SECTION 8.2 – Distribution of the Sample Proportion

Definition

Suppose that a random sample of size n is obtained from a population in which each individual either does or does not have a certain characteristic. The sample proportion, denoted $\hat{p}$ (read "p-hat"), is given by

$$\hat{p} = \frac{x}{n}$$

where x is the number of individuals in the sample with the specified characteristic. The sample proportion, $\hat{p}$, is a statistic that estimates the population proportion, p.

Sampling Distribution of $\hat{p}$

For a simple random sample of size n with a population proportion p,

- The mean of the sampling distribution of $\hat{p}$ is $\mu_{\hat{p}} = p$.
- The standard deviation of the sampling distribution of $\hat{p}$ is $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$.
- The shape of the sampling distribution of $\hat{p}$ is approximately normal provided $np(1-p) \geq 10$.

Note: We also require that the sampled values must be independent of each other; that is, one outcome cannot affect the success or failure of any other outcome. So, when sampling from finite populations, this means that the sample size can be no more than 5% of the population size $(n \leq 0.05N)$

The z-Score of a Sample Proportion

$$z = \frac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}}$$

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☺ **Exercises**:

1) Describe the sampling distribution of $\hat{p}$, if $N = 25{,}000$, $n = 300$, $p = 0.7$.

2) A simple random sample of size $n = 200$ is obtained from a population whose size is $N = 25{,}000$ and whose population proportion with a specified characteristic is $p = 0.65$.

a) Describe the sampling distribution of $\hat{p}$.

b) What is the probability of obtaining $x = 136$ or more individuals with the characteristic? That is, what is $P(\hat{p} \geq 0.68)$?

c) What is the probability of obtaining $x = 118$ or fewer individuals with the characteristic? That is, what is $P(\hat{p} \leq 0.59)$?

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☺ **Exercises**:

3) **Credit Cards.** According to creditcard.com, 29% of adults do not own a credit card.

a) Suppose a random sample of 500 adults is asked, "Do you own a credit card?" Describe the sampling distribution of $\hat{p}$, the proportion of adults who do not own a credit card.

b) What is the probability that in a random sample of 500 adults more than 30% do not own a credit card?

c) What is the probability that in a random sample of 500 adults between 25% and 30% do not own a credit card?

d) Would it be unusual for a random sample of 500 adults to result in 125 or fewer who do not own a credit card? Why?