**Definition**

Suppose that a random sample of size *n* is obtained from a population in which each individual either does or does not have a certain characteristic. The sample proportion, denoted  (read “p-hat”), is given by

where *x* is the number of individuals in the sample with the specified characteristic. The sample proportion, , is a statistic that estimates the population proportion, *p*.

**Sampling Distribution of**

For a simple random sample of size *n* with a population proportion *p*,

* The mean of the sampling distribution of is .
* The standard deviation of the sampling distribution of is .
* The shape of the sampling distribution of is approximately normal provided *np*(1 – *p*) ≥ 10.

*Note:* We also require that the sampled values must be independent of each other; that is, one outcome

cannot affect the success or failure of any other outcome. So, when sampling from finite populations, this means that the sample size can be no more than 5% of the population size (*n* ≤ 0.05*N*).

**The z-Score of a Sample Proportion**

or equivalently, from the sampling distribution of above,

Since and .

**☺ Exercises**:

**1)** Describe the sampling distribution of , if *N* = 25,000, *n* = 300, *p* = 0.7.

* First, check that the shape of the sampling distribution of is approximately normal by verifying that *np*(1 – *p*) ≥ 10.

So, since , then is approximately normal.

* Next, check that the sampled values are independent of each other by showing *n* ≤ 0.05*N*.

So, since , then the sampled values are independent of each other.

* The mean is . The standard deviation is

Thus, is approximately normally distributed with mean and .

**2)** A simple random sample of size *n* = 200 is obtained from a population whose size is *N* = 25,000 and whose population proportion with a specified characteristic is *p* = 0.65.

**a)** Describe the sampling distribution of .

Thus, is approximately normally distributed. Thus, the sampled values are independent of each other.

The mean is . The standard deviation is

Thus, is approximately normally distributed with mean and .

**b)** Find?

**c)** Find ?

**☺ Extra Practice and Exercises with Answers**:

**3)** **Credit Cards.** According to creditcard.com, 29% of adults do not own a credit card.

**a)**  Suppose a random sample of 500 adults is asked, “Do you own a credit card?” Describe the

sampling distribution of , the proportion of adults who do not own a credit card.

Thus, is approximately normally distributed. Thus, the sampled values are independent of each other.

The mean is . The standard deviation is

Thus, is approximately normally distributed with mean and .

**b)**  What is the probability that in a random sample of 500 adults more than 30% do not own

a credit card?

**c)** What is the probability that in a random sample of 500 adults between 25% and 30% do

not own a credit card?

**d)**  Would it be unusual for a random sample of 500 adults to result in 125 or fewer who do

not own a credit card? Why?

Yes, it is unusual, since 0.0244 < 5%.