<u>Math 160 – Project #2</u>

Chi-Square Analysis using 

(20 points)

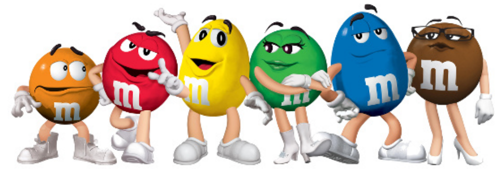
Name (Last, First): ________________________________

ID: ________________________________

SCORE: __________ **CLASS/DAYS/TIME:** ____________________________

At the 5% significance level, do the data provide sufficient evidence to conclude that the Mars color distribution for "Plain" M&M's is different than the distribution obtained by Wicklin?

<u>Step 1</u>: State the null and alternative hypotheses. **(4 pts)**



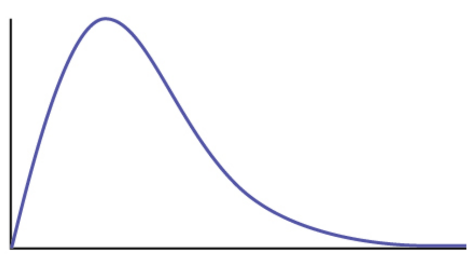
<u>Step 2</u>: Calculate the observed and expected frequencies **<u>only</u>** for each possible value of the variable. Also, indicate the size of the bag as well as the factory your M&M's were obtained from. **(2 pts)**

M&M's Plain / Milk Chocolate		***SIZE BAG (oz./lbs) =*** _________		***FACTORY CODE =*** __________	
COLOR	**Observed Frequency** ***O***	**Expected Frequency** ***E***	**Difference** ***O – E***	**Square of Difference** **$(O-E)^2$**	**χ^2 Subtotal** **$(O-E)^2/E$**
Brown					
Blue					
Orange					
Green					
Red					
Yellow					
Total			**Value of χ^2 Test Statistic →**		

<u>Step 3</u>: List and verify whether the expected frequencies satisfy the required assumptions in order to use the test. **(2 pts)**

<u>Step 4</u>: Fill in the rest of the table. What is the value of the χ^2 test statistic? **(6 pts)** ________________

<u>Step 5</u>: Sketch the χ^2 density curve and state the critical value along with the region of rejection. **(2 pts)**

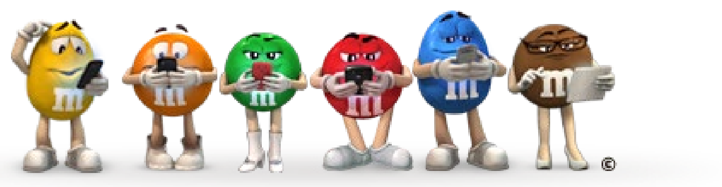


<u>Step 6</u>: Decision on H_0. **(2 pts)**

<u>Step 7</u>: Interpret the results of the hypotheses test. **(2 pts)**

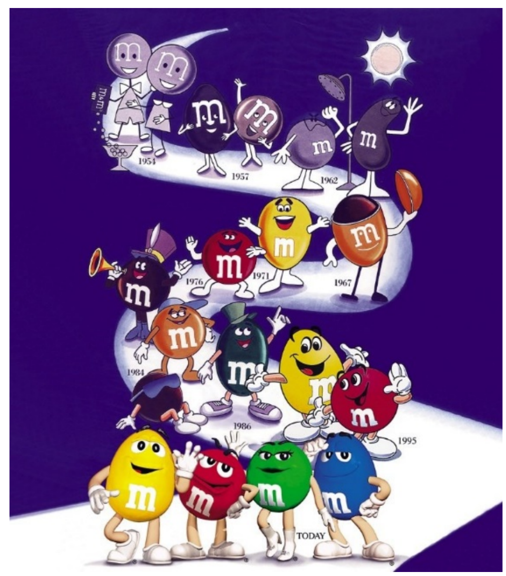
Chi-Square Analysis using M&M's

Have you ever wondered why the package of M&M's you just bought never seems to have enough of your favorite color? Have you ever wondered why you get more of a particular color over others? Is the distribution of the colors of M&M's in a package really different from one package to the next, or does the Mars Company do something to ensure that each package gets the correct number of each color of M&Ms?

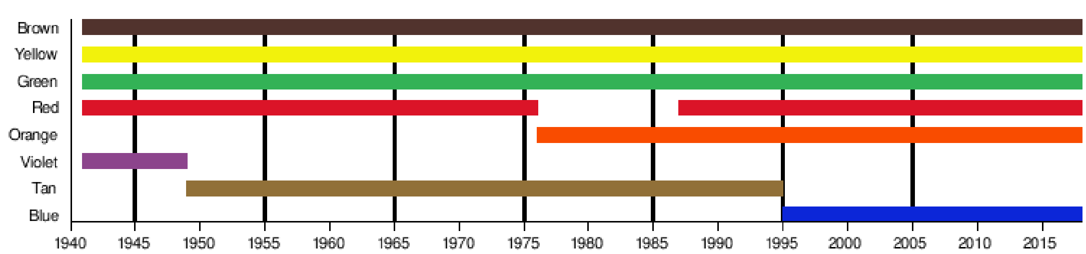


<u>The distribution of colors for plain M&M's over the years</u>

The "plain" M&M candies (now called "milk chocolate M&M's") are produced by the Mars, Inc. company. The distribution of colors in M&M's has a long and colorful history. The colors and proportions occasionally change, and the distribution is different for other varieties, such as peanut, crispy, minis, caramel, peanut butter, almond, and pretzel. Here is a brief timeline highlighting the history as well as the color changes:



- **<u>1941:</u>** Plain/Milk Chocolate M&M's were introduced. There were only 5 colored candies: Brown, Yellow, Green, Red, & Violet.
- **<u>1948:</u>** The violet color was replaced by tan.
- **<u>1954:</u>** Peanut M&M's were introduced.
- **<u>1976:</u>** Red M&M's were discontinued and replaced by orange. This was in response to the "Red Dye Scare". At the time, "Red Dye #2" and "Red Dye #4" had been discovered to be a carcinogen. Although Mars used "Red Dye #40" in their candies and not the two mentioned above, the company changed colors to alleviate any customer concerns.
- **<u>1987:</u>** Red M&M's are brought back. Orange stays.
- **<u>1995:</u>** The tan color is replaced by a more vivid color. In a promotional campaign, the public is asked to vote for the replacement color. Ten million vote; blue wins in a landslide.
- **<u>Late 1990s:</u>** The M&M web site lists the distribution of colors. Circa 1997, the color distribution was 30% brown, 20% yellow, 20% red, 10% orange, 10% green, and 10% blue. Statisticians rejoice and publish many papers on the topic and K-12 educators uses M&M's to engage in the learning of statistics.
- **<u>2008:</u>** Mars changes the color distribution to 24% blue, 20% orange, 16% green, 14% yellow, 13% red, 13% brown. Sometime later, the proportions were removed from the web site and have not been restored.
- **<u>2017 to present:</u>** What is the current distribution of colors today?



Chi-Square Analysis using M&M's

The last time the Mars Co. advertised the distribution of colors for M&M's plain candies was sometime after 2008. There is statistical evidence to suggest that the distribution of colors has changed from what was listed in 2008. In 2017, a computer programmer and statistician at the Cary, North Carolina-based statistical software company SAS, named Rick Wicklin, went out to find if indeed this was the case. SAS is one of the biggest corporate sponsors of M&M's. So, Rick was able to obtain the claimed proportions from the Mars Company and wrote about in his blog. There are two main factories where M&M's are made. It turns out that depending on which factory your package was produced in, creates two different distributions of colors. The distributions by factory are shown below:

Claimed Distribution of Colors in Plain Milk Chocolate "M&M's"

	Hackettstown, New Jersey HKP	Cleveland, Tennessee CLV
Brown	**12.5%**	**12.4%**
Blue	**25%**	**20.7%**
Orange	**25%**	**20.5%**
Green	**12.5%**	**19.8%**
Red	**12.5%**	**13.1%**
Yellow	**12.5%**	**13.5%**

One way that we could determine if the Mars Co. color distribution is true to its word is to sample a package of M&M's and do a statistical test on the findings. The type of statistical test we need must allow us to determine if any differences between our observed measurements (counts of colors from our M&Ms sample) and our expected (what the Mars Co. claims) are simply due to chance or some other reason (i.e. the Mars company's sorters, usually robots, aren't doing an efficient job of putting the correct number of M&M's in each package). We will be deciding whether to reject or not reject a null hypothesis like in a standard one-sample mean hypothesis test, but since we have several different categories to consider at the same time, in this case, one for each color, we need to use a different probability distribution. In this case, we will use the Chi-Square distribution, which is non-normal and skewed to the right. The test that uses this distribution is known as a "Chi-Square (χ^2) Test" or sometimes referred to a "Goodness of Fit Test".

Null Hypothesis (H_o): *The M&M's color distribution is the same as the distribution obtained by Wicklin*

This also means that there will be "no difference" from the values predicted in the sample. To test this hypothesis we will need to calculate the χ^2 Test Statistic, which is calculated by:

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

where *O* is the observed (actual count) and *E* is the expected number for each color category.

The main thing to note about the χ^2 Test Statistic is that, when all else is equal, the value of χ^2 increases as the difference between the observed and expected values increase.

Chi-Square Analysis using M&M's

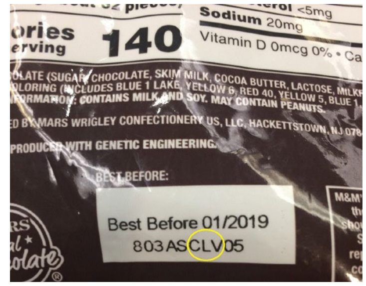
Procedure:

1) Obtain any size bag (excluding trial size) of M&M's Plain / Milk Chocolate flavor. (AS TEMPTING AS IT MAY BE, PLEASE DO NOT EAT ANY OF YOUR DATA UNTIL STEP 6 IS COMPLETED)

2) Separate the M&M's into color categories.

3) Record the actual number of M&M's of each color in the first column of the table on your worksheet labeled "Observed Frequency".

4) You need to look on the packaging for the manufacturing code, which is usually stamped inside a rectangle. In the middle of the code will be the letters HKP or CLV. HKP means the package was produced in the Hackettstown, New Jersey, plant. CLV means the package was produced in the Cleveland, Tennessee, plant. An example of a bag marked CLV is shown to the right.



5) Determine the expected number of M&M's of each color. Be sure to use the appropriate percentages on page 3, based off where your package of M&M's came from in Step #4. Remember *E*=*np*, where *n* is the total number of M&M's in your bag(s) and *p* is the proportion based on the corresponding color. Record the data in the second column of the table on your worksheet labeled "Expected Frequency". Round to the nearest hundred*th*.

6) In order to perform a χ^2 Goodness-of-Fit Test, the following two assumptions must be met:

1. All expected frequencies are 1 or greater.
2. At most 20% of the expected frequencies are less than 5.

If these assumptions are not met, an additional bag of the same type and size must also be used as part of the sample. This may happen on a rare occasion with the 1.5-1.75 ounce packages usually found near the registers at a grocery store. The expected frequencies will then need to be re-calculated. Once that is done, check and see if the assumptions now have been met. If not, repeat the process again, until the assumptions have been met. (Now, you may eat your data!)

7) Complete the rest of the table on your worksheet, rounding to the nearest hundred*th*. The very last entry is the value of the χ^2-test statistic! (See the grey shaded cell with thick black border.)

Now you must determine the probability that the difference between the observed and expected values occurred simply by chance. The procedure is to compare the calculated χ^2 Test Statistic to the appropriate value from Table VIII. Notice we need the degrees of freedom. For this statistical test the degrees of freedom equals one less the number of classes (i.e., color categories). Hence,

degrees of freedom = number of classes – 1

or

***df* = *k* – 1**

The reason why it is important to consider degrees of freedom is that the value of the χ^2 Test Statistic is calculated as the sum of the squared deviations for all classes. The natural increase in the value of χ^2 with an increase in classes must be taken into account.

Chi-Square Analysis using M&M's

So, using Table VIII, scan across the row corresponding to your degrees of freedom. The degrees of freedom is 5, since there were 6 categories. Values of the χ^2 are given for several different probabilities, ranging from 0.995 on the left to 0.001 on the right. Note that the χ^2 value increases as the probability decreases. If your exact χ^2 value is not listed in the table, then estimate the probability or use a graphing calculator.

Degree of Freedom	Area to the Right of Critical Value										Degree of Freedom
	0.995	0.99	0.975	0.95	0.90		0.10	0.05	0.025	0.01	0.005	
5	0.412	0.554	0.831	1.145	1.610		9.236	11.070	12.833	15.086	16.750	5

Notice that a χ^2 value as large as 0.412 would be expected by chance in 99.5% of the cases, whereas one as large as 16.75 would only be expected by chance in 0.5% of the cases. Stated another way, it is more likely that you'll get a little deviation from the expected (thus a lower χ^2 value) than a large deviation from the expected. The column that we need to concern ourselves with is the one under "0.05", since the significance level, α, we will be using is 5%. Statisticians and scientists, in general, are willing to say that if their probability of getting the observed deviation from the expected results by chance is greater than 0.05 (5%), then there is insufficient evidence against the null hypothesis. In other words, there is really no difference in actual ratios…any differences we see between what Mars claims and what is actually in a bag of M&M's just happened by chance sampling error. Five percent! That is not much, but it's good enough for a statistician or a scientist.

If, however, the probability of getting the observed deviation from the expected results by chance is less than 0.05 (5%) then we should reject the null hypothesis. In other words, for our study, there is a significant difference in M&M color ratios between actual store-bought bags of M&M's and what the Mars Co. claims are the actual ratios. Stated another way…any differences we see between what Mars claims and what is actually in a bag of M&M's did not just happen by chance or sampling error.

Good Fit at 5%	Poor Fit at 5%
Do Not Reject H_o	Reject H_o
$\chi^2 < 11.07$	$\chi^2 \geq 11.07$
←	→

Degree of Freedom	Area to the Right of Critical Value										Degree of Freedom
	0.995	0.99	0.975	0.95	0.90		0.10	0.05	0.025	0.01	0.005	
5	0.412	0.554	0.831	1.145	1.610		9.236	11.070	12.833	15.086	16.750	5

If the χ^2 value …

- … is smaller than the critical value for the indicated degrees of freedom → Do Not Reject H_o
  Conclusion: There is not sufficient evidence to conclude that the M&M's color distribution is different than the distribution obtained by Wicklin. Perhaps, the variation in color percentages is due to chance (random) variation.

- … is larger than the critical value for the indicated degrees of freedom → Reject H_o
  Conclusion: There is sufficient evidence to conclude that the M&M's color distribution is different than the distribution obtained by Wicklin. The sorting machines need to be recalibrated or the distribution has been changed since.

Note: If your sampling size is small, a null hypothesis might be retained simply because there are not enough data to reject it. This is why we use the phrase "cannot reject the null" as opposed to the phrase "accept the null". If the goal is to not reject the null hypothesis, the most scientifically valid approach is to use as large a sample as possible. This prevents the possible criticism that the null hypothesis was retained only because the sample was not large enough to provide conclusive evidence.