SECTION 5.2 – The Addition Rule and Complements

Mutually Exclusive (or Disjoint)
Two or more events are mutually exclusive (or disjoint) events if and only if no two of them have outcomes in common.

Addition Rule for Mutually Exclusive Events
If E and F are mutually exclusive (or disjoint) events, then

$$P(E \text{ or } F) = P(E) + P(F)$$

The General Addition Rule
For any two events E and F,

$$P(E \text{ or } F) = P(E) + P(F) - P(E \text{ and } F)$$

Complement of an Event
Let S denote the sample space of a probability experiment and let E denote an event. The complement of E, denoted E^C, is all outcomes in the sample space S that are not outcomes in the event E.

Complement Rule
If E represents any event and E^C represents the complement of E, then

$$P(E^C) = 1 - P(E) \quad \text{or} \quad P(E) = 1 - P(E^C)$$

equivalently,

$$P(\text{not } E) = 1 - P(E) \quad \text{or} \quad P(E) = 1 - P(\text{not } E)$$

☺ **Exercises**:

1) Suppose $P(E) = 0.55$ and $P(F) = 0.35$. Find $P(E \text{ or } F)$, if $P(E \text{ and } F) = 0.15$.

2) Suppose $P(E) = 0.30$ and $P(F) = 0.50$. Find $P(E \text{ and } F)$, if $P(E \text{ or } F) = 0.75$.

3) Suppose $P(E) = 0.20$ and $P(F) = 0.30$. Find $P(E \text{ or } F)$, if E and F are mutually exclusive.

4) Suppose $P(E) = 0.35$ and $P(F) = 0.55$. Find $P(E \text{ and } F)$, if E and F are mutually exclusive.

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☺ **Exercises**:

5) **Family Structure.** The following probability model shows the distribution of family structure among families with at least one child younger than 18 years of age in 2013.

Family Structure	Probability
Two married parents, first marriage	0.46
Two married parents, one or both remarried	0.15
Single Parent	0.34
No parent at home	0.05

a) Verify that this is a probability model.

b) What is the probability that a randomly selected family with at least one child younger than 18 years of age in 2013 had two married parents in their first marriage?

c) What is the probability that a randomly selected family with at least one child younger than 18 years of age in 2013 had two married parents?

d) What is the probability that a randomly selected family with at least one child younger than 18 years of age in 2013 had at least one parent at home?

6) **Social Media.** Harris Interactive conducted a survey in which they asked adult Americans (18 years or older) whether they use social media (Facebook, Twitter, and so on) regularly. The following contingency table is based on the results of the survey.

	18–34	35–44	45–54	55+	Total
Use social media	117	89	83	49	**338**
Do not use social media.	33	36	57	66	**192**
Total	**150**	**125**	**140**	**115**	**530**

a) If an adult American is randomly selected, what is the probability he or she uses social media?

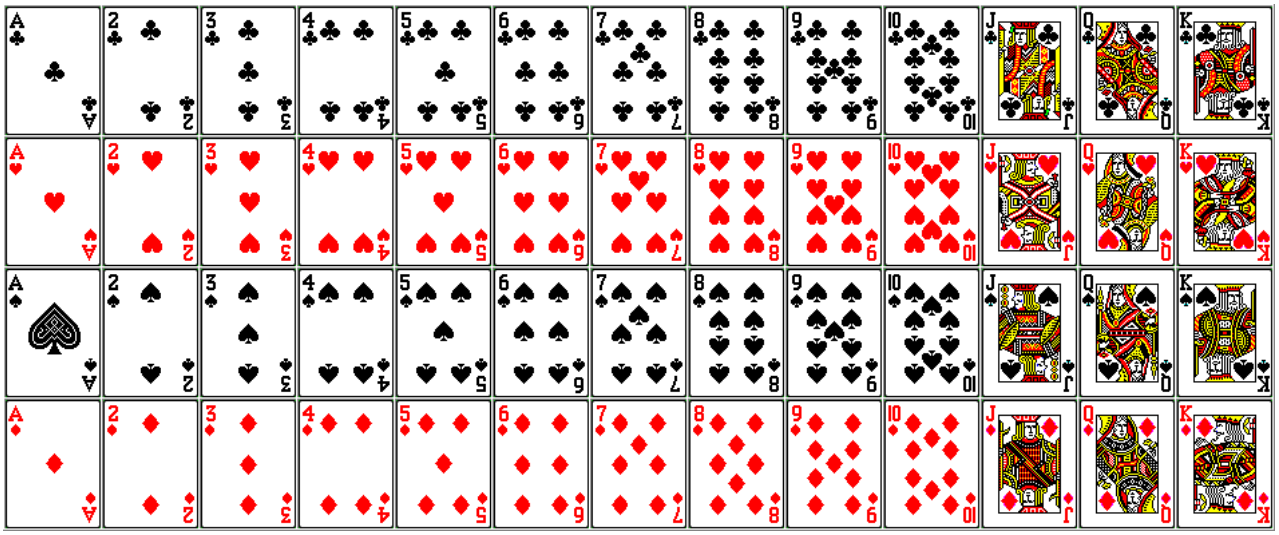
b) If an adult American is randomly selected, what is the probability he or she is 45 to 54 years of age?

c) If an adult American is randomly selected, what is the probability he or she is a 35– to 44–year old social media user?

d) If an adult American is randomly selected, what is the probability he or she is 35 to 44 years old or uses social media?

SECTION 5.2 – The Addition Rule and Complements

The Sample Space for a standard (no jokers) deck of cards.



☺ **Exercises**:

Experiment – Draw a card from a standard deck of cards.

7) What is the probability that you draw a heart? _____

8) What is the probability that you draw a red card? _____

9) What is the probability that you draw a '7'? _____

10) What is the probability that you draw a '5' of clubs? _____

11) What is the probability that you draw a '9' of moons? _____

12) What is the probability that you do not draw an ace? _____

13) What is the probability that you draw a '6' or a queen? _____

14) What is the probability that you draw an ace or spades? _____

15) What is the probability that you draw a face card or heart? _____

16) What is the probability that you draw a red card or a '3'? _____

17) What is the probability that you draw an even numbered card or a king? _____

18) For fun only: What is the probability that you draw a one-eyed jack? _____

19) For fun only: What is the probability that you draw a suicide king? _____

SECTION 5.2 – The Addition Rule and Complements

☺ **Extra Practice Problems**:

Experiment – Draw a card from a standard deck of cards.

20) What is the probability that you draw a club? _____

21) What is the probability that you draw a '9'? _____

22) What is the probability that you draw a '9' of clubs? _____

23) What is the probability that you draw a '9' or a club? _____

24) What is the probability that you draw a club or spade? _____

25) What is the probability that you draw a club and spade? _____

26) What is the probability that you draw a '2' or a heart? _____

27) What is the probability that you draw an ace or diamond? _____

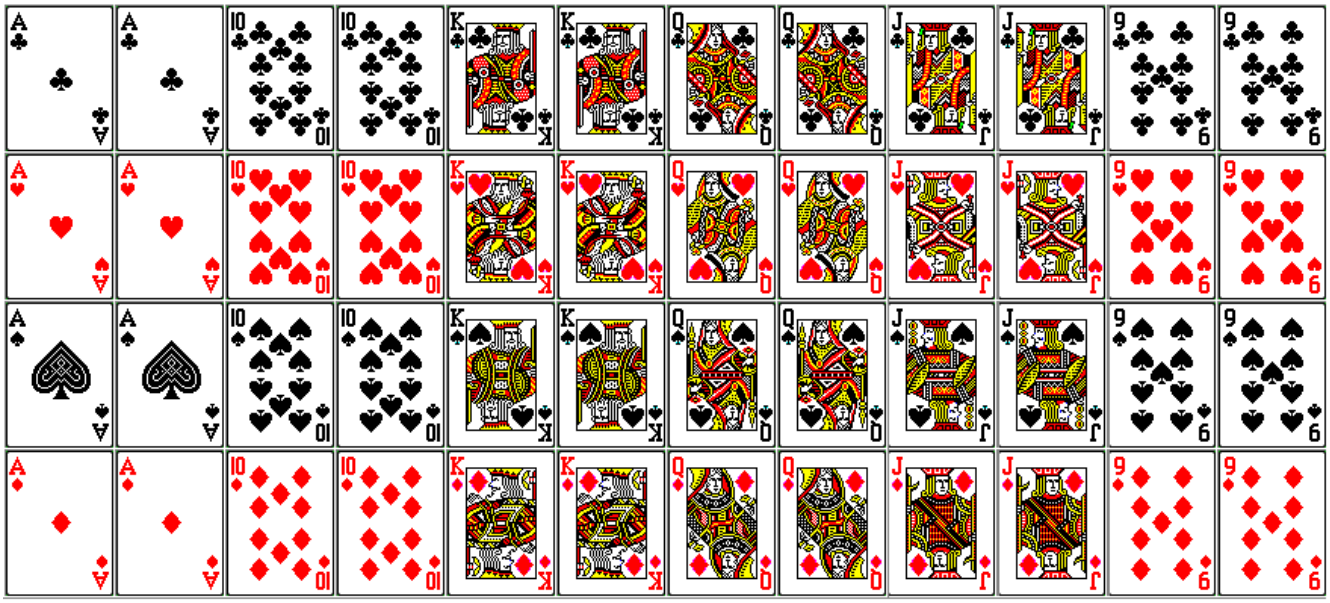
28) What is the probability that you draw a face card or red card? _____

29) For fun only: The Scots consider the Nine of Diamonds to be an unlucky card, and its nickname goes back to the 19th century. The Seven of Diamonds is known as the 'Beer Card' and originated in Denmark. The Four of Clubs known as the 'Devil's Bedpost' is derived from the design of the pips on the card, which can be imagined to be the four posts of a bed. The Ace of Spades is commonly referred to as 'The Death Card'. What the probability that you draw an 'unlucky card' or the 'Beer Card' or the 'Devil's Bedpost' or the 'Death Card'?

_____

SECTION 5.2 – The Addition Rule and Complements

Pinochle is a trick-taking, Ace-Ten card game typically for two to four players and is played with a 48-card deck. It is derived from the French card game Bezique or Bésigue. Players score points by trick-taking and also by forming combinations of cards into melds. Each hand is played in three phases: bidding, melds, and tricks. The standard game today is called "Partnership Auction Pinochle." A Pinochle deck consists of two copies of each of the 9, 10, jack, queen, king, and ace cards of all four suits, for 48 cards per deck. Aces are always considered high. Pinochle follows a nonstandard card ordering. The complete ordering from highest to lowest is A, 10, K, Q, J, 9. The sample space is shown below.



☺ **Exercises**:

Experiment – Draw a card from a Pinochle deck of cards.

30) How many cards are there in a Pinochle deck? _____

31) What is the probability that you draw a heart? _____

32) What is the probability that you draw a red card? _____

33) What is the probability that you draw a '9'? _____

34) What is the probability that you draw a '10' of diamonds? _____

35) What is the probability that you draw a '7'? _____

36) What is the probability that you draw a 'face' card? _____

37) What is the probability that you draw a red ace? _____

38) Under Pinochle rules, what is the probability that you draw a card higher than a king? _____

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☺ **Solutions to Exercises and Extra Practice Problems:**

1) 0.75; **2)** 0.05; **3)** 0.50; **4)** 0; **5a)** 0.46 + 0.15 + 0.34 + 0.05 = 1; **5b)** 0.46; **5c)** 0.61; **5d)** 0.95;

6a) 0.638; **6b)** 0.264; **6c)** 0.168; **6d)** 0.706; **7)** $\frac{13}{52}=\frac{1}{4}$; **8)** $\frac{26}{52}=\frac{1}{2}$; **9)** $\frac{4}{52}=\frac{1}{13}$; **10)** $\frac{1}{52}$; **11)** 0;

12) $\frac{48}{52}=\frac{12}{13}$; **13)** $\frac{8}{52}=\frac{2}{13}$; **14)** $\frac{16}{52}=\frac{4}{13}$; **15)** $\frac{22}{52}=\frac{11}{26}$; **16)** $\frac{28}{52}=\frac{7}{13}$; **17)** $\frac{24}{52}=\frac{6}{13}$; **18)** $\frac{2}{52}=\frac{1}{26}$; **19)** $\frac{1}{52}$;

20) $\frac{13}{52}=\frac{1}{4}$; **21)** $\frac{4}{52}=\frac{1}{13}$; **22)** $\frac{1}{52}$; **23)** $\frac{16}{52}=\frac{4}{13}$; **24)** $\frac{26}{52}=\frac{1}{2}$; **25)** 0; **26)** $\frac{16}{52}=\frac{4}{13}$; **27)** $\frac{16}{52}=\frac{4}{13}$;

28) $\frac{32}{52}=\frac{8}{13}$; **29)** $\frac{4}{52}=\frac{1}{13}$; **30)** 48; **31)** $\frac{12}{48}=\frac{1}{4}$; **32)** $\frac{24}{48}=\frac{1}{2}$; **33)** $\frac{8}{48}=\frac{1}{6}$; **34)** $\frac{2}{48}=\frac{1}{24}$; **35)** 0;

36) $\frac{24}{48}=\frac{1}{2}$; **37)** $\frac{4}{48}=\frac{1}{12}$; **38)** $\frac{16}{48}=\frac{1}{3}$.