**Probability Density Function (pdf)**

A **probability density function** is an equation used to compute probabilities of continuous random variables. It must satisfy the following two properties:

**1.** The total area under the graph of the equation over all possible values of the random variable must equal 1.

**2.** The height of the graph of the equation must be greater than or equal to 0 for all possible values of the random variable.

**Relationship between areas and probabilities**

The area under the graph of a density function over an interval represents the probability of observing a value of the random variable in that interval.

**Definition** – A continuous random variable is normally distributed, or has a normal probability distribution, if

 its relative frequency histogram has the shape of a normal curve.



Note: Inflection points are the points on the curve where the curvature (or concavity) of the graph changes.

**Properties of the Normal Density Curve**

**1.** It is symmetric about its mean, *μ*.

**2.** Because mean = median = mode, the curve has a single peak and the highest point occurs at *x* = *μ*.

**3.** It has inflection points at *μ* – *σ* and *μ* + *σ*.

**4.** The area under the curve is 1.

**5.** The area under the curve to the right of *μ* equals the area

 Under the curve to the left of *μ*, which equals .

**6.** As *x* increases without bound (gets larger and larger), the

graph approaches, but never reaches, the horizontal axis.

 As *x* decreases without bound (gets more and more negative),

 the graph approaches, but never reaches, the horizontal axis.

**7.** The Empirical Rule:

 Approximately 68% of the area under the normal

 curve is between *x = μ* – *σ* and *x = μ* + *σ*;

 approximately 95% of the area under the normal

 curve is between *x = μ* – 2*σ* and *x = μ* + 2*σ*;

 approximately 99.7% of the area under the normal

 curve is between *x = μ* – 3*σ* and *x = μ* + 3*σ*.

**Examples and comparisons of normal distributions**



Here, we can see that increasing the mean from 0 to 3 caused the graph to shift three units to the right but maintained its shape.

Here, we can see that increasing the standard deviation from 1 to 2 caused the graph to become flatter and more spread out but maintained its location of center.

**Area Under a Normal Curve**

Suppose that a random variable *X* is normally distributed with mean *μ* and standard deviation *σ*. The area under the normal curve for any interval of values of the random variable *X* represents either

• the proportion of the population with the characteristic described by the interval of values or

• the probability that a randomly selected individual from the population will have the characteristic described by the interval of values.

**☺ Exercises**:

**1)** **Uniform Distribution**. The reaction time *X* (in minutes) of a certain chemical process follows a uniform probability distribution with 5 ≤ *X* ≤ 10.

**a)** Draw the graph of the density curve.

**b)** What is the probability that the reaction time is between 6 and 8 minutes?

**c)** What is the probability that the reaction time is between 5 and 8 minutes?

**d)** What is the probability that the reaction time is less than 6 minutes?

**☺ Exercises**:

*For Exercises #2 and #3, determine whether or not the histogram indicates that a normal distribution could be*

*used as a model for the variable.*



**2)** **Waiting in Line.** The relative frequency histogram

represents the waiting times (in minutes) to ride the

Demon Roller Coaster for 2000 randomly selected

people on a Saturday afternoon in the summer.

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**3)** **Incubation Times.** The relative frequency histogram

represents the incubation times of a random sample

of Rhode Island Red hens’ eggs.



**4)** The graph of a normal curve is given. Use the

graph to identify the values of *μ* and *σ.*

**5)** The graph of a normal curve is given. Use the

graph to identify the values of *μ* and *σ.*

**☺ Exercises**:

**6)** Draw a normal curve with *μ* = 30 and *σ =* 10. Label the mean and the inflection points.

**7)** **Refrigerators.** The lives of refrigerators are normally distributed with mean, *μ* = 14 years and standard deviation *σ =* 2.5 years.

*Source: Based on information from Consumer Reports.*



**a)** Draw a normal curve with the parameters labeled.

**b)** Shade the region that represents the proportion of refrigerators that last for more than 17 years.

**c)** Suppose the area under the normal curve to the right of *x* = 17 is 0.1151. What is the probability that refrigerator will last more than 17 years?

**d)** What is the probability that refrigerator will last less than 17 years?