Section 10.3 – Hypothesis Tests for a Population Mean

Testing Hypotheses Regarding a Population Mean, μ

Use steps 1 through 5, provided that

- the sample is obtained by simple random sampling or from a randomized experiment.
- the sampled values are independent of each other, ($n < 0.05N$).
- the sample has no outliers and the population from which the sample is drawn is normally distributed, or the sample size, n, is large ($n \geq 30$).

Step 1 – Determine the null and alternative hypotheses. The hypotheses can be structured in one of three ways:

Two- Tailed	Left-Tailed	Right-Tailed
$H_0 : \mu = \mu_0$	$H_0 : \mu = \mu_0$	$H_0 : \mu = \mu_0$
$H_1 : \mu \neq \mu_0$	$H_1 : \mu < \mu_0$	$H_1 : \mu > \mu_0$

Note: μ_0 is the assumed value of the population mean.

Step 2 – Select a level of significance, α, depending on the seriousness of making a Type I error.

Step 3 – Compute the **test statistic** $t_0 = \dfrac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$

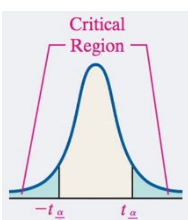
which follows Student's t-distribution with $n - 1$ degrees of freedom.

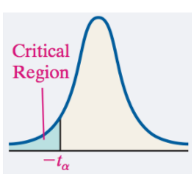
CLASSICAL APPROACH OR *P*-VALUE APPROACH

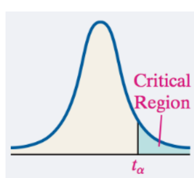
CLASSICAL APPROACH

Use Table VII to determine the critical value .

$\pm t_{\alpha/2}$	or	$-t_\alpha$	or	$+t_\alpha$
(Two-tailed)		(Left-tailed)		(Right-tailed)





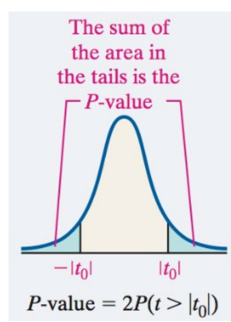


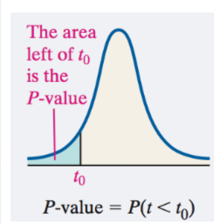
Step 4 – Compare the critical value with the test statistic.

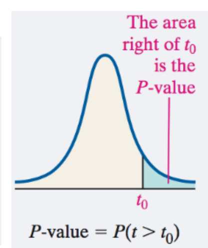
Two- Tailed	Left-Tailed	Right-Tailed
If $t_0 < -t_{\frac{\alpha}{2}}$ or $t_0 > t_{\frac{\alpha}{2}}$ reject the null hypothesis	If $t_0 < -t_\alpha$ reject the null hypothesis	If $t_0 > t_\alpha$ reject the null hypothesis

P-VALUE APPROACH

By Hand:
Use Table VII to approximate the *P*-value.







By Technology: Use a statistical spreadsheet or calculator with statistical capabilities to obtain the *P*-value.

Step 4 – If *P*-value $\leq \alpha$, reject the null hypothesis.

Step 5 – State the conclusion.

Section 10.3 – Hypothesis Tests for a Population Mean

☺ **Exercises**:

Use Table VII to approximate the P-value when testing a hypothesis regarding a population mean, μ.

1) Right-tailed test, $n = 8$, $t = 1.792$

2) Right-tailed test, $n = 25$, $t = 2.338$

3) Left-tailed test, $n = 39$, $t = -3.912$

4) Left-tailed test, $n = 49$, $t = -3.111$

5) Two-tailed test, $n = 15$, $t = 1.482$

Section 10.3 – Hypothesis Tests for a Population Mean

☺ **Exercises**:

In Exercises #6 through #8, use either the classical or P-value approach or both.

6) **Are Women Getting Taller?** In 1990, the mean height of women 20 years of age or older was 63.7 inches. Suppose that a random sample of 45 women who are 20 years of age or older today results in a mean height of 63.9 inches. Conduct the appropriate hypothesis test to assess whether women are taller today using a significance level of 0.05. (Assume that $s = 0.85$).

Source: Centers for Disease Control and Prevention's Advance Data Report, No. 347.

Assumptions:

Step 1: Determine the hypotheses

Step 2: Select the level of significance

Step 3: Compute the test statistic

Step 4: Decide on whether to reject or not reject H_0

Step 5: State the conclusion

Section 10.3 – Hypothesis Tests for a Population Mean

☺ **Exercises**:

7) **SAT Verbal Scores.** Do students who learned English as well as another language simultaneously score worse on the SAT Critical Reading exam than the general population of test takers? The mean score among all test takers on the SAT Critical Reading exam is 501. A random sample of 100 test takers who learned English as well as another language simultaneously had a mean SAT Critical Reading score of 485 with a standard deviation of 116. Do these results suggest that students who learn English as well as another language simultaneously score worse on the SAT Critical Reading exam? Use a significance level of 10%.

Assumptions:

Step 1: Determine the hypotheses

Step 2: Select the level of significance

Step 3: Compute the test statistic

Step 4: Decide on whether to reject or not reject H_0

Step 5: State the conclusion

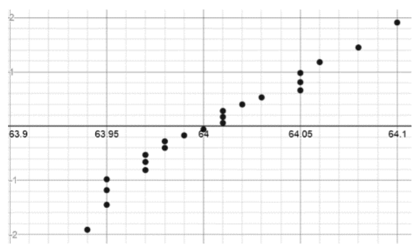
Section 10.3 – Hypothesis Tests for a Population Mean

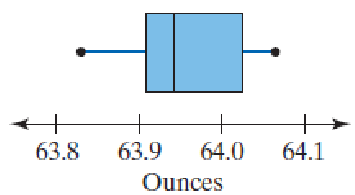
☺ **Exercises**:

8) **Filling Bottles.** A certain brand of apple juice is supposed to have 64 ounces of juice. Because the penalty for underfilling bottles is severe, the target mean amount of juice is 64.05 ounces. However, the filling machine is not precise, and the exact amount of juice varies from bottle to bottle. The quality-control manager wishes to verify that the mean amount of juice in each bottle is 64.05 ounces so that she can be sure that the machine is not over- or underfilling. She randomly samples 22 bottles of juice, measures the content, and obtains the following data:

64.05	64.05	64.03	63.97	63.95	64.02
64.01	63.99	64.00	64.01	64.06	63.94
63.98	64.05	63.95	64.01	64.08	64.01
63.95	63.97	64.10	63.98		

a) Because the sample size is small, she must verify that the amount of juice is normally distributed and the sample does not contain any outliers. The normal probability plot and boxplot are shown. Are the conditions for testing the hypothesis satisfied?





b) Should the assembly line be shut down so that the machine can be recalibrated? Use a 0.01 level of significance. (Note: The sample mean is $\overline{x} = 64.01$ and the sample standard deviation is $s = 0.045$.)

Assumptions:

Step 1: Determine the hypotheses

Step 2: Select the level of significance

Step 3: Compute the test statistic

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☺ **Exercises (continued)**:

Step 4: Decide on whether to reject or not reject H_0

Step 5: State the conclusion

☺ **Exercises (continued):**

c) Explain why a level of significance of $\alpha = 0.01$ might be more reasonable than $\alpha = 0.10$. [Hint: Consider the consequences of incorrectly rejecting the null hypothesis.]