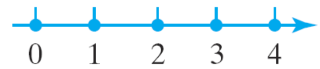
SECTION 6.1 – Discrete Random Variables

Random Variable

A **random variable** is a numerical measure of the outcome of a probability experiment, so its value is determined by chance. Random variables are typically denoted using capital letters such as *X*.

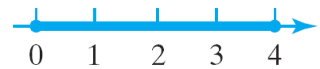
Discrete Random Variable

A **discrete random variable** has either a finite or countable number of values. The values of a discrete random variable can be plotted on a number line with space between each point.



Continuous Random Variable

A **continuous random variable** has infinitely many values. The values of a continuous random variable can be plotted on a line in an uninterrupted fashion.



Probability Distribution

The **probability distribution** of a discrete random variable *X* provides the possible values of the random variable and their corresponding probabilities. A probability distribution can be in the form of a table, graph, or mathematical formula.

Rules for a Discrete Probability Distribution

1. $\sum P(x) = 1$
2. $0 \le P(x) \le 1$

The Mean of a Discrete Random Variable

The **mean of a discrete random variable** is given by the formula

$$\mu_X = \sum \left[x \cdot P(x) \right]$$

where *x* is the value of the random variable and $P(x)$ is the probability of observing the value *x*.

Interpretation of the Mean of a Discrete Random Variable

Suppose an experiment is repeated *n* independent times and the value of the random variable *X* is recorded. As the number of repetitions of the experiment increases, the mean value of the *n* trials will approach μ_X, the mean of the random variable *X*. In other words, let x_1 be the value of the random variable *X* after the first experiment, x_2 be the value of the random variable *X* after the second experiment, and so on. Then

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

The difference between $\bar{x}$ and μ_X gets closer to 0 as *n* increases.

Note: The interpretation of **expected value** and **expectation** is the same as the interpretation of the mean of a discrete random variable.

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Standard Deviation of a Discrete Random Variable

The **standard deviation of a discrete random variable** X is given by

Conceptual Formula

$$\sigma_X = \sqrt{\sum\left[(x-\mu_X)^2 \cdot P(x)\right]}$$

Computational Formula

$$\sigma_X = \sqrt{\sum\left[x^2 \cdot P(x)\right] - \mu_X^2}$$

where x is the value of the random variable, μ_X is the mean of the random variable, and $P(x)$ is the probability of observing a value of the random variable.

☺ **Exercises**:

1) *Class Activity*: The data will be collected by each member of the class. Each student will write on the board, the total number of natural siblings they have. Let the random variable X represent the number of siblings each student has.

a) Draw a discrete probability distribution for the random variable X.

x	$P(x)$
0	
1	
2	
3	
4	
5	
6	
7	
8	

b) What are the possible values that the random variable X can take on?

c) Find $P(X=2)$ or $P(2)$.

d) Find $P(X \geq 3)$.

e) Find $P(X < 2)$.

f) Find $P(4 \leq X \leq 6)$.

In Exercises #2 and #3, determine whether the distribution is a discrete probability distribution. If not, state why.

2)

X	$P(x)$
0	0.1
1	0.5
2	0.05
3	0.25
4	0.1

3)

X	$P(x)$
6	0.35
12	0.10
18	0.25
24	0.05
30	0.45

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☺ **Exercises**:

4) **Pascal's Triangle.** Pascal's Triangle is a triangular array of numbers in which those at the ends of the rows are 1 and each of the others is the sum of the nearest two numbers in the row above (the apex, 1, being at the top). The first 7 lines are shown in the figure below:

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

Let the random variable X represent any number in any row of the first 7 lines in Pascal's Triangle.

a) Draw a discrete probability distribution for the random variable X.

x	$P(x)$

b) What is the probability that a randomly selected number in any row is 1?

c) What is the probability that a randomly selected number is greater than 6?

☺ **Exercises**:

5) What is the mean number of siblings for Exercise #1?

x	$P(x)$	$x \cdot P(x)$

6) What is the standard deviation for the number of siblings for Exercise #1.

x	$P(x)$	x^2	$x^2 \cdot P(x)$

☺ **Exercises**:

7) **Roulette**. Roulette is a common game of gambling found in Casinos. An American roulette wheel contains 38 numbers: 18 are red, 18 are black, and 2 are green. When the roulette wheel is spun, the ball is equally likely to land on any of the 38 numbers. Suppose that you bet $1 on red. If the ball lands on a red number, you win $1; otherwise you lose your $1. Let the random variable *X* be the amount you win on your $1 bet. Then the probability distribution is as follows:

Winnings	Probability
$1	0.474
–$1	0.526

a) Verify that the probability distribution is correct.

b) Compute and interpret the expected value of the game from the player's point of view.

c) Suppose over the course of one hour, a player can expect to play 30 games. How much should a player expect to win or lose over the course of three hours? (Assume the player continues to bet $1 on red each and every time).

8) **Blackjack.** Blackjack is a popular casino game in which a player is dealt two cards where the value of the card corresponds to the number on the card, face cards are worth ten, and aces are worth either one or eleven. The object is to get as close to 21 as possible without going over and have cards whose value exceeds that of the dealer. A blackjack is an ace and a ten in two cards. (Ten is a 10, J, Q, or K). It pays 1.5 times the bet. The dealer plays last and must draw a card with sixteen and hold with seventeen or more. The following distribution shows the winnings and probability for $20 bet. In cases where the dealer and player have the same value, there is a tie (called a "push").

Winnings	Probability
$0	0.0848
$30	0.0483
$20	0.376
–$20	0.4909

Source: *The Wizard of Odds*: https://wizardofodds.com/games/blackjack/appendix/4.
Note: The distribution above assumes the player is following basic strategy in a cut card game. The probabilities were obtained by a simulation of about ten billion hands played.

a) Compute and interpret the expected value of the game from the player's point of view.

b) Suppose over the course of one hour, a player can expect to play 40 hands. How much should a player expect to win or lose over the course of four hours?