{2} &= 3^y \\ \log_3\left(\frac{x-1}{2}\right) &= y \\ f^{-1}(x) &= \log_3\left(\frac{x-1}{2}\right) \end{aligned}$$

85. Since $3^x = 10$, $x = \log_3(10) = \dfrac{\ln(10)}{\ln(3)} \approx 2.0959$.

86. Solving for x, we find

$$\begin{aligned} 4^{2x} &= 12 \\ 2x\ln(4) &= \ln(12) \\ x &= \frac{\ln(12)}{2\ln(4)} \\ x &\approx 0.8962. \end{aligned}$$

87. Since $\log_3(x) = 1.876$, $x = 3^{1.876} \approx 7.8538$.

88. Solving for x, we obtain

$$\begin{aligned} 5^{2.7} &= x+2 \\ 5^{2.7}-2 &= x \\ x &\approx 75.1292. \end{aligned}$$

89. After taking the natural logarithm of both sides,
we have

$$\begin{aligned} \ln(5^x) &= \ln(8^{x+1}) \\ x\cdot\ln(5) &= (x+1)\cdot\ln(8) \\ x\cdot\ln(5) &= x\cdot\ln(8)+\ln(8) \\ x\cdot(\ln(5)-\ln(8)) &= \ln(8) \\ x &= \frac{\ln(8)}{\ln(5)-\ln(8)} \\ x &\approx -4.4243. \end{aligned}$$

90. After taking the natural logarithm of both sides, we have

$$\begin{aligned} \ln(3^x) &= \ln(e^{x+1}) \\ x\cdot\ln(3) &= (x+1)\cdot\ln(e) \\ x\cdot\ln(3) &= (x+1)\cdot 1 \\ x\cdot(\ln(3)-1) &= 1 \\ x &= \frac{1}{\ln(3)-1} \\ x &\approx 10.1407. \end{aligned}$$

91. True, since $\log_3(81) = 4$ and $2 = \log_3(9)$.

92. False, $\log(81) = \log(9^2) = 2\cdot\log(9) \neq (\log(9))^2$.

93. False, since $\ln(3^2) = 2\cdot\ln(3) \neq (\ln(3))^2$.

94. False, since $\ln\left(\frac{5}{8}\right) = \ln(5)-\ln(8) \neq \frac{\ln(5)}{\ln(8)}$.

95. True, since $4\cdot\log_2(8) = 4\cdot 3 = 12$.

96. True, since $\log(8.2\times 10^{-9}) =$ $\log(8.2)+\log(10^{-9}) = \log(8.2)-9$.

97. False, since $3+\log(6) = \log(10^3)+\log(6) =$ $\log(6000)$.

98. True, since $\log_2(16) = 4$, $\log_2(4) = 2$, and $\frac{4}{2} = 4-2$.

99. False, since $\log_2(16) = 4$, $\log_2(8) = 3$, and $\frac{3}{4} \neq 3-4$.

100. True, since $\ln(e^w) = w$ for all w.

101. False, since $\log_2(25) = 2\log_2(5) =$ $2\cdot\frac{\log(5)}{\log(2)} \neq 2\cdot\log(5)$.

102. False, since $\log_3(5/7) = \log_3(5)-\log_3(7) \neq$ $\log(5)-\log(7)$.

103. True, because of the base-changing formula.

104. True, because of the base-changing formula.

105. If H_B^+ is the hydrogen ion concentration of liquid B then $10\cdot H_B^+$ is the hydrogen ion concentration of liquid A. The pH of A is

$$-\log(10\cdot H_B^+) = -1-\log(H_B^+)$$

i.e. the pH of A is one less than the pH of B.

106. Subtracting P, we obtain

$$\begin{aligned} A-P &= Ce^{-kt} \\ \frac{A-P}{C} &= e^{-kt} \\ \ln\left(\frac{A-P}{C}\right) &= -kt \\ -\frac{1}{k}\cdot\ln\left(\frac{A-P}{C}\right) &= t. \end{aligned}$$

107. The value at the end of 18 years is

$$50,000\left(1+\frac{0.05}{4}\right)^{18\cdot 4} \approx \$122,296.01.$$

108. The value at the end of 12.25 years is

$$30,000\cdot e^{0.0618(12.25)} \approx \$63,959.33.$$

109. Let t be the number of years.

$$\begin{aligned} 50,000\left(1+\frac{0.05}{4}\right)^{4t} &= 100,000 \\ (1.0125)^{4t} &= 2 \\ 4t &= \log_{1.0125}(2) \\ t &= \frac{1}{4}\cdot\frac{\ln(2)}{\ln(1.0125)} \\ t &\approx 13.9 \text{ years} \end{aligned}$$

It doubles in $4 \cdot 13.9 \approx 56$ quarters.

110. Let t be the number of years.

$$\begin{aligned} 30,000e^{0.0618t} &= 60,000 \\ e^{0.0618t} &= 2 \\ 0.0618t &= \ln(2) \\ t &\approx 11.2159 \text{ years} \end{aligned}$$

It doubles in 11 yr, 79 days.

111. The present amount is $A = 25 \cdot e^0 = 25$ g. After $t = 1000$ years, the amount left is $A = 25 \cdot e^{-0.32} \approx 18.15$ g.

To find the half-life, let $A = 12.5$.

$$\begin{aligned} 25 \cdot e^{-0.00032t} &= 12.5 \\ e^{-0.00032t} &= 0.5 \\ -0.00032t &= \ln(0.5) \\ t &\approx 2166 \end{aligned}$$

The half-life is 2166 years.

112.

$$\begin{aligned} 800 \cdot e^{0.06t} &= 1000 \cdot \left(1+\frac{0.05}{12}\right)^{12t} \\ e^{0.06t} &\approx 1.25(1.0041667)^{12t} \\ 0.06t &\approx \ln(1.25(1.0041667)^{12t}) \end{aligned}$$

$$0.06t \approx \ln(1.25) + 12t \cdot \ln(1.0041667)$$

$$0.06t \approx \ln(1.25) + 0.0498965 \cdot t$$

$$\begin{aligned} 0.0101035t &\approx \ln(1.25) \\ t &\approx 22.08577 \text{ yrs.} \end{aligned}$$

The investments are equal after 22 yr, 1 mo.

113. Let $f(t) = 10,000$.

$$\begin{aligned} 40,000 \cdot (1 - e^{-0.0001t}) &= 10,000 \\ 1 - e^{-0.0001t} &= 0.25 \\ 0.75 &= e^{-0.0001t} \\ \ln(0.75) &= -0.0001t \\ t &\approx 2877 \text{ hr} \end{aligned}$$

It takes 2877 hours to learn 10,000 words.

114.

a) $P = 31.5(0.935)^8 \approx 18\%$

b) If $P = 2\%$, then

$$\begin{aligned} 31.5(0.935)^i &= 2 \\ 0.935^i &= \frac{2}{31.5} \\ i\ln(0.935) &= \ln\left(\frac{2}{31.5}\right) \\ i &= \frac{\ln\left(\frac{2}{31.5}\right)}{\ln(0.935)} \\ i &\approx 41.0 \end{aligned}$$

The per capita income is \$41,000.

115. In the following equations, we solve for x.

$$\begin{aligned} 1026\left(\frac{25,005}{64}\right)^x &= 19.2 \\ \left(\frac{25,005}{64}\right)^x &= \frac{19.2}{1026} \\ x\ln\left(\frac{25,005}{64}\right) &= \ln\left(\frac{19.2}{1026}\right) \\ x &= \frac{\ln\left(\frac{19.2}{1026}\right)}{\ln\left(\frac{25,005}{64}\right)} \end{aligned}$$

$$x \approx -0.667 \quad \text{or} \quad x \approx -\frac{2}{3}$$

In the following equations, we obtain y.

$$\begin{aligned} \frac{25005}{2240}\left(\frac{35+\frac{1}{12}}{100}\right)^y &= 258.51 \\ \left(\frac{35+\frac{1}{12}}{100}\right)^y &= \frac{258.51(2240)}{25005} \end{aligned}$$

$$\begin{aligned} y &= \frac{\ln\left(\frac{258.51(2240)}{25005}\right)}{\ln\left(\frac{35+\frac{1}{12}}{100}\right)} \\ y &\approx -3 \end{aligned}$$

Next, we solve for z.

$$\begin{aligned} 13.5\left(\frac{25005}{64}\right)^z &= 1.85 \\ \left(\frac{25005}{64}\right)^z &= \frac{1.85}{13.5} \\ z\ln\left(\frac{25005}{64}\right) &= \ln\left(\frac{1.85}{13.5}\right) \\ z &= \frac{\ln\left(\frac{1.85}{13.5}\right)}{\ln\left(\frac{25005}{64}\right)} \\ z \approx -0.333 &\quad \text{or } z \approx -\frac{1}{3} \end{aligned}$$

116.

a) If on the nth bet a gambler gets her/his first win, then the gambler has a net gain of \$2.

b) $n \approx 20$

c) If the nth bet is more than \$1 million, then

$$\begin{aligned} 2^n &> 10^6 \\ n\ln(2) &> 6\ln(10) \\ n &> \frac{6\ln(10)}{\ln(2)} \\ n &\approx 19.9. \end{aligned}$$

The first bet that exceeds \$1,000,000 is the 20th bet.

Thinking Outside the Box XLIV

We begin by noting that

$$\left(\begin{array}{c}\text{area of}\\ \text{two crescents}\end{array}\right) + \left(\begin{array}{c}\text{area of}\\ \text{largest semicircle}\end{array}\right) =$$

$$\left(\begin{array}{c}\text{area of two}\\ \text{smaller circles}\end{array}\right) + \left(\begin{array}{c}\text{area of the}\\ \text{right triangle}\end{array}\right) =$$

Let x and y be the sides of the right triangle where crescents A and B intersect the triangle, respectively. Then the hypotenuse of the right triangle is $\sqrt{x^2+y^2}$.

Recall, the area of a semicircle with diameter d is $\pi d^2/8$. Thus, the sum of the areas of the two smaller semicircles with diameters x and y is equal to the area of the largest semicircle with diameter $\sqrt{x^2+y^2}$.

If we subtract the areas of the circles from the equation above, then we obtain

$$\left(\begin{array}{c}\text{area of}\\ \text{two crescents}\end{array}\right) = \left(\begin{array}{c}\text{area of the}\\ \text{right triangle}\end{array}\right).$$

Hence, the ratio of the total area of the two crescents to the area of the triangle is 1.

Chapter 4 Test

1. 3 **2.** -2 **3.** 6.47 **4.** $\sqrt{2}$

5. $f^{-1}(x) = e^x$

6. Replace $f(x)$ by y, interchange x and y, solve for y, and replace y by $f^{-1}(x)$.

$$\begin{aligned} y &= 8^{x+1} - 3 \\ x &= 8^{y+1} - 3 \\ x+3 &= 8^{y+1} \\ \log_8(x+3) &= y+1 \\ \log_8(x+3) - 1 &= y \\ f^{-1}(x) &= \log_8(x+3) - 1 \end{aligned}$$

7. $\log(x) + \log(y^3) = \log(xy^3)$

8. $\ln(\sqrt{x-1}) - \ln(33) = \ln\left(\frac{\sqrt{x-1}}{33}\right)$

9. $\log_a(2^2 \cdot 7) = \log_a(2^2) + \log_a(7) = 2 \cdot \log_a(2) + \log_a(7)$

10. $\log_a\left(\frac{7}{2}\right) = \log_a(7) - \log_a(2)$

11.

$$\begin{aligned} \log_2(x^2 - 2x) &= 3 \\ x^2 - 2x &= 2^3 \\ x^2 - 2x - 8 &= 0 \\ (x-4)(x+2) &= 0 \\ x &= 4, -2 \end{aligned}$$

But $\log_2(-2)$ is undefined, so $x = 4$.

12.

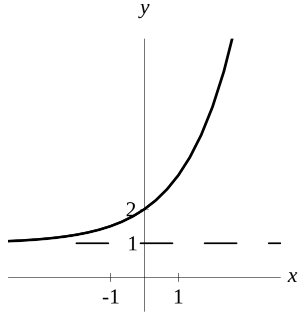
$$\begin{aligned} \log\left(\frac{10x}{x+2}\right) &= \log(3^2) \\ \frac{10x}{x+2} &= 9 \\ 10x &= 9x + 18 \\ x &= 18 \end{aligned}$$

13.

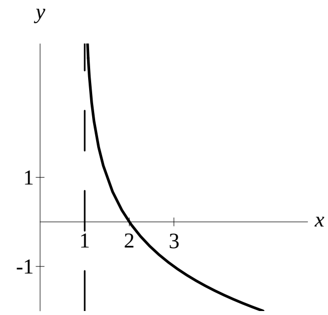
$$\begin{aligned} \ln(3^x) &= \ln(5^{x-1}) \\ x \cdot \ln(3) &= (x-1) \cdot \ln(5) \\ x \cdot \ln(3) &= x \cdot \ln(5) - \ln(5) \\ x(\ln(3) - \ln(5)) &= -\ln(5) \\ x(\ln(5) - \ln(3)) &= \ln(5) \\ x &= \frac{\ln(5)}{\ln(5) - \ln(3)} \\ x &\approx 3.1507 \end{aligned}$$

14. By the definition of a logarithm, we obtain $x - 1 = 3^{5.46}$. Then $x = 1 + 3^{5.46} \approx 403.7931$.

15. Domain $(-\infty, \infty)$, range $(1, \infty)$, increasing, asymptote $y = 1$



16. Domain $(1, \infty)$, range $(-\infty, \infty)$, decreasing, asymptote $x = 1$



17. $(1, 0)$

18. Compounded quarterly, the investment is worth

$$2000\left(1 + \frac{0.08}{4}\right)^{80} \approx \$9750.88.$$

Compounded continuously, the investment is worth

$$2000 \cdot e^{0.08(20)} \approx \$9906.06.$$

19. The amount of power at the end of $t = 200$ days is $P = 50 \cdot e^{-200/250} \approx 22.5$ watts.

To find the half-life, let $P = 25$.

$$\begin{aligned} 50 \cdot e^{-t/250} &= 25 \\ e^{-t/250} &= 0.5 \\ -\frac{t}{250} &= \ln(0.5) \\ t &\approx 173.3 \end{aligned}$$

The half-life is 173.3 days.

The operational life for a power of $P = 9$ watts is given by

$$\begin{aligned} 50 \cdot e^{-t/250} &= 9 \\ e^{-t/250} &= \frac{9}{50} \\ -\frac{t}{250} &= \ln\left(\frac{9}{50}\right) \\ t &\approx 428.7 \text{ days.} \end{aligned}$$

20.

$$\begin{aligned} 4,000 \cdot \left(1 + \frac{0.06}{4}\right)^{4t} &= 10,000 \\ (1.015)^{4t} &= 2.5 \\ 4t &= \log_{1.015}(2.5) \\ t &\approx 15.38576 \text{ years} \\ t &\approx 61.5 \text{ quarters} \end{aligned}$$

21. Substituting $t = 100$, we obtain

$$\begin{aligned} -50 \cdot \ln(1-p) &= 100 \\ \ln(1-p) &= -2 \\ 1-p &= e^{-2} \\ p &= 1 - e^{-2} \\ p &\approx 0.86. \end{aligned}$$

The level reached after 100 hr is $p = 0.86$. When $p = 1$, then $t = -50\ln(0)$ which is undefined, so it is impossible to master MGM.

Tying It All Together

1. Since $x - 3 = \pm 2$, we get $x = 3 \pm 2 = 1, 5$.

2. Since $\log((x-3)^2) = \log(4)$, we obtain

$$\begin{aligned} (x-3)^2 &= 4 \\ x-3 &= \pm 2 \\ x &= 3 \pm 2 \\ x &= 5, 1. \end{aligned}$$

Checking $x = 1$, one gets $2\log(-2)$ which is undefined, so $x = 5$.

3. Using the definition of a logarithm, we obtain $x - 3 = 2^4$. The solution is $x = 16 + 3 = 19$.

4. Since $2^{x-3} = 2^2$, we have $x - 3 = 2$. So $x = 5$.

5. Square both sides of $\sqrt{x-3} = 4$. Then $x - 3 = 16$. The solution is $x = 19$.

6. An equivalent equation is $x - 3 = \pm 4$. Thus, $x = 3 \pm 4 = -1, 7$.

7. Completing the square, we obtain

$$\begin{aligned} x^2 - 4x + 4 &= -2 + 4 \\ (x-2)^2 &= 2 \\ x - 2 &= \pm\sqrt{2} \\ x &= 2 \pm \sqrt{2}. \end{aligned}$$

8. Since $2^{x-3} = 2^{2x}$, $x - 3 = 2x$. So $x = -3$.

9. Raise both sides to the third power.

$$\begin{aligned} (\sqrt[3]{x-5})^3 &= 5^3 \\ x - 5 &= 125 \\ x &= 130 \end{aligned}$$

10. By using the definition of a logarithm, we have $x = \log_2(3)$.

11.

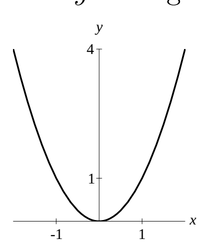
$$\begin{aligned} \log(4x - 12) &= \log(x) \\ 4x - 12 &= x \\ 3x &= 12 \\ x &= 4 \end{aligned}$$

12. Use synthetic division with $c = -1$.

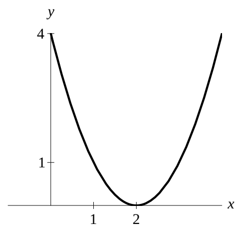
-1	1	-4	1	6
		-1	5	-6
	1	-5	6	0

The quotient factors as $x^2 - 5x + 6 = (x-3)(x-2)$. The solutions are $x = -1, 2, 3$.

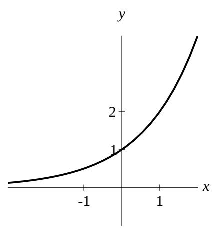
13. Parabola $y = x^2$ goes through $(0, 0)$,$(\pm 1, 1)$



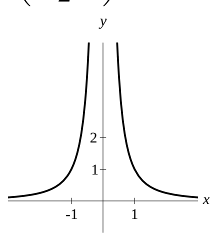
14. Parabola $y = (x-2)^2$ goes through $(0,4)$, $(1,1)$, $(3,1)$



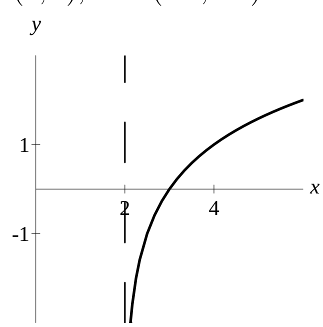
15. $y = 2^x$ goes through $\left(-1, \dfrac{1}{2}\right)$, $(0,1)$, $(1,2)$



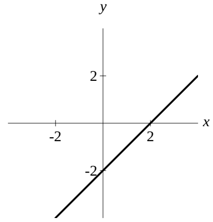
16. $y = x^{-2}$ goes through $(\pm 1, 1)$, $\left(\pm 2, \dfrac{1}{4}\right)$, and $\left(\pm\dfrac{1}{2}, 4\right)$



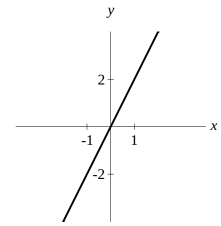
17. $y = \log_2(x-2)$ goes through $(3,0)$, $(4,1)$, and $(2.5,-1)$



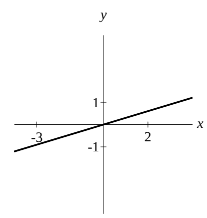
18. Line $y = x - 2$ goes through $(0,-2)$, $(2,0)$



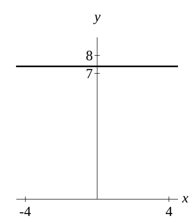
19. Line $y = 2x$ goes through $(0,0)$, $(1,2)$



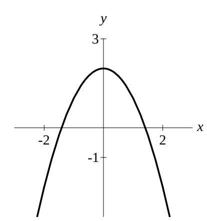
20. Line $y = x \cdot \log(2)$ goes through $(0,0)$, $(1, \log(2))$



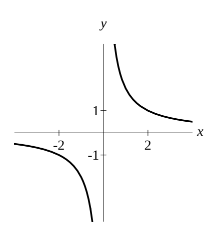
21. Horizontal line $y = e^2$ goes through $(0, e^2)$



22. Parabola $y = 2 - x^2$ goes through $(0,2)$, $(\pm 1, 1)$



23. $y = \dfrac{2}{x}$ is symmetric about the origin and goes through $(1,2)$, $(2,1)$, $\left(\dfrac{1}{2}, 4\right)$, $\left(4, \dfrac{1}{2}\right)$



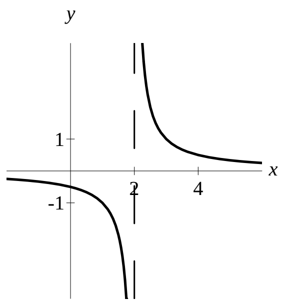
24. The graph of

$$y = \frac{1}{x-2}$$

is obtained by shifting the graph

$$y = \frac{1}{x}$$

to the right by 2 units.



25. $f^{-1}(x) = 3x$

26. $f^{-1}(x) = -\log_3(x)$ since $f(x) = 3^{-x}$

27. $f^{-1}(x) = x^2 + 2 \quad \text{for } x \geq 0$

28. $f^{-1}(x) = \sqrt[3]{x-2} + 5$

29. $f^{-1}(x) = (10^x + 3)^2$ **30.** $\{(1,3),(4,5)\}$

31. $f^{-1}(x) = 5 + \dfrac{1}{x-3}$

32. $f^{-1}(x) = (\ln(3-x))^2$ for $x \leq 2$

33. $(p \circ m)(x) = e^{x+5}$, domain $(-\infty, \infty)$, range $(0, \infty)$

34. $(p \circ q)(x) = e^{\sqrt{x}}$, domain $[0, \infty)$, range $[1, \infty)$

35. $(q \circ p \circ m)(x) = q\left(e^{x+5}\right) = \sqrt{e^{x+5}}$, domain $(-\infty, \infty)$, range $(0, \infty)$

36. $(m \circ r \circ q)(x) = m\left(\ln(\sqrt{x})\right) = \ln(\sqrt{x}) + 5$, domain $(0, \infty)$, range $(-\infty, \infty)$

37. $(p \circ r \circ m)(x) = p\left(\ln(x+5)\right) = e^{\ln(x+5)}$, domain $(-5, \infty)$, range $(0, \infty)$

38. $(r \circ q \circ p)(x) = r\left(\sqrt{e^x}\right) = \ln\left(\sqrt{e^x}\right) = \dfrac{1}{2}x$, domain $(-\infty, \infty)$, range $(-\infty, \infty)$

39. $F(x) = (f \circ g \circ h)(x)$

40. $H(x) = (g \circ h \circ f)(x)$

41. $G(x) = (h \circ g \circ f)(x)$

42. $M(x) = (h \circ f \circ g)(x)$

43. origin

44. circle

45. rise, run

46. point-slope form

47. slope-intercept form

48. perpendicular

49. parallel

50. quadratic

51. quadratic

52. equivalent

Concepts of Calculus

1. a.

h	0.1	0.01	0.001	0.0001
$(e^h - 1)/h$	1.051709	1.005017	1.0005002	1.0000500

b. No, since if $h = 0$ in $\dfrac{e^h - 1}{h}$ we get an undefined term $\dfrac{0}{0}$.

c. Note, the values of $\dfrac{e^h - 1}{h}$ approaches 1 as h approaches 0 as seen in part a. Thus, we say

$$\lim_{h \to 0} \frac{e^h - 1}{h} = 1.$$

2. Let $f(x) = e^x$.

a.

$$\frac{f(x+h) - f(x)}{h} = \frac{e^{x+h} - e^x}{h}$$

$$= \frac{e^x(e^h - 1)}{h}$$

$$\frac{f(x+h) - f(x)}{h} = e^x \left(\frac{e^h - 1}{h}\right)$$

b. By using the answer in part a, we obtain

$$\begin{aligned}\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} &= \lim_{h\to 0}\left(e^x\left(\frac{e^h-1}{h}\right)\right)\\ &= e^x\lim_{h\to 0}\left(\frac{e^h-1}{h}\right)\end{aligned}$$

And, by using the answer to Exercise 1(c) we obtain

$$\begin{aligned}\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} &= e^x\lim_{h\to 0}\left(\frac{e^h-1}{h}\right)\\ &= e^x(1)\\ \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} &= e^x\end{aligned}$$

c. By using the answer to part b, we find $f'(x)=e^x$.

3. Let $f(x)=e^{2x}$.

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{e^{2(x+h)}-e^{2x}}{h}\\ &= \frac{e^{2x}(e^{2h}-1)}{h}\\ \frac{f(x+h)-f(x)}{h} &= 2e^{2x}\left(\frac{e^{2h}-1}{2h}\right)\end{aligned}$$

By using Exercise 1(c), we can conclude that

$$\lim_{h\to 0}\frac{e^{2h}-1}{2h}=1.$$

Consequently, we obtain

$$\begin{aligned}\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} &= \lim_{h\to 0}\left(2e^{2x}\left(\frac{e^{2h}-1}{2h}\right)\right)\\ &= 2e^{2x}\lim_{h\to 0}\left(\frac{e^{2h}-1}{2h}\right)\\ &= 2e^{2x}\cdot 1\\ \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} &= 2e^{2x}.\end{aligned}$$

Thus, we find $f'(x)=2e^{2x}$.

4. Let $f(x)=e^{3x}$.

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{e^{3(x+h)}-e^{3x}}{h}\\ &= \frac{e^{3x}(e^{3h}-1)}{h}\\ \frac{f(x+h)-f(x)}{h} &= 3e^{3x}\left(\frac{e^{3h}-1}{3h}\right)\end{aligned}$$

By using Exercise 1(c), we can conclude that

$$\lim_{h\to 0}\frac{e^{3h}-1}{3h}=1.$$

Consequently, we obtain

$$\begin{aligned}\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} &= \lim_{h\to 0}\left(3e^{3x}\left(\frac{e^{3h}-1}{3h}\right)\right)\\ &= 3e^{3x}\lim_{h\to 0}\left(\frac{e^{3h}-1}{3h}\right)\\ &= 3e^{3x}\cdot 1\\ \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} &= 3e^{3x}.\end{aligned}$$

Hence, we find $f'(x)=3e^{3x}$.

5. Based on the answers to Exercises 2-4, it can be shown that if $f(x)=e^{nx}$ where n is a positive integer, then $f'(x)=ne^{nx}$. In fact, we have

$$\begin{aligned}\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} &= \lim_{h\to 0}\frac{e^{n(x+h)}-e^{nx}}{h}\\ &= \lim_{h\to 0}\frac{e^{nx}(e^{nh}-1)}{h}\\ &= \lim_{h\to 0}\left(ne^{nx}\frac{e^{nh}-1}{nh}\right)\\ &= ne^{nx}\lim_{h\to 0}\frac{e^{nh}-1}{nh}\\ &= ne^{nx}\cdot 1\\ \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} &= ne^{nx}.\end{aligned}$$

Therefore, we have $f'(x)=ne^{nx}$.