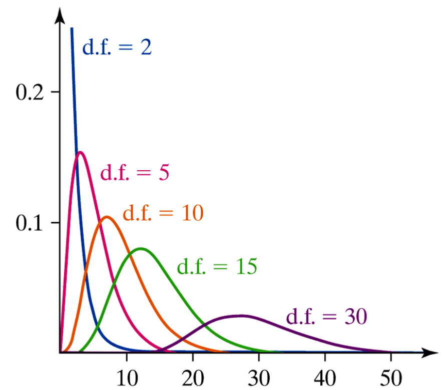
SECTION 12.1 – Goodness-of-Fit-Test

Characteristics of the Chi-Square Distribution

1. It is not symmetric. It is right-skewed for smaller degrees of freedom.
2. Its shape depends on the degrees of freedom, just like Student's t-distribution.
3. As the number of degrees of freedom increases, it becomes more symmetric, as illustrated in the figure below.
4. The values of χ^2 are nonnegative. That is, the values of χ^2 are greater than or equal to 0.



Goodness-of-Fit-Test

A **goodness-of-fit test** is an inferential procedure used to determine whether a frequency distribution follows a specific distribution.

Expected Counts

Suppose there are n independent trials of an experiment with $k \geq 3$ mutually exclusive possible outcomes. Let p_1 represent the probability of observing the first outcome and E_1 represent the expected count of the first outcome; p_2 represent the probability of observing the second outcome and E_2 represent the expected count of the second outcome; and so on. The expected counts for each possible outcome are given by

$$E = \mu_i = np_i \quad \text{for} \quad i = 1, 2, \ldots, k$$

Test Statistic for Goodness-of-Fit Tests

Let O_i represent the observed counts of category i, E_i represent the expected counts of category i, k represent the number of categories, and n represent the number of independent trials of an experiment. Then the formula

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}, \qquad i = 1, 2, \ldots, k$$

approximately follows the chi-square distribution with $k - 1$ degrees of freedom, provided that

1. all expected frequencies are greater than or equal to 1 (all $E_i \geq 1$) and
2. no more than 20% of the expected frequencies are less than 5.

Note: $E = np_i$ for $i = 1, 2, \ldots, k$

SECTION 12.1 – Goodness-of-Fit-Test

The Goodness-of-Fit Test

To test hypothesis regarding a distribution, use the steps that follow.

Step 1 – Determine the null and alternative hypotheses.

H_0 : The random variable follows a certain distribution
H_1 : The random variable does not follow the distribution in the null hypothesis.

Step 2 – Decide on a level of significance, α, depending on the seriousness of making a Type I error.

Step 3 –

a) Calculate the expected counts, E_i, for each of the k categories: $E = np_i$ for $i = 1, 2, \ldots, k$, where n is the number of trials and p_i is the probability of the ith category, assuming that the null hypothesis is true.

b) Verify that the requirements for the goodness-of-fit test are satisfied.

1. All expected counts are greater than or equal to 1 (all $E_i \geq 1$).
2. No more than 20% of the expected counts are less than 5.

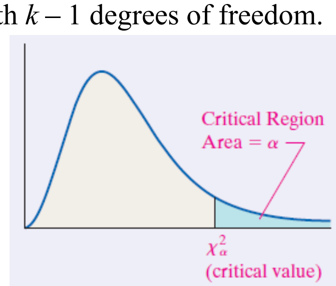
c) Compute the **test statistic** $\chi_0^2 = \sum \frac{(O_i - E_i)^2}{E_i}$.

Note: O_i is the observed count for the ith category.

CLASSICAL APPROACH OR ***P*-VALUE APPROACH**

CLASSICAL APPROACH

d) Determine the critical value. All goodness-of-fit tests are right-tailed tests, so the critical value is χ^2 a with $k - 1$ degrees of freedom.

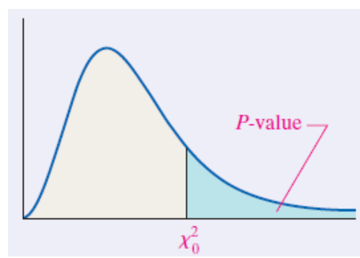


Step 4 – Compare the critical value to the statistic.
If $\chi_0^2 > \chi_\alpha^2$, reject the null hypothesis.

***P*-VALUE APPROACH**

By Hand:

d) Use Table VIII to approximate the *P*-value by determining the area under the chi-square distribution with $k - 1$ degrees of freedom to the right of the test statistic.



By Technology: Use a statistical spreadsheet or calculator with statistical capabilities to obtain the *P*-value.

Step 4 – If *P*-value $\leq \alpha$, reject the null hypothesis.

Step 5 – State the conclusion.

SECTION 12.1 – Goodness-of-Fit-Test

☺ **Exercises**:

Use Table VIII to determine the required χ^2-values. Illustrate your work graphically.

1) For a χ^2-curve with 22 degrees of freedom, find the χ^2-value that has area 0.01 to its right.

2) For a χ^2 curve with df = 4, determine $\chi^2_{0.005}$.

3) Determine the expected counts for each outcome. Also, what is the degrees of freedom for the chi-square test statistic.

n = 500	A	B	C	D
p_i	0.2	0.1	0.45	0.25
Expected Counts				

SECTION 12.1 – Goodness-of-Fit-Test

☺ **Exercises**:

4) **Bicycle Deaths.** A researcher wanted to determine whether bicycle deaths were uniformly distributed over the days of the week. She randomly selected 210 deaths that involved a bicycle, recorded the day of the week on which the death occurred, and obtained the following results (the data are based on information obtained from the Insurance Institute for Highway Safety).

Day	Frequency
Sunday	18
Monday	37
Tuesday	17
Wednesday	29
Thursday	36
Friday	43
Saturday	30

Is there reason to believe that bicycle fatalities occur with equal frequency with respect to day of the week at the $\alpha = 0.05$ level of significance?

Day	Observed Frequency O	Expected Frequency E	Difference $O - E$	Square of Difference $(O - E)^2$	χ^2 Subtotal $(O - E)^2/E$
Sunday					
Monday					
Tuesday					
Wednesday					
Thursday					
Friday					
Saturday					
Total			**Value of χ^2 Test Statistic →**		

SECTION 12.1 – Goodness-of-Fit-Test

☺ **Exercises**:

5) The following table shows the ABO blood type distribution for the population averages of the United States.

Blood Type	Percentage
O	44%
A	42%
B	10%
AB	4%

A local hospital serviced a random sample of 187 patients on a particular day. The ABO blood types were obtained and recorded in the table to the right:

Blood Type	Frequency
O	67
A	83
B	29
AB	8

At the 1% significance level, does it appear that the data provide sufficient evidence to conclude that the distribution of the patient's ABO blood-types are different than the nation-wide distribution?

Blood Type	Observed Frequency O	Expected Frequency E	Difference $O - E$	Square of Difference $(O - E)^2$	χ^2 Subtotal $(O - E)^2/E$
O					
A					
B					
AB					
Total			**Value of χ^2 Test Statistic →**		