SECTION 5.4 – Conditional Probability and the General Multiplication Rule

Conditional Probability

The notation $P(F \mid E)$ is read "the probability of event F given event E." It is the probability that the event F occurs, given that the event E has occurred.

Conditional Probability Rule

If E and F are any two events with $P(E) > 0$, then

$$P(F \mid E) = \frac{P(E \text{ and } F)}{P(E)} = \frac{N(E \text{ and } F)}{N(E)}$$

Note: Two events E and F are independent $P(E \mid F) = P(E)$ or equivalently, if $P(F \mid E) = P(F)$.

General Multiplication Rule

The probability that two events E and F both occur is

$$P(E \text{ and } F) = P(E) \cdot P(F \mid E)$$

☺ **Example #1**:

Suppose you roll a single die. What is the probability that you roll a '6', given that the outcome is even?

Solution #1 → The sample space for rolling a single die is {1, 2, 3, 4, 5, 6}, a total of 6 outcomes. Since it is given that the outcome is even, then the revised sample space is now {2, 4, 6}. So, there are now 3 outcomes, one of which is the value '6'. Thus, the probability is $\frac{1}{3}$.

Solution #2 → Let F = the event that the outcome is a '6'.

Let E = the event that the outcome is even. Then, $P(E) = \frac{1}{2}$.

Then, $P(E \text{ and } F) = \frac{1}{6}$, since the probability of a roll of '6' and 'even' is $\frac{1}{6}$.

Using the formula in the conditional probability rule,

$$P(F \mid E) = \frac{P(E \text{ and } F)}{P(E)} = \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{1}{3}$$

So, the probability using the rule is again $\frac{1}{3}$.

☺ **Example #2**:

Suppose you roll two standard dice. What is the probability that you roll a sum of seven, given that the sum is not twelve?

Solution → There are 36 different outcomes for rolling a two dice. Since it is given that the outcome is not a '12', then, there is only one outcome where a '12' is possible. So, the revised sample space is reduced to 35 different outcomes. There are still 6 different ways to obtain a '7'. Thus, the probability is $\frac{6}{35}$.

Using the formula in the conditional probability rule, $P(F \mid E) = \frac{P(E \text{ and } F)}{P(E)} = \frac{\frac{6}{36}}{\frac{35}{36}} = \frac{6}{35}$

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☺ **Exercises**:

1) Suppose you roll a single die. What is the probability that you roll a '4', given that the outcome is not a '5'?

2) Suppose you roll two standard dice. What is the probability that you roll a sum of two, given that neither die has a '6' showing?

3) Suppose you roll two standard dice. What is the probability that you roll a sum of eight, given that the sum is even?

4) Suppose you draw a card from a standard deck. What is the probability that you draw a heart, given that the card is not a club?

5) Suppose you draw a card from a standard deck. What is the probability that you draw a red card, given that the card is not a club?

6) Suppose you draw a card from a standard deck. What is the probability that you draw a diamond, given that the card is a black card?

7) Suppose you draw a card from a standard deck. What is the probability that you draw an even-numbered card, given that the card is not an ace or a face card?

8) An urn contains 5 red marbles and 5 blue marbles. A marble is drawn without replacement. A second marble is then drawn. What is the probability that you draw a red marble on the second draw, given that a red marble was obtained on the first draw?

9) An urn contains 4 green marbles and 7 yellow marbles. A marble is drawn without replacement. A second marble is then drawn. What is the probability that you draw both green marbles?

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☺ **Exercises**:

10) An urn contains 4 green marbles and 7 yellow marbles. A marble is drawn without replacement. A second marble is then drawn. What is the probability that you draw one green and one yellow marble?

11) The following is a contingency table providing a cross-classification of known worldwide reported shark attacks for the entire 2019 calendar year by country and lethality of attack. The data comes from the Global Shark Attack File (GSAF), a spreadsheet of human/shark interactions, compiled by the *Shark Research Institute*.

Country	Fatal	Nonfatal	Total
United States	3	51	54
Australia	1	20	21
South Africa	0	5	5
Réunion Island	4	0	4
Mexico	0	4	4
New Zealand	0	3	3
Bahamas	1	1	2
New Caledonia	1	1	2
French Polynesia	0	2	2
Papua New Guinea	1	0	1
Brazil	0	1	1
China	0	1	1
Israel	0	1	1
Seychelles	0	1	1
Total	11	91	102

For a randomly selected attack from that year:

a) What is the probability that the attack was from United States?

b) What is the probability that the attack was fatal?

c) What is the probability that the attack was from the United States and fatal?

d) What is the probability that the attack was from the United States, given that the attack was fatal?

e) What is the probability that the attack was fatal, given that the attack was from United States?

f) What is the probability that the attack was from Réunion, given that the attack was fatal?

g) What is the probability that the attack was fatal, given the attack was from Réunion Island?