SECTION 6.2 – The Binomial Probability Distribution

Criteria for a Binomial Probability Experiment

An experiment is said to be a binomial experiment if

1. The experiment is performed a fixed number of times. Each repetition of the experiment is called a trial.
2. The trials are independent. This means that the outcome of one trial will not affect the outcome of the other trials.
3. For each trial, there are two mutually exclusive (disjoint) outcomes: success or failure.
4. The probability of success is the same for each trial of the experiment.

Note: Repeated trials of an experiment that satisfy the above conditions are also called Bernoulli trials.

Binomial Random Variable

Let the random variable X be the number of successes in n trials of a binomial experiment. Then X is called a **binomial random variable**.

Notation Used in the Binomial Probability Distribution

- There are n independent trials of the experiment.
- Let p denote the probability of success for each trial so that $q = 1 - p$ is the probability of failure for each trial.
- Let X denote the number of successes in n independent trials of the experiment. So, $0 \le x \le n$.

Binomial Probability Distribution Function

The probability of obtaining x successes in n independent trials of a binomial experiment is given by

$$P(x) = {}_nC_x \cdot p^x \cdot q^{n-x}, \qquad x = 0, 1, 2, \ldots, n.$$

where p is the probability of success and $q = 1 - p$ is the probability of failure.

Note: ${}_nC_x = \dfrac{n!}{(n-x)!x!}$ and $n! = n(n-1)(n-2)\cdots(3)(2)(1)$. Also, $0! = 1$.

Mean (or Expected Value) and Standard Deviation of a Binomial Random Variable

A binomial experiment with n independent trials and probability of success p has a mean and standard deviation given by the formulas

$$\mu_X = np \quad \text{and} \quad \sigma_X = \sqrt{npq}$$

When a Binomial Distribution is Approximately Bell Shaped

For a fixed p, as the number of trials n in a binomial experiment increases, the probability distribution of the random variable X becomes bell shaped. As a rule of thumb, if $np(1-p) \ge 10$, the probability distribution will be approximately bell shaped.

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☺ **Exercises**:

For Exercises #1 through #4, determine which of the following probability experiments represents a binomial experiment. If the probability experiment is not a binomial experiment, state why.

1) A random sample of 30 cars in a used car lot is obtained, and their mileages recorded.

2) A poll of 1200 registered voters is conducted in which the respondents are asked whether they believe Congress should reform Social Security.

3) Three cards are selected from a standard 52-card deck with replacement. The number of kings selected is recorded.

4) In a town with 400 citizens, 100 randomly selected citizens are asked to identify their religion. The number who identify with a Christian religion is recorded.

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5) Roll a standard die 3 times. Let the random variable X represent the number of times a 5 is rolled. (So, a success is when a 5 is rolled in a single trial.)

a) What is the probability of a success?

b) What is the probability of a failure?

c) What is the probability of rolling exactly one 5?

d) What is the probability of rolling exactly two 5's?

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☺ **Example #1**:
A multiple-choice quiz of four questions is given. Each question has five possible answers. Let the random variable *X* represent the number of questions answered correctly. If a student was to guess on all the answers, find the binomial distribution of *X*.

Solution →
The random variable *X* takes on the values 0, 1, 2, 3, and 4, the possible number of successes in four trials. The probability of each value occurring is computed by using binomial trials with:

P(success) = *P*(answering the question correctly) → $p = \frac{1}{5}$

P(failure) = *P*(answering the question incorrectly) → $q = \frac{4}{5}$ (Since $q = 1 - p = 1 - \frac{1}{5} = \frac{4}{5}$)

x	*P*(*x*)
0	$P(0) = {}_4C_0 \cdot \left(\frac{1}{5}\right)^0 \cdot \left(\frac{4}{5}\right)^4 = 1 \cdot 1 \cdot (.4096) = .4096$
1	$P(1) = {}_4C_1 \cdot \left(\frac{1}{5}\right)^1 \cdot \left(\frac{4}{5}\right)^3 = 4 \cdot (.2) \cdot (.512) = .4096$
2	$P(2) = {}_4C_2 \cdot \left(\frac{1}{5}\right)^2 \cdot \left(\frac{4}{5}\right)^2 = 6 \cdot (.04) \cdot (.64) = .1536$
3	$P(3) = {}_4C_3 \cdot \left(\frac{1}{5}\right)^3 \cdot \left(\frac{4}{5}\right)^1 = 4 \cdot (.008) \cdot (.80) = .0256$
4	$P(4) = {}_4C_4 \cdot \left(\frac{1}{5}\right)^4 \cdot \left(\frac{4}{5}\right)^0 = 1 \cdot (.0016) \cdot 1 = .0016$

Notice that the probabilities in the binomial distribution sum to 1. Now, we can ask follow-up questions:

a) What is the probability that a student answers exactly 3 out of 4 questions correct?
Answer: $P(3) = .0256$

b) What is the probability that a student answers at least 3 out of 4 questions correct?
Answer: $P(x \geq 3) = P(3) + P(4) = .0256 + .0016 = .0272$

c) What is the probability that a student answers at most 1 out of 4 questions correct?
Answer: $P(x \leq 1) = P(0) + P(1) = .4096 + .4096 = .8192$

d) What is the probability that a student answers at least 1 question correct?
Answer: $P(x \geq 1) = 1 - P(0) = 1 - .4096 = .5904$

e) What is the mean number of questions a student will answer correctly?
Answer: Since this a binomial distribution, we can use the shortcut here: $\mu_X = np = (4)(\frac{1}{5}) = 0.8$

f) What is the standard deviation for the number of questions a student will answer correctly?
Answer: Again, since this a binomial, use the shortcut : $\sigma_X = \sqrt{npq} = \sqrt{(4)(\frac{1}{5})(\frac{4}{5})} = \sqrt{0.64} = 0.8$

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☺ **Exercises**:

6) A coin is tossed six times. Find the probability of its landing on tails exactly three times.

7) The probability that a student is accepted to a prestigious college is 0.35. If 8 students from the same school apply, what is the probability that at most 2 are accepted?

8) A certain drug was developed, tested, and found to be effective 70% of the time. Find the probability of successfully administering the drug to at least nine of ten patients.

9) As of August 13th, 2018, Mike Trout of the Los Angeles Angels has a lifetime batting average of .306. (This is the probability of getting a hit). Suppose he batted 5 times in one game.

a) Find the probability distribution of the random variable X, the number of hits possible in five at-bats.

b) If Mike batted 5 times in one game, what is the probability that he gets between 1 and 3 hits inclusive?

c) Find the mean number of hits in five at-bats.

d) Find the standard deviation for the number of hits in five at-bats.

☺ **Answers to selected Exercises:**

6) 0.3125; **7)** 0.4278 **8)** 0.1493 **9b)** 0.8059 **9c)** 1.53 **9d)** 1.03