**Hypothesis**

A **hypothesis** is a statement regarding a characteristic of one or more populations.

**Hypothesis Testing**

**Hypothesis testing** is a procedure, based on sample evidence and probability, used to test statements regarding a characteristic of one or more populations.

**Steps in Hypothesis Testing**

**1.** Make a statement regarding the nature of the population.

**2.** Collect evidence (sample data) to test the statement.

**3.** Analyze the data to assess the plausibility of the statement.

**Null Hypothesis**

The **null hypothesis**, denoted *H*0 (read “H-naught”), is a statement to be tested. The null hypothesis is a statement of no change, no effect, or no difference and is assumed true until evidence indicates otherwise.

**Alternative Hypothesis**

The **alternative hypothesis**, denoted *H*1 (read “H-one”), is a statement that we are trying to find evidence to support. (Sometimes *H*a is used instead of *H*1).

**Possibilities for the Alternative Hypothesis**

*H*0: parameter = some value

*H*1: parameter ≠ some value

**1.** Equal hypothesis versus not equal hypothesis **(two-tailed test)**

*H*0: parameter = some value

*H*1: parameter < some value

**2.** Equal versus less than **(left-tailed test)**

*H*0: parameter = some value

*H*1: parameter > some value

**3.** Equal versus greater than **(right-tailed test)**

**Four Outcomes from Hypothesis Testing**

**1.** Reject the null hypothesis when the alternative hypothesis is true. This decision would be correct.

**2.** Do not reject the null hypothesis when the null hypothesis is true. This decision would be correct.

**3.** Reject the null hypothesis when the null hypothesis is true. This decision would be incorrect. This

type of error is called a **Type I error**.

**4.** Do not reject the null hypothesis when the alternative hypothesis is true. This decision would be

incorrect. This type of error is called a **Type II error**.

|  |  |
| --- | --- |
|  | Reality |
| *H*0 is True | *H*0 is False |
| Conclusion | Do not reject *H*0 | Correct decision | Type IIerror |
| Reject *H*0 | Type Ierror | Correct decision |

*α* = *P*(Type I error) = *P*(rejecting *H*0 when *H0* is true)

*β* = *P*(Type II error) = *P*(not rejecting *H*0 when *H1* is true)

**☺ Example #1**:

Here is an example of applying Type I & Type II errors using a man on trial for murder:

A man goes to trial, where he is being tried for the murder of his friend.

We can put this case in a hypothesis testing framework. The hypotheses being tested are:

*H*0 : Not Guilty

*H*1 : Guilty

A **Type I error** is committed if we reject, when it is true.

In other words, he did not kill his friend, but was found guilty and is punished for a crime he did not really commit.

A **Type II error** is committed if we fail to reject, when it is false.

In other words, if the man actually did kill his friend, but was found not guilty and was not punished.

**Significance Level**

The **level of significance**, *α*, is the probability of making a Type I error.

**Relation Between Type I and Type II Error Probabilities**

For a fixed sample size, the smaller we specify the significance level, *α*, the larger will be the probability, *β*, of not rejecting a false null hypothesis.

**Possible Conclusions for a Hypothesis Test**

Suppose that a hypothesis test is conducted at a small significance level.

* If the null hypothesis is rejected, we conclude that the alternative hypothesis is true.
* If the null hypothesis is not rejected, we conclude that the data do not provide sufficient evidence to support the alternative hypothesis.

*Note:* When the null hypothesis is rejected in a hypothesis test, we say there is sufficient to support the statement in the alternative hypothesis. When the null hypothesis is not rejected in a hypothesis test, we say that there is not sufficient evidence to support the statement in an alternative hypothesis. We never say accept.

**☺ Exercises**:

*In Exercises #1 through #3, the null and alternative hypotheses are given. Determine whether the hypothesis test is left-tailed, right tailed, or two-tailed.*

**1)** *H*0 : *p* = 0.2

Left-Tailed

*H*1 : *p* < 0.2

**2)** *H*0 : *µ* = 76.2

Right-Tailed

 *H*1 : *µ* > 76.2

**3)** *H*0 : *p* = 0.35

Two-Tailed

*H*1 : *p* ≠ 0.35

**☺ Exercises**:

*In Exercise #4 through #6,*

***a)*** *determine the null and alternative hypotheses*

***b)*** *explain what it would mean to make a Type I error.*

***c)*** *explain what it would mean to make a Type II error.*

**4)** **Pizza.** Historically, the time to order and deliver a pizza at Jimbo’s pizza was 48 minutes. Jim the owner, implements a new system for ordering and delivering pizzas that he believes will reduce the time required to get a pizza to his customers.

*H*0: *µ* = 48

*H*1: *µ* < 48

Making a Type I error would mean concluding that the mean time to order and deliver a pizza is less than 48 minutes when, in fact, the mean time to order and deliver a pizza is 48 minutes.

Making a Type II error would mean concluding that the mean time to order and deliver a pizza is 48 minutes when, in fact, the mean time to order and deliver a pizza is less than 48 minutes.

**5)** **Overweight.** According to the Centers for Disease Control and Prevention, 19.6% of children aged 6 to 11 years are overweight. A school nurse thinks that the percentage of 6- to 11-year-olds who are overweight is higher in her school district.

*H*0: *p* = 0.196

*H*1: *p* > 0.196

Making a Type I error would mean concluding that the percentage of overweight 6- to 11-year olds in the nurse’s school district is higher than 19.6% when, in fact, the percentage of overweight 6- to 11-year olds in the nurse’s school district is 19.6%.

Making a Type II error would mean concluding that the percentage of overweight 6- to 11-year olds in the nurse’s school district is 19.6% when, in fact, the percentage of overweight 6- to 11-year olds in the nurse’s school district is greater 19.6%.

**6)** **Credit-Card Balances.** In 2022, U.S. consumers had a mean credit-card balance of $5,910. A researcher believes that this amount has changed since then.

**Source: https://www.creditcards.com/statistics/credit-card-debt-statistics-1276/**

*H*0: *µ* = $5,910

*H*1: *µ* ≠ $5,910

Making a Type I error would mean concluding that the mean credit-card balance for U.S. consumers has changed since 2022 when, in fact, the mean credit-card balance for U.S. consumers has not changed since 2022.

Making a Type II error would mean concluding that the mean credit-card balance for U.S. consumers has not changed since 2022 when, in fact, the mean credit-card balance for U.S. consumers has changed since 2022.

**☺ Exercises**:

**7)** **Popcorn Consumption**. According to popcorn.org, the mean consumption of popcorn annually by Americans is 43 quarts. The marketing division of popcorn.org unleashes an aggressive campaign designed to get Americans to consume even more popcorn.

*H*0 : *µ* = 43

*H*1 : *µ* > 43

1. Determine the null and alternative hypotheses that would

be used to test the effectiveness of the marketing campaign.

**b)** A sample of 800 Americans provides enough evidence to conclude that the marketing campaign was effective. Provide a statement that should be put out by the marketing department.

There is sufficient evidence to conclude that, after the marketing campaign, the mean consumption of popcorn annually by Americans is greater than 43 quarts.

**c)** Suppose, in fact, that the mean annual consumption of popcorn after the marketing campaign is 42.1 quarts. Has a Type I or Type II error been made by the marketing department? If they tested the hypothesis at the *α* = 0.05 level of significance, what is the probability of making a Type I error?

Type I error, because the null hypothesis was rejected when, in fact, the null hypothesis was true; The probability of making a Type I error is 0.05 or 5%.

**8)** **Migraines.** According to the Centers for Disease Control, 15.2% of American adults experience migraine headaches. Stress is a major contributor to the frequency and intensity of headaches. A massage therapist feels that she has a technique that can reduce the frequency and intensity of migraine headaches.

*Source: The Centers for Disease Control.*

*H*0 : *p* = 0.152

*H*1 : *p* < 0.152

1. Determine the null and alternative hypotheses that would be

used to test the effectiveness of the massage therapist’s techniques.

**b)** A sample of 500 American adults who participated in the massage therapist’s program results in data that indicate that the null hypothesis should not be rejected. Provide a statement that supports the massage therapist’s program.

There is not sufficient evidence to conclude that the therapist’s technique reduces the frequency and intensity of migraine headaches in American adults below 15.2%.

**c)** Suppose, in fact, that the percentage of patients in the program who experience migraine headaches is less than 15.2%. Was a Type I or Type II error committed?

Type II error, because the null hypothesis was not rejected when, in fact, the null hypothesis was false.