SECTION 10.1 – The Language of Hypothesis Testing

Hypothesis

A **hypothesis** is a statement regarding a characteristic of one or more populations.

Hypothesis Testing

Hypothesis testing is a procedure, based on sample evidence and probability, used to test statements regarding a characteristic of one or more populations.

Steps in Hypothesis Testing

1. Make a statement regarding the nature of the population.
2. Collect evidence (sample data) to test the statement.
3. Analyze the data to assess the plausibility of the statement.

Null Hypothesis

The **null hypothesis**, denoted H_0 (read "H-naught"), is a statement to be tested. The null hypothesis is a statement of no change, no effect, or no difference and is assumed true until evidence indicates otherwise.

Alternative Hypothesis

The **alternative hypothesis**, denoted H_1 (read "H-one"), is a statement that we are trying to find evidence to support. (Sometimes H_a is used instead of H_1).

Possibilities for the Alternative Hypothesis

1. Equal hypothesis versus not equal hypothesis **(two-tailed test)**

H_0: parameter = some value
H_1: parameter ≠ some value

2. Equal versus less than **(left-tailed test)**

H_0: parameter = some value
H_1: parameter < some value

3. Equal versus greater than **(right-tailed test)**

H_0: parameter = some value
H_1: parameter > some value

Four Outcomes from Hypothesis Testing

1. Reject the null hypothesis when the alternative hypothesis is true. This decision would be correct.
2. Do not reject the null hypothesis when the null hypothesis is true. This decision would be correct.
3. Reject the null hypothesis when the null hypothesis is true. This decision would be incorrect. This type of error is called a **Type I error**.
4. Do not reject the null hypothesis when the alternative hypothesis is true. This decision would be incorrect. This type of error is called a **Type II error**.

		Reality	
		H_0 is True	H_0 is False
Conclusion	Do not reject H_0	Correct decision	Type II error
	Reject H_0	Type I error	Correct decision

$\alpha = P$(Type I error) $= P$(rejecting H_0 when H_0 is true)
$\beta = P$(Type II error) $= P$(not rejecting H_0 when H_1 is true)

SECTION 10.1 – The Language of Hypothesis Testing

☺ **Example #1**:

Here is an example of applying Type I & Type II errors using a man on trial for murder:

A man goes to trial, where he is being tried for the murder of his friend.

We can put this case in a hypothesis testing framework. The hypotheses being tested are:

$$H_0 : \text{Not Guilty}$$
$$H_1 : \text{Guilty}$$

A **Type I error** is committed if we reject, when it is true.
In other words, he did not kill his friend, but was found guilty and is punished for a crime he did not really commit.

A **Type II error** is committed if we fail to reject, when it is false.
In other words, if the man actually did kill his friend, but was found not guilty and was not punished.

Significance Level
The **level of significance**, α, is the probability of making a Type I error.

Relation Between Type I and Type II Error Probabilities
For a fixed sample size, the smaller we specify the significance level, α, the larger will be the probability, β, of not rejecting a false null hypothesis.

Possible Conclusions for a Hypothesis Test
Suppose that a hypothesis test is conducted at a small significance level.

- If the null hypothesis is rejected, we conclude that the alternative hypothesis is true.
- If the null hypothesis is not rejected, we conclude that the data do not provide sufficient evidence to support the alternative hypothesis.

Note: When the null hypothesis is rejected in a hypothesis test, we say there is sufficient to support the statement in the alternative hypothesis. When the null hypothesis is not rejected in a hypothesis test, we say that there is not sufficient evidence to support the statement in an alternative hypothesis. We never say accept.

☺ **Exercises**:

In Exercises #1 through #3, the null and alternative hypotheses are given. Determine whether the hypothesis test is left-tailed, right tailed, or two-tailed.

1) $H_0 : p = 0.2$
$H_1 : p < 0.2$

2) $H_0 : \mu = 76.2$
$H_1 : \mu > 76.2$

3) $H_0 : p = 0.35$
$H_1 : p \neq 0.35$

SECTION 10.1 – The Language of Hypothesis Testing

☺ **Exercises**:

In Exercise #4 through #6,

a) *determine the null and alternative hypotheses*
b) *explain what it would mean to make a Type I error.*
c) *explain what it would mean to make a Type II error.*

4) **Pizza.** Historically, the time to order and deliver a pizza at Jimbo's pizza was 48 minutes. Jim the owner, implements a new system for ordering and delivering pizzas that he believes will reduce the time required to get a pizza to his customers.

5) **Overweight.** According to the Centers for Disease Control and Prevention, 19.6% of children aged 6 to 11 years are overweight. A school nurse thinks that the percentage of 6- to 11-year-olds who are overweight is higher in her school district.

6) **Credit-Card Debt.** According to creditcard.com, the mean outstanding credit-card debt of college undergraduates was $3173 in 2010. A researcher believes that this amount has changed since then.

SECTION 10.1 – The Language of Hypothesis Testing

☺ **Exercises**:

7) **Popcorn.** Consumption According to popcorn.org, the mean consumption of popcorn annually by Americans is 54 quarts. The marketing division of popcorn.org unleashes an aggressive campaign designed to get Americans to consume even more popcorn.

a) Determine the null and alternative hypotheses that would be used to test the effectiveness of the marketing campaign.

b) A sample of 800 Americans provides enough evidence to conclude that the marketing campaign was effective. Provide a statement that should be put out by the marketing department.

c) Suppose, in fact, that the mean annual consumption of popcorn after the marketing campaign is 53.4 quarts. Has a Type I or Type II error been made by the marketing department? If they tested the hypothesis at the $\alpha = 0.05$ level of significance, what is the probability of making a Type I error?

8) **Migraines.** According to the Centers for Disease Control, 15.2% of American adults experience migraine headaches. Stress is a major contributor to the frequency and intensity of headaches. A massage therapist feels that she has a technique that can reduce the frequency and intensity of migraine headaches.

Source: The Centers for Disease Control.

a) Determine the null and alternative hypotheses that would be used to test the effectiveness of the massage therapist's techniques.

b) A sample of 500 American adults who participated in the massage therapist's program results in data that indicate that the null hypothesis should not be rejected. Provide a statement that supports the massage therapist's program.

c) Suppose, in fact, that the percentage of patients in the program who experience migraine headaches is less than 15.2%. Was a Type I or Type II error committed?