Section 10.2 – Hypothesis Tests for a Population Proportion

Statistically Significant
When observed results are unlikely under the assumption that the null hypothesis is true, we say the result is **statistically significant** and we reject the null hypothesis.

Hypothesis Testing Using the Classical Approach (or Critical Value Approach)
If the sample proportion is too many standard deviations from the proportion stated in the null hypothesis, we reject the null hypothesis.

***P*-value**
A ***P*-value** is the probability of observing a sample statistic as extreme or more extreme than one observed under the assumption that the statement in the null hypothesis is true. Put another way, the *P*-value is the likelihood or probability that a sample will result in a statistic such as the one obtained if the null hypothesis is true.

Hypothesis Testing Using the *P*-Value Approach
If the probability of getting a sample proportion as extreme or more extreme than the one obtained is small under the assumption the statement in the null hypothesis is true, reject the null hypothesis.

Guidelines for using the *P*-value to assess the evidence against the null hypothesis.

P-value	Evidence against H_0
$P > 0.10$	Weak or none
$0.05 < P \le 0.10$	Moderate
$0.01 < P \le 0.05$	Strong
$P \le 0.01$	Very strong

Section 10.2 – Hypothesis Tests for a Population Proportion

<u>Testing Hypotheses Regarding a Population Proportion, *p*</u>

Use steps 1 through 5, provided that

- the sample is obtained by simple random sampling or the data result from a randomized experiment.
- the sampled values are independent of each other, ($n \le 0.05N$).
- $np_0(1 - p_0) \ge 10$, (approximately normal)

<u>Step 1</u> – Determine the null and alternative hypotheses. The hypotheses can be structured in one of three ways:

Two- Tailed	Left-Tailed	Right-Tailed
$H_0 : p = p_0$	$H_0 : p = p_0$	$H_0 : p = p_0$
$H_1 : p \ne p_0$	$H_1 : p < p_0$	$H_1 : p > p_0$

Note: p_0 is the assumed value of the population proportion.

<u>Step 2</u> – Select a level of significance, α, depending on the seriousness of making a Type I error.

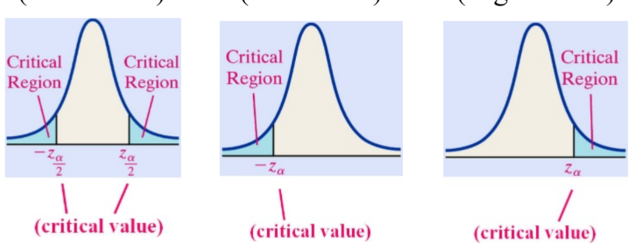
<u>Step 3</u> – Compute the **test statistic** $z_0 = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$.

CLASSICAL APPROACH OR ***P*-VALUE APPROACH**

CLASSICAL APPROACH

Use Table V to determine the critical value.

$\pm z_{\alpha/2}$ (Two-tailed) or $-z_\alpha$ (Left-tailed) or $+z_\alpha$ (Right-tailed)

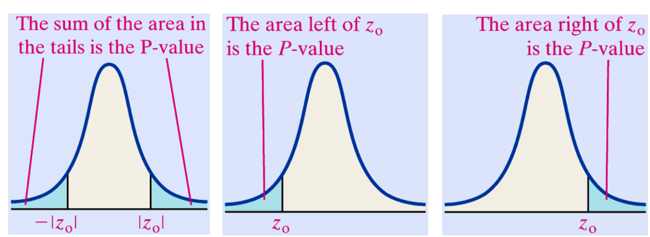


<u>Step 4</u> – Compare the critical value with the test statistic.

Two- Tailed	Left-Tailed	Right-Tailed
If $z_0 < -z_{\frac{\alpha}{2}}$ or $z_0 > z_{\frac{\alpha}{2}}$ reject the null hypothesis	If $z_0 < -z_\alpha$ reject the null hypothesis	If $z_0 > z_\alpha$ reject the null hypothesis

***P*-VALUE APPROACH**

By Hand:
Use Table V to determine the *P*-value.



By Technology: Use a statistical spreadsheet or calculator with statistical capabilities to obtain the *P*-value.

<u>Step 4</u> – If *P*-value $\le \alpha$, reject the null hypothesis.

<u>Step 5</u> – State the conclusion.

SECTION 10.2 – Hypothesis Tests for a Population Proportion

☺ **Exercises**:

In Exercises #1 through #3, use either the classical or P-value approach or both.

1) **Gender.** Globally the long-term proportion of newborns who are male is 51.46%. A researcher believes that currently under the severe economic conditions today, that the proportion of boys has changed. To test this belief randomly selected birth records of 5,000 babies this year were examined. It was found in the sample that 2,627 of the newborns were boys. Determine whether there is sufficient evidence, at the 10% level of significance, to support the researcher's belief.

Assumptions:

Step 1: Determine the hypotheses

Step 2: Select the level of significance

Step 3: Compute the test statistic

Step 4: Decide on whether to reject or not reject H_0

Step 5: State the conclusion

SECTION 10.2 – Hypothesis Tests for a Population Proportion

☺ **Exercises**:

2) **Poverty in the U.S.** According to the Federal Poverty Measure 12% of the U.S. population lives in poverty. The governor of a certain state believes that the proportion there is lower. In a sample of size 1,550, it was determined that 163 were impoverished according to the federal measure. Test whether the true proportion of the state's population that is impoverished is less than 12%, at the 1% level of significance.

Assumptions:

Step 1: Determine the hypotheses

Step 2: Select the level of significance

Step 3: Compute the test statistic

Step 4: Decide on whether to reject or not reject H_0

Step 5: State the conclusion

SECTION 10.2 – Hypothesis Tests for a Population Proportion

☺ **Exercises**:

3) **Births to Unmarried Women.** In 2016 The National Center for Health Statistics reported that 39.8% of all births were to unmarried women. Suppose that a random sample this year of 1,000 births showed 444 to unmarried women. If appropriate, test whether the population proportion has increased since 2016, using a level of significance $\alpha = 0.10$.

Source: https://www.cdc.gov/nchs/pressroom/sosmap/unmarried/unmarried.htm

Assumptions:

Step 1: Determine the hypotheses

Step 2: Select the level of significance

Step 3: Compute the test statistic

Step 4: Decide on whether to reject or not reject H_0

Step 5: State the conclusion