Chapter 8

COUNTING PRINCIPLES; FURTHER PROBABILITY TOPICS

8.1 The Multiplication Principle; Permutations

Your Turn 1

Each of the four digits can be one of the ten digits $0, 1, 2, \ldots, 10,$ so there are $10 \cdot 10 \cdot 10 \cdot 10$ or 10,000 possible sequences. If no digit is repeated, there are 10 choices for the first place, 9 for the second, 8 for the third, and 7 for the fourth, so there are $10 \cdot 9 \cdot 8 \cdot 7 = 5040$ possible sequences.

Your Turn 2

Any of the 8 students could be first in line. Then there are 7 choices for the second spot, 6 for the third, and so on, with only one student remaining for the last spot in line. There are $8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 40{,}320$ different possible lineups.

Your Turn 3

The teacher has 8 ways to fill the first space (say the one on the left), 7 choices for the next book, and so on, leaving 4 choices for the last book on the right. So the number of possible arrangements is $8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 = 6720.$

Your Turn 4

There are 6 letters. If we use 3 of the 6, the number of permutations is

$$P(6, 3) = \frac{6!}{(6-3)!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2}$$
$$= 6 \cdot 5 \cdot 4 = 120.$$

Your Turn 5

If the panel sits in a row, there are $4!$ or 24 ways of arranging the four class groups. Within the groups, there are 2! ways of arranging the freshmen, 2! ways of arranging the sophomores, 2! ways of arranging the juniors, and 3! ways of arranging the seniors. Using the multiplication principle, the number of ways of seating the panel with the classes together is $24 \cdot 2! \cdot 2! \cdot 2! \cdot 3! = 24 \cdot 2 \cdot 2 \cdot 2 \cdot 6 = 1152.$

Your Turn 6

The word *Tennessee* contains 9 letters, consisting of 1 t, 4 e's, 2 n's and 2 s's. Thus the number of possible arrangements is

$$\frac{9!}{1!4!2!2!} = 3780.$$

Your Turn 7

The student has $4 + 5 + 3 + 2 = 14$ pairs of socks, so if the pairs were distinguishable there would be 14! possible selections for the next two weeks. But since the pairs of each color are identical, the number of distinguishable selections is

$$\frac{14!}{4!5!3!2!} = 2{,}522{,}520.$$

8.1 Exercises

1. $6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$

2. $7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5040$

3. $$15! = 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \approx 1.308 \times 10^{12}$$

4. $$16! = 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \approx 2.092 \times 10^{13}$$

5. $$P(13, 2) = \frac{13!}{(13-2)!} = \frac{13!}{11!} = \frac{13 \cdot 12 \cdot 11!}{11!} = 156$$

6. $$P(12, 3) = \frac{12!}{(12-3)!} = \frac{12!}{9!} = \frac{12 \cdot 11 \cdot 10 \cdot 9!}{9!} = 1320$$

7. $P(38,17) = \frac{38!}{(38-17)!} = \frac{38!}{21!}$
$\approx 1.024 \times 10^{25}$

8. $P(33,19) = \frac{33!}{(33-19)!} = \frac{33!}{14!}$
$\approx 9.960 \times 10^{25}$

9. $P(n,0) = \frac{n!}{(n-0)!} = \frac{n!}{n!} = 1$

10. $P(n,n) = \frac{n!}{(n-n)!} = \frac{n!}{0!} = \frac{n!}{1} = n!$

11. $P(n,1) = \frac{n!}{(n-1)!} = \frac{n(n-1)!}{(n-1)!} = n$

12. $P(n,n-1) = \frac{n!}{[n-(n-1)]!} = \frac{n!}{(n-n+1)!}$
$= \frac{n!}{1!} = n!$

13. By the multiplication principle, there will be $6 \cdot 3 \cdot 2 = 36$ different home types available.

14. By the multiplication principle, there will be $3 \cdot 8 \cdot 7 = 168$ different meals possible.

15. There are 4 choices for the first name and 5 choices for the middle name, so, by the multiplication principle, there are $4 \cdot 5 = 20$ possible arrangements.

16. The number of ways to choose a slate of 3 officers is

$$P(16,3) = \frac{16!}{(16-3)!} = \frac{16!}{13!} = \frac{16 \cdot 15 \cdot 14 \cdot 13!}{13!} = 3360.$$

19. There is exactly one 3-letter subset of the letters A, B, and C, namely A, B, and C.

20. In Example 6, there are only 3 unordered 2-letter subsets of letters A, B, and C. They are AB, AC, and BC.

21. **(a)** initial

This word contains 3 i's, 1 n, 1 t, 1 a, and 1 ℓ. Use the formula for distinguishable permutations with $n = 7, n_1 = 3, n_2 = 1, n_3 = 1, n_4 = 1,$ and $n_5 = 1.$

$$\frac{n!}{n_1!n_2!n_3!n_4!n_5!} = \frac{7!}{3!1!1!1!1!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3!}{3!} = 840$$

There are 840 distinguishable permutations of the letters.

(b) little

Use the formula for distinguishable permutetions with $n = 6, n_1 = 2, n_2 = 1, n_3 = 2,$ and $n_4 = 1.$

$$\frac{6!}{2!1!2!1!} = \frac{6!}{2!2!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 2 \cdot 1} = 180$$

There are 180 distinguishable permutations.

(c) decreed

Use the formula for distinguishable permutations with $n = 7, n_1 = 2, n_2 = 3, n_3 = 1,$ and $n_4 = 1.$

$$\frac{7!}{2!3!1!1!} = \frac{7!}{2!3!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3!}{2 \cdot 1 \cdot 3!} = 420$$

There are 420 distinguishable permutations.

22. Use the formula for distinguishable permutations. The number of different "words" is

$$\frac{n!}{n_1!n_2!n_3!n_4!} = \frac{13!}{5!4!2!2!} = 540{,}540.$$

23. **(a)** The 9 books can be arranged in $P(9,9) = 9! = 362{,}880$ ways.

(b) The blue books can be arranged in 4! ways, the green books can be arranged in 3! ways, and the red books can be arranged in 2! ways. There are 3! ways to choose the order of the 3 groups of books. Therefore, using the multiplication principle, the number of possible arrangements is

$$4!3!2!3! = 24 \cdot 6 \cdot 2 \cdot 6 = 1728.$$

(c) Use the formula for distinguishable permutations with $n = 9, n_1 = 4, n_2 = 3,$ and $n_3 = 2$. The number of distinguishable arrangements is

$$\frac{9!}{4!3!2!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4!}{4! \cdot 6 \cdot 2} = 1260.$$

(d) There are 4 choices for the blue book, 3 for the green book, and 2 for the red book. The total number of arrangements is

$$4 \cdot 3 \cdot 2 = 24.$$

(e) From part (d) there are 24 ways to select a blue, red, and green book if the order does not matter. There 3! ways to choose the order. Using the multiplication principle, the number of possible ways is

$$24 \cdot 3! = 24 \cdot 6 = 144.$$

24. **(a)** Since there are 14 distinguishable objects to be arranged, use permutations. The number of arrangements is

$$P(14, 14) = 14! = 87{,}178{,}291{,}000 \text{ or } 8.7178291 \times 10^{10}.$$

(b) There are 3! ways to arrange the pyramids among themselves, 4! ways to arrange the cubes, and 7! ways to arrange the spheres. We must also consider the number of ways to arrange the order of the three groups of shapes. This can be done in 3! ways. Using the multiplication principle, the number of arrangements is

$$3!4!7!3! = 6 \cdot 24 \cdot 5040 \cdot 6 = 4{,}354{,}560.$$

(c) In this case, all of the objects that are the same shape are indistinguishable. Use the formula for distinguishable permutations. The number of distinguishable arrangements is

$$\frac{n!}{n_1!n_2!n_3!} = \frac{14!}{3!4!7!} = 120{,}120.$$

(d) There are 3 choices for the pyramid, 4 for the cube, and 7 for the sphere. The total number of ways is

$$3 \cdot 4 \cdot 7 = 84.$$

(e) From part (d) there are 84 ways if the order does not matter. There are 3! ways to choose the order. Using the multiplication principle, the number of possible ways is

$$84 \cdot 3!\cdot = 84 \cdot 6 = 504.$$

25. $10! = 10 \cdot 9!$

To find the value of 10!, multiply the value of 9! by 10.

26. $$451! = 451 \cdot 450! \approx 451 \cdot 1.7333687 \times 10^{1000} \approx 781.7493 \times 10^{1000} = 7.817493 \times 10^{1002}$$

27. **(a)** The number 13! has 2 factors of five so there must be 2 ending zeros in the answer.

(b) The number 27! has 6 factors of five (one each in 5, 10, 15, and 20 and two factors in 25), so there must be 6 ending zeros in the answer.

(c) The number 75! has $15 + 3 = 18$ factors of five (one each in $5, 10, \ldots, 75$ and two factors each in 25, 50, and 75), so there must be 18 ending zeros in the answer.

28. **(a)** Since 12! has two 5s, there are two ending zeros in the answer to 12!. Thus, $12! \neq 479{,}001{,}610$.

(b) Since 23! Has four 5s, there are four ending zeros in the answer to 23!. Thus, $23! \neq 25{,}852{,}016{,}740{,}000{,}000{,}000{,}000$.

(c) Since 15! has three 5s, there are three ending zeros in the answer to 15!. Thus, $15! \neq 1{,}307{,}643{,}680{,}000$.

(d) Since 14! has two 5s, there are two ending zeros in the answer to 14!. Using a calculator, 14! is approximated at 8.71782912E10. Since the last two digits are zero, $14! = 87{,}178{,}291{,}200$.

29. $$P(4, 4) = \frac{4!}{(4-4)!} = \frac{4!}{0!}$$

If $0! = 0$, then $P(4,4)$ would be undefined.

30. Use the multiplication principle. There are

$$(4)(53)^3(18)(27)(12)(48)(3)(2)(2)^4 = 1.600 \times 10^{13}$$

different possible bags.

31. **(a)** By the multiplication principle, since there are 7 pastas and 6 sauces, the number of different bowls is $7 \cdot 6 = 42$.

(b) If we exclude the two meat sauces there are 4 sauces left and the number of bowls is now $7 \cdot 4 = 28$.

32. Ranking 5 investments out of 9 amounts to finding the number of permutations of 9 elements taken 5 at a time, which is

$$P(9,5) = \frac{9!}{(9-5)!} = \frac{9!}{4!} = 15{,}120.$$

33. **(a)** Since there are 11 slots, the 11 commercials can be arranged in $11! = 39{,}916{,}800$ ways.

(b) Use the multiplication principle. We can put either stores or restaurants first (2 choices). Then there are 6! orders for the stores and 5! orders for the restaurants, so the number of groupings is $2 \cdot 6! \cdot 5! = 172{,}800$.

(c) Since the number of restaurants is one more than the number of stores, a restaurant must come first. This eliminates the first choice in part (b), but we still can order the restaurants and the stores freely within each category, so the answer is $6! \cdot 5! = 86{,}400$.

34. 5 of the 12 drugs can be administered in

$$P(12,5) = \frac{12!}{(12-5)!} = \frac{12!}{7!} = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7!}{7!} = 95{,}040$$

different sequences.

35. If each species were to be assigned 3 initials, since there are 26 different letters in the alphabet, there could be $26^3 = 17{,}576$ different 3-letter designations. This would not be enough. If 4 initials were used, the biologist could represent $26^4 = 456{,}976$ different species, which is more than enough. Therefore, the biologist should use at least 4 initials.

36. **(a)** The total number of presentations is $5 + 5 + 2 = 12$, so the presentations can be scheduled in $12! = 479{,}001{,}600$ orders.

(b) If we group the presentations for a given subject together, these three groups can present in 3! orders, and within each group there are 5!, 5! and 2! possible orders. So the number of possible orders is $3! \cdot 5! \cdot 5! \cdot 2! = 172{,}800$.

(c) There are only 2 physics presentations, so there are just 2 ways of selecting the first and last presentation. The remaining 10 presentations can come in any order, so the number of possible orders is $2 \cdot 10! = 7{,}257{,}600$.

37. The number of ways to seat the people is

$$P(6,6) = \frac{6!}{0!} = \frac{6!}{1} = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720.$$

38. A ballot would consist of a list of the 3 candidates for office 1 and a list of the 6 candidates for office 2. The number of ways to list candidates for office 1 is $P(3,3) = 3! = 6$. The number of ways to list candidates for office 2 is $P(6,6) = 6! = 720$. There are two ways to choose which office goes first. By the multiplication principle, the number of different ballots is

$$6 \cdot 720 \cdot 2 = 8640.$$

39. The number of possible batting orders is

$$P(19,9) = \frac{19!}{(19-9)!} = \frac{19!}{10!} = 33{,}522{,}128{,}640 \approx 3.352 \times 10^{10}.$$

40. The number of ways to select the 4 officers is

$$P(35,4) = \frac{35!}{(35-4)!} = \frac{35!}{31!} = \frac{35 \cdot 34 \cdot 33 \cdot 32 \cdot 31!}{31!} = 1{,}256{,}640.$$

41. **(a)** The number of ways 5 works can be arranged is

$$P(5,5) = 5! = 120.$$

(b) If one of the 2 overtures must be chosen first, followed by arrangements of the 4 remaining pieces, then

$$P(2,1) \cdot P(4,4) = 2 \cdot 24 = 48$$

is the number of ways the program can be arranged.

42. **(a)** Pick one of the 5 traditional numbers followed by an arrangement of the remaining total of 7. The program can be arranged in

$$P(5,1) \cdot P(7,7) = 5 \cdot 7! = 25{,}200$$

different ways.

(b) Pick one of the 3 original Cajun compositions to play last, preceded by an arrangement of the remaining total of 7. This program can be arranged in

$$P(7,7) \cdot P(3,1) = 7! \cdot 3 = 15{,}120$$

different ways.

43. **(a)** There are 4 tasks to be performed in selecting 4 letters for the call letters. The first task may be done in 2 ways, the second in 25, the third in 24, and the fourth in 23. By the multiplication principle, there will be

$$2 \cdot 25 \cdot 24 \cdot 23 = 27{,}600$$

different call letter names possible.

(b) With repeats possible, there will be

$$2 \cdot 26 \cdot 26 \cdot 26 = 2 \cdot 26^3 \quad \text{or} \quad 35{,}152$$

call letter names possible.

(c) To start with W or K, make no repeats, and end in R, there will be

$$2 \cdot 24 \cdot 23 \cdot 1 = 1104$$

possible call letter names.

44. **(a)** There are 5 odd digits: 1, 3, 5, 7, and 9. There are 7 decisions to be made, one for each digit; there are 5 choices for each digit. Thus, $5^7 = 78{,}125$ phone numbers are possible.

(b) The first digit has 9 possibilities, since 0 is not allowed; the middle 5 digits each have 10 choices; the last digit must be 0. Thus, there are

$$9 \cdot 10^5 \cdot 1 = 900{,}000$$

possible phone numbers.

(c) Solve as in part (b), except that the last *two* digits must be 0; therefore there are

$$9 \cdot 10^4 \cdot 1 \cdot 1 = 90{,}000$$

possible phone numbers.

(d) There are no choices for the first three digits; thus,

$$1^3 \cdot 10^4 = 10{,}000$$

phone numbers are possible.

(e) The first digit cannot be 0; in the absence of repetitions there are 9 choices for the second digit, and the choices decrease by one for each subsequent digit. The result is

$$9 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 = 544{,}320$$

phone numbers.

45. **(a)** Our number system has ten digits, which are 1 through 9 and 0.

There are 3 tasks to be performed in selecting 3 digits for the area code. The first task may be done in 8 ways, the second in 2, and the third in 10. By the multiplication principle, there will be

$$8 \cdot 2 \cdot 10 = 160$$

different area codes possible.

There are 7 tasks to be performed in selecting 7 digits for the telephone number. The first task may be done in 8 ways, and the other 6 tasks may each be done in 10 ways. By the multiplication principle, there will be

$$8 \times 10^6 = 8{,}000{,}000$$

different telephone numbers possible within each area code.

(b) Some numbers, such as 911, 800, and 900, are reserved for special purposes and are therefore unavailable for use as area codes.

(c) There are 8 choices for the first digit, since it cannot be 0 or 1. Since restrictions are eliminated for the second digit, there are 10 possibilities for each of the second and third digits. Thus, the total number of area codes would be

$$8 \cdot 10 \cdot 10 = 800.$$

46. Under the old system there were $2^{32} = 4.295 \times 10^9$ IP addresses available. Under the new system there are $2^{128} = 3.403 \times 10^{38}$ IP addresses available

47. **(a)** There were

$$26^3 \cdot 10^3 = 17{,}576{,}000$$

license plates possible that had 3 letters followed by 3 digits.

(b) There were

$$10^3 \cdot 26^3 = 17{,}576{,}000$$

new license plates possible when plates were also issued having 3 digits followed by 3 letters.

(c) The number of possible new license plates added by the format change in 1980 was $(10)(1000)(26^3) = 175{,}760{,}000.$

48. Since a social security number has 9 digits with no restrictions, there are

$$10^9 = 1{,}000{,}000{,}000 \text{ (1 billion)}$$

different social security numbers. Yes, this is enough for every one of the 318 million people in the United States to have a social security number.

49. If there are no restrictions on the digits used, there would be

$$10^5 = 100{,}000$$

different 5-digit zip codes possible.
If the first digit is not allowed to be 0, there would be

$$9 \cdot 10^4 = 90{,}000$$

zip codes possible.

50. Since a zip code has nine digits with no restrictions, there are

$$10^9 = 1{,}000{,}000{,}000$$

different 9-digit zip codes.

51. There are 3 possible identical shapes on each card.

There are 3 possible shapes for the identical shapes.

There are 3 possible colors.

There are 3 possible styles.

Therefore, the total number of cards is $3 \cdot 3 \cdot 3 \cdot 3 = 81$.

52. Since a 20-sided die is rolled 12 times, the number of possible games is

$$20^{12} \quad \text{or} \quad 4.096 \times 10^{15} \text{ games.}$$

53. There are 3 possible answers for the first question and 2 possible answers for each of the 19 other questions. The number of possible objects is

$$3 \cdot 2^{19} = 1{,}572{,}864.$$

20 questions are not enough.

54. **(a)** The number of different circuits is $P(9,9)$ since we do not count the city he is starting in.

$$P(9,9) = 9! = 362{,}880$$

is the number of different circuits.

(b) He must check half of the circuits since, for each circuit, there is a corresponding one in the reverse order. Therefore,

$$\frac{1}{2}(362{,}880) = 181{,}440$$

circuits should be checked.

(c) No, it would not be feasible.

55. **(a)** Since the starting seat is not counted, the number of arrangements is

$$P(19,19) = 19! \approx 1.216451 \times 10^{17}.$$

(b) Since the starting bead is not counted and the necklace can be flipped, the number of arrangements is

$$\frac{P(14,14)}{2} = \frac{14!}{2} = 43{,}589{,}145{,}600.$$

8.2 Combinations

Your Turn 1

Use the combination formula.

$$C(10,4) = \frac{10!}{6!4!} = 210$$

Your Turn 2

Since the group of students contains either 3 or 4 students out of 15, it can be selected in $C(15,3) + C(15,4)$ ways.

$$\begin{aligned} C(15,3) + C(15,4) &= \frac{15!}{12!3!} + \frac{15!}{11!4!} \\ &= 455 + 1365 \\ &= 1820 \end{aligned}$$

Your Turn 3

(a) Use permutations.

$$P(10,4) = 10 \cdot 9 \cdot 8 \cdot 7 = 5040$$

(b) Use combinations.

$$C(15,3) = \frac{15!}{12!3!} = 455$$

(c) Use combinations.

$$C(8,2) = \frac{8!}{6!2!} = 28$$

(d) Use combinations and permutations. First pick 4 rooms; this is an unordered selection:

$$C(6,4) = \frac{6!}{2!4!} = 15$$

Now assign the patients to the rooms; this is an ordered selection:

$$P(4,4) = 4! = 24$$

The number of possible assignments is $15 \cdot 24 = 360$.

Your Turn 4

The committee is an unordered selection.

$$C(20,3) = \frac{20!}{17!3!} = 1140$$

If the selection includes assignment to one of the three offices we have an ordered selection.

$$P(20,3) = 20 \cdot 19 \cdot 18 = 6840$$

Your Turn 5

There are $C(4,2)$ ways to select 2 aces from the 4 aces in the deck and $C(48,3)$ ways to select the 3 remaining cards from the 48 non-aces. Now use the multiplication principle.

$$C(4,2) \cdot C(48,3) = 6 \cdot 17{,}296 = 103{,}776$$

8.2 Warmup Exercises

W1. $P(5,3) = 5 \cdot 4 \cdot 3 = 60$

W2. There are 3 a's and two n's, one b and one s, so the number of distinguishable permutations is

$$\frac{7!}{(3!)(2!)(1!)(1!)} = 420.$$

8.2 Exercises

3. To evaluate $C(8, 3)$, use the formula

$$C(n, r) = \frac{n!}{(n-r)!r!}$$

with $n = 8$ and $r = 3$.

$$\begin{aligned} C(8,3) &= \frac{8!}{(8-3)!3!} \\ &= \frac{8!}{5!3!} \\ &= \frac{8 \cdot 7 \cdot 6 \cdot 5!}{5! \cdot 3 \cdot 2 \cdot 1} = 56 \end{aligned}$$

4. $$\begin{aligned} C(12,5) &= \frac{12!}{(12-5)!5!} = \frac{12!}{7!5!} \\ &= \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7!}{7! \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} \\ &= 792 \end{aligned}$$

5. To evaluate $C(44, 20)$, use the formula

$$C(n, r) = \frac{n!}{(n-r)!r!}$$

with $n = 44$ and $r = 20$.

$$\begin{aligned} C(44,20) &= \frac{44!}{(44-20)!20!} \\ &= \frac{44!}{24!20!} \\ &= 1.761 \times 10^{12} \end{aligned}$$

6. $$\begin{aligned} C(40,18) &= \frac{40!}{(40-18)!18!} \\ &= \frac{40!}{22!18!} \\ &\approx 1.134 \times 10^{11} \end{aligned}$$

7. $$\begin{aligned} C(n,0) &= \frac{n!}{(n-0)!0!} \\ &= \frac{n!}{n! \cdot 1} \\ &= 1 \end{aligned}$$

8. $$C(n,n) = \frac{n!}{(n-n)!n!} = \frac{n!}{0!n!} = \frac{n!}{1 \cdot n!} = 1$$

9. $$\begin{aligned} C(n,1) &= \frac{n!}{(n-1)!1!} \\ &= \frac{n(n-1)!}{(n-1)! \cdot 1} \\ &= n \end{aligned}$$

10. $$\begin{aligned} C(n,n-1) &= \frac{n!}{[n-(n-1)]!(n-1)!} \\ &= \frac{n!}{(n-n+1)!(n-1)!} \\ &= \frac{n(n-1)!}{1!(n-1)!} \\ &= n \end{aligned}$$

11. There are 13 clubs, from which 6 are to be chosen. The number of ways in which a hand of 6 clubs can be chosen is

$$C(13,6) = \frac{13!}{7!6!} = 1716.$$

12. There are 26 red cards in a deck, so there are $C(26, 6)$ ways to select a hand of 6 red cards.

$$C(26,6) = \frac{26!}{20!6!} = 230,230$$

13. **(a)** There are

$$C(5,2) = \frac{5!}{3!2!} = \frac{5 \cdot 4 \cdot 3!}{3! \cdot 2 \cdot 1} = 10$$

different 2-card combinations possible.

(b) The 10 possible hands are

$$\{1,2\}, \{2,3\}, \{3,4\}, \{4,5\}, \{1,3\},$$
$$\{2,4\}, \{3,5\}, \{1,4\}, \{2,5\}, \{1,5\}.$$

Of these, 7 contain a card numbered less than 3.

14. **(a)** The number of ways to select a committee of 4 from a club with 31 members is

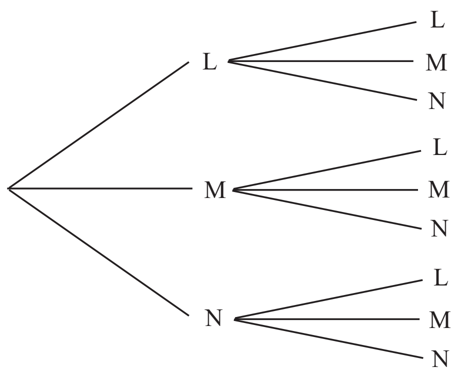
$$C(31,4) = 31,465.$$

(b) If the committee must have at least 1 member and at most 3 members, it must have 1, 2, or 3 members. The number of committees is

$$\begin{aligned} &C(31,1) + C(31,2) + C(31,3) \\ &= 31 + 465 + 4495 \\ &= 4991. \end{aligned}$$

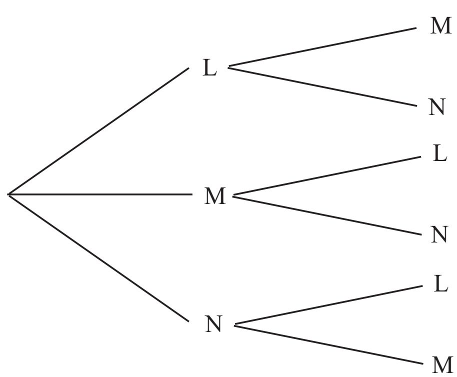
15. Choose 2 letters from {L, M, N}; order is important.

(a)



There are 9 ways to choose 2 letters if repetition is allowed.

(b)



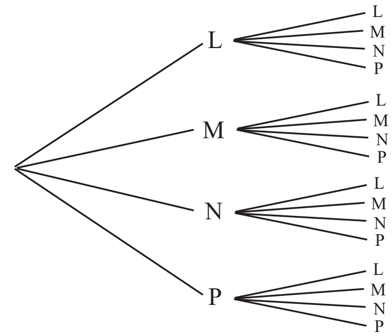
There are 6 ways to choose 2 letters if no repeats are allowed.

(c) The number of combinations of 3 elements taken 2 at a time is

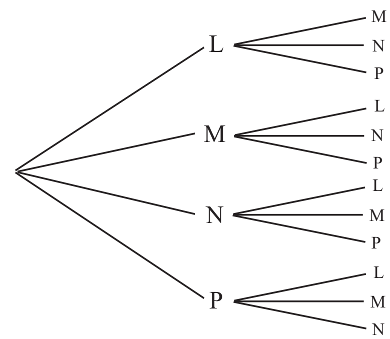
$$C(3,2) = \frac{3!}{1!2!} = 3.$$

This answer differs from both parts (a) and (b).

16. **(a)** With repetition permitted, the tree diagram shows 16 different pairs.



(b) If repetition is not permitted, one branch is missing from each of the clusters of second branches, for a total of 12 different pairs.



(c) Find the number of combinations of 4 elements taken 2 at a time.

$$C(4,2) = 6$$

No repetitions are allowed, so the answer cannot equal that for part (a). However, since order does not matter, our answer is only half of the answer for part (b). For example, LM and ML are distinct in (b) but not in (c). Thus, the answer differs from both (a) and (b).

17. Order does not matter in choosing members of a committee, so use combinations rather than permutations.

(a) The number of committees whose members are all men is

$$\begin{aligned} C(9,5) &= \frac{9!}{4!5!} \\ &= \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5!}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 5!} \\ &= 126. \end{aligned}$$

(b) The number of committees whose members are all women is

$$\begin{aligned} C(11,5) &= \frac{11!}{6!5!} \\ &= \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6!}{6! \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} \\ &= 462. \end{aligned}$$

(c) The 3 men can be chosen in

$$C(9,3) = \frac{9!}{6!3!} = \frac{9 \cdot 8 \cdot 7 \cdot 6!}{6! \cdot 3 \cdot 2 \cdot 1} = 84 \text{ ways.}$$

The 2 women can be chosen in

$$C(11,2) = \frac{11!}{9!2!} = \frac{11 \cdot 10 \cdot 9!}{9! \cdot 2 \cdot 1} = 55 \text{ ways.}$$

Using the multiplication principle, a committee of 3 men and 2 women can be chosen in

$$84 \cdot 55 = 4620 \text{ ways.}$$

18. Since order is not important, the answers are combinations.

(a) If there are at least 4 women, there will be either 4 women and 1 man or 5 women and no men. The number of such committees is

$$C(11,4)C(9,1) + C(11,5)C(9,0) = 2970 + 462 = 3432.$$

(b) If there are no more than 2 men, there will be either no men and 5 women, 1 man and 4 women, or 2 men and 3 women. The number of such committees is

$$C(9,0)C(11,5) + C(9,1)C(11,4) + C(9,2)C(11,3) = 462 + 2970 + 5940 = 9372.$$

19. Order is important, so use permutations. The number of ways in which the children can find seats is

$$P(12,11) = \frac{12!}{(12-11)!} = \frac{12!}{1!} = 12! = 479{,}001{,}600.$$

20. Order does not matter, so use combinations.

(a) The 3 students who will take part in the course can be chosen in

$$C(14,3) = \frac{14!}{11!3!} = \frac{14 \cdot 13 \cdot 12 \cdot 11!}{11! \cdot 3 \cdot 2 \cdot 1} = 364 \text{ ways.}$$

(b) The 9 students who will not take part in the course can be chosen in

$$C(14,9) = \frac{14!}{3!11!} = 364 \text{ ways.}$$

21. Since order does not matter, the answers are combinations.

(a) $C(16,2) = \frac{16!}{14!2!} = \frac{16 \cdot 15 \cdot 14!}{14! \cdot 2 \cdot 1} = 120$

120 samples of 2 marbles can be drawn.

(b) $C(16,4) = 1820$

1820 samples of 4 marbles can be drawn.

(c) Since there are 9 blue marbles in the bag, the number of samples containing 2 blue marbles is

$$C(9,2) = 36.$$

22. Since order does not matter, use combinations.

(a) There are

$$C(26,3) = 2600$$

possible samples of 3 apples.

(b) There are

$$C(7,3) = 35$$

possible samples of 3 rotten apples.

(c) There are

$$C(7,1)C(19,2) = 1197$$

possible samples with exactly 1 rotten apple.

23. Since order does not matter, use combinations.

(a) $C(5,3) = \frac{5!}{2!3!} = \frac{5 \cdot 4 \cdot 3!}{2 \cdot 1 \cdot 3!} = 10$

There are 10 possible samples with all black jelly beans.

(b) There is only 1 red jelly bean, so there are no samples in which all 3 are red.

(c) $C(3,3) = 1$

There is 1 sample with all yellow.

(d) $C(5,2)C(1,1) = 10 \cdot 1 = 10$

There are 10 samples with 2 black and 1 red.

(e) $C(5,2)C(3,1) = 10 \cdot 3 = 30$

There are 30 samples with 2 black and 1 yellow.

(f) $C(3,2)C(5,1) = 3 \cdot 5 = 15$

There are 15 samples with 2 yellow and 1 black.

(g) There is only 1 red jelly bean, so there are no samples containing 2 red jelly beans.

24. Since order is important, use a permutation. The plants can be arranged in

$$P(9,5) = 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 = 15{,}120$$

different ways.

25. Show that $C(n,r) = C(n,n-r)$.

Work with each side of the equation separately.

$$C(n,r) = \frac{n!}{r!(n-r)!}$$

$$C(n,n-r) = \frac{n!}{(n-r)![n-(n-r)]!} = \frac{n!}{(n-r)!r!}$$

Since both results are the same, we have shown that

$$C(n,r) = C(n,n-r).$$

26. (a) We will call the digit on the left the first digit.

No fives: There are 8 choices for the first digit and 9 choices for the second and third digits, giving $8(9^2)$ possibilities.

Two fives: If the non-5 digit goes in the first place, it can have one of eight values; if it goes in the second or third place it can have one of 9 values, so these are 8 + 9 + 9 possibilites.

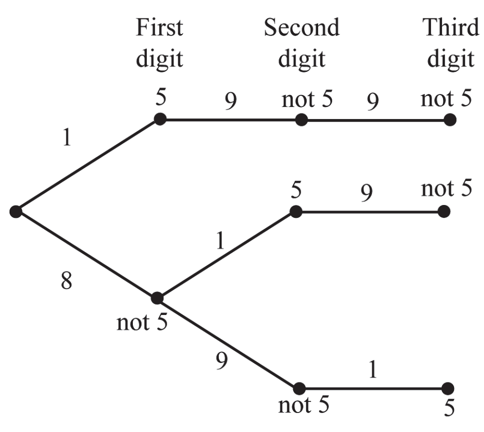
Three fives: For 3 fives there is only one possibility, namely the number 555.

Thus out of the 900 numbers from 100 to 999,

$$900 - 8(9^2) - (8+9+9) - 1 = 225$$

contain exactly one 5.

(b) Multiply the numbers of possibilities along each path in the tree diagram and add these products.



$$(1)(9)(9) + (8)(1)(9) + (8)(9)(1) = 225$$

27. Use combinations since order does not matter.

(a) First consider how many pairs of circles there are. This number is

$$C(6,2) = \frac{6!}{2!4!} = 15.$$

Each pair intersects in two points. The total number of intersection points is $2 \cdot 15 = 30$.

(b) The number of pairs of circles is

$$C(n,2) = \frac{n!}{(n-2)!2!} = \frac{n(n-1)(n-2)!}{(n-2)! \cdot 2} = \frac{1}{2}n(n-1).$$

Each pair intersects in two points. The total number of points is

$$2 \cdot \frac{1}{2}n(n-1) = n(n-1).$$

28. There are 7 digits. The number of cases with the same number of dots on both sides is $C(7,1) = 7$. The number of cases with a different number of dots on each side is $C(7,2) = \frac{7!}{5!2!} = 21$. The total number of dominoes that can be formed is $C(7,1) + C(7,2) = 7 + 21 = 28$.

29. Since the assistants are assigned to different managers, this amounts to an ordered selection of 3 from 8.

$$P(8,3) = 8 \cdot 7 \cdot 6 = 336$$

30. Since order is not important, use combinations.

$$C(50,5) = \frac{50!}{45!5!} = 2{,}118{,}760$$

31. Order is important in arranging a schedule, so use permutations.

(a) $P(6,6) = \frac{6!}{0!} = 6! = 720$

She can arrange her schedule in 720 ways if she calls on all 6 prospects.

(b) $P(6,4) = \frac{6!}{2!} = 360$

She can arrange her schedule in 360 ways if she calls on only 4 of the 6 prospects.

32. Since order is not important, use combinations.

(a) Since 3 workers are to be chosen from a group of 9, the number of possible delegations is

$$C(9,3) = 84.$$

(b) Since a particular worker must be in the delegation, the first person can only be chosen in 1 way. The two others must be selected from the 8 workers who are not the worker who must be included. The number of different delegations is

$$1 \cdot C(8, 2) = 28.$$

(c) We must count those delegations with exactly 1 woman (1 woman and 2 men), those with exactly 2 women (2 women and 1 man), and those with 3 women. The number of delegations including at least 1 woman is

$$C(4,1)C(5,2) + C(4,2)C(5,1) + C(4,3)$$
$$= 4 \cdot 10 + 6 \cdot 5 + 4 = 74.$$

33. There are 2 types of meat and 6 types of extras. Order does not matter here, so use combinations.

(a) There are $C(2,1)$ ways to choose one type of meat and $C(6,3)$ ways to choose exactly three extras. By the multiplication principle, there are

$$C(2,1)C(6,3) = 2 \cdot 20 = 40$$

different ways to order a hamburger with exactly three extras.

(b) There are

$$C(6,3) = 20$$

different ways to choose exactly three extras.

(c) "At least five extras" means "5 extras or 6 extras." There are $C(6,5)$ different ways to choose exactly 5 extras and $C(6,6)$ ways to choose exactly 6 extras, so there are

$$C(6,5) + C(6,6) = 6 + 1 = 7$$

different ways to choose at least five extras.

34. There are no restrictions as to whether the scoops have to be different flavors.

(a) The number of different double-scoop cones will be

$$31 \cdot 31 = 961.$$

(b) The number of different triple-scoop cones will be

$$31 \cdot 31 \cdot 31 = 31^3 = 29,791.$$

(c) There are

$$C(31,2) = \frac{31!}{29!2!} = 465$$

ways to make double-scoop cones with two different flavors. In addition, there would be 31 ways to make double-scope cones with the same flavors. Therefore,

$$465 + 31 = 496$$

double-scoop cones can be made if order doesn't matter.

(d) There is one mono-flavor triple-scoop cone. For two-flavor cones, we choose one of 31 flavors to be the single scoop, leaving 30 flavors for the non-unique scoops. Finally, there are $C(31,3)$ ways of building a cone with three different flavors. Thus there are

$$1 + (31)(30) + \frac{(31)(30)(29)}{3!} = 5456$$

different triple-scoop cones if order doesn't matter.

35. Select 8 of the 16 smokers and 8 of the 22 non-smokers; order does not matter in the group, so use combinations. There are

$$C(16,8)C(22,8) = 4,115,439,900$$

different ways to select the study group.

36. Since the plants are selected at random, that is, order does not matter, the answers are combinations.

(a) She is selecting 4 plants out of 11 plants. The number of ways in which this can be done is

$$C(11,4) = 330.$$

(b) She is selecting 2 of the 6 wheat plants and 2 of the 5 other plants. The number of ways in which this can be done is

$$C(6,2)C(5,2) = 150.$$

37. Order does not matter in choosing a delegation, so use combinations. This committee has $5 + 4 = 9$ members.

(a) There are

$$C(9,3) = \frac{9!}{6!3!} = \frac{9 \cdot 8 \cdot 7 \cdot 6!}{6! \cdot 3 \cdot 2 \cdot 1} = 84 \text{ possible delegations.}$$

(b) To have all Democrats, the number of possible delegations is

$$C(5,3) = 10.$$

(c) To have 2 Democrats and 1 Republican, the number of possible delegations is

$$C(5,2)C(4,1) = 10 \cdot 4 = 40.$$

(d) We have previously calculated that there are 84 possible delegations, of which 10 consist of all Democrats. Those 10 delegations are the only ones with no Republicans, so the remaining $84 - 10 = 74$ delegations include at least one Republican.

38. Since order is important, use permutations.

$$P(10, 4) = \frac{10!}{(10-4)!} = \frac{10!}{6!} = 5040$$

different committees are possible.

39. Order does not matter in choosing the panel, so use combinations.

$$C(45, 3) = \frac{45!}{42!\,3!} = \frac{45 \cdot 44 \cdot 43 \cdot 42!}{3 \cdot 2 \cdot 1 \cdot 42!} = 14{,}190$$

The publisher was wrong. There are 14,190 possible three judge panels.

40. Since order does not matter, use combinations.

$$C(52, 13) = \frac{52!}{(52-13)!\,13!} = \frac{52!}{39!\,13!} = 635{,}013{,}559{,}600$$

41. Since the cards are chosen at random, that is, order does not matter, the answers are combinations.

(a) There are 4 queens and 48 cards that are not queens. The total number of hands is

$$C(4, 4)C(48, 1) = 1 \cdot 48 = 48.$$

(b) Since there are 12 face cards (3 in each suit), there are 40 nonface cards. The number of ways to choose no face cards (all 5 nonface cards) is

$$C(40, 5) = \frac{40!}{35!\,5!} = 658{,}008.$$

(c) If there are exactly 2 face cards, there will be 3 nonface cards. The number of ways in which the face cards can be chosen is $C(12, 2)$, while the number of ways in which the nonface cards can be chosen is $C(40, 3)$. Using the multiplication principle, the number of ways to get this result is

$$C(12, 2)C(40, 3) = 66 \cdot 9880 = 652{,}080.$$

(d) If there are at least 2 face cards, there must be either 2 face cards and 3 nonface cards, 3 face cards and 2 nonface cards, 4 face cards and 1 nonface card, or 5 face cards. Use the multiplication principle as in part (c) to find the number of ways to obtain each of these possibilities. Then add these numbers. The total number of ways to get at least 2 face cards is

$$\begin{aligned} &C(12, 2)C(40, 3) + C(12, 3)C(40, 2) + C(12, 4)C(40, 1) + C(12, 5) \\ &= 66 \cdot 9880 + 220 \cdot 780 + 495 \cdot 40 + 792 \\ &= 652{,}080 + 171{,}600 + 19{,}800 + 792 \\ &= 844{,}272. \end{aligned}$$

(e) The number of ways to choose 1 heart is $C(13, 1)$, the number of ways to choose 2 diamonds is $C(13, 2)$, and the number of ways to choose 2 clubs is $C(13, 2)$. Using the multiplication principle, the number of ways to get this result is

$$C(13, 1)C(13, 2)C(13, 2) = 13 \cdot 78 \cdot 78 = 79{,}092.$$

42. **(a)** List the possibilities for each suit.

5, 6, 7, 8, 9; 5, 6, 7, 8, 10; 5, 6, 7, 9, 10;
5, 6, 8, 9, 10; 5, 7, 8, 9, 10; 6, 7, 8, 9, 10

There are 6 possibilities for each suit and there are 4 suits, so there are $4 \cdot 6 = 24$ possibilities.

(b) There are 6 cards of each suit from 5 to 10. Select 5 of the 6 cards. There are

$$4 \cdot C(6, 5) = 4 \cdot 6 = 24 \text{ possibilities.}$$

43. Since order does not matter, use combinations.

2 good hitters: $C(5, 2)C(4, 1) = 10 \cdot 4 = 40$

2 good hitters: $C(5, 3)C(4, 0) = 10 \cdot 1 = 10$

The total number of ways is $40 + 10 = 50$.

44. Since the hitters are being chosen at random, that is, order does not matter, the answers are combinations.

(a) The coach will choose 2 of the 6 good hitters and 1 of the 8 poor hitters. Using the multiplication principle, this can be done in

$$C(6, 2)C(8, 1) = 120 \text{ ways.}$$

(b) The coach will choose 3 of the 6 good hitters. This can be done in

$$C(6,3) = 20 \text{ ways.}$$

(c) The coach must choose either 2 good hitters and 1 poor hitter or 3 good hitters. Add the results from parts (a) and (b). This can be done in

$$C(6,2)C(8,1) + C(6,3)$$
$$= 140 \text{ ways.}$$

45. Since order does not matter, use combinations.

(a) There are

$$C(20,5) = 15{,}504$$

different ways to select 5 of the orchids.

(b) If 2 special orchids must be included in the show, that leaves 18 orchids from which the other 3 orchids for the show must be chosen. This can be done in

$$C(18,3) = 816$$

different ways.

46. In the lottery, 6 different numbers are to be chosen from the 99 numbers.

(a) There are

$$C(99,6) = \frac{99!}{93!\,6!}$$
$$= 1{,}120{,}529{,}256$$

different ways to choose 6 numbers if order is not important.

(b) There are

$$P(99,6) = \frac{99!}{93!}$$
$$= 806{,}781{,}064{,}320$$

different ways to choose 6 numbers if order matters.

47. Since order is not important, use combinations. To pick 5 of the 6 winning numbers, we must also pick 1 of the 93 losing numbers. Therefore, the number of ways to pick 5 of the 6 winning numbers is

$$C(6,5)C(93,1) = 6 \cdot 93 = 558.$$

48. (a) The number of ways to form the two committees assuming the nominating committee is formed first is

$$C(19,7)C(12,5) = 39{,}907{,}296.$$

(b) The number of ways to form the two committees assuming the public relations committee is formed first is

$$C(19,5)C(14,7) = 39{,}907{,}296.$$

(c) The number of indistinguishable ways can be thought of as first choosing the 7 places occupied by the red shirts out of 19 possible places in the line up and then choosing the 5 places occupied by yellow shirts out of the remaining 12 places. The number of ways is

$$C(19,7)C(12,5) = 39{,}907{,}296.$$

This is the same calculation as in part (a).

49. (a) The number of different committees possible is

$$C(5,2) + C(5,3) + C(5,4) + C(5,5)$$
$$= 10 + 10 + 5 + 1 = 26.$$

(b) The total number of subsets is

$$2^5 = 32.$$

The number of different committees possible is

$$2^5 - C(5,1) - C(5,0)$$
$$= 32 - 5 - 1$$
$$= 26.$$

50. Three distinct letters can be selected in $C(26, 3)$ ways, and once they are selected there is only one way to arrange them in alphabetical order. There are $10 \cdot 10 \cdot 10 = 1000$ ways to select the three digits. So the total number of plates is

$$\frac{26!}{23!\,3!} \cdot 1000 = 2{,}600{,}000.$$

51. (a) If the letters can be repeated, there are 26^6 choices of 6 letters, and there are 10 choices for the digit, giving

$$26^6 \cdot 10 = 3{,}089{,}157{,}760 \text{ passwords.}$$

(b) For nonrepeating letters we have $P(26,6) \cdot 10$ or 1,657,656,000 passwords.

52. (a) There can be 5, 4, 3, 2, 1, or no toppings. The total number of possibilities for the first pizza is

$$C(11,5) + C(11,4) + C(11,3) + C(11,2)$$
$$+ C(11,1) + C(11,0)$$
$$= 462 + 330 + 165 + 55 + 11 + 1$$
$$= 1024.$$

The total number of possibilities for the toppings on two pizzas is

$$1024 \cdot 1024 = 1{,}048{,}576.$$

(b) In part (a), we found that if the order of the two pizzas matters, there are

$$1024^2 = 1{,}048{,}576$$

possibilities. If we had a list of all of these possibilities and if the order of the pizzas doesn't matter, we must eliminate all of the possibilities that involve the same two pizzas. There are 1024 such items on the list, one of each of the possibilities for one pizza. Therefore, the number of items on the list that have a duplicate is

$$1{,}048{,}576 - 1024 = 1{,}047{,}552.$$

To eliminate duplicates, we eliminate the second listing of each of these, that is,

$$\frac{1{,}047{,}552}{2} = 523{,}776.$$

Subtracting this from the number of possibilities on the list, we see that if the order of the two pizzas doesn't matter, the number of possibilities is

$$1{,}048{,}576 - 523{,}776 = 524{,}800.$$

53. **(a)** A pizza can have 3, 2, 1, or no toppings. The number of possibilities is

$$\begin{aligned}&C(17,3) + C(17,2) + C(17,1) + C(17,0)\\&= 680 + 136 + 17 + 1\\&= 834.\end{aligned}$$

There are also four speciality pizzas, so the number of different pizzas is $834 + 4 = 838.$

(b) The number of 4forAll Pizza possibilities if all four pizzas are different is

$$C(838,4) = 20{,}400{,}978{,}015.$$

The number of 4forAll Pizza possibilities if there are three different pizzas (2 pizzas are the same and the other 2 are different) is

$$\begin{aligned}838 \cdot C(837,2) &= 838 \cdot 349{,}866\\&= 293{,}187{,}708.\end{aligned}$$

The number of 4forAll Pizza possibilities if there are two different pizzas (3 pizzas are the same or 2 pizzas and 2 pizzas are the same) is

$$\begin{aligned}&838 \cdot 837 + C(838,2)\\&= 701{,}406 + 350{,}703\\&= 1{,}052{,}109.\end{aligned}$$

The number of 4forAll Pizza possibilities if all four are the same is 838. The total number of 4forAll Pizza possibilities is

$$\begin{aligned}&20{,}400{,}978{,}015 + 293{,}187{,}708\\&+ 1{,}052{,}109 + 838\\&= 20{,}695{,}218{,}670\end{aligned}$$

(c) Using the described method, there would be 837 vertical lines and 4 X's or 841 objects, so the total number is

$$C(841,4) = 20{,}695{,}218{,}670.$$

54. **(a)**
$$\begin{aligned}&C(8,1) + C(8,2) + C(8,3) + C(8,4)\\&\quad + C(8,5) + C(8,6) + C(8,7) + C(8,8)\\&= 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1\\&= 255\end{aligned}$$

There are 255 breakfasts that can be made.

An easier way to compute this number is to notice that we could assemble a breakfast by deciding, for each of the 8 items, whether to include it. This gives 2^8 or 256 breakfasts. But one of these is the "empty breakfast" containing no items. Since no one wants to eat this breakfast, we discard it and are left with $256 - 1 = 255$ breakfasts.

(b) She has 2 choices. For the first choice she has 4 items. For the second choice she has 4 items.

$$4 \cdot 4 = 16$$

She can make 16 breakfasts.

(c) She has $C(4,2)$ choices of cereal mix and $C(4,3)$ choices of add-in mix. Her total number of choices is

$$C(4,2)C(4,3) = 6 \cdot 4 = 24.$$

(d) He has

$$\begin{aligned}&C(4,1) + C(4,2) + C(4,3) + C(4,4)\\&= 4 + 6 + 4 + 1 = 15\end{aligned}$$

choices of cereal mix and

$$\begin{aligned}&C(4,1) + C(4,2) + C(4,3) + C(4,4)\\&= 4 + 6 + 4 + 1 = 15\end{aligned}$$

choices of add-in mix. His total number of breakfasts is

$$15 \cdot 15 = 225.$$

(e)
$$\begin{aligned}&C(7,0) + C(7,1) + C(7,2)\\&\quad + C(7,3) + C(7,4)\\&\quad + C(7,5) + C(7,6) + C(7,7)\\&= 1 + 7 + 21 + 35 + 35\\&\quad + 21 + 7 + 1\\&= 128\end{aligned}$$

She has 128 different cereals.

55. **(a)** $C(9,3) = \frac{9!}{6!\,3!} = 84$

(b) $9 \cdot 9 \cdot 9 = 729$

(d) $C(17,3) = \frac{17!}{14!\,3!} = 680$

(e) First pick the two boneless buffalo wing flavors; there are $C(5,2) = 10$ ways of doing this. Then we still have 7 non-wing options, plus the 5 buffalo chicken wings available for the third item. So our total is $10 \cdot 12 = 120$.

56. Consider the first conference with five teams in each of the three divisions. The three winners from the three divisions can result in $5 \cdot 5 \cdot 5$, or 5^3, ways. Then, of the remaining $15 - 3 = 12$ teams, the three wild card teams can be chosen in $12 \cdot 11 \cdot 10$ ways. And since the order of the teams is not relevant, there are $\frac{5^3 \cdot 12 \cdot 11 \cdot 10}{6}$ ways to choose the teams from the first conference.

In the second conference, the situation is the same with the single exception being that one division has six, not five, teams. Therefore, there are $6 \cdot 5 \cdot 5 = 6 \cdot 5^2$ ways to choose the three division winners and $13 \cdot 12 \cdot 11$ ways to choose the wild card teams. Therefore, there are $\frac{6.5^2 \cdot 13 \cdot 12 \cdot 11}{6}$ ways to choose the six teams from the second conference.

Finally, the number of ways the six teams can be selected from the first conference and the six teams can be selected from the second conference is

$$\frac{5^3 \cdot 12 \cdot 11 \cdot 10}{6} \cdot \frac{6 \cdot 5^2 \cdot 13 \cdot 12 \cdot 11}{6} = 1{,}179{,}750{,}000.$$

57. **(a)** The number of ways the names can be arranged is

$$18! \approx 6.402 \times 10^{15}.$$

(b) 4 lines consist of a 3 syllable name repeated, followed by a 2 syllable name and then a 4 syllable name. Including order, the number of arrangements is

$$10 \cdot 4 \cdot 4 \cdot 9 \cdot 3 \cdot 3 \cdot 8 \cdot 2 \cdot 2 \cdot 7 \cdot 1 \cdot 1 = 2{,}903{,}040.$$

2 lines consist of a 3 syllable name repeated, followed by two more 3 syllable names. Including order, the number of arrangements is

$$6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720.$$

The number of ways the similar 4 lines can be arranged among the 6 total lines is

$$C(6,4) = 15.$$

The number of arrangements that fit the pattern is

$$2{,}903{,}040 \cdot 720 \cdot 15 \approx 3.135 \times 10^{10}.$$

58. **(a)** The number of ways the judges can be selected is

$$C(12,9) = 220.$$

(b) The number of different sets of scores is

$$C(12,9)C(12,9) = 220^2 = 48{,}400.$$

8.3 Probability Applications of Counting Principles

Your Turn 1

Using method 2,

$$P(\text{1 NY, 1 Chicago}) = \frac{C(3,1)C(1,1)}{C(6,2)} = \frac{1}{5}.$$

Your Turn 2

Since 8 of the nurses are men, $22 - 8 = 14$ of them are women. We choose 4 nurses, 2 men and 2 women.

$$P(\text{2 men among 4 selected}) = \frac{C(8,2)C(14,2)}{C(22,4)} = \frac{2548}{7315} \approx 0.3483$$

Your Turn 3

The probability that the container will be shipped is the probability of selecting 3 working engines for testing when there are $12 - 4 = 8$ working engines in the container. This is

$$P(\text{all 3 work}) = \frac{C(8,3)}{C(12,3)} = \frac{56}{220} = \frac{14}{55}.$$

The probability that at least one defective engine is in the batch is

$$P(\text{at least one defective}) = 1 - \frac{14}{55} = \frac{41}{55} \approx 0.7455.$$

Your Turn 4

$$P(\text{2 aces, 2 kings, 1 other}) = \frac{C(4,2)C(4,2)C(44,1)}{C(52,5)} = \frac{6 \cdot 6 \cdot 44}{2{,}598{,}960} \approx 0.0006095$$

Your Turn 5

If the slips are chosen without replacement, there are $P(7, 3) = 7 \cdot 6 \cdot 5 = 210$ ordered selections. Only one of these spells "now" so $P(\text{now}) = \frac{1}{210}$. If the slips are chosen with replacement there are 7 choices for each and thus $7 \cdot 7 \cdot 7 = 343$ selections. Again, only one of these spells "now" so in this case $P(\text{now}) = \frac{1}{343}$.

Your Turn 6

Using method 1 we compute the number of ways to arrange 14 pieces of fruit which come in 4 kinds: 2 kiwis, 3 apricots, 4 pineapples and 5 coconuts. Assuming the pieces of each kind of fruit are indistinguishable (all kiwis look alike, and so on), the number of arrangements is $\frac{14!}{2!3!4!5!} = 2,522,520$. If we keep the four kinds together (all kiwis next to each other, and so on) there are $4! = 24$ arrangements of the kinds that keep them together. So

$$P(\text{all of same kind together}) = \frac{24}{2,522,520} \approx 9.514 \times 10^{-6}.$$

8.3 Warmup Exercises

W1. There are 4 choices for the suit and then C(13, 6) ways of choosing from this suit, so there are

$$(4)\frac{13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 6864$$

ways of choosing the cards.

W2. $C(59,3) \cdot C(41,3) = 346,545,940$

8.3 Exercises

1. There are $C(11, 3)$ samples of 3 apples.

$$C(11, 3) = \frac{11 \cdot 10 \cdot 9}{3 \cdot 2 \cdot 1} = 165$$

There are $C(7, 3)$ samples of 3 red apples.

$$C(7, 3) = \frac{7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1} = 35$$

Thus,

$$P(\text{all red apples}) = \frac{35}{165} = \frac{7}{33}.$$

2. There are $C(11,3) = 165$ ways to select 3 of the 11 apples, while there are $C(4,3) = 4$ ways to select 3 yellow ones. Hence,

$$P(3\,\text{yellow}) = \frac{C(4, 3)}{C(11, 3)} = \frac{4}{165}.$$

3. There are $C(4, 2)$ samples of 2 yellow apples.

$$C(4, 2) = \frac{4 \cdot 3}{2 \cdot 1} = 6$$

There are $C(7,1) = 7$ samples of 1 red apple. Thus, there are $6 \cdot 7 = 42$ samples of 3 in which 2 are yellow and 1 red. Thus,

$$P(2\,\text{yellow and 1 red apple}) = \frac{42}{165} = \frac{14}{55}.$$

4. "More red than yellow" means 2 or 3 red. There are $C(7, 2)$ ways to choose 2 red apples and $C(4, 1)$ ways to pick a yellow; hence, there are $C(7, 2)C(4, 1) = 84$ ways to choose 2 red. Since there are $C(7, 3) = 35$ ways to pick 3 red, we have $84 + 35 = 119$ ways to have more red than yellow. Therefore,

$$P(\text{more red}) = \frac{119}{C(11, 3)} = \frac{119}{165}.$$

For Exercises 5 through 10 the number of possible 5-member committees is $C(20, 5) = 15,504$.

5. $$P(\text{all men}) = \frac{C(9, 5)}{C(20, 5)} = \frac{126}{15,504} \approx 0.008127$$

6. $$P(\text{all women}) = \frac{C(11, 5)}{C(20, 5)} = \frac{462}{15,504} \approx 0.02980$$

7. $$P(3\text{ men, 2 women}) = \frac{C(9, 3)C(11, 2)}{C(20, 5)} = \frac{4620}{15,504} \approx 0.2980$$

8. $$P(2\text{ men, 3 women}) = \frac{C(9, 2)C(11, 3)}{C(20, 5)} = \frac{5940}{15,504} \approx 0.3831$$

9. $$P(\text{at least 4 women}) = P(4\text{ women}) + P(5\text{ women}) = \frac{C(11, 4)C(9, 1) + C(11, 5)C(9, 0)}{C(20, 5)} = \frac{3532}{15,504} \approx 0.2214$$

10.

$$\begin{aligned}&P(\text{no more than 2 men})\\&= P(\text{no men}) + P(1\text{ man}) + P(2\text{ men})\\&= \frac{C(9,0)C(11,5) + C(9,1)C(11,4) + C(9,2)C(11,3)}{C(20,5)}\\&= \frac{9372}{15{,}504} \approx 0.6045\end{aligned}$$

11. The number of 2-card hands is

$$C(52,2) = \frac{52 \cdot 51}{2 \cdot 1} = 1326.$$

12. There are $C(4,2) = 6$ ways to pick 2 aces out of $C(52,2)$ ways to pick 2 cards; hence,

$$\begin{aligned}P(2\text{ aces}) &= \frac{C(4,2)}{C(52,2)} = \frac{6}{1326}\\&= \frac{1}{221} \approx 0.0045.\end{aligned}$$

13. There are $C(52,2) = 1326$ different 2-card hands. The number of 2-card hands with exactly one ace is

$$C(4,1)C(48,2) = 4 \cdot 48 = 192.$$

The number of 2-card hands with two aces is

$$C(4,2) = 6.$$

Thus there are 198 hands with at least one ace. Therefore,

$$\begin{aligned}&P(\text{the 2-card hand contains an ace})\\&= \frac{198}{1326} = \frac{33}{221} \approx 0.149.\end{aligned}$$

14. There are $C(13,2) = 78$ ways to pick 2 spades; hence,

$$P(2\text{ spades}) = \frac{78}{1326} = \frac{1}{17} \approx 0.059.$$

15. There are $C(52,2) = 1326$ different 2-card hands. There are $C(13,2) = 78$ ways to get a 2-card hand where both cards are of a single named suit, but there are 4 suits to choose from. Thus,

$$\begin{aligned}&P(\text{two cards of same suit})\\&= \frac{4 \cdot C(13,2)}{C(52,2)} = \frac{312}{1326} = \frac{4}{17} \approx 0.235.\end{aligned}$$

16. There are $C(12,2) = 66$ ways to pick 2 face cards; hence,

$$P(2\text{ face cards}) = \frac{66}{1326} = \frac{11}{221} \approx 0.0498.$$

17. There are $C(52,2) = 1326$ different 2-card hands. There are 12 face cards in a deck, so there are 40 cards that are not face cards. Thus,

$$\begin{aligned}&P(\text{no face cards})\\&= \frac{C(40,2)}{C(52,2)} = \frac{780}{1326} = \frac{130}{221} \approx 0.588.\end{aligned}$$

18. Ace, 2, 3, 4, 5, 6, 7, and 8 are the cards in each suit that are “not higher than 8,” for a total of 32, so

$$\begin{aligned}&P(\text{no card higher than 8})\\&= \frac{C(32,2)}{C(52,2)} = \frac{496}{1326} = \frac{248}{663} \approx 0.374.\end{aligned}$$

19. There are 26 choices for each slip pulled out, and there are 5 slips pulled out, so there are

$$26^5 = 11{,}881{,}376$$

different “words” that can be formed from the letters. If the “word” must be “chuck,” there is only one choice for each of the 5 letters (the first slip must contain a “c,” the second an “h,” and so on). Thus,

$$\begin{aligned}&P(\text{word is “chuck”})\\&= \frac{1^5}{26^5} = \left(\frac{1}{26}\right)^5 \approx 8.417 \times 10^{-8}.\end{aligned}$$

20. Only the first letter is specified; the other 4 can be any letter. The probability of starting with the letter p is

$$\frac{1}{26} \approx 0.038.$$

21. There are $26^5 = 11{,}881{,}376$ different “words” that can be formed. If the “word” is to have no repetition of letters, then there are 26 choices for the first letter, but only 25 choices for the second (since the letters must all be different), 24 choices for the third, and so on. Thus,

$$\begin{aligned}&P(\text{all different letters})\\&= \frac{26 \cdot 25 \cdot 24 \cdot 23 \cdot 22}{26^5}\\&= \frac{1 \cdot 25 \cdot 24 \cdot 23 \cdot 22}{26^4}\\&= \frac{303{,}600}{456{,}976}\\&= \frac{18{,}975}{28{,}561} \approx 0.6644.\end{aligned}$$

22. There are 26^5 possible 5-letter "words," and 23^5 "words" that do not contain x, y, or z. Hence,

$$P(\text{no x, y, or z}) = \frac{23^5}{26^5} = \frac{6,436,343}{11,881,376} \approx 0.5417.$$

25. $P(\text{at least 2 presidents have the same birthday})$
$= 1 - P(\text{no 2 presidents have the same birthday})$

The number of ways that 43 people can have the same or different birthdays is $(365)^{43}$. The number of ways that 43 people can have all different birthdays is the number of permutations of 365 things taken 43 at a time or $P(365, 43)$. Thus,

$$P(\text{at least 2 presidents have the same birthday}) = 1 - \frac{P(365, 43)}{365^{43}}.$$

(Be careful to realize that the symbol P is sometimes used to indicate permutations and sometimes used to indicate probability; in this solution, the symbol is used both ways.)

26. Using the result from Example 6, the probability that at least 2 people in a group of n people have the same birthday is

$$1 - \frac{P(365, n)}{(365)^n}.$$

Therefore, the probability that at least 2 of the 100 U.S. Senators have the same birthday is

$$1 - \frac{P(365, 100)}{(365)^{100}}.$$

27. Since there are 435 members of the House of Representatives, and there are only 365 days in a year, it is a certain event that at least 2 people will have the same birthday. Thus,

$P(\text{at least 2 members have the same birthday}) = 1.$

28. There are $C(n, 2)$ ways to pick which pair is to have the same birthday. One member of the pair has 365 choices of a birthday, the other only 1. The other $n - 2$ people have 364, 363, 362, etc., choices. Thus, the probability is

$$C(n, 2) \cdot \frac{P(365, n-1)}{(365)^n}.$$

29. $P(\text{matched pair})$

$$\begin{aligned} &= P(\text{2 black or 2 brown or 2 blue}) \\ &= P(\text{2 black}) + P(\text{2 brown}) + 2(\text{2 blue}) \\ &= \frac{C(9,2)}{C(17,2)} + \frac{C(6,2)}{C(17,2)} + \frac{C(2,2)}{C(17,2)} \\ &= \frac{36}{136} + \frac{15}{136} + \frac{1}{136} \\ &= \frac{52}{136} = \frac{13}{34} \end{aligned}$$

30. The number of orders of the three types of birds is $P(3, 3)$. The number of arrangements of the crows is $P(3, 3)$, of the bluejays is $P(4, 4)$, and of the starlings is $P(5, 5)$. The total number of arrangements of all the birds is $P(12, 12)$.

$$P(\text{all birds of same type are sitting together}) = \frac{P(3,3) \cdot P(3,3) \cdot P(4,4) \cdot P(5,5)}{P(12,12)} \approx 2.165 \times 10^{-4}$$

31. There are 6 letters so the number of possible spellings (counting duplicates) is $6! = 720$. Since the letter l is repeated 2 times and the letter t is repeated 2 times, the spelling "little" will occur $2!2! = 4$ times. The probability that "little" will be spelled is $\frac{4}{720} = \frac{1}{180}$.

32. There are 11 letters so the number of possible spellings (counting duplicates) is $11! = 39,916,800$. Since the letter s is repeated 4 times, the letter s is repeated 4 times, and the letter p is repeated 2 times, the spelling "Mississippi" will occur $4!4!2! = 1152$ times. The probability that "Mississippi" will be spelled is

$$\frac{1152}{39,916,800} \approx 0.0000289.$$

33. Each of the 4 people can choose to get off at any one of the 7 floors, so there are 7^4 ways the four people can leave the elevator. The number of ways the people can leave at different floors is the number of permutations of 7 things (floors) taken 4 at a time or

$$P(7, 4) = 7 \cdot 6 \cdot 5 \cdot 4 = 840.$$

The probability that no 2 passengers leave at the same floor is

$$\frac{P(7,4)}{7^4} = \frac{840}{2401} \approx 0.3499.$$

Thus, the probability that at least 2 passengers leave at the same floor is

$$1 - 0.3499 = 0.6501.$$

(Note the similarity of this problem and the "birthday problem.")

34. Let x = the total number of balls. Since the probability of picking 5 balls which all are blue is $\frac{1}{2}$, we can see that $x > 5$. (If $x = 5$, the probability would be 1.) Let's look at the number of blue balls needed. If there were 6 blue balls, $C(6,5) = 6$ and $C(x,5) = 12$, since the probability is $\frac{1}{2}$. Since x must be larger than the number of blue balls, $x \geq 7$. But since $C(7,5) = 21$,

$$C(x,5) \geq 21 \neq 12.$$

If there were 7 blue balls, $C(7,5) = 21$ and $C(x,5) = 42$. Since $x \geq 8$,

$$C(x,5) \geq 56 \neq 42.$$

If there were 8 blue balls, $C(8,5) = 56$ and $C(x,5) = 112$. Since $x \geq 9$,

$$C(x,5) \geq 126 \neq 112.$$

If there were 9 blue balls, $C(9,5) = 126$ and $C(x,5) = 252$. Since $x \geq 10$, $C(x,5) \geq 252$, and x must be 10.

Therefore, there were 10 balls, 9 of them blue,

$$P(\text{all 5 blue}) = \frac{C(9,5)}{C(10,5)} = \frac{1}{2}.$$

35.
$$\begin{aligned} &P(\text{at least one \$100-bill})\\ &= P(1\ \$100\text{-bill}) + P(2\ \$100\text{-bills})\\ &= \frac{C(2,1)C(4,1)}{C(6,2)} + \frac{C(2,2)C(4,0)}{C(6,2)}\\ &= \frac{8}{15} + \frac{1}{15} = \frac{9}{15} = \frac{3}{5} \end{aligned}$$

$$P(\text{no \$100-bill}) = \frac{C(2,0)\,C(4,2)}{C(6,2)} = \frac{6}{15} = \frac{2}{5}$$

It is more likely to get at least one $100-bill.

36. There are 11 ways to choose 1 typewriter from the shipment of 11. Since 2 of the 11 are defective, there are 9 ways to choose 1 nondefective typewriter. Thus,

$$P(\text{1 drawn from the 11 is not defective}) = \frac{9}{11}.$$

37. There are $C(9,2)$ possible ways to choose 2 nondefective typewriters out of the $C(11,2)$ possible ways of choosing any 2. Thus,

$$P(\text{no defective}) = \frac{C(9,2)}{C(11,2)} = \frac{36}{55}.$$

38. There are $C(11,3)$ ways to choose 3 typewriters.

$$C(11,3) = \frac{11!}{3!\,8!} = \frac{11 \cdot 10 \cdot 9}{3 \cdot 2 \cdot 1} = 165$$

There are $C(9,3)$ ways to choose 3 nondefective typewriters.

$$C(9,3) = \frac{9!}{3!\,6!} = \frac{9 \cdot 8 \cdot 7}{3 \cdot 2 \cdot 1} = 84$$

Thus,

$$\begin{aligned} &P(\text{3 drawn from the 9 are nondefective})\\ &= \frac{84}{165} = \frac{28}{55}. \end{aligned}$$

39. There are $C(9,4)$ possible ways to choose 4 nondefective typewriters out of the $C(11,4)$ possible ways of choosing any 4. Thus,

$$P(\text{no defective}) = \frac{C(9,4)}{C(11,4)} = \frac{126}{330} = \frac{21}{55}.$$

40. There are $C(12,4) = 495$ different ways to choose 4 engines for testing from the crate of 12. A crate will not be shipped if any one of the 4 in the sample is defective. If there are 2 defectives in the crate, then there are $C(10,4) = 210$ ways of choosing a sample with no defectives. Thus,

$$\begin{aligned} &P(\text{shipping a crate with 2 defectives})\\ &= \frac{210}{495} = \frac{14}{33} \approx 0.424. \end{aligned}$$

41. There are $C(12,5) = 792$ ways to pick a sample of 5. It will be shipped if all 5 are good. There are $C(10,5) = 252$ ways to pick 5 good ones, so

$$P(\text{all good}) = \frac{252}{792} = \frac{7}{22} \approx 0.318.$$

42. If Melanie is first and Boyd last, the remaining 7 can be in any order. There are 9! possible orders, so

$$P(\text{Melanie first, Boyd last}) = \frac{7!}{9!} = \frac{1}{72}.$$

43.
$$\begin{aligned} &P(\text{not Scottsdale customer})\\ &= \frac{\left[\begin{array}{l}\text{number of choices of 4 out of the}\\ \text{5 non-Scottsdale customers}\end{array}\right]}{\text{number of choices of 4 out of the 6 customers}}\\ &= \frac{C(5,4)}{C(6,4)} = \frac{5}{15} = \frac{1}{3} \end{aligned}$$

44. There are $P(5,5)$ different orders of the names. Only one of these would be in alphabetical order. Therefore, $P(5,5) - 1$ are not in alphabetical order. Thus,

$P(\text{not in alphabetical order})$

$$= \frac{P(5,5) - 1}{P(5,5)} = \frac{120 - 1}{120} = \frac{119}{120}.$$

45. There are 20 people in all, so the number of possible 5-person committees is $C(20,5) = 15{,}504$. Thus, in parts (a)-(g), $n(S) = 15{,}504$.

(a) There are $C(10,3)$ ways to choose the 3 men and $C(10,2)$ ways to choose the 2 women. Thus,

$P(3 \text{ men and } 2 \text{ women})$

$$= \frac{C(10,3)C(10,2)}{C(20,5)} = \frac{120 \cdot 45}{15{,}504}$$

$$= \frac{225}{646} \approx 0.348.$$

(b) There are $C(6,3)$ ways to choose the 3 Miwoks and $C(9,2)$ ways to choose the 2 Pomos. Thus,

$P(\text{exactly 3 Miwoks and 2 Pomos})$

$$= \frac{C(6,3)C(9,2)}{C(20,5)} = \frac{20 \cdot 36}{15{,}504}$$

$$= \frac{15}{323} \approx 0.046.$$

(c) Choose 2 of the 6 Miwoks, 2 of the 5 Hoopas, and 1 of the 9 Pomos. Thus,

$P(\text{2 Miwoks, 2 Hoopas, and a Pomo})$

$$= \frac{C(6,2)C(5,2)C(9,1)}{C(20,5)} = \frac{15 \cdot 10 \cdot 9}{15{,}504}$$

$$= \frac{225}{2584} \approx 0.087.$$

(d) There cannot be 2 Miwoks, 2 Hoopas, and 2 Pomos, since only 5 people are to be selected. Thus,

$P(\text{2 Miwoks, 2 Hoopas, and 2 Pomos}) = 0.$

(e) Since there are more women then men, there must be 3, 4, or 5 women.

$P(\text{more women than men})$

$$= \frac{C(10,3)C(10,2) + C(10,4)C(10,1) + C(10,5)C(10,0)}{C(20,5)}$$

$$= \frac{7752}{15{,}504} = \frac{1}{2}$$

(f) Choose 3 of 5 Hoopas and any 2 of the 15 non-Hoopas.

$P(\text{exactly 3 Hoopas})$

$$= \frac{C(5,3)C(15,2)}{C(20,5)}$$

$$= \frac{175}{2584} \approx 0.068$$

(g) There can be 2 to 5 Pomos, the rest chosen from the 11 non-Pomos.

$P(\text{at least 2 Pomos})$

$$= \frac{C(9,2)C(11,3) + C(9,3)C(11,2) + C(9,4)C(11,1) + C(9,5)C(11,0)}{C(20,5)}$$

$$= \frac{503}{646} \approx 0.779$$

46. (a) $P(\text{first person}) = \frac{5}{40} = \frac{1}{8}$

(b) $P(\text{last person}) = \frac{5(39!)}{40!}$

$$= \frac{5(39!)}{40(39!)}$$

$$= \frac{5}{40} = \frac{1}{8}$$

(c) No, everybody has the same chance.

47. There are 21 books, so the number of selection of any 6 books is

$$C(21,6) = 54{,}264.$$

(a) The probability that the selection consisted of 3 Hughes and 3 Morrison books is

$$\frac{C(9,3)C(7,3)}{C(21,6)} = \frac{85 \cdot 35}{54{,}264}$$

$$= \frac{2940}{54{,}264} \approx 0.0542.$$

(b) A selection containing exactly 4 Baldwin books will contain 2 of the 16 books by the other authors, so the probability is

$$\frac{C(5,4)C(16,2)}{C(21,6)} = \frac{5 \cdot 120}{54{,}264}$$

$$= \frac{600}{54{,}264} \approx 0.0111.$$

(c) The probability of a selection consisting of 2 Hughes, 3 Baldwin, and 1 Morrison book is

$$\frac{C(9,2)C(5,3)C(7,1)}{C(21,6)} = \frac{30 \cdot 10 \cdot 7}{54{,}264}$$

$$= \frac{2520}{54{,}264}$$

$$\approx 0.0464.$$

(d) A selection consisting of at least 4 Hughes books may contain 4, 5, or 6 Hughes books, with any remaining books by the other authors. Therefore, the probability is

$$\frac{\begin{pmatrix} C(9,4)C(12,2) + C(9,5)C(12,1) \\ + C(9,6)C(12,0) \end{pmatrix}}{C(21,6)} = \frac{126 \cdot 66 + 126 \cdot 12 + 84}{54,264} = \frac{8316 + 1512 + 84}{54,264} = \frac{9912}{54,264} \approx 0.1827.$$

(e) Since there are 9 Hughes books and 5 Baldwin books, there are 14 books written by males. The probability of a selection with exactly 4 books written by males is

$$\frac{C(14,2)C(7,2)}{C(21,6)} = \frac{1001 \cdot 21}{54,264} = \frac{21,021}{54,264} \approx 0.3874.$$

(f) A selection with no more than 2 books written by Baldwin may contain 0, 1, or 2 books by Baldwin, with the remaining books by the other authors. Therefore, the probability is

$$\frac{C(5,0)C(16,6) + C(5,1)C(16,5) + C(5,2)C(16,4)}{C(21,6)} = \frac{8008 + 5 \cdot 4368 + 10 \cdot 1820}{54,264} = \frac{8008 + 21,840 + 18,200}{54,264} = \frac{48,048}{54,264} \approx 0.8854.$$

48. There are $C(52,5)$ different 5-card poker hands. There are 4 royal flushes, one for each suit. Thus,

$$P(\text{royal flush}) = \frac{4}{C(52,5)} = \frac{4}{2,598,960} = \frac{1}{649,740} \approx 1.539 \times 10^{-6}.$$

49. A straight flush could start with an ace, $2, 3, 4, \ldots, 7, 8,$ or 9. This gives 9 choices in each of 4 suits, so there are 36 choices in all. Thus,

$$P(\text{straight flush}) = \frac{36}{C(52,2)} = \frac{36}{2,598,960} \approx 0.00001385 = 1.385 \times 10^{-5}.$$

50. The four of a kind can be chosen in 13 ways and then is matched with 1 of the remaining 48 cards to make a 5-card hand containing four of a kind. Thus, there are $13 \cdot 48 = 624$ poker hands with four of a kind. It follows that

$$P(\text{four of a kind}) = \frac{624}{C(52,2)} = \frac{624}{2,598,960} = \frac{1}{4165} \approx 2.401 \times 10^{-4}.$$

51. A straight could start with an ace, 2, 3, 4, 5, 6, 7, 8, 9, or 10 as the low card, giving 40 choices. For each succeeding card, only the suit may be chosen. Thus, the number of straights is

$$40 \cdot 4^4 = 10,240.$$

But this also counts the straight flushes, of which there are 36 (see Exercise 49), and the 4 royal flushes. There are thus 10,200 straights that are not also flushes, so

$$P(\text{straight}) = \frac{10,200}{2,598,960} \approx 0.0039.$$

52. There are 13 different values with 4 cards of each value. The total number of possible three of a kinds is then $13 \cdot C(4,3)$. The other 2 cards must be chosen from the remaining 48 cards of different value. However, these 2 cards must be different. Thus, for the last 2 cards, there are 48 cards to choose from, but the cards must not have the same value. The number of possibilities for the last 2 cards is $C(48,2) - 12 \cdot C(4,2)$, and

$$P(\text{three of a kind}) = \frac{13 \cdot C(4,3)\,[C(48,2) - 12 \cdot C(4,2)]}{C(52,5)} \approx 0.0211.$$

53. There are 13 different values of cards and 4 cards of each value. Choose 2 values out of the 13 for the values of the pairs. The number of ways to select the 2 values is $C(13, 2)$. The number of ways to select a pair for each value is $C(4, 2)$. There are $52 - 8 = 44$ cards that are neither of these 2 values, so the number of ways to select the fifth card is $C(44,1)$. Thus,

$$P(\text{two pairs}) = \frac{C(13,2)\,C(4,2)\,C(4,2)\,C(44,1)}{C(52,5)} = \frac{123{,}552}{2{,}598{,}960} \approx 0.0475.$$

54. There are 13 different values with 4 cards of each value. The total number of possible pairs is $13 \cdot C(4,2)$. The remaining 3 cards must be chosen from the 48 cards of different value. However, among these 3 we cannot have 3 of a kind nor can we have 2 of a kind.

$$P(\text{one pair}) = \frac{13 \cdot C(4,2)[C(48,3) - 12 \cdot C(4,3) - 12 \cdot C(4,2) \cdot C(44,1)]}{C(52,5)} \approx 0.4226$$

55. There are $C(52,13)$ different 13-card bridge hands. Since there are only 13 hearts, there is exactly one way to get a bridge hand containing only hearts. Thus,

$$P(\text{only hearts}) = \frac{1}{C(52,13)} \approx 1.575 \cdot 10^{-12}.$$

56. The hand can have exactly 3 aces or 4 aces. There are $C(4,3) = 4$ ways to pick exactly 3 aces, and there are $C(48,10)$ ways to pick the other 10 cards. Also, there is only $C(4,4) = 1$ way to pick 4 aces, and there are $C(48,9)$ ways to pick the other 9 cards. Hence,

$$P(3\text{ aces}) = \frac{C(4,3)\,C(48,10) + C(4,4)\,C(48,9)}{C(52,13)} \approx 0.0438.$$

57. There are $C(4,2)$ ways to obtain 2 aces, $C(4,2)$ ways to obtain 2 kings, and $C(44,9)$ ways to obtain the remaining 9 cards. Thus,

P(exactly 2 aces and exactly 2 kings)

$$= \frac{C(4,2)\,C(4,2)\,C(44,9)}{C(52,13)} \approx 0.0402.$$

58. The number of ways of choosing 3 suits is $P(4,3)$. The number of ways of choosing 6 of one suit is $C(13,6)$, 4 of another is $C(13,4)$ and 3 of another is $C(13,3)$. Thus,

P(6 of one suit, 4 of another, and 3 of another)

$$= \frac{P(4,3)C(13,6)C(13,4)C(13,3)}{C(52,13)} \approx 0.0133.$$

Order is important in this problem because 6 spades, 4 hearts, and 3 clubs would be different than 6 hearts, 4 clubs, and 3 spades.

For Exercises 59 through 65, use the fact that the number of 7-card selections is $C(52, 7)$.

59. Pick a kind for the pair: 13 choices
Choose 2 suits out of the 4 suits for this kind: $C(4, 2)$
Then pick 5 kinds out of the 12 remaining: $C(12, 5)$
For each of these 5 kinds, pick one of the 4 suits: 4^5
The product of these factors gives the numerator and the denominator is $C(52, 7)$.

$$\frac{13 \cdot C(4, 2) \cdot C(12, 5) \cdot 4^5}{C(52, 7)} \approx 0.4728$$

60. Pick the 2 kinds for the 2 pairs: $C(13, 2)$
For each of these kinds, choose 2 of the 4 suits: two factors of $C(4, 2)$
There are now 3 cards remaining which must be 3 of the 11 kinds remaining: $C(11, 3)$
For each of these 3 kinds we pick one of the 4 suits: 4^3
The product of these factors gives the numerator and the denominator is $C(52, 7)$.

$$\frac{C(13, 2) \cdot [C(4, 2)]^2 \cdot C(11, 3) \cdot 4^3}{C(52, 7)} \approx 0.2216$$

61. Pick a kind for the three-of-a-kind: 13 choices
Choose 3 suits out of the 4 suits for this kind: $C(4, 3)$
There are now 4 cards remaining which must be 4 of the 12 kinds remaining: $C(12, 4)$
For each of these 4 kinds we pick one of the 4 suits: 4^4
The product of these factors gives the numerator and the denominator is $C(52, 7)$.

$$\frac{13 \cdot C(4, 3) \cdot C(12, 4) \cdot 4^4}{C(52, 7)} \approx 0.0493$$

62. Pick a kind for the four-of-a-kind: 13 choices
Choose 4 suits out of the 4 suits for this kind: $C(4, 4)$ (there's only one way of doing this!)
There are now 3 cards remaining which must be 3 of the 12 kinds remaining: $C(12, 3)$
For each of these 3 kinds we pick one of the 4 suits: 4^3
The product of these factors gives the numerator and the denominator is $C(52, 7)$.

$$\frac{13 \cdot C(4, 4) \cdot C(12, 3) \cdot 4^3}{C(52, 7)} \approx 0.0014$$

63. Pick a suit: 4
Now we either get exactly 5 of this suit and 2 of other suits: $C(13,5) \cdot C(39,2)$
…or exactly 6 of this suit and 1 of another suit: $C(13,6) \cdot C(39,1)$
…or all 7 of our chosen suit: $C(13,7)$. We now add these options over our usual denominator.

$$\frac{4 \cdot [C(13,5) \cdot C(39,2) + C(13,6) \cdot C(39,1) + C(13,7)]}{C(52,7)} \approx 0.0306$$

64. Pick 2 kinds: $C(13,2)$
Pick either 3 of the first kind and 2 of the second and 2 not of either of these 2 kinds: $C(4,3) \cdot C(4,2) \cdot C(44,2)$
…or pick 2 of the first kind and 3 of the second and 2 not of either of these 2 kinds: $C(4,2) \cdot C(4,3) \cdot C(44,2)$
…or pick 3 of each kind and 1 of a different kind: $C(4,3) \cdot C(4,3) \cdot C(44,1)$
Noting that the first and second expressions above are the same, we can assemble the numerator over the usual denominator.

$$\frac{C(13,2) \cdot [2 \cdot C(4,3) \cdot C(4,2) \cdot C(44,2) + C(4,3) \cdot C(4,3) \cdot C(44,1)]}{C(52,7)} \approx 0.0269$$

65. We need at least 3 hearts out of 5 cards. This can happen in three ways:
3 hearts, 2 non-hearts: $C(11,3) \cdot C(39,2)$
4 hearts, 1 non-heart: $C(11,4) \cdot C(39,1)$
5 hearts: $C(11,5)$
Add these options over the usual denominator.

$$\frac{C(11,3) \cdot C(39,2) + C(11,4) \cdot C(39,1) + C(11,5)}{C(52,7)} \approx 0.0640$$

66. There are $C(99,6) = 1{,}120{,}529{,}256$ different ways to pick 6 numbers from 1 to 99, but there is only 1 way to win; the 6 numbers you pick must exactly match the 6 winning numbers, without regard to order. Thus,

$$P(\text{win the big prize}) = \frac{1}{1{,}120{,}529{,}256} \approx 8.924 \times 10^{-10}.$$

67. To find the probability of picking 5 of the 6 lottery numbers correctly, we must recall that the total number of ways to pick the 6 lottery numbers is $C(99, 6) = 1{,}120{,}529{,}256$. To pick 5 of the 6 winning numbers, we must also pick 1 of the 93 losing numbers. Therefore, the number of ways of picking 5 of the 6 winning numbers is

$$C(6,5)\, C(93,1) = 558.$$

Thus, the probability of picking 5 of the 6 numbers correctly is

$$\frac{C(6,5)\, C(93,1)}{C(99,6)} \approx 4.980 \times 10^{-7}.$$

68. Let A be the event of drawing four royal flushes in a row all in spades, and B be the event of meeting four strangers all with the same birthday. Then,

$$P(A) = \left[\frac{1}{C(52,5)}\right]^4.$$

For four people, the number of possible birthdays is 365^4. Of these there are 365 which are the same (that is, all birthdays January 1 or January 2 or January 3, etc.).

$$P(B) = \frac{365}{365^4} = \frac{1}{365^3}$$

Therefore,

$$P(A \cap B) = \left[\frac{1}{C(52,5)}\right]^4 \frac{1}{365^3} \approx 4.507 \times 10^{-34}.$$

No, this probability is much smaller than that of winning the lottery.

69. The probability of picking six numbers out of 49 is

$$P(\text{6 out of 49}) = \frac{1}{C(49,6)} = \frac{1}{13{,}983{,}816}$$

The probability of picking of picking five numbers out of 52 is

$$P(\text{5 out of 52}) = \frac{1}{C(52,5)} = \frac{1}{2{,}598{,}960}$$

The probability of winning the lottery when picking five out of 52 is higher.

70. **(a)** The number of ways to select 5 numbers between 1 and 55 is $C(55,5) = 3{,}478{,}761$ and there are 42 ways to select the bonus number.

$$P(\text{winning jackpot}) = \frac{1}{3{,}478{,}761 \cdot 42} = \frac{1}{146{,}107{,}962}$$

(b) The number of selections you would make over 138 years is

$$138 \cdot 365 \cdot 24 \cdot 60 = 72{,}582{,}480.$$

The probability that none of the selections win is

$$\left(\frac{146{,}107{,}961}{146{,}107{,}962}\right)^{72{,}482{,}480} \approx 0.6085.$$

Therefore, the probability of winning is $1 - 0.6085 = 0.3915$. This calculator answer is slightly inaccurate because of rounding in the calculator's arithmetic. The correct answer to four places is 0.3911.

71. **(a)** The number of ways to select 6 numbers between 1 and 49 is $C(49,6) = 13{,}983{,}816$. The number of ways to select 3 of the 6 numbers, while not selecting the bonus number is

$$C(6,3)\,C(42,3) = 20 \cdot 11{,}480 = 229{,}600.$$

The probability of winning fifth prize is

$$\frac{229{,}600}{13{,}983{,}816} \approx 0.01642.$$

(b) The number of ways to select 2 of the 6 numbers plus the bonus number is

$$C(6,2)\,C(1,1)\,C(42,3) = 15 \cdot 1 \cdot 11{,}480 = 172{,}200.$$

The probability of winning sixth prize is

$$\frac{172{,}200}{13{,}983{,}816} \approx 0.01231.$$

72. P(saying "math class is tough")

$$= \frac{C(1,1)\,C(269,3)}{C(270,4)} \approx 0.0148$$

No, it is not correct. The correct figure is 1.48%.

73. **(a)** There were 28 games played in the season, since the numbers in the "Won" column have a sum of 28 (and the numbers in the "Lost" column have a sum of 28).

(b) Assuming no ties, each of the 28 games had 2 possible outcomes; either Team A won and Team B lost, or else Team A lost and Team B won. By the multiplication principle, this means that there were

$$2^{28} = 268{,}435{,}456$$

different outcomes possible.

(c) Any one of the 8 teams could have been the one that won all of its games, any one of the remaining 7 teams could have been the one that won all but one of its games, and so on, until there is only one team left, and it is the one that lost all of its games. By the multiplication principle, this means that there were

$$8! = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 40{,}320$$

different "perfect progressions" possible.

(d) Thus,

P("perfect progression" in an 8-team league)

$$= \frac{8!}{2^{28}} \approx 0.0001502 = 1.502 \times 10^{-4}.$$

(e) If there are n teams in the league, then the "Won" column will begin with $n-1$, followed by $n-2$, then $n-3$, and so on down to 0. It can be shown that the sum of these n numbers is $\frac{n(n-1)}{2}$, so there are $2^{n(n-1)/2}$ different win/lose progressions possible. The n teams can be ordered in $n!$ different ways, so there are $n!$ different "perfect progressions" possible. Thus,

P("perfect progression" in an n-team league)

$$= \frac{n!}{2^{n(n-1)/2}}.$$

74. **(a)** $\dfrac{C(103,19)}{C(162,19)} \approx 9.366 \times 10^{-5}$

(b) $\dfrac{C(48,19)}{C(81,19)} \approx 7.622 \times 10^{-6}$

75. **(a)** There are only 4 ways to win in just 4 calls: the 2 diagonals, the center column, and the center row. There are $C(75,4)$ combinations of 4 numbers that can occur. The probability that a person will win bingo after just 4 numbers are called is $\frac{4}{C(75,4)} \approx 3.291 \times 10^{-6}$.

(b) There is only 1 way to get an L. It can occur in as few as 9 calls. There are $C(75,9)$ combinations of 9 numbers that can occur in 9 calls , so the probability of an L in 9 calls is $\frac{1}{C(75,9)} \approx 7.962 \times 10^{-12}$.

(c) There is only 1 way to get an X-out. It can ocur in as few as 8 calls. There are $C(75,8)$ combinations of 8 numbers that can occur. The probability that an X-out occurs in 8 calls is $\frac{1}{C(75,8)} \approx 5.927 \times 10^{-11}$.

(d) Four columns contain a permutation of 15 numbers taken 5 at a time. One column contains a permutation of 15 numbers taken 4 at a time. The number of distinct cards is
$P(15,5)^4 \cdot P(15,4) \approx 5.524 \times 10^{26}$.

76. (a) $P(\text{6 blues drawn} \mid \text{red removed}) = \left(\frac{3}{5}\right)^6$

and

$P(\text{6 blues drawn} \mid \text{blue removed}) = \left(\frac{2}{5}\right)^6$.

Since the box initially contained equal numbers of red and blue balls,

$P(\text{red removed}) = P(\text{blue removed}) = \frac{1}{2}$.

By Bayes' Theorem,

$$\begin{aligned} &P(\text{red removed} \mid \text{6 blues drawn}) \\ &= \left[P(\text{6 blues drawn} \mid \text{red removed}) \cdot P(\text{red removed})\right] \\ &\quad \div \left[\begin{array}{l} P(\text{6 blues drawn} \mid \text{red removed}) \cdot P(\text{red removed}) \\ \quad + P(\text{6 blues drawn} \mid \text{blue removed}) \cdot P(\text{blue removed}) \end{array}\right] \\ &= \frac{\left(\frac{3}{5}\right)^6 \cdot \frac{1}{2}}{\left(\frac{3}{5}\right)^6 \cdot \frac{1}{2} + \left(\frac{2}{5}\right)^6 \cdot \frac{1}{2}} \\ &= \frac{3^6}{3^6 + 2^6} \approx 0.9193 \end{aligned}$$

(b) $P(\text{44 blues in 80} \mid \text{red removed})$

$$= C(80, 44) \cdot \left(\frac{3}{5}\right)^{44} \left(\frac{2}{5}\right)^{36}$$

and

$P(\text{44 blues in 80} \mid \text{blue removed})$

$$= C(80, 44) \cdot \left(\frac{2}{5}\right)^{44} \left(\frac{3}{5}\right)^{36}$$

(See the next section for an explanation of this probability). By Bayes' Theorem,

$P(\text{red removed} \mid \text{44 blues in 80})$

$$\begin{aligned} &= \left[P(\text{44 blues in 80} \mid \text{red removed}) \cdot P(\text{red removed})\right] \\ &\quad \div \left[\begin{array}{l} P(\text{44 blues in 80} \mid \text{red removed}) \cdot P(\text{red removed}) \\ \quad + P(\text{44 blues in 80} \mid \text{blue removed}) \cdot P(\text{blue removed}) \end{array}\right] \\ &= \frac{C(80, 44) \cdot \left(\frac{3}{5}\right)^{44} \left(\frac{2}{5}\right)^{36} \cdot \frac{1}{2}}{C(80, 44) \cdot \left(\frac{3}{5}\right)^{44} \left(\frac{2}{5}\right)^{36} \cdot \frac{1}{2} + C(80, 44) \cdot \left(\frac{2}{5}\right)^{44} \left(\frac{3}{5}\right)^{36}} \\ &\approx 0.9624 \end{aligned}$$

8.4 Binomial Probability

Your Turn 1

$$\begin{aligned} P(\text{exactly 2 of 6}) &= C(6,2)(0.59)^2(0.41)^4 \\ &= 15(0.3481)(0.0283) \\ &\approx 0.1475 \end{aligned}$$

Your Turn 2

$$\begin{aligned} &P(\text{at least one incorrect charge}) \\ &\quad = 1 - P(\text{no incorrect charges in 4}) \\ &\quad = 1 - C(4,0)(0.29)^0(0.71)^4 \\ &\quad \approx 1 - 0.2541 \\ &\quad = 0.7459 \end{aligned}$$

Your Turn 3

$$\begin{aligned} &P(\text{at most 3 incorrect charges in 6}) \\ &\quad = P(0) + P(1) + P(2) + P(3) \\ &\quad = C(6,0)(0.29)^0(0.71)^6 + C(6,1)(0.29)^1(0.71)^5 \\ &\qquad + C(6,2)(0.29)^2(0.71)^4 + C(6,3)(0.29)^3(0.71)^3 \\ &\quad \approx 0.9372 \end{aligned}$$

8.4 Warmup Exercises

W1. $C(100,10) \approx 1.731 \times 10^{13}$

W2. $C(32,8) = 10{,}518{,}300$

8.4 Exercises

1. This is a Bernoulli trial problem with

$P(\text{success}) = P(\text{girl}) = \frac{1}{2}$. The probability of exactly x successes in n trials is

$$C(n,x)p^x(1-p)^{n-x},$$

where p is the probability of success in a single trial. We have $n = 5, x = 2$, and $p = \frac{1}{2}$

Note that

$$1 - p = 1 - \frac{1}{2} = \frac{1}{2}.$$

$$\begin{aligned} &P(\text{exactly 2 girls and 3 boys}) \\ &= C(5,2)\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^3 \\ &= \frac{10}{32} = \frac{5}{16} \approx 0.313 \end{aligned}$$

2. This is a Bernoulli trial problem with $P(\text{success}) = P(\text{girl}) = \frac{1}{2}$. The probability of exactly x successes in n trials is

$$C(n,x)p^x(1-p)^{n-x},$$

where p is the probability of success in a single trial.

Here $n = 5$, $x = 3$, and $p = \frac{1}{2}$.

$$\begin{aligned} &P(\text{exactly 3 girls and 2 boys}) \\ &= C(5,3)\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2 \\ &= 10\left(\frac{1}{32}\right) = \frac{5}{16} \approx 0.313 \end{aligned}$$

3. We have $n = 5$, $x = 0$, $p = \frac{1}{2}$, and $1 - p = \frac{1}{2}$.

$$\begin{aligned} P(\text{no girls}) &= C(5,0)\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^5 \\ &= \frac{1}{32} \approx 0.031 \end{aligned}$$

4. We have $n = 5$, $x = 0$ is the number of boys, and $P = \frac{1}{2}$ is the probability of having a boy.

$$P(\text{no boys}) = C(5,0)\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^5 = \frac{1}{32} \approx 0.031$$

5. "At least 4 girls" means either 4 or 5 girls.

$$\begin{aligned} &P(\text{at least 4 girls}) \\ &= C(5,4)\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^1 + C(5,5)\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^0 \\ &= \frac{5}{32} + \frac{1}{32} = \frac{6}{32} = \frac{3}{16} \approx 0.188 \end{aligned}$$

6. We have 3, 4, or 5 boys, so

$$\begin{aligned} &P(\text{at least 3 boys}) \\ &= C(5,3)\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2 + C(5,4)\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^1 \\ &\qquad + C(5,5)\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^0 \\ &= \frac{10}{32} + \frac{5}{32} + \frac{1}{32} = \frac{16}{32} = \frac{1}{2}. \end{aligned}$$

7.
$$\begin{aligned} &P(\text{no more than 3 boys}) \\ &= 1 - P(\text{at least 4 boys}) \\ &= 1 - P(\text{4 boys or 5 boys}) \\ &= 1 - [P(\text{4 boys}) + P(\text{5 boys})] \\ &= 1 - \left(\frac{5}{32} + \frac{1}{32}\right) \\ &= 1 - \frac{6}{32} \\ &= 1 - \frac{3}{16} = \frac{13}{16} \approx 0.813 \end{aligned}$$

8.
$$\begin{aligned} &P(\text{no more than 4 girls}) \\ &= 1 - P(\text{5 girls}) \\ &= 1 - C(5,5)\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^0 \\ &= 1 - \frac{1}{32} = \frac{31}{32} \approx 0.969 \end{aligned}$$

9. On one roll, $P(1) = \frac{1}{6}$. We have $n = 12$, $x = 12$, and $p = \frac{1}{6}$. Note that $1 - p = \frac{5}{6}$. Thus,

$$\begin{aligned} P(\text{exactly 12 ones}) &= C(12,12)\left(\frac{1}{6}\right)^{12}\left(\frac{5}{6}\right)^0 \\ &\approx 4.594 \times 10^{-10}. \end{aligned}$$

10. We have $n = 12$, $x = 6$, and $p = \frac{1}{6}$, so

$$\begin{aligned} P(\text{exactly 6 ones}) &= C(12,6)\left(\frac{1}{6}\right)^6\left(\frac{5}{6}\right)^6 \\ &\approx 0.0066. \end{aligned}$$

11.
$$\begin{aligned} P(\text{exactly 1 one}) &= C(12,1)\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^{11} \\ &\approx 0.2692 \end{aligned}$$

12. We have $n = 12$, $x = 2$, and $p = \frac{1}{6}$, so

$$\begin{aligned} P(\text{exactly 2 ones}) &= C(12,2)\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^{10} \\ &\approx 0.2961. \end{aligned}$$

13. "No more than 3 ones" means 0, 1, 2, or 3 ones. Thus,

$$\begin{aligned} &P(\text{no more than 3 ones}) \\ &= P(0\text{ ones}) + P(1\text{ one}) + P(2\text{ ones}) + P(3\text{ ones}) \\ &= C(12,0)\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^{12} + C(12,1)\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^{11} \\ &\quad + C(12,2)\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^{10} + C(12,3)\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^{9} \\ &\approx 0.8748. \end{aligned}$$

14. "No more than 1 one" means 0 one or 1 one. Thus,

$$\begin{aligned} P(\text{no more than 1 one}) &= P(0\text{ one}) + P(1\text{ one}) \\ &= C(12,0)\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^{12} + C(12,1)\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^{11} \\ &\approx 0.3813. \end{aligned}$$

17.

$$\begin{aligned} &C(n,r) + C(n,r+1) \\ &= \frac{n!}{r!(n-r)!} + \frac{n!}{(r+1)![n-(r+1)]!} \\ &= \frac{n!(r+1)}{r!(r+1)(n-r)!} + \frac{n!(n-r)}{(r+1)![n-(r+1)]!(n-r)} \\ &= \frac{rn! + n!}{(r+1)!(n-r)!} + \frac{n(n!) - rn!}{(r+1)!(n-r)!} \\ &= \frac{rn! + n! + n(n!) - rn!}{(r+1)!(n-r)!} \\ &= \frac{n!(n+1)}{(r+1)!(n-r)!} \\ &= \frac{(n+1)!}{(r+1)![(n+1)-(r+1)]!} \\ &= C(n+1,\ r+1) \end{aligned}$$

18. Since these two crib deaths cannot be assumed to be independent events, the use of binomial probabilities is not applicable and thus the probabilities that are computed are not correct.

19. Since the potential callers are not likely to have birthdates that are distributed evenly throughout the twentieth century, the use of binomial probabilities is not applicable and thus, the probabilities that are computed are not correct.

For Exercises 20 through 24 we define a success to be the event that a customer is charged incorrectly. In this situation, $n = 15, p = \frac{1}{30}$ and $1 - p = \frac{29}{30}$.

20.

$$\begin{aligned} &P(3\text{ incorrect charges}) \\ &= C(15,3)\left(\frac{1}{30}\right)^3\left(\frac{29}{30}\right)^{12} \\ &\approx 0.0112 \end{aligned}$$

21.

$$\begin{aligned} &P(0\text{ incorrect charges}) \\ &= C(15,0)\left(\frac{1}{30}\right)^0\left(\frac{29}{30}\right)^{15} \\ &\approx 0.6014 \end{aligned}$$

22.

$$\begin{aligned} &P(\text{at least 1 incorrect charges}) \\ &= 1 - P(\text{no incorrect charges}) \\ &= 1 - 0.6014 \text{ (from Exercise 25)} \\ &\approx 0.3986 \end{aligned}$$

23.

$$\begin{aligned} &P(\text{at least 2 incorrect charges}) \\ &= 1 - P(0\text{ or 1 incorrect charges}) \\ &= 1 - C(15,0)\left(\frac{1}{30}\right)^0\left(\frac{29}{30}\right)^{15} \\ &\quad - C(15,1)\left(\frac{1}{30}\right)^1\left(\frac{29}{30}\right)^{14} \\ &\approx 0.0876 \end{aligned}$$

24.

$$\begin{aligned} &P(\text{at most 2 incorrect charges}) \\ &= C(15,0)\left(\frac{1}{30}\right)^0\left(\frac{29}{30}\right)^{15} + C(15,1)\left(\frac{1}{30}\right)^1\left(\frac{29}{30}\right)^{14} \\ &\quad + C(15,2)\left(\frac{1}{30}\right)^2\left(\frac{29}{30}\right)^{13} \\ &\approx 0.9875 \end{aligned}$$

For Exercises 25 through 28, we define a success to be the event that the family hardly ever pays off the balance. In this situation, $n = 20, p = 0.254$ and $1 - p = 0.746$.

25.

$$\begin{aligned} P(6) &= C(20,6)(0.254)^6(0.746)^{14} \\ &\approx 0.1721 \end{aligned}$$

26.

$$\begin{aligned} P(9) &= C(20,9)(0.254)^9(0.746)^{11} \\ &\approx 0.0294 \end{aligned}$$

27.

$$\begin{aligned} &P(\text{at least 4}) \\ &= 1 - C(20,0)(0.254)^0(0.746)^{20} - C(20,1)(0.254)^1(0.746)^{19} \\ &\quad - C(20,2)(0.254)^2(0.746)^{18} - C(20,3)(0.254)^3(0.746)^{17} \\ &\approx 0.7868 \end{aligned}$$

28.

$$
\begin{aligned}
&P(\text{at most } 5)\\
&= C(20,0)(0.254)^0(0.746)^{20} + C(20,1)(0.254)^1(0.746)^{19}\\
&\quad + C(20,2)(0.254)^2(0.746)^{18} + C(20,3)(0.254)^3(0.746)^{17}\\
&\quad + C(20,4)(0.254)^4(0.746)^{16} + C(20,5)(0.254)^5(0.746)^{15}\\
&\approx 0.6009
\end{aligned}
$$

29. $n = 20, p = 0.05, x = 0$

$$
\begin{aligned}
&P(0 \text{ defective transistors}) = C(20,0)(0.05)^0(0.95)^{20}\\
&\approx 0.3585
\end{aligned}
$$

30. We have $n = 20, p = 0.05,$ and $1 - p = 0.95.$ Thus,

$$
\begin{aligned}
&P(\text{at most 2 defective transistors})\\
&= P(\text{none defective}) + P(\text{one defective})\\
&\quad + P(\text{two defective})\\
&= C(20,0)(0.05)^0(0.95)^{20} + C(20,1)(0.05)^1(0.95)^{19}\\
&\quad + C(20,2)(0.05)^2(0.95)^{18}\\
&\approx 0.9245.
\end{aligned}
$$

31. Let success mean producing a defective item. Then we have $n = 75, p = 0.05,$ and $1 - p = 0.95.$

(a) If there are exactly 5 defective items, then $x = 5.$ Thus,

$$
\begin{aligned}
&P(\text{exactly 5 defective})\\
&= C(75,5)(0.05)^5(0.95)^{70}\\
&\approx 0.1488.
\end{aligned}
$$

(b) If there are no defective items, then $x = 0.$ Thus,

$$
\begin{aligned}
&P(\text{none defective})\\
&= C(75,0)(0.05)^0(0.95)^{75}\\
&\approx 0.0213.
\end{aligned}
$$

(c) If there is at least 1 defective item, then we are interested in $x \geq 1.$ We have

$$
\begin{aligned}
&P(\text{at least one defective})\\
&= 1 - P(x = 0)\\
&\approx 1 - 0.021\\
&= 0.9787.
\end{aligned}
$$

32. $n = 58, p = 0.7$

(a) The probability that all 58 people like the product is

$$
\begin{aligned}
P(\text{all } 58) &= C(58,58)(0.7)^{58}(0.3)^0\\
&\approx 1.037 \times 10^{-9}.
\end{aligned}
$$

(b) The probability that from 28 to 30 people (inclusive) like the product is

$$
\begin{aligned}
&P(\text{exactly } 28) + P(\text{exactly } 29) + P(\text{exactly } 30)\\
&= C(58,28)(0.7)^{28}(0.3)^{30} + C(58,29)(0.7)^{29}(0.3)^{29}\\
&\quad + C(58,30)(0.7)^{30}(0.3)^{28}\\
&\approx 0.0024.
\end{aligned}
$$

33. **(a)** Since 80% of the "good nuts" are good, 20% of the "good nuts" are bad. Let's let success represent "getting a bad nut." Then 0.2 is the probability of success in a single trial. The probability of 8 successes in 20 trials is

$$
\begin{aligned}
&C(20,8)(0.2)^8(1 - 0.2)^{20-8}\\
&= C(20,8)(0.2)^8(0.8)^{12}\\
&\approx 0.0222
\end{aligned}
$$

(b) Since 60% of the "blowouts" are good, 40% of the "blowouts" are bad. Let's let success represent "getting a bad nut." Then 0.4 is the probability of success in a single trial. The probability of 8 successes in 20 trials is

$$
\begin{aligned}
&C(20,8)(0.4)^8(1 - 0.4)^{20-8}\\
&= C(20,8)(0.4)^8(0.6)^{12}\\
&\approx 0.1797
\end{aligned}
$$

(c) The probability that the nuts are "blowouts" is

$$
\frac{\left(\begin{array}{c}\text{Probability of "Blowouts"}\\ \text{having 8 bad nuts out of 20}\end{array}\right)}{\left(\begin{array}{c}\text{Probability of "Good Nuts" or "Blowouts"}\\ \text{having 8 bad nuts of 20}\end{array}\right)}
$$

$$
= \frac{0.3\left[C(20,8)(0.4)^8(0.6)^{12}\right]}{0.7\left[C(20,8)(0.2)^8(0.8)^{12}\right] + 0.3\left[C(20,8)(0.4)^8(0.6)^{12}\right]}
$$

$\approx 0.7766.$

34. $n = 20,\ p = 0.05$

$$
\begin{aligned}
&P(\text{fewer than } 3) = P(0) + P(1) + P(2)\\
&= C(20,0)(0.05)^0(0.95)^{20} + C(20,1)(0.05)^1(0.95)^{19}\\
&\quad + C(20,2)(0.05)^2(0.95)^{18}\\
&\approx 0.9245
\end{aligned}
$$

The answer is **e**.

35. $n = 15, p = 0.85$

$$P(\text{all } 15) = C(15,15)(0.85)^{15}(0.15)^{0} \approx 0.0874$$

36. $n = 15, p = 0.85$

$$P(\text{none}) = C(15,0)(0.85)^{0}(0.15)^{15} \approx 4.379 \times 10^{-13}$$

37. $n = 15, p = 0.85$

$$\begin{aligned} P(\text{not all}) &= 1 - P(\text{all } 15) \\ &= 1 - C(15,15)(0.85)^{15}(0.15)^{0} \\ &\approx 0.9126 \end{aligned}$$

38. $n = 15, p = 0.85$

$$\begin{aligned} P(\text{more than half}) &= P(8) + P(9) + P(10) + P(11) + P(12) \\ &\quad + P(13) + P(14) + P(15) \\ &= C(15,8)(0.85)^{8}(0.15)^{7} + C(15,9)(0.85)^{9}(0.15)^{6} \\ &\quad + C(15,10)(0.85)^{10}(0.15)^{5} + C(15,11)(0.85)^{11}(0.15)^{4} \\ &\quad + C(15,12)(0.85)^{12}(0.15)^{3} + C(15,13)(0.85)^{13}(0.15)^{2} \\ &\quad + C(15,14)(0.85)^{14}(0.15)^{1} + C(15,15)(0.85)^{15}(0.15)^{0} \\ &\approx 0.9994 \end{aligned}$$

39. $n = 100, p = 0.012, x = 2$

$$\begin{aligned} &P(\text{exactly 2 sets of twins}) \\ &= C(100,2)(0.012)^{2}(0.988)^{98} \\ &\approx 0.2183 \end{aligned}$$

40. We have $n = 100, p = 0.012,$ and $1 - p = 0.988.$

Thus,

$$\begin{aligned} P(\text{more than half}) &= P(x = 0) + P(x = 1) + P(x = 2) \\ &= C(100,0)(0.012)^{0}(0.988)^{100} + C(100,1)(0.012)^{1}(0.988)^{99} \\ &\quad + C(100,2)(0.012)^{2}(0.988)^{98} \\ &= (0.012)^{0}(0.988)^{100} + 100(0.012)^{1}(0.0988)^{99} \\ &\quad + 4950(0.012)^{2}(0.988)^{98} \\ &\approx 0.8805. \end{aligned}$$

41. **(a)** $\dfrac{1}{2^{19}} = 1.907 \times 10^{-6}$

(b) We distribute the 40 millon births that occur in 10 years evenly among the 5700 hospitals, and then divide the births in any one hospital into strings of 19 consecutive births. Any one hospital has

$$\frac{40,000,000}{(5700)(19)} = 369.344$$

or about 369 such strings. Thus the probability of a boy-string in any one of the hospitals is

$$\frac{40,000,000}{(5700)(19)}\left(1.907 \times 10^{-6}\right) = 7.043 \times 10^{-4}$$

or about 7×10^{-4}.

(c) The probability of a boy-string in at least one hospital is 1 minus the probability of no string in any hospital, which is

$$1 - (1 - 7.043 \times 10^{-4})^{5700} = 0.982$$

or about 0.98.

(d) Boys have a birth probability greater than 0.5.

42. **(a)** The probability of at least 54 male births out of 84 can be found using the binomial probability function on a calculator, and it is about 0.0058. The probability of at least 201 male births out of 346 is given by the calculator as 0.001526.

(b) The male percentage is higher in the Clay County data, but the total number of births in Clay County is much smaller, so in fact the Macon County data is more surprising (less probable).

(c) We are interested in the probability that the male number would be this high or higher, since any higher value also have been surprising. Any particular number of male births is very improbable, so the probability of exactly 54 male births is not a good measure of how surprising this event is.

43. $n = 53, p = 0.042$

(a) The probability that exactly 5 men are color-blind is

$$P(5) = C(53,5)(0.042)^{5}(0.958)^{48} \approx 0.0478.$$

(b) The probability that no more than 5 men are color-blind is

$P(\text{no more than 5 men are color-blind})$
$= C(53,0)(0.042)^0(0.958)^{53}$
$+ C(53,1)(0.042)^1(0.958)^{52}$
$+ C(53,2)(0.042)^2(0.958)^{51}$
$+ C(53,3)(0.042)^3(0.958)^{50}$
$+ C(53,4)(0.042)^4(0.958)^{49}$
$+ C(53,5)(0.042)^5(0.958)^{48}$
$\approx 0.9767.$

(c) The probability that at least 1 man is color-blind is

$1 - P(\text{0 men are color-blind})$
$= 1 - C(53,0)(0.042)^0(0.958)^{53}$
$\approx 0.8971.$

44. **(a)** $P(\text{10 or more}) = 1 - P(\text{less than 10})$
$= 1 - [P(0) + P(1) + P(2) + \ldots + P(9)]$
$= 1 - [C(100,0)(0.073)^0(0.927)^{100}$
$+ C(100,1)(0.073)^1(0.927)^{99}$
$+ C(100,2)(0.073)^2(0.927)^{98}$
$+ C(100,3)(0.073)^3(0.927)^{97}$
$+ C(100,4)(0.073)^4(0.927)^{96}$
$+ C(100,5)(0.073)^5(0.927)^{95}$
$+ C(100,6)(0.073)^6(0.927)^{94}$
$+ C(100,7)(0.073)^7(0.927)^{93}$
$+ C(100,8)(0.73)^8(0.927)^{92}$
$+ C(100,9)(0.073)^9(0.927)^{91}]$
$\approx 1 - [0.00051 + 0.00402 + 0.01567$
$+ 0.04031 + 0.07698 + 0.11639$
$+ 0.14512 + 0.15346$
$+ 0.14049 + 0.11309]$
$= 1 - 0.80604 = 0.19396$

The probability that 10 or more will experience nausea/vomiting is about 0.1940.

(b) $P(\text{10 or more}) = 1 - P(\text{less than 10})$

$= 1 - [P(0) + P(1) + P(2) + \ldots + P(9)]$
$= 1 - [C(100,0)(0.71)^0(0.929)^{100}$
$+ C(100,1)(0.071)^1(0.929)^{99}$
$+ C(100,2)(0.71)^2(0.929)^{98}$
$+ C(100,3)(0.071)^3(0.929)^{97}$
$+ C(100,4)(0.71)^4(0.929)^{96}$
$+ C(100,5)(0.071)^5(0.929)^{95}$
$+ C(100,6)(0.71)^6(0.929)^{94}$
$+ C(100,7)(0.071)^7(0.929)^{93}$
$+ C(100,8)(0.71)^8(0.929)^{92}$
$+ C(100,9)(0.071)^9(0.929)^{91}$
$= 1 - 0.82765$
$= 0.17235$

The probability that 10 or more will experience nausea/vomiting is about 0.1724.

45. **(a)** Since the probability of a particular band matching is 1 in 4 or $\frac{1}{4}$, the probability that 5 bands match is $\left(\frac{1}{4}\right)^5 = \frac{1}{1024}$ or 1 chance in 1024.

(b) The probability that 20 bands match is $\left(\frac{1}{4}\right)^{20} \approx \frac{1}{1.1\times 10^{12}}$ or about 1 chance in 1.1×10^{12}.

(c) If 20 bands are compared, the probability that 16 or more bands match is

$P(\text{at least 16})$
$= P(16) + P(17) + P(18) + P(19) + P(20)$

$$= C(20,16)\left(\frac{1}{4}\right)^{16}\left(\frac{3}{4}\right)^4 + C(20,17)\left(\frac{1}{4}\right)^{17}\left(\frac{3}{4}\right)^3$$
$$+ C(20,18)\left(\frac{1}{4}\right)^{18}\left(\frac{3}{4}\right)^2 + C(20,19)\left(\frac{1}{4}\right)^{19}\left(\frac{3}{4}\right)^1$$
$$+ C(20,20)\left(\frac{1}{4}\right)^{20}\left(\frac{3}{4}\right)^0$$

$$= 4845\left(\frac{1}{4}\right)^{16}\left(\frac{3}{4}\right)^{4} + 1140\left(\frac{1}{4}\right)^{17}\left(\frac{3}{4}\right)^{3} + 190\left(\frac{1}{4}\right)^{18}\left(\frac{3}{4}\right)^{2} + 20\left(\frac{1}{4}\right)^{19}\left(\frac{3}{4}\right)^{1} + \left(\frac{1}{4}\right)^{20}\cdot 1$$

$$= \left(\frac{1}{4}\right)^{16}\left[4845\left(\frac{81}{256}\right) + 1140\left(\frac{1}{4}\right)\left(\frac{27}{64}\right) + 190\left(\frac{1}{16}\right)\left(\frac{9}{16}\right) + 20\left(\frac{1}{64}\right)\left(\frac{3}{4}\right) + \frac{1}{256}\right]$$

$$= \left(\frac{1}{4}\right)^{16}\left(\frac{392{,}445 + 30{,}780 + 1710 + 60 + 1}{256}\right)$$

$$= \frac{424{,}996}{4^{20}} = \frac{1}{\frac{4^{20}}{424{,}996}}$$

$$\approx \frac{1}{2{,}587{,}110}$$

or about 1 chance in 2.587×10^6.

46. **(a)** $n = 4, p = 0.25$

$$P(\text{at least 1}) = 1 - P(\text{none}) = 1 - C(4,0)(0.25)^0(0.75)^4 \approx 0.6836$$

(b) $n = 3, p = 0.25$

$$P(\text{at least 1}) = 1 - P(\text{none}) = 1 - C(3,0)(0.25)^0(0.75)^3 \approx 0.5781$$

(c) The assumption of independence is likely not justified because the bacteria would be present in groups of eggs from the same source.

47. $n = 4800, p = 0.001$

$$P(\text{more than 1}) = 1 - P(1) - P(0) = 1 - C(4800,1)(0.001)^1(0.999)^{4799} - C(4800,0)(0.001)^0(0.999)^{4800} \approx 0.9523$$

48. First, find the probability that one out of 30 vials is ineffective, given that the shipment came from company X.

$n = 30, p = 0.1, x = 1$

$$P(1) = C(30,1)(0.01)^1(0.9)^{29} \approx 0.1413$$

Next, find the probability that one out of 30 vials is ineffective, given that the shipment came from company Y.

$n = 30, p = 0.02, x = 1$

$$P(1) = C(30,1)(0.02)^1(0.98)^{29} \approx 0.3340$$

Use Bayes' Theorem to find the probability that shipment came from company X. Let A be the event that the shipment came from company X and B be the event that one vial out of thirty is ineffective.

$$P(A) = \frac{1}{5} = 0.2$$

$$P(B) \approx \frac{1}{5}(0.1413) + \frac{4}{5}(0.3340) \approx 0.2955$$

$$P(B|A) \approx 0.1413$$

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} \approx \frac{0.11413 \cdot 0.2}{0.2955} \approx 0.10$$

The answer is **a**.

49. First, find the probability that one group of ten has at least 9 participants complete the study. $n = 10, P = 0.8,$

$$P(\text{at least 9 complete}) = P(9) + P(10) = C(10,9)(0.8)^9(0.2)^1 + C(10,10)(0.8)^{10}(0.2)^0 \approx 0.3758$$

The probability that 2 or more drop out in one group is $1 - 0.3758 = 0.6242$. Thus, the probability that at least 9 participants complete the study in one of the two groups, but not in both groups, is

$$(0.3758)(0.6242) + (0.6242)(0.3758) \approx 0.469.$$

The answer is e.

50. We define a success to be the event that a woman would prefer to work part-time if money were not a concern.

In this situation, $n = 10, x =$ number of successes, $p = 0.60$, and $1 - p = 0.40$.

$$P(\text{at least 3}) = P(x = 0,1, \text{ or } 2)$$
$$= 1 - P(x = 0) - P(x = 1) - P(x = 2)$$
$$= 1 - C(10,0)(0.60)^0(0.40)^{10} - C(10,1)(0.60)^1(0.40)^9 - C(10,2)(0.60)^2(0.40)^8$$
$$\approx 0.9877$$

51. $n = 12, x = 7, p = 0.83$

$$P(7) = C(12,7)(0.83)^7(0.17)^5 \approx 0.0305$$

52. $n = 12, x = 9, p = 0.83$

$$P(9) = C(12,9)(0.83)^9(0.17)^3 \approx 0.2021$$

53. $n = 12, p = 0.83$

$$P(\text{at least } 9) = P(9) + P(10) + P(11) + P(12)$$
$$= C(12,9)(0.83)^9(0.17)^3 + C(12,10)(0.83)^{10}(0.17)^2 + C(12,11)(0.83)^{11}(0.17)^1 + C(12,12)(0.83)^{12}(0.17)^0$$
$$\approx 0.8676$$

54. $n = 12, p = 0.83$

$$P(\text{at most } 9) = 1 - P(\text{at least } 10) = 1 - P(10) - P(11) - P(12)$$
$$= 1 - C(12,10)(0.83)^{10}(0.17)^2 - C(12,11)(0.83)^{11}(0.17)^1 - C(12,12)(0.83)^{12}(0.17)^0$$
$$\approx 0.3344$$

55. $n = 10, p = 0.322, 1 - p = 0.678$

(a) $P(2) = C(10,2)(0.322)^2(0.678)^8 \approx 0.2083$

(b) $P(3 \text{ or fewer}) = C(10,0)(0.322)^0(0.678)^{10} + C(10,1)(0.322)^1(0.678)^9 + C(10,2)(0.322)^2(0.678)^8 + C(10,3)(0.322)^3(0.678)^7$
≈ 0.5902

(c) If exactly 5 *do not* belong to a minority, then exactly $10 - 5 = 5$ *do* belong to a minority, and this probability is

$$P(5) = C(10,5)(0.322)^5(0.678)^5 \approx 0.1250.$$

(d) If 6 or more *do not* belong to a minority, then at most 4 *do* belong to a minority, and this probability is $P(\text{at most } 4)$

$$P(\text{at most } 4) = C(10,0)(0.322)^0(0.678)^{10} + C(10,1)(0.322)^1(0.678)^9 + C(10,2)(0.322)^2(0.678)^8 + C(10,3)(0.322)^3(0.678)^7 + C(10,4)(0.322)^4(0.678)^6$$
$$\approx 0.8095$$

56. (a) No, the results only indicated that 84% of college students believe they need to cheat to get ahead in the world today. It says nothing about whether or not they cheat.

(b)

$$P(90 \text{ or more}) = P(90) + P(91) + \ldots + P(100)$$
$$= C(100,90)(0.84)^{90}(0.16)^{10} + C(100,91)(0.84)^{91}(0.16)^9 + C(100,92)(0.84)^{92}(0.16)^8 + C(100,93)(0.84)^{93}(0.16)^7$$
$$+ C(100,94)(0.84)^{94}(0.16)^6 + C(100,95)(0.84)^{95}(0.16)^5 + C(100,96)(0.84)^{96}(0.16)^4 + C(100,97)(0.84)^{97}(0.16)^3$$
$$+ C(100,98)(0.84)^{98}(0.16)^2 + C(100,99)(0.84)^{99}(0.16)^1 + C(100,100)(0.84)^{100}(0.16)^0$$
$$\approx 0.02915 + 0.01682 + 0.00864 + 0.00390 + 0.00152 + 0.00051 + 0.00014 + 0.00003 + 0 + 0 + 0$$
$$= 0.06071$$

The probability that 90 or more will answer affirmatively to the question is about 0.0607.

57. (a) Using the binomcdf function on a graphing calculator, we find

$$P(\text{at least } 30) = 1 - P(29 \text{ or fewer}) = 1 - \text{binomcdf}(40, 0.74, 29) \approx 1 - 0.4740 = 0.5260$$

(b) Using the binomcdf function on a graphing calculator, we find

$$P(\text{at least } 30) = 1 - P(29 \text{ or fewer}) = 1 - \text{binomcdf}(40, 0.83, 29) \approx 1 - 0.0657 = 0.9343$$

58. (a) $n = 1613, p = \dfrac{1174}{644{,}066}$

$$P(\text{at least } 9) = 1 - P(\text{at most } 8) = 1 - \text{binomcdf}(1613, 1174/644066, 8) \approx 0.0033$$

(b) $n = 7146, p = \dfrac{1174}{644{,}066}$

$$P(\text{at least } 28) = 1 - P(\text{at most } 27) = 1 - \text{binomcdf}(7146, 1174/644066, 27) \approx 2.076 \times 10^{-4}$$

(c) $n = 7146, p = \dfrac{1174}{644{,}066}$

$$P(\text{at most } 54) = \text{binomcdf}(62572, 1174/644066, 54) \approx 2.799 \times 10^{-10}$$

59. (a) Suppose the National League wins the series in four games. Then they must win all four games and $P = C(4,4)(0.5)^4(0.5)^0 = 0.0625$. Since the probability that the American League wins the series in four games is equally likely, the probability the series lasts four games is $2(0.0625) = 0.125$.

Suppose the National League wins the series in five games. Then they must win exactly three of the previous four games and $P = C(4,3)(0.5)^3(0.5)^1 \cdot (0.5) = 0.125$. Since the probability that the American League wins the series in five games is equally likely, the probability the series lasts five games is $2(0.125) = 0.25$. Suppose the National League wins the series in six games. Then they must win exactly three of the previous five games and

$P = C(5,3)(0.5)^3(0.5)^2 \cdot (0.5) = 0.15625$.

Since the probability that the American League wins the series in six games is equally likely, the probability the series lasts six games is $2(0.15625) = 0.3125$. Suppose the National League wins the series in seven games. Then they must win exactly three of the previous six games and

$P = C(6,3)(0.5)^3(0.5)^3 \cdot (0.5) = 0.15625$.

Since the Probability that the American League wins the series in seven games is equally likely, the probability the series last seven games is $2(0.15625) = 0.3125$.

(b) Suppose the better team wins the series in four games. Then they must win all four games and $P = C(4,4)(0.73)^4(0.27)^0 \approx 0.2840$. Suppose the other team wins the series in four games. Then they must win all four games and

$P = C(4,4)(0.27)^4(0.73)^0 \approx 0.0053$. The probability the series lasts four games is the sum of two probabilities, 0.2893.

Suppose the better team wins the series in five games. Then they must win exactly three of the previous four games and

$P = C(4,3)(0.73)^3(0.27)^1 \cdot (0.73) \approx 0.3067$.

Suppose the other team wins the series in five games. Then they must win exactly three of the previous four games and

$C(4,3)(0.27)^3(0.73)^1 \cdot (0.27) \approx 0.0155$. The probability the series lasts five games is the sum of the two probabilities, 0.3222.

Suppose the better team wins the series in six games. Then they must win exactly three of the previous five games and

$P = C(5,3)(0.73)^3(0.27)^2 \cdot (0.73) \approx 0.2070$.

Suppose the other team wins the series in six games. Then they must win exactly three of the previous five games and

$P = C(5,3)(0.27)^3(0.73)^2 \cdot (0.27) \approx 0.0283$.

The probability the series lasts six games is the sum of the two probabilities, 0.2353.

Suppose the better team wins the series in seven games. Then they must win exactly three of the previous six games and

$P = C(6,3)(0.73)^3(0.27)^3 \cdot (0.73) \approx 0.1118$.

Suppose the other team wins the series in seven games. Then they must win exactly three of the previous six games and

$P = C(6,3)(0.27)^3(0.73)^3 \cdot (0.27) \approx 0.0413$.

The probability the series lasts seven games is the sum of the two probabilities, 0.1531.

8.5 Probability Distributions; Expected Value

Your Turn 1

$$P(x = 0) = \frac{C(3,0)C(9,2)}{C(12,2)} = \frac{6}{11}$$

$$P(x = 1) = \frac{C(3,1)C(9,1)}{C(12,2)} = \frac{9}{22}$$

$$P(x = 2) = \frac{C(3,2)C(9,0)}{C(12,2)} = \frac{1}{22}$$

The distribution is shown in the following table:

x	0	1	2
$P(x)$	6/11	9/22	1/22

Your Turn 2

Let the random variable x represent the number of tails.

$$P(x = 0) = C(3,0)\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

$$P(x = 1) = C(3,1)\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^2 = \frac{3}{8}$$

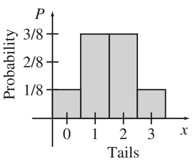
$$P(x = 2) = C(3,2)\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^1 = \frac{3}{8}$$

$$P(x = 3) = C(3,3)\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^0 = \frac{1}{8}$$

The distribution is shown n the following table:

x	0	1	2	3
$P(x)$	1/8	3/8	3/8	1/8

Here is the histogram.



Your Turn 3

The expected payback is

$$955\left(\frac{1}{1000}\right) + 495\left(\frac{1}{1000}\right) + 245\left(\frac{1}{1000}\right) + (-5)\left(\frac{997}{1000}\right)$$
$$= \frac{-3250}{1000}$$
$$= -3.25 \quad \text{or} \quad -\$3.25.$$

Your Turn 4

Let the random variable x represents the number of male engineers in the sample of 5.

$$P(x = 0) = C(5,0)(0.809)^0(0.191)^5 \approx 0.00025$$
$$P(x = 1) = C(5,1)(0.809)^1(0.191)^4 \approx 0.00538$$
$$P(x = 2) = C(5,2)(0.809)^2(0.191)^3 \approx 0.04560$$
$$P(x = 3) = C(5,3)(0.809)^3(0.191)^2 \approx 0.19316$$
$$P(x = 4) = C(5,4)(0.809)^4(0.191)^1 \approx 0.40907$$
$$P(x = 5) = C(5,5)(0.816)^5(0.184)^0 \approx 0.34653$$
$$E(x) \approx (0)(0.0025) + (1)(0.00538) + (2)(0.04560) + (3)(0.19316) + (4)(0.40907) + (5)(0.34653)$$
$$= 4.0420$$

In fact the exact value of the expectation can be computed quickly using the formula $E(x) = np$. For this example, $n = 5$ and $p = 0.809$ so $np = (5)(0.809) = 4.045$.

Your Turn 5

The expected number of girls in a family of a dozen children is $12\left(\frac{1}{2}\right) = 6$.

8.5 Warmup Exercises

W1. This is a binomial experiment. The probability of a six on one roll is 1/6.
The probability of exactly n sixes in 5 rolls is

$$C(5, n)\left(\frac{1}{6}\right)^n\left(\frac{5}{6}\right)^{5-n}$$

so for $n = 0, 1, \ldots, 5$ these probabilities are approximately 0.4019, 0.4019, 0.1608, 0.0322, 0.0032, and 0.0001.

W2. This is a binomial experiment. The probability of drawing a spade is 1/4.
The probability of exactly n spades in 4 draws is

$$C(4, n)\left(\frac{1}{4}\right)^n\left(\frac{3}{6}\right)^{4-n}$$

so for $n = 0, 1, \ldots, 4$ these probabilities are approximately 0.3164, 0.4219, 0.2109, 0.0469, and 0.0039.

8.5 Exercises

1. Let x denote the number of heads observed. Then x can take on 0, 1, 2, 3, or 4 as values. The probabilities are as follows.

$$P(x = 0) = C(4,0)\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^4 = \frac{1}{16}$$
$$P(x = 1) = C(4,1)\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^3 = \frac{4}{16} = \frac{1}{4}$$
$$P(x = 2) = C(4,2)\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2 = \frac{6}{16} = \frac{3}{8}$$
$$P(x = 3) = C(4,3)\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^1 = \frac{4}{16} = \frac{1}{4}$$
$$P(x = 4) = C(4,4)\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^0 = \frac{1}{16}$$

Therefore, the probability distribution is as follows.

Number of Heads	0	1	2	3	4
Probability	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

2. There are 36 outcomes. We count the number of ways each sum can be obtained, and then divide by 36 to get each probability.

Number of Points	2	3	4	5	6	7	8	9	10	11	12
Ways to Get This Total	1	2	3	4	5	6	5	4	3	2	1
Probability	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$

3. Let x denote the number of aces drawn. Then x can take on values 0, 1, 2, or 3. The probabilities are as follows.

$$P(x = 0) = C(3,0)\left(\frac{48}{52}\right)\left(\frac{47}{51}\right)\left(\frac{46}{50}\right) \approx 0.7826$$

$$P(x = 1) = C(3,1)\left(\frac{4}{52}\right)\left(\frac{48}{51}\right)\left(\frac{47}{50}\right) \approx 0.2042$$

$$P(x = 2) = C(3,2)\left(\frac{4}{52}\right)\left(\frac{3}{51}\right)\left(\frac{48}{50}\right) \approx 0.0130$$

$$P(x = 3) = C(3,3)\left(\frac{4}{52}\right)\left(\frac{3}{51}\right)\left(\frac{2}{50}\right) \approx 0.0002$$

Therefore, the probability distribution is as follows.

Number of Aces	0	1	2	3
Probability	0.7826	0.2042	0.0130	0.0002

4. Use combinations to find the probabilities of drawing 0, 1, and 2 black balls.

$$P(0) = \frac{C(2,0)C(4,2)}{C(6,2)} = \frac{6}{15} = \frac{2}{5}$$

$$P(1) = \frac{C(2,1)C(4,1)}{C(6,2)} = \frac{8}{15}$$

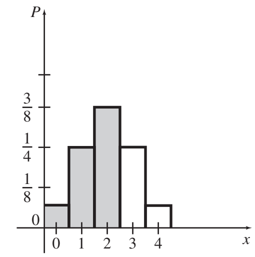
$$P(2) = \frac{C(2,2)C(4,0)}{C(6,2)} = \frac{1}{15}$$

Number of Black Balls	0	1	2
Probability	$\frac{2}{5}$	$\frac{8}{15}$	$\frac{1}{15}$

5. Use the probabilities that were calculated in Exercise 1. Draw a histogram with 5 rectangles, corresponding to $x = 0, x = 1, x = 2, x = 3,$ and $x = 4.$ $P(x \le 2)$ corresponds to

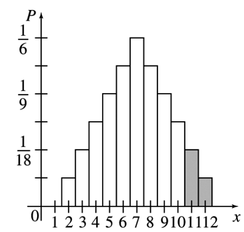
$$P(x = 0) + P(x = 1) + P(x = 2),$$

so shade the first 3 rectangles in the histogram.



6. $P(x \ge 11)$

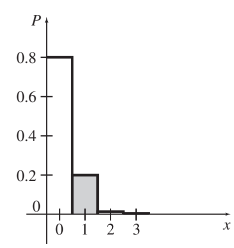
Use the probabilities that were calculated in Exercise 2, and shade the regions corresponding to $x = 11$ and $x = 12.$



7. Use the probabilities that were calculated in Exercise 3. Draw a histogram with 4 rectangles, corresponding to $x = 0, x = 1, x = 2,$ and $x = 3.$ $P(\text{at least one ace}) = P(x \ge 1)$ corresponds to

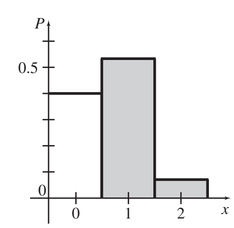
$$P(x = 1) + P(x = 2) + P(x = 3),$$

so shade the last 3 rectangles.



8. $P(\text{at least one black ball})$

Use the probabilities that were calculated in Exercise 4, and shade the regions corresponding to $x = 1$ and $x = 2.$



9. $E(x) = 2(0.1) + 3(0.4) + 4(0.3) + 5(0.2)$
$= 3.6$

10. $E(y) = 4(0.4) + 6(0.4) + 8(0.05) + 10(0.15)$
$= 5.9$

11. $E(z) = 9(0.14) + 12(0.22) + 15(0.38)$
$+ 18(0.19) + 21(0.07)$
$= 14.49$

12. $E(x) = 30(0.31) + 32(0.29) + 36(0.26)$
$+38(0.09) + 44(0.05)$
$= 33.56$

13. It is possible (but not necessary) to begin by writing the histogram's data as a probability distribution, which would look as follows.

x	1	2	3	4
$P(x)$	0.2	0.3	0.1	0.4

The expected value of x is

$$E(x) = 1(0.2) + 2(0.3) + 3(0.1) + 4(0.4) = 2.7.$$

14. $P(2) = 0.2, P(4) = 0.3, P(6) = 0.2,$
$P(8) = 0.1,$ and $P(10) = 0.2.$
$E(x) = 2(0.2) + 4(0.3) + 6(0.2) + 8(0.1)$
$+10(0.2)$
$= 5.6$

15. The expected value of x is

$$E(x) = 6(0.1) + 12(0.2) + 18(0.4) + 24(0.2) + 30(0.1) = 18.$$

16. Notice that the probability of all values is 0.2.

$$E(x) = 0.2(10 + 20 + 30 + 40 + 50) = 0.2(150) = 30$$

17. Using the data from Example 5, the expected winnings for Mary are

$$E(x) = -1.2\left(\frac{1}{4}\right) + 1.2\left(\frac{1}{4}\right) + 1.2\left(\frac{1}{4}\right) + (-1.2)\left(\frac{1}{4}\right) = 0.$$

Yes, it is still a fair game if Mary tosses and Donna calls.

18.

	Possible	Results
Result of toss	H	H
Call	H	T
Caller wins?	Yes	No
Probability	$\frac{1}{2}$	$\frac{1}{2}$

(a) Yes, this is still a fair game, since the probability of Donna matching is still $\frac{1}{2}$.

(b) If Donna calls heads, her expected gain (since she will match with probability $= 1$) is

$$1(1.20) = \$1.20.$$

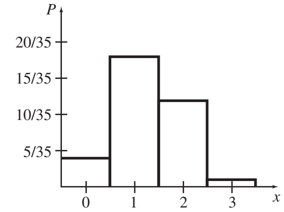
(c) If Donna calls tails, her expected gain (since she will lose with probability $= 1$) is

$$1(-1.20¢) = -\$1.20.$$

19. **(a)**

Number of Yellow Marbles	Probability
0	$\frac{C(3,0)\,C(4,3)}{C(7,3)} = \frac{4}{35}$
1	$\frac{C(3,1)\,C(4,2)}{C(7,3)} = \frac{18}{35}$
2	$\frac{C(3,2)\,C(4,1)}{C(7,3)} = \frac{12}{35}$
3	$\frac{C(3,3)\,C(4,0)}{C(7,3)} = \frac{1}{35}$

Draw a histogram with four rectangles corresponding to $x = 0, 1, 2,$ and 3.



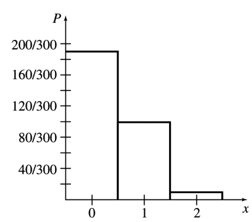
(b) Expected number of yellow marbles

$$= 0\left(\frac{4}{35}\right) + 1\left(\frac{18}{35}\right) + 2\left(\frac{12}{35}\right) + 3\left(\frac{1}{35}\right)$$

$$= \frac{45}{35} = \frac{9}{7} \approx 1.286$$

20. (a) Use combinations to set up the probability distribution. In each case, there are 5 rotten apples and 20 good apples to pick from. There are a total of $C(25, 2)$ ways to choose any two apples.

Number of Rotten Apples	Probability	Simplified
0	$\frac{C(5,0)C(20,2)}{C(25,2)}$	$\frac{190}{300}$
1	$\frac{C(5,1)C(20,1)}{C(25,2)}$	$\frac{100}{300}$
2	$\frac{C(5,2)C(20,0)}{C(25.2)}$	$\frac{10}{300}$



(b) We therefore have

$$E(x) = 0\left(\frac{190}{300}\right) + 1\left(\frac{100}{300}\right) + 2\left(\frac{10}{300}\right)$$

$$= \frac{120}{300} = 0.4.$$

21. (a) Let x be the number of times 1 is rolled. Since the probability of getting a 1 on any single roll is $\frac{1}{6}$, the probability of any other outcome is $\frac{5}{6}$. Use combinations since the order of outcomes is not important.

$$P(x = 0) = C(4,0)\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^4 = \frac{625}{1296}$$

$$P(x = 1) = C(4,1)\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^3 = \frac{125}{324}$$

$$P(x = 2) = C(4,2)\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^2 = \frac{25}{216}$$

$$P(x = 3) = C(4,3)\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^1 = \frac{5}{324}$$

$$P(x = 4) = C(4,4)\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^0 = \frac{1}{1296}$$

x	0	1	2	3	4
$P(x)$	$\frac{625}{1296}$	$\frac{125}{324}$	$\frac{25}{216}$	$\frac{5}{324}$	$\frac{1}{1296}$

(b)
$$E(x) = 0\left(\frac{625}{1296}\right) + 1\left(\frac{125}{324}\right) + 2\left(\frac{25}{216}\right) + 3\left(\frac{5}{324}\right) + 4\left(\frac{1}{1296}\right)$$

$$= \frac{2}{3}$$

22. The probability that the delegation contains no liberals and 3 conservatives is

$$\frac{C(5,0)C(6,3)}{C(11,3)} = \frac{1 \cdot 6}{11} = \frac{20}{165}.$$

Similarly, use combinations to calculate the remaining probabilities for the probability distribution.

(a) Let x represent the number of liberals on the delegation. The probability distribution of x is as follows.

x	0	1	2	3
$P(x)$	$\frac{20}{165}$	$\frac{75}{165}$	$\frac{60}{165}$	$\frac{10}{165}$

The expected value is

$$E(x) = 0\left(\frac{20}{165}\right) + 1\left(\frac{75}{165}\right) + 2\left(\frac{60}{165}\right) + 3\left(\frac{10}{165}\right)$$

$$= \frac{225}{165} = \frac{15}{11} \approx 1.3636 \text{ liberals.}$$

(b) Let y represent the number of conservatives on the committee. The probability distribution of y is as follows.

y	0	1	2	3
$P(y)$	$\frac{10}{165}$	$\frac{60}{165}$	$\frac{75}{165}$	$\frac{20}{165}$

The expected value is

$$E(y) = 0\left(\frac{10}{165}\right) + 1\left(\frac{60}{165}\right) + 2\left(\frac{75}{165}\right) + 3\left(\frac{20}{165}\right) = \frac{270}{165} = \frac{18}{11} \approx 1.6364 \text{ conservatives.}$$

23. Set up the probability distribution.

Number of Women	0	1	2
Probability	$\frac{C(3,0)\,C(5,2)}{C(8,2)}$	$\frac{C(3,1)\,C(5,1)}{C(8,2)}$	$\frac{C(3,2)\,C(5,0)}{C(8,2)}$
Simplified	$\frac{5}{14}$	$\frac{15}{28}$	$\frac{3}{28}$

$$E(x) = 0\left(\frac{5}{14}\right) + 1\left(\frac{15}{28}\right) + 2\left(\frac{3}{28}\right) = \frac{21}{28} = \frac{3}{4} = 0.75$$

24. Let x represent the number of junior members on the committee. Use combinations to find the probability of 0, 1, 2, 3, and 4 junior members.

$C(30, 4) = 27{,}405;\ C(10, 0)C(20, 4) = 4845;$

$C(10, 1)C(20, 3) = 11{,}400;\ C(10, 2)C(20, 2) = 8550;$

$C(10, 3)C(20, 1) = 2400;\ C(10, 4)C(20, 0) = 210;$

The probability distribution of x is as follows.

x	0	1	2	3	4
$P(x)$	$\frac{323}{1827}$	$\frac{760}{1827}$	$\frac{190}{609}$	$\frac{160}{1827}$	$\frac{2}{261}$

$$E(x) = 0\left(\frac{323}{1827}\right) + 1\left(\frac{760}{1827}\right) + 2\left(\frac{190}{609}\right) + 3\left(\frac{160}{1827}\right) + 4\left(\frac{2}{261}\right) = \frac{4}{3}.$$

The expected number of junior members is $1\frac{1}{3}$.

25. Set up the probability distribution as in Exercise 20.

Number of Women	Probability	Simplified
0	$\frac{C(13,0)\,C(39,2)}{C(52,2)}$	$\frac{741}{1326}$
1	$\frac{C(13,1)\,C(39,1)}{C(52,2)}$	$\frac{507}{1326}$
2	$\frac{C(13,2)\,C(39,0)}{C(52,2)}$	$\frac{78}{1326}$

$$E(x) = 0\left(\frac{741}{1326}\right) + 1\left(\frac{507}{1326}\right) + 2\left(\frac{78}{1326}\right) = \frac{663}{1326} = \frac{1}{2}$$

26. The probability of drawing 2 diamonds is

$$\frac{C(13,2)}{C(52,2)} = \frac{78}{1326},$$

and the probability of not drawing 2 diamonds is

$$1 - \frac{78}{1326} = \frac{1248}{1326}.$$

Let x represent your net winnings. Then the expected value of the game is

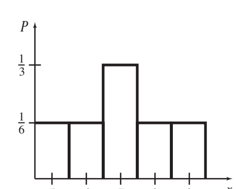
$$E(x) = 4.5\left(\frac{78}{1326}\right) + (-0.5)\left(\frac{1248}{1326}\right) = -\frac{273}{1326} \approx -\$0.21 \text{ or } -21¢.$$

No, the game is not fair since your expected winnings are not zero.

29. **(a)** First list the possible sums, 5, 6, 7, 8, and 9, and find the probabilities for each. The total possible number of results are $4 \cdot 3 = 12$. There are two ways to draw a sum of 5 (2 then 3, and 3 then 2). The probability of 5 is $\frac{2}{12} = \frac{1}{6}$. There are two ways to draw a sum of 6 (2 then 4, and 4 then 2). The probability of 6 is $\frac{2}{12} = \frac{1}{6}$. There are four ways to draw a sum of 7 (2 then 5, 3 then 4, 4 then 3, and 5 then 2). The probability of 7 is $\frac{4}{12} = \frac{1}{3}$. There are two ways to draw a sum of 8 (3 then 5, and 5 then 3). The probability of 8 is $\frac{2}{12} = \frac{1}{6}$. There are two ways to draw a sum of 9 (4 then 5, and 5 then 4).The probability of 9 is $\frac{2}{12} = \frac{1}{6}$. The distribution is as follows.

Sum	5	6	7	8	9
Probability	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$

(b)



(c) The probability that the sum is even is $\frac{1}{6}+\frac{1}{6}=\frac{1}{3}$. Thus the odds are 1 to 2.

(d) $$E(x)=\frac{1}{6}(5)+\frac{1}{6}(6)+\frac{1}{3}(7)+\frac{1}{6}(8)+\frac{1}{6}(9)=7$$

30. The expected value is

$$\begin{aligned}E(x) &= 0(0.02)+1(0.06)+2(0.16)+3(0.25)\\&\quad+4(0.32)+5(0.13)+6(0.06)\\&= 3.42 \text{ complaints per day.}\end{aligned}$$

31. We first compute the amount of money the company can expect to pay out for each kind of policy. The sum of these amounts will be the total amount the company can expect to pay out. For a single \$100,000 policy, we have the following probability distribution.

	Pay	Don't Pay
Outcome	\$100,000	\$100,000
Probability	0.0012	0.9998

$$\begin{aligned}E(\text{payoff}) &= 100{,}000(0.0012)+0(0.9998)\\&= \$120\end{aligned}$$

For all 100 such policies, the company can expect to pay out

$$100(120)=\$12{,}000.$$

For a single \$50,000 policy,

$$\begin{aligned}E(\text{payoff}) &= 50{,}000(0.0012)+0(0.9998)\\&= \$60.\end{aligned}$$

For all 500 such policies, the company can expect to pay out

$$500(60)=\$30{,}000.$$

Similarly, for all 1000 policies of \$10,000, the company can expect to pay out

$$1000(12)=\$12{,}000.$$

Thus, the total amount the company can expect to pay out is

$$\$12{,}000+\$30{,}000+\$12{,}000=\$54{,}000.$$

32. Let x represent the benefit amount.

x	4000	3000	2000	1000	0
$P(x)$	0.4	$0.6\cdot 0.4=0.24$	$0.6^2\cdot 0.4=0.144$	$0.6^3\cdot 0.4=0.0864$	$1-0.8704=0.1296$

$$\begin{aligned}E(x) &= 4000(0.4)+3000(0.24)+2000(0.144)\\&\quad+1000(0.0864)+0(0.1296)\\&= 2694.4\end{aligned}$$

The answer is e.

33. **(a)** Expected number of good nuts in 50 "blow outs" is

$$E(x)=50(0.60)=30.$$

(b) Since 80% of the "good nuts" are good, 20% are bad. Expected number of bad nuts in 50 "good nuts" is

$$E(x)=50(0.20)=10.$$

34.

Account Number	Expected Value	Exist. Vol. + Exp. Value	Class
1	\$ 2500	\$17,000	C
2	---------	\$40,000	C
3	\$ 2000	\$22,000	C
4	\$ 1000	\$51,000	B
5	\$25,000	\$30,000	C
6	\$60,000	\$60,000	A
7	\$16,000	\$46,000	B

35. The tour operator earns \$1050 if 1 or more tourists do not show up. The tour operator earns \$950 if all tourists show up. The probability that all tourists show up is $(0.98)^{21}\approx 0.6543.$ The expected revenue is
$1050(0.3457)+950(0.6543)=984.57$

The answer is e.

36. Let x represent the number of offspring. We have the following probability distribution.

x	0	1	2	3	4
$P(x)$	0.29	0.23	0.18	0.16	0.14

$$\begin{aligned}E(x) &= 0(0.29)+1(0.23)+2(0.18)\\&\quad+3(0.16)+4(0.14)\\&= 1.63\end{aligned}$$

37. **(a)** Expected cost of Amoxicillin:

$E(x) = 0.75(\$59.30) + 0.25(\$96.15) = \$68.51$

Expected cost of Cefaclor:

$E(x) = 0.90(\$69.15) + 0.10(\$106.00) = \$72.84$

(b) Amoxicillin should be used to minimize total expected cost.

38. Calculate the probability and payment for the number of days of hospitalization.

X	$P(x)$	Payment
1	$\frac{5}{15} = \frac{1}{3}$	\$100
2	$\frac{4}{15}$	\$200
3	$\frac{3}{15} = \frac{1}{5}$	\$300
4	$\frac{2}{15}$	\$325
5	$\frac{1}{15}$	\$350

Expected payment is

$$100\left(\frac{1}{3}\right) + 200\left(\frac{4}{15}\right) + 300\left(\frac{1}{5}\right) + 325\left(\frac{2}{15}\right) + 350\left(\frac{1}{15}\right) \approx 213.$$

The answer is d.

39. $E(x) = 250(0.74) = 185$

We would expect 38 low-birth-weight babies to graduate from high school.

40. Expected number of a group of 500 college students that say they need to cheat is

$E(x) = 500(0.84) = 420.$

41. **(a)** Using binomial probability, $n = 48, x = 0,$ $p = 0.0976.$

$$P(0) = C(48,0)(0.0976)^0(0.9024)^{48} \approx 0.007230$$

(b) Using combinations, the probability is

$$\frac{C(74,48)}{C(82,48)} \approx 5.094 \times 10^{-4}.$$

(c) Using binomial probability, $n = 6, x = 5,$ $p = 0.1.$

$$P(0) = C(6,5)(0.1)^5(0.9)^1 + (0.1)^6$$
$$= 5.5 \times 10^{-5}$$

(d) Using binomial probability, $n = 6, p = 0.1.$

$P(\text{at least } 2)$

$$= 1 - C(6,0)(0.1)^0(0.9)^6 - C(6,1)(0.1)^1(0.9)^5$$
$$\approx 0.1143$$

42. **(a)** Let x represent the amount of damage in millions of dollars. For seedling, the expected value is

$$E(x) = 0.038(335.8) + 0.143(191.1) + 0.392(100) + 0.255(46.7) + 0.172(16.3)$$
$$\approx \$94.0 \text{ million.}$$

For not seedling, the expected value is

$$E(x) = 0.054(335.8) + 0.206(191.1) + 0.480(100) + 0.206(46.7) + 0.054(16.3)$$
$$\approx \$116.0 \text{ million.}$$

(b) Seed, since the total expected damage is less with that option.

43. **(a)** The six possibilities and their scores are as follows:

$P(\text{neither 1 nor 5}) = \left(\frac{4}{6}\right)\left(\frac{4}{6}\right)$ score: 0

$P(\text{exactly one 5}) = 2\left(\frac{1}{6}\right)\left(\frac{4}{6}\right)$ score: 50

$P(\text{exactly one 1}) = 2\left(\frac{1}{6}\right)\left(\frac{4}{6}\right)$ score: 100

$P(\text{one 5, one 1}) = 2\left(\frac{1}{6}\right)\left(\frac{1}{6}\right)$ score: 150

$P(\text{two 5s}) = \left(\frac{1}{6}\right)\left(\frac{1}{6}\right)$ score: 100

$P(\text{two 1s}) = \left(\frac{1}{6}\right)\left(\frac{1}{6}\right)$ score: 200

The expected score is

$$0 + (50)\left[2\left(\frac{1}{6}\right)\left(\frac{4}{6}\right)\right] + (100)\left[2\left(\frac{1}{6}\right)\left(\frac{4}{6}\right)\right] + (150)\left[2\left(\frac{1}{6}\right)\left(\frac{1}{6}\right)\right] + (100)\left[\left(\frac{1}{6}\right)\left(\frac{1}{6}\right)\right] + (200)\left[\left(\frac{1}{6}\right)\left(\frac{1}{6}\right)\right] = 50$$

(b) The ten possibilities and their scores are

$P(\text{neither 1 nor 5}) = \left(\frac{4}{6}\right)^3$ score: 0

$P(\text{exactly one 5}) = 3\left(\frac{1}{6}\right)\left(\frac{4}{6}\right)^2$ score: 50

$P(\text{exactly one 1}) = 3\left(\frac{1}{6}\right)\left(\frac{4}{6}\right)^2$ score: 100

$P(\text{one 5, one 1}) = 6\left(\frac{1}{6}\right)^2\left(\frac{4}{6}\right)$ score: 150

$P(\text{two 5s}) = 3\left(\frac{1}{6}\right)^2\left(\frac{4}{6}\right)$ score: 100

$P(\text{two 1s}) = 3\left(\frac{1}{6}\right)^2\left(\frac{4}{6}\right)$ score: 200

$P(\text{two 5s, one 1}) = 3\left(\frac{1}{6}\right)^2\left(\frac{1}{6}\right)$ score: 200

$P(\text{one 5, two 1s}) = 3\left(\frac{1}{6}\right)^2\left(\frac{1}{6}\right)$ score: 250

$P(\text{three 5s}) = \left(\frac{1}{6}\right)^3$ score: 150

$P(\text{three 1s}) = \left(\frac{1}{6}\right)^3$ score: 300

The expected score is

$$0 + (50)\left[3\left(\frac{1}{6}\right)\left(\frac{4}{6}\right)^2\right] + (100)\left[3\left(\frac{1}{6}\right)\left(\frac{4}{6}\right)^2\right]$$
$$+ (150)\left[6\left(\frac{1}{6}\right)^2\left(\frac{4}{6}\right)\right] + (100)\left[3\left(\frac{1}{6}\right)^2\left(\frac{4}{6}\right)\right]$$
$$+ (200)\left[3\left(\frac{1}{6}\right)^2\left(\frac{4}{6}\right)\right] + (200)\left[3\left(\frac{1}{6}\right)^2\left(\frac{1}{6}\right)\right]$$
$$(250)\left[3\left(\frac{1}{6}\right)^2\left(\frac{1}{6}\right)\right] + 150\left[\left(\frac{1}{6}\right)^3\right] + 300\left[\left(\frac{1}{6}\right)^3\right] = 75$$

(c) We can subtract the last two terms from the expected value sum in (b) and in their place add the term

$$(1000 + 200 + 300 + 400 + 500 + 600)\left(\frac{1}{6}\right)^3$$

which yields an expected value of

$$75 - 450\left(\frac{1}{6}\right)^3 + 3000\left(\frac{1}{6}\right)^3 \approx 86.81$$

44. **(a)** We define a success to be the event that a letter was delivered the next day. In this situation, $n = 10;\ x = 0, 1, 2, 3, ..., 10;$ $p = 0.83,$ and $1 - p = 0.17.$

Number of Letters Delivered the Next Day	Probability
0	$C(10,0)(0.83)^0(0.17)^{10} \approx 0.0000$
1	$C(10,1)(0.83)^1(0.17)^9 \approx 0.0000$
2	$C(10,2)(0.83)^2(0.17)^8 \approx 0.0000$
3	$C(10,3)(0.83)^3(0.17)^7 \approx 0.0003$
4	$C(10,4)(0.83)^4(0.17)^6 \approx 0.0024$
5	$C(10,5)(0.83)^5(0.17)^5 \approx 0.0141$
6	$C(10,6)(0.83)^6(0.17)^4 \approx 0.0573$
7	$C(10,7)(0.83)^7(0.17)^3 \approx 0.1600$
8	$C(10,8)(0.83)^8(0.17)^2 \approx 0.2929$
9	$C(10,9)(0.83)^9(0.17)^1 \approx 0.3178$
10	$C(10,10)(0.83)^{10}(0.17)^0 \approx 0.1552$

(b) $P(\text{4 or fewer letters would be delivered})$
$$= P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3) + P(x = 4)$$
$$\approx 0.0027$$

(d) Expected number of letters delivered next day
$$\approx 0(0.0000) + 1(0.0000) + 2(0.0000) + 3(0.0003) + 4(0.0024) + 5(0.0141) + 6(0.0573) + 7(0.1600) + 8(0.2929) + 9(0.3178) + 10(0.1552)$$
$$\approx 8.3$$

45. Below is the probability distribution of x, which stands for the person's payback.

x	\$398	\$78	−\$2
$P(x)$	$\frac{1}{500} = 0.002$	$\frac{3}{500} = 0.006$	$\frac{497}{500} = 0.994$

The expected value of the person's winnings is

$$E(x) = 398(0.002) + 78(0.006) + (-2)(0.994) \approx -\$0.72 \text{ or } -72¢.$$

Since the expected value of the payback is not 0, this is not a fair game.

46. Reduce each price by the 50¢ cost of the raffle ticket, and multiply by the corresponding probability.

$$E(x) = 999.50\left(\frac{1}{10,000}\right) + 299.50\left(\frac{2}{10,000}\right) + 9.50\left(\frac{20}{10,000}\right) + (-0.50)\left(\frac{9977}{10,000}\right)$$
$$= \frac{-3200}{10,000} = -\$0.32 \text{ or } -32¢$$

No, this is not a fair game. In a fair game the expected value is 0.

47. There are 13 possible outcomes for each suit. That would make $13^4 = 28,561$ total possible outcomes. In one case, you win \$5000 (minus the \$1 cost to play the game). In the other 28,560, cases, you lose your dollar.

$$E(x) = 4999\left(\frac{1}{28,561}\right) + (-1)\left(\frac{28,560}{28,561}\right)$$
$$= -82¢$$

48. The probability of getting exactly 3 of the 4 selections correct and winning this game is

$$C(4,3)\left(\frac{1}{13}\right)^3\left(\frac{12}{13}\right)^1 \approx 0.001681.$$

The probability of losing is 0.998319. If you win, your payback is \$199. Otherwise, you lose \$1 (win −\$1). If x represents your payback, then the expected value is

$$E(x) = 199(0.001681) + (-1)(0.998319)$$
$$= 0.334519 - 0.998319$$
$$= -0.6638 \approx -\$0.66 \text{ or } -66¢.$$

49. There are $18 + 20 = 38$ possible outcomes. In 18 cases you win a dollar and in 20 you lose a dollar; hence,

$$E(x) = 1\left(\frac{18}{38}\right) + (-1)\left(\frac{20}{38}\right)$$
$$= -\frac{1}{19}, \text{ or about } -5.3¢.$$

50. In this form of roulette,

$$P(\text{even}) = \frac{18}{37} \text{ and } P(\text{noneven}) = \frac{19}{37}.$$

If an even number comes up, you win \$1. Otherwise, you lose \$1 (win −\$1). If x represents your payback, then the expected value is

$$E(x) = 1\left(\frac{18}{37}\right) + (-1)\left(\frac{19}{37}\right)$$
$$= -\frac{1}{37} \approx -\$0.027 \text{ or } -2.7¢.$$

51. You have one chance in a thousand of winning \$500 on a \$1 bet for a net return of \$499. In the 999 other outcomes, you lose your dollar.

$$E(x) = 499\left(\frac{1}{1000}\right) + (-1)\left(\frac{999}{1000}\right)$$
$$= -\frac{500}{1000} = -50¢$$

52. In this form of the game Keno,

$$P(\text{your number comes up}) = \frac{20}{80} = \frac{1}{4}$$

and

$$P(\text{your number doesn't come up}) = \frac{60}{80} = \frac{3}{4}.$$

If your number comes up, you win \$2.20. Otherwise, you lose \$1 (win −\$1). If x represents your payback, then the expected value is

$$E(x) = 2.20\left(\frac{1}{4}\right) - 1\left(\frac{3}{4}\right) = 0.55 - 0.75$$
$$= -\$0.20 \text{ or } -20¢.$$

53. Let x represent the payback. The probability distribution is as follows.

x	$P(x)$
100,000	$\frac{1}{2,000,000}$
40,000	$\frac{2}{2,000,000}$
10,000	$\frac{2}{2,000,000}$
0	$\frac{1,999,995}{2,000,000}$

The expected value is

$$E(x) = 100{,}000\left(\frac{1}{2{,}000{,}000}\right) + 40{,}000\left(\frac{2}{2{,}000{,}000}\right) + 10{,}000\left(\frac{2}{2{,}000{,}000}\right) + 0\left(\frac{1{,}999{,}995}{2{,}000{,}000}\right)$$
$$= 0.05 + 0.04 + 0.01 + 0$$
$$= \$0.10 = 10¢.$$

Since the expected payback is 10¢, if entering the context costs 100¢, then it would be worth it to enter. The expected net return is $-\$0.90$.

54. At any one restaurant, your expected winnings are

$$E(x) = 100{,}000\left(\frac{1}{176{,}402{,}500}\right) + 25{,}000\left(\frac{1}{39{,}200{,}556}\right) + 5000\left(\frac{1}{17{,}640{,}250}\right) + 1000\left(\frac{1}{1{,}568{,}022}\right) + 100\left(\frac{1}{288{,}244}\right) + 5\left(\frac{1}{7056}\right) + 1\left(\frac{1}{588}\right)$$
$$= 0.00488.$$

Going to 25 restaurants gives you expected earnings of $25(0.00488) = 0.122$. Since you spent \$1, you lose 87.8¢ on the average, so your expected value is $-87.8¢$.

55. **(a)** The possible scores are 0, 2, 3, 4, 5, 6. Each score has a probability of $\frac{1}{6}$.

$$E(x) = 0\left(\frac{1}{6}\right) + 2\left(\frac{1}{6}\right) + 3\left(\frac{1}{6}\right) + 4\left(\frac{1}{6}\right) + 5\left(\frac{1}{6}\right) + 6\left(\frac{1}{6}\right)$$
$$= \frac{1}{6}(20) = \frac{10}{3}$$

(b) The possible scores are

0 which has a probability of $\frac{11}{36}$,

4 which has a probability of $\frac{1}{36}$,

5 which has a probability of $\frac{2}{36}$,

6 which has a probability of $\frac{3}{36}$,

7 which has a probability of $\frac{4}{36}$,

8 which has a probability of $\frac{5}{36}$,

9 which has a probability of $\frac{4}{36}$,

10 which has a probability of $\frac{3}{36}$,

11 which has a probability of $\frac{2}{36}$,

12 which has a probability of $\frac{1}{36}$.

$$E(x) = 0\left(\frac{11}{36}\right) + 4\left(\frac{1}{36}\right) + 5\left(\frac{2}{36}\right) + 6\left(\frac{3}{36}\right) + 7\left(\frac{4}{36}\right) + 8\left(\frac{5}{36}\right) + 9\left(\frac{4}{36}\right) + 10\left(\frac{3}{36}\right) + 11\left(\frac{2}{36}\right) + 12\left(\frac{1}{36}\right)$$
$$= \frac{4}{36} + \frac{10}{36} + \frac{18}{36} + \frac{28}{36} + \frac{40}{36} + \frac{36}{36} + \frac{30}{36} + \frac{22}{36} + \frac{12}{36}$$
$$= \frac{200}{36} = \frac{50}{9}$$

(c) If a single die does not result in a score of zero, the possible scores are 2, 3, 4, 5, 6 with each of these having a probability of $\frac{1}{5}$.

$$E(x) = 2\left(\frac{1}{5}\right) + 3\left(\frac{1}{5}\right) + 4\left(\frac{1}{5}\right) + 5\left(\frac{1}{5}\right) + 6\left(\frac{1}{5}\right)$$
$$= \frac{1}{5}(20) = 4$$

Thus, if a player rolls n dice the expected average score is

$$n \cdot E(x) = n \cdot 4 = 4n.$$

(d) If a player rolls n dice, a nonzero score will occur whenever each die rolls a number other than 1. For each die there are 5 possibilities so the possible scoring ways for n dice is 5^n. When rolling one die there are 6 possibilities so the possible outcomes for n dice is 6^n. The probability of rolling a scoring set of dice is $\frac{5^n}{6^n}$; thus the expected value of the player's score when rolling n dice is $E(x) = \frac{5^n(4n)}{6^n}$.

56. **(a)** Expected value of a two-point conversion:
$E(x) = 2(0.478) = 0.956$

Expected value of an extra-point kick:
$E(x) = 1(0.996) = 0.996$

(b) Since the expected value of an extra-point kick is greater than the expected value of a two-point conversion, the extra-point kick will maximize the number of points scored over the long run.

57. Let x represent the number of hits. Since $p = 0.331, 1 - p = 0.669.$

$$P(0) = C(4,0)(0.331)^0(0.669)^4 = 0.2003$$

$$P(1) = C(4,1)(0.331)^1(0.669)^3 = 0.3964$$

$$P(2) = C(4,2)(0.331)^2(0.669)^2 = 0.2942$$

$$P(3) = C(4,3)(0.331)^3(0.669)^1 = 0.0970$$

$$P(4) = C(4,4)(0.331)^4(0.669)^0 = 0.0120$$

The distribution is shown in the following table.

x	0	1	2	3	4
$P(x)$	0.2003	0.3964	0.2942	0.0970	0.0120

The expected number of hits is $np = (4)(0.331) = 1.324.$

Chapter 8 Review Exercises

1. True

2. True

3. True

4. True

5. False: The probability of at least two occurrences is the probability of two or more occurrences.

6. True

7. True

8. False: Binomial probability applies to trials with exactly two outcomes.

9. True

10. False: For example, the random variable that assigns 0 to a head and 1 to a tail has expected value $1/2$ for a fair coin.

11. True

12. False: The expected value of a fair game is 0.

13. 6 shuttle vans can line up at the airport in

$$P(6,6) = 6! = 720$$

different ways.

14. Since order makes a difference, use permutations.

$$P(6,3) = \frac{6!}{(6-3)!} = \frac{6!}{3!} = 6 \cdot 5 \cdot 4 = 120$$

There are 120 variations in first-, second- and third-place finishes.

15. 3 oranges can be taken from a bag of 12 in

$$C(12,3) = \frac{12!}{9!3!} = \frac{12 \cdot 11 \cdot 10}{3 \cdot 2 \cdot 1} = 220$$

different ways.

16. Since the order of selection is not important, use combinations.

$$C(10,4) = \frac{10!}{6!4!} = 210$$

17. **(a)** The sample will include 1 of the 2 rotten oranges and 2 of the 10 good oranges. Using the multiplication principle, this can be done in

$$C(2,1)C(10,2) = 2 \cdot 45 = 90 \text{ ways.}$$

(b) The sample will include both of the 2 rotten oranges and 1 of the 10 good oranges. This can be done in

$$C(2,2)C(10,1) = 1 \cdot 10 = 10 \text{ ways.}$$

(c) The sample will include 0 of the 2 rotten oranges and 3 of the 10 good oranges. This can be done in

$$C(2,0)C(10,3) = 1 \cdot 120 = 120 \text{ ways.}$$

(d) If the sample contains at most 2 rotten oranges, it must contain 0, 1, or 2 rotten oranges. Adding the results from parts (a), (b), and (c), this can be done in

$$90 + 10 + 120 = 220 \text{ ways.}$$

18. **(a)** Exactly 3 males means 3 males out of 6 and 1 female out of 4:
$C(6,3)C(4,1) = 20 \cdot 4 = 80$

(b) No males means all 4 females must be selected, and there is 1 way of doing this.

(c) At least 2 males means 2, 3 or 4 males:

$$\begin{aligned} &C(6,2)C(4,2) + C(6,3)C(4,1) \\ &\quad + C(6,4)C(4,0) \\ &= 90 + 80 + 15 \\ &= 185 \end{aligned}$$

19. **(a)** $P(5,5) = 5! = 120$

(b) $P(4,4) = 4! = 24$

20. **(a)** There are 2! ways to arrange the landscapes, 3! ways to arrange the puppies, and 2 choices whether landscape or puppies come first. Thus, the pictures can be arranged in

$$2!3!\cdot 2 = 24$$

different ways.

(b) The pictures must be arranged puppy, landscape, puppy, landscape, puppy. Arrange the puppies in 3! or 6 ways. Arrange the landscapes in 2! or 2 ways. In this scheme, the pictures can be arranged in $6 \cdot 2 = 12$ different ways.

21. (a) The order within each list is not important. Use combinations and the multiplication principle. The choice of three items from column A can be made in $C(8, 3)$ ways, and the choice of two from column B can be made in $C(6, 2)$ ways. Thus, the number of possible dinners is

$$C(8,3)C(6,2) = 56 \cdot 15 = 840.$$

(b) There are

$$C(8,0) + C(8,1) + C(8,2) + C(8,3)$$

ways to pick up to 3 items from column A. Likewise, there are

$$C(6,0) + C(6,1) + C(6,2)$$

ways to pick up to 2 items from column B. We use the multiplication principle to obtain

$$\begin{aligned}&[C(8,0) + C(8,1) + C(8,2) + C(8,3)]\\ &\quad \cdot [C(6,0) + C(6,1) + C(6,2)]\\ &= (1 + 8 + 28 + 56)(1 + 6 + 15)\\ &= 93(22) = 2046.\end{aligned}$$

Since we are assuming that the diner will order at least one item, subtract 1 to exclude the dinner that would contain no items. Thus, the number of possible dinners is 2045.

22. (a) There are $7 \cdot 5 \cdot 4 = 140$ different groups of 3 representatives possible.

(b) $7 \cdot 5 \cdot 4 = 140$ is the number of groups with 3 representatives. For 2 representatives, the number of groups is

$$7 \cdot 5 + 7 \cdot 4 + 5 \cdot 4 = 83.$$

For 1 representative, the number of groups is

$$7 + 5 + 4 = 16.$$

The total number of these groups is

$$140 + 83 + 16 = 239$$

groups.

25. There are $C(13, 3)$ ways to choose the 3 balls and $C(4, 3)$ ways to get all black balls. Thus,

$$\begin{aligned}P(\text{all black}) &= \frac{C(4,3)}{C(13,3)} = \frac{4}{286}\\ &= \frac{2}{143} \approx 0.0140.\end{aligned}$$

26. It is impossible to draw 3 blue balls, since there are only 2 blue balls in the basket; hence,

$$P(\text{all blue balls}) = 0.$$

27. There are $C(4, 2)$ ways to get 2 black balls and $C(7, 1)$ ways to get 1 green ball. Thus,

$$\begin{aligned}P(\text{2 black and 1 green}) &= \frac{C(4,2)C(7,1)}{C(11,3)}\\ &= \frac{(6.7)}{286} = \frac{42}{286} = \frac{21}{143} \approx 0.1469.\end{aligned}$$

28. $P(\text{exactly 2 black balls})$

$$= \frac{C(4,2)C(9,1)}{C(13,3)} = \frac{54}{286} = \frac{27}{143} \approx 0.1888$$

29. There are $C(2,1)$ ways to get 1 blue ball and $C(11,2)$ ways to get 2 nonblue balls. Thus,

$$\begin{aligned}P(\text{exactly 1 blue}) &= \frac{C(2,1)C(11,2)}{C(13,3)}\\ &= \frac{2 \cdot 55}{286} = \frac{110}{286} = \frac{5}{13} \approx 0.3846.\end{aligned}$$

30. $P(\text{2 green balls and 1 blue ball})$

$$= \frac{C(7,2)C(2,1)}{C(13,3)} = \frac{42}{286} = \frac{21}{143} \approx 0.1469$$

31. This is a Bernoulli trial problem with

$P(\text{success}) = P(\text{girl}) = \frac{1}{2}$. Here, $n = 6, p = \frac{1}{2}$, and $x = 3$.

$$\begin{aligned}P(\text{exactly 3 girls}) &= C(6,3)\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^3\\ &= \frac{20}{64} = \frac{5}{16} \approx 0.313\end{aligned}$$

32. Let x represent the number of girls. We have $n = 6, x = 6, p = \frac{1}{2}$, and $1 - p = \frac{1}{2}$, so

$$P(\text{all girls}) = C(6,6)\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^0 = \frac{1}{64} \approx 0.016.$$

33. $P(\text{at least 4 girls})$

$$= P(4\text{ girls}) + P(5\text{ girls}) + P(6\text{ girls})$$
$$= C(6,4)\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^2 + C(6,5)\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^1 + C(6,6)\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^0$$
$$= \frac{22}{64} = \frac{11}{32} \approx 0.344$$

34. Let x represent the number of boys, and then $p = \frac{1}{2}$ and $1 - p = \frac{1}{2}$. We have

$P(\text{no more than 2 boys})$

$$= P(x \le 2)$$
$$= P(x = 0) + P(x = 1) + P(x = 2)$$
$$= C(6,0)\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^6 + C(6,1)\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^5 + C(6,2)\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^4$$
$$= \frac{11}{32} \approx 0.344.$$

35. $P(\text{both red})$

$$= \frac{C(26,2)}{C(52,2)} = \frac{325}{1326} = \frac{25}{102} \approx 0.245$$

36. $$P(2\text{ spades}) = \frac{C(13,2)}{C(52,2)} = \frac{78}{1326} = \frac{1}{17} \approx 0.059$$

37. $P(\text{at least 1 card is a spade})$

$$= 1 - P(\text{neither is a spade})$$
$$= 1 - \frac{C(39,2)}{C(52,2)} = 1 - \frac{741}{1326}$$
$$= \frac{585}{1326} = \frac{15}{34} \approx 0.441$$

38. $P(\text{exactly 1 face card})$

$$= \frac{C(12,1)C(40,1)}{C(52,2)} = \frac{480}{1326} = \frac{80}{221} \approx 0.3620$$

39. There are 12 face cards and 40 nonface cards in an ordinary deck.

$P(\text{at least 1 face card})$

$$= P(1\text{ face card}) + P(2\text{ face cards})$$
$$= \frac{C(12,1)C(40,1)}{C(52,2)} + \frac{C(12,2)}{C(52,2)}$$
$$= \frac{480}{1326} + \frac{66}{1326}$$
$$= \frac{546}{1326} \approx 0.4118$$

40. $P(\text{at most 1 queen})$

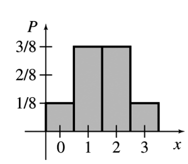
$$= P(0\text{ queens}) + P(1\text{ queen})$$
$$= \frac{C(48,2)}{C(52,2)} + \frac{C(4,1)C(48,1)}{C(52,2)}$$
$$= \frac{1128}{1326} + \frac{192}{1326}$$
$$= \frac{1320}{1326} = \frac{220}{221} \approx 0.9955$$

41. This is a Bernoulli trial problem.

(a) $P(\text{success}) = P(\text{head}) = \frac{1}{2}$. Hence, $n = 3$ and $p = \frac{1}{2}$.

Number of Heads	Probability
0	$C(3,0)\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^3 = 0.125$
1	$C(3,1)\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^2 = 0.375$
2	$C(3,2)\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^1 = 0.375$
3	$C(3,3)\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^0 = 0.125$

(b)



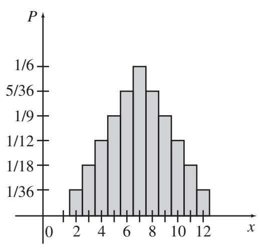
(c) $$E(x) = 0(0.125) + 1(0.375) + 2(0.375) + 3(0.125) = 1.5$$

42. **(a)** There are $n = 36$ possible outcomes. Let x represent the sum of the dice, and note that the possible values of x are the whole numbers from 2 to 12. The probability distribution is as follows.

x	2	3	4	5	6
$P(x)$	$\frac{1}{36}$	$\frac{2}{36} = \frac{1}{18}$	$\frac{3}{36} = \frac{1}{12}$	$\frac{4}{36} = \frac{1}{9}$	$\frac{5}{36}$

x	7	8	9	10	11	12
$P(x)$	$\frac{6}{36} = \frac{1}{6}$	$\frac{5}{36}$	$\frac{4}{36} = \frac{1}{9}$	$\frac{3}{36} = \frac{1}{12}$	$\frac{2}{36} = \frac{1}{18}$	$\frac{1}{36}$

(b) The histogram consists of 11 rectangles.



(c) The expected value is

$$E(x) = 2\left(\frac{1}{36}\right) + 3\left(\frac{2}{36}\right) + 4\left(\frac{3}{36}\right) + 5\left(\frac{4}{36}\right) + 6\left(\frac{5}{36}\right) + 7\left(\frac{6}{36}\right) + 8\left(\frac{5}{36}\right) + 9\left(\frac{4}{36}\right) + 10\left(\frac{3}{36}\right) + 11\left(\frac{2}{36}\right) + 12\left(\frac{1}{36}\right)$$
$$= \frac{252}{36} = 7.$$

43. The probability that corresponds to the shaded region of the histogram is the total of the shaded areas, that is,

$$1(0.3) + 1(0.2) + 1(0.1) = 0.6.$$

44. The probability that corresponds to the shaded region of the histogram is the total of the shaded areas, that is,

$$1(0.1) + 1(0.3) + 1(0.2) = 0.6.$$

45. The probability of rolling a 6 is $\frac{1}{6}$, and your net winnings would be \$2. The probability of rolling a 5 is $\frac{1}{6}$, and your net winnings would be \$1. The probability of rolling something else is $\frac{4}{6}$, and your net winnings would be $-\$2$. Let x represent your winnings. The expected value is

$$E(x) = 2\left(\frac{1}{6}\right) + 1\left(\frac{1}{6}\right) + (-2)\left(\frac{4}{6}\right) = -\frac{5}{6} \approx -\$0.833 \text{ or } -83.3¢.$$

This is not a fair game since the expected value is not 0.

46. Let x represent the number of girls. The probability distribution is as follows.

x	0	1	2	3	4	5
$P(x)$	$\frac{1}{32}$	$\frac{5}{32}$	$\frac{10}{32}$	$\frac{10}{32}$	$\frac{5}{32}$	$\frac{1}{32}$

The expected value is

$$E(x) = 0\left(\frac{1}{32}\right) + 1\left(\frac{5}{32}\right) + 2\left(\frac{10}{32}\right) + 3\left(\frac{10}{32}\right) + 4\left(\frac{5}{32}\right) + 5\left(\frac{1}{32}\right) = \frac{80}{32} = 2.5 \text{ girls.}$$

47. **(a)**

Number of Aces	Probability
0	$\frac{C(4,0)\,C(48,3)}{C(52,3)} = \frac{17{,}296}{22{,}100}$
1	$\frac{C(4,1)\,C(48,2)}{C(52,3)} = \frac{4512}{22{,}100}$
2	$\frac{C(4,2)\,C(48,1)}{C(52,3)} = \frac{288}{22{,}100}$
3	$\frac{C(4,3)\,C(48,0)}{C(52,3)} = \frac{4}{22{,}100}$

$$E(x) = 0\left(\frac{17{,}296}{22{,}100}\right) + 1\left(\frac{4512}{22{,}100}\right) + 2\left(\frac{288}{22{,}100}\right) + 3\left(\frac{4}{22{,}100}\right)$$
$$= \frac{5100}{22{,}100} = \frac{51}{221} = \frac{3}{13} \approx 0.231$$

(b)

Number of Clubs	Probability
0	$\dfrac{C(13,0)C(39,3)}{C(52,3)} = \dfrac{9139}{22{,}100}$
1	$\dfrac{C(13,1)C(39,2)}{C(52,3)} = \dfrac{9633}{22{,}100}$
2	$\dfrac{C(13,2)C(39,1)}{C(52,3)} = \dfrac{3042}{22{,}100}$
3	$\dfrac{C(13,3)C(39,0)}{C(52,3)} = \dfrac{286}{22{,}100}$

$$E(x) = 0\left(\frac{9139}{22{,}100}\right) + 1\left(\frac{9633}{22{,}100}\right) + 2\left(\frac{3042}{22{,}100}\right) + 3\left(\frac{286}{22{,}100}\right)$$
$$= \frac{16{,}575}{22{,}100} = \frac{3}{4} = 0.75$$

48. $P(3\text{ clubs}) = \dfrac{C(13,3)}{C(52,3)} = \dfrac{286}{22{,}100} \approx 0.0129$

Thus,

$P(\text{win}) = 0.0129$ and
$P(\text{lose}) = 1 - 0.0129 = 0.9871.$

Let x represent the amount you should pay. Your net winnings are $100 - x$ if you win and $-x$ if you lose. If it is a fair game, your expected winnings will be 0. Thus, $E(x) = 0$ becomes

$$\begin{aligned} 0.0129(100 - x) + 0.9871(-x) &= 0 \\ 1.29 - 0.0129x - 0.9871x &= 0 \\ 1.29 - x &= 0 \\ x &= 1.29. \end{aligned}$$

You should pay $1.29.

49. We define a success to be the event that a student flips heads and is on the committee. In this situation, $n = 6$; $x = 1, 2, 3, 4,$ or 5; $p = \frac{1}{2}$; and $1 - p = \frac{1}{2}$.

$$\begin{aligned} &P(x = 1,2,3,4,\text{ or }5) \\ &= 1 - P(x = 6) - P(x = 0) \\ &= 1 - C(6,6)\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^0 - C(6,0)\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^6 \\ &= 1 - \frac{1}{64} - \frac{1}{64} = \frac{62}{64} = \frac{31}{32} \end{aligned}$$

50. At most two students means 0, 1 or 2 students.

$$\begin{aligned} &P(\text{at most } 2) \\ &= C(6,0)\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^6 + C(6,1)\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^5 \\ &\qquad + C(6,2)\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^4 \\ &= \frac{C(6,0) + C(6,1) + C(6,2)}{2^6} \\ &= \frac{1 + 6 + 15}{64} = \frac{22}{64} = \frac{11}{32} \end{aligned}$$

51. **(a)** Given a set with n elements, the number of subsets of size

0 is $C(n, 0) = 1$,
1 is $C(n, 1) = n$,
2 is $C(n, 2) = \dfrac{n(n-1)}{2}$, and
n is $C(n, n) = 1$.

(b) The total number of subsets is

$C(n, 0) + C(n, 1) + C(n, 2) + \cdots + C(n, n).$

(d) Let $n = 4$.

$$C(4,0) + C(4,1) + C(4,2) + C(4,3) + C(4,4)$$
$$= 1 + 4 + 6 + 4 + 1 = 16 = 2^4 = 2^n$$

Let $n = 5$.

$$C(5,0) + C(5,1) + C(5,2) + C(5,3) + C(5,4) + C(5,5)$$
$$= 1 + 5 + 10 + 10 + 5 + 1 = 32 = 2^5 = 2^n$$

(e) The sum of the elements in row n of Pascal's triangle is 2^n.

52. **(a)**

First Card	Second Card	Number of Possibilities for Third Card
1	2	7
1	3	6
1	4	5
1	5	4
1	6	3
1	7	2
1	8	1
2	3	6
2	4	5
2	5	4
2	6	3
2	7	2
2	8	1
3	4	5
3	5	4

3	6	3
3	7	2
3	8	1
4	5	4
4	6	3
4	7	2
4	8	1
5	6	3
5	7	2
5	8	1
6	7	2
6	8	1
7	8	1

The sum of the numbers in the third column is 84.

(b) There are 4 even digits and 5 odd digits.

$$P(\text{all even}) = \frac{C(4,3)\,C(5,0)}{C(9,3)} = \frac{4}{84} = \frac{1}{21}$$

(c) There are 7 possibilities for three consecutive digits:

1, 2, 3
2, 3, 4
3, 4, 5
4, 5, 6
5, 6, 7
6, 7, 8
7, 8, 9.

$$P(\text{consecutive integers}) = \frac{7}{C(9,3)} = \frac{7}{84} = \frac{1}{12}$$

(d) Refer to column 3 from part (a). The sum of the numbers when the first card is 4 is $1 + 2 + 3 + 4 = 10$.

$$P(x = 4) = \frac{10}{84} = \frac{5}{42}$$

(e) From part (a), we see that the first card x ranges from 1 to 7. If k is an integer such that $1 \le k \le 7$, the number of possibilities is

$$1 + 2 + \cdots + [9 - (k + 1)] = \frac{[9 - (k + 1)][9 - (k + 1) + 1]}{2} = \frac{(8 - k)(9 - k)}{2}.$$

The probability of $x = k$ is given by

$$P(x = k) = \frac{\frac{(9-k)(8-k)}{2}}{C(9,3)} = \frac{(9 - k)(8 - k)}{2} \cdot \frac{1}{84} = \frac{(9 - k)(8 - k)}{168}.$$

The expected value of x is

$$E(x) = 1\left(\frac{8 \cdot 7}{168}\right) + 2\left(\frac{7 \cdot 6}{168}\right) + 3\left(\frac{6 \cdot 5}{168}\right) + 4\left(\frac{5 \cdot 4}{168}\right) + 5\left(\frac{4 \cdot 3}{168}\right) + 6\left(\frac{3 \cdot 2}{168}\right) + 7\left(\frac{2 \cdot 1}{168}\right) = \frac{420}{168} = \frac{5}{2}.$$

53. Use the multiplication principle.

$$3 \cdot 8 \cdot 2 = 48$$

54. Since the jobs are considered the same in parts (a), (b) and (c), use combinations for these parts. Since the jobs are not the same for part (d), use permutations for part (d).

(a) $C(12,4) = 495$

(b) If she hires only qualified applicants she is choosing 4 from 9.

$$C(9,4) = 126$$

(c) If she hires at most one unqualified applicant, either all 4 are qualified (126 possibilities according to (b)), or exactly 3 are qualified. The number of ways of selecting exactly 3 qualified and 1 unqualified is $C(9,3)\,C(3,1) = 252$. Thus the number of ways of getting at most 1 unqualified applicant is

$$126 + 252 = 378.$$

(d) Since the jobs are not all the same, use permutations.
$P(12, 4) = 12 \cdot 11 \cdot 10 \cdot 9 = 11{,}880$

55. $n = 12, x = 0, p = \frac{1}{6}$

$$P(0) = C(12,0)\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^{12} \approx 0.1122$$

56. $n = 12, x = 12, p = \frac{1}{6}$

$$P(12) = C(12,12)\left(\frac{1}{6}\right)^{12}\left(\frac{5}{6}\right)^0 \approx 4.594 \times 10^{-10}$$

57. $n = 12, x = 10, p = \frac{1}{6}$

$$P(10) = C(12,10)\left(\frac{1}{6}\right)^{10}\left(\frac{5}{6}\right)^2 \approx 7.580 \times 10^{-7}$$

58. $n = 12, x = 2, p = \frac{1}{6}$

$$P(2) = C(12,2)\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^{10} \approx 0.2961$$

59. $n = 12, p = \frac{1}{6}$

$$\begin{aligned} P(\text{at least } 2) &= 1 - P(\text{at most } 1) \\ &= 1 - P(0) - P(1) \\ &= 1 - C(12,0)\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^{12} \\ &\quad - C(12,1)\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^{11} \\ &\approx 0.6187 \end{aligned}$$

60. $n = 12, p = \frac{1}{6}$

$$\begin{aligned} P(\text{at most } 3) &= P(0) + P(1) + P(2) + P(3) \\ &= C(12,0)\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^{12} \\ &\quad + C(12,1)\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^{11} \\ &\quad + C(12,2)\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^{10} \\ &\quad + C(12,3)\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^9 \\ &\approx 0.8748 \end{aligned}$$

61. The expected value is $\frac{1}{6}(12) = 2$.

62. $$\begin{aligned} E(x) &= 26{,}000(0.7) + (-9000)(0.3) \\ &= 15{,}500 \end{aligned}$$

The expected profit is \$15,500.

63. Observe that for $a + b = 7$,
$P(a)P(b) = \left(\frac{1}{2^{a+1}}\right)\left(\frac{1}{2^{b+1}}\right) = \frac{1}{2^{a+b+2}} = \frac{1}{2^9}$.
The probability that exactly seven claims will be received during a given two-week period is

$$\begin{aligned} &P(0)P(7) + P(1)P(6) + P(2)P(5) \\ &\quad + P(3)P(4) + P(4)P(3) + P(5)P(2) \\ &\quad + P(6)P(1) + P(7)P(0) \\ &= 8\left(\frac{1}{2^9}\right) = \frac{1}{64}. \end{aligned}$$

The answer is d.

64. $P(0) = \frac{1}{2}; P(1) = \frac{1}{6}; P(2) = \frac{1}{12}; P(3) = \frac{1}{20};$
$P(4) = \frac{1}{30}$

The probability of at most 4 claims is

$$\begin{aligned} &P(0) + P(1) + P(2) + P(3) + P(4) \\ &= \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} \\ &= \frac{5}{6}. \end{aligned}$$

The probability of at least one claim and at most 4 claims is

$$\begin{aligned} &P(1)+P(2)+P(3)+P(4)\\ &=\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\frac{1}{30}\\ &=\frac{1}{3}. \end{aligned}$$

The probability of at least one claim given that there have been at most 4 claims is

$$\frac{\frac{1}{3}}{\frac{5}{6}}=\frac{2}{5}.$$

The answer is b.

65. Denote by S the event that a product is successful.

Denote by U the event that a product is unsuccessful.

Denote by Q the event of passing quality control.

We must calculate the conditional probabilities $P(S|Q)$ and $P(U|Q)$ using Bayes' Theorem in order to calculate the expected net profit (in millions).

$$E=40P(S|Q)-15P(U|Q).$$
$$P(S)=P(U)=0.5$$
$$P(Q|S)=0.8, P(Q|U)=0.25$$

$$\begin{aligned} P(S|Q)&=\frac{P(S)\cdot P(Q|S)}{P(S)\cdot P(Q|S)+P(U)\cdot P(Q|U)}\\ &=\frac{0.5(0.8)}{0.5(0.8)+0.5(0.25)}\\ &=\frac{0.4}{0.4+0.125}=0.762 \end{aligned}$$

$$\begin{aligned} P(U|Q)&=\frac{P(U)\cdot P(Q|U)}{P(U)\cdot P(Q|U)+P(S)\cdot P(Q|S)}\\ &=\frac{0.125}{0.525}=0.238 \end{aligned}$$

Therefore,

$$\begin{aligned} E&=40P(S|Q)-15P(U|Q)\\ &=40(0.762)-15(0.238)\\ &\approx 27. \end{aligned}$$

So the expected net profit is \$27 million, or the correct answer is e.

66. If a box is good (probability 0.9) and the merchant samples an excellent piece of fruit from that box (probability 0.80), then he will accept the box and earn a \$200 profit on it.

If a box is bad (probability 0.1) and he samples an excellent piece of fruit from the box (probability 0.30), then he will accept the box and earn a $-\$1000$ profit on it.

If the merchant ever samples a nonexcellent piece of fruit, he will not accept the box. In this case he pays nothing and earns nothing, so the profit will be \$0.

Let x represent the merchant's earnings. Note that

$$\begin{aligned} 0.9(0.80)&=0.72,\\ 0.1(0.30)&=0.03,\\ \text{and } 1-(0.72+0.03)&=0.25. \end{aligned}$$

The probability distribution is as follows.

x	200	–1000	0
$P(x)$	0.72	0.03	0.25

The expected value when the merchant samples the fruit is

$$\begin{aligned} E(x)&=200(0.72)+(-1000)(0.03)+0(0.25)\\ &=144-30+0=\$114. \end{aligned}$$

We must also consider the case in which the merchant does not sample the fruit. Let x again represent the merchant's earnings. The probability distribution is as follows.

x	200	–1000
$P(x)$	0.9	0.1

The expected value when the merchant does not sample the fruit is

$$\begin{aligned} E(x)&=200(0.9)+(-1000)(0.1)\\ &=180-100\\ &=\$80. \end{aligned}$$

Combining these two results, the expected value of the right to sample is $\$144-\$80=\$34$, which corresponds to choice c.

67. Let $I(x)$ represent the airline's net income if x people show up.

$$\begin{aligned} I(0)&=0\\ I(1)&=400\\ I(2)&=2(400)=800\\ I(3)&=3(400)=1200\\ I(4)&=3(400)-400=800\\ I(5)&=3(400)-2(400)=400\\ I(6)&=3(400)-3(400)=0 \end{aligned}$$

Let $P(x)$ represent the probability that x people will show up. Use the binomial probability formula to find the values of $P(x)$.

$$P(0) = C(6,0)(0.6)^0(0.4)^6 = 0.0041$$
$$P(1) = C(6,1)(0.6)^1(0.4)^5 = 0.0369$$
$$P(2) = C(6,2)(0.6)^2(0.4)^4 = 0.1382$$
$$P(3) = C(6,3)(0.6)^3(0.4)^3 = 0.2765$$
$$P(4) = C(6,4)(0.6)^4(0.4)^2 = 0.3110$$
$$P(5) = C(6,5)(0.6)^5(0.4)^1 = 0.1866$$
$$P(6) = C(6,6)(0.6)^6(0.4)^0 = 0.0467$$

(a) $$\begin{aligned} E(I) &= 0(0.0041) + 400(0.0369) \\ &\quad + 800(0.1382) + 1200(0.2765) \\ &\quad + 800(0.3110) + 400(0.1866) \\ &\quad + 0(0.0467) \\ &= \$780.56 \end{aligned}$$

(b) $n = 3$

x	0	1	2	3
Income	0	100	200	300
$P(x)$	0.064	0.288	0.432	0.216

$$\begin{aligned} E(I) &= 0(0.064) + 400(0.288) + 800(0.432) \\ &\quad + 1200(0.216) \\ &= \$720 \end{aligned}$$

On the basis of all the calculations, the table given in the exercise is completed as follows.

x	Income	$P(x)$
0	0	0.004
1	400	0.037
2	800	0.038
3	1200	0.276
4	800	0.311
5	400	0.187
6	0	0.047

$n = 4$

x	1	1	2	3	4
Income	0	400	800	1200	800
$P(x)$	0.0256	0.1536	0.3456	0.3456	0.1296

$$\begin{aligned} E(I) &= 0(0.0256) + 400(0.1536) \\ &\quad + 800(0.3456) + 1200(0.3456) \\ &\quad + 800(0.1296) \\ &= \$856.32 \end{aligned}$$

$n = 5$

x	Income	$P(x)$
0	0	0.01024
1	400	0.0768
2	800	0.2304
3	1200	0.3456
4	800	0.2592
5	400	0.07776

$$\begin{aligned} E(I) &= 0(0.01024) + 400(0.0768) \\ &\quad + 800(0.2304) + 1200(0.3456) \\ &\quad + 800(0.2596) + 400(0.07776) \\ &= \$868.22 \end{aligned}$$

Since $E(I)$ is greatest when $n = 5$, the airlines should book 5 reservations to maximize revenue.

68. **(a)** $$\begin{aligned} &P(10 \text{ or more}) \\ &= 1 - P(\text{less than } 10) \\ &= 1 - [P(0) + P(1) + \ldots + P(9)] \\ &= 1 - [C(50,0)(0.23)^0(0.77)^{50} \\ &\quad + C(50,1)(0.23)^1(0.77)^{49} \\ &\quad + C(50,2)(0.23)^2(0.77)^{48} \\ &\quad + C(50,3)(0.23)^3(0.77)^{47} \\ &\quad + C(50,4)(0.23)^4(0.77)^{46} \\ &\quad + C(50,5)(0.23)^5(0.77)^{45} \\ &\quad + C(50,6)(0.23)^6(0.77)^{44} \\ &\quad + C(50,7)(0.23)^7(0.77)^{43} \\ &\quad + C(50,8)(0.23)^8(0.77)^{42} \\ &\quad + C(50,9)(0.23)^9(0.77)^{41}] \\ &\approx 1 - [0 + 0.00003 + 0.00025 \\ &\quad + 0.00110 + 0.00387 \\ &\quad + 0.01064 + 0.02383 \\ &\quad + 0.04474 + 0.07183 \\ &\quad + 0.10013] \\ &= 1 - 0.25614 \\ &= 0.74386 \end{aligned}$$

or about 0.7439.

(b) The expected number of 50 patients to experience nausea is
$E(x) = 50(0.23) = 11.5$ or about 12 patients.

(c) $P(10 \text{ or fewer})$

$$= P(0) + P(1) + P(1) + P(2) + \cdots + P(10)$$
$$= C(50,0)(0.1)^0(0.9)^{50} + C(50,1)(0.1)^1(0.9)^{49} + C(50,2)(0.1)^2(0.9)^{48} + C(50,3)(0.1)^3(0.9)^{47} + C(50,4)(0.1)^4(0.9)^{46} + C(50,5)(0.1)^5(0.9)^{45} + C(50,6)(0.1)^6(0.9)^{44} + C(50,7)(0.1)^7(0.9)^{43} + C(50,8)(0.1)^8(0.9)^{42} + C(50,9)(0.1)^9(0.9)^{41} + C(50,10)(0.1)^{10}(0.9)^{40}$$
$$\approx 0.00515 + 0.02863 + 0.07794 + 0.13857 + 0.18090 + 0.18492 + 0.15410 + 0.10763 + 0.06428 + 0.03333 + 0.01518$$
$$= 0.99063$$

or about 0.9906.

(d) The probability that a person experiencing nausea is taking Prozac is

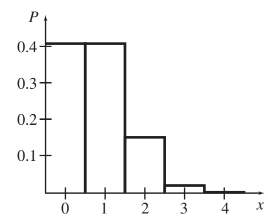
$$\frac{E(\text{people who take Prozac and experience nausea})}{E(\text{people who experience nausea})} = \frac{500(0.23)}{500(0.23) + 500(0.10)} = \frac{115}{115 + 50} = \frac{115}{165} \approx 0.6970.$$

69. $C(40, 5)\left(\frac{1}{8}\right)^5\left(\frac{7}{8}\right)^{35} \approx 0.1875$

70. **(a)** Define a success to be the event that an orange M&M is selected. In this situation, $n = 4; x = 0,1,2,3,$ or $4; p = 0.2;$ and $1 - p = 0.8.$

Number of Orange M&M's	Probability
0	$C(4,0)(0.2)^0(0.8)^4 = 0.4096$
1	$C(4,1)(0.2)^1(0.8)^3 = 0.4096$
2	$C(4,2)(0.2)^2(0.8)^2 = 0.1536$
3	$C(4,3)(0.2)^3(0.8)^1 = 0.0256$
4	$C(4,4)(02)^4(0.8)^0 = 0.0016$

(b) Draw a histogram with 5 rectangles.



(c) Expected number of orange M&M's $= np = 4(0.2) = 0.8$

71. This is a set of binomial trials with $n = 5, p = 0.48,$ and $1 - p = 0.52.$

(a) $P(0 \text{ schools}) = C(5,0)(0.48)^0(0.52)^5 \approx 0.0380$

$P(1 \text{ school}) = C(5,1)(0.48)^1(0.52)^4 \approx 0.1755$

$P(2 \text{ schools}) = C(5,2)(0.48)^2(0.52)^3 \approx 0.3240$

$P(3 \text{ schools}) = C(5,3)(0.48)^3(0.52)^2 \approx 0.2990$

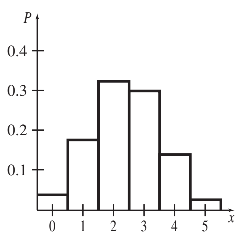
$P(4 \text{ schools}) = C(5,4)(0.48)^4(0.52)^1 \approx 0.1380$

$P(5 \text{ schools}) = C(5,5)(0.48)^5(0.52)^0 \approx 0.0255$

The distribution is shown in the following table.

Number of schools	Probability
0	0.0380
1	0.1755
2	0.3240
3	0.2990
4	0.1380
5	0.0255

(b)

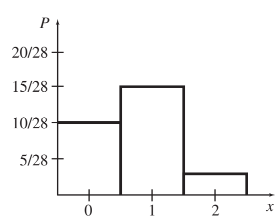


(c) Expected number of schools $= np = 5(0.48) = 2.4.$

72. **(a)**

Number of African-Americans	Probability
0	$\frac{C(2,0)\,C(6,3)}{C(8,3)} = \frac{20}{56} = \frac{10}{28}$
1	$\frac{C(2,1)\,C(6,2)}{C(8,3)} = \frac{30}{56} = \frac{15}{28}$
2	$\frac{C(2,2)\,C(6,1)}{C(8,3)} = \frac{6}{56} = \frac{3}{28}$

(b) Draw a histogram with 3 rectangles.



(c) Expected number of African-Americans

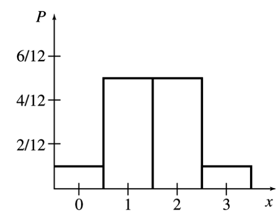
$$= 0\left(\frac{10}{28}\right) + 1\left(\frac{15}{28}\right) + 2\left(\frac{3}{28}\right)$$

$$= \frac{21}{28} = \frac{3}{4}$$

73. **(a)**

Number Who Did Not Do Homework	Probability
0	$\frac{C(3,0)C(7,5)}{C(10,5)} = \frac{21}{252} = \frac{1}{12}$
1	$\frac{C(3,1)C(7,4)}{C(10,5)} = \frac{105}{252} = \frac{5}{12}$
2	$\frac{C(3,2)C(7,3)}{C(10,5)} = \frac{105}{252} = \frac{5}{12}$
3	$\frac{C(3,3)C(7,2)}{C(10,5)} = \frac{21}{252} = \frac{1}{12}$

(b) Draw a histogram with four rectangles.



(c) Expected number who did not do homework

$$= 0\left(\frac{1}{12}\right) + 1\left(\frac{5}{12}\right) + 2\left(\frac{5}{12}\right) + 3\left(\frac{1}{12}\right)$$

$$= \frac{18}{12} = \frac{3}{2}$$

74. The probability that out of 53 games Chesbro's team would win 41 games and lose 12.

75. It costs $2(0.49 + 0.04) = 1.06$ to play the game.

x	\$1998.94	–\$1.06
$P(x)$	$\frac{1}{8000}$	$\frac{7999}{8000}$

$$E(x) = \$1998.94\left(\frac{1}{8000}\right) - \$1.06\left(\frac{7999}{8000}\right)$$

$$= -\$0.81$$

76. **(a)** After a specific number is chosen, each of the 10,000 numbers (0000 to 9999) has an equally likely chance of being chosen. Therefore, the probability that the same specific number is chosen again is $\frac{1}{10{,}000}$.

(b) The probability that any specific number is chosen is $\frac{1}{10{,}000}$. Since the selections are independent, the probability that any specific number, in this case 3199, is chosen twice in one day is $\left(\frac{1}{10{,}000}\right)^2 = \frac{1}{100{,}000{,}000}$.

77. (a) The probability of the outcome 000 is 0.001, so the expected number of occurrences of this outcome in 32 years of play is $(32)(365)(0.001) = 11.68$.

(b) In 10 years of play the expected number of wins for 000 is $(10)(365)(0.001) \approx 3.65$.

78. (a) First, we assume 4 numbers are picked. You win if 2, 3, or 4 numbers match.

$P(\text{win})$

$$= \frac{\begin{pmatrix} C(20,2)C(60,2) + C(20,3)C(60,1) \\ + C(20,4)C(60,0) \end{pmatrix}}{\begin{pmatrix} C(20,0)C(60,4) + C(20,1)C(60,3) \\ + C(20,2)C(60,2) + C(20,3)C(60,1) \\ + C(20,4)C(60,0) \end{pmatrix}}$$

$$= \frac{409{,}545}{1{,}581{,}580} \approx \frac{1}{3.86}$$

Next, we assume 5 numbers are picked. You win if 3, 4, or 5 numbers match.

$P(\text{win})$

$$= \frac{\begin{pmatrix} C(20,3)C(60,2) + C(20,4)C(60,1) \\ + C(20,5)C(60,0) \end{pmatrix}}{\begin{pmatrix} C(20,0)C(60,5) + C(20,1)C(60,4) \\ + C(20,2)C(60,3) + C(20,3)C(60,2) \\ + C(20,4)C(60,1) + C(20,5)C(60,0) \end{pmatrix}}$$

$$= \frac{2{,}324{,}004}{24{,}040{,}016} \approx \frac{1}{10.34}$$

(b) Expected value when you pick 4

$$= 1 \cdot \frac{C(20,2)C(60,2)}{1{,}581{,}580} + 5 \cdot \frac{C(20,3)C(60,1)}{1{,}581{,}580} + 55 \cdot \frac{C(20,4)C(60,0)}{1{,}581{,}580} - 1(1)$$

$\approx -\$0.4026.$

Expected value when you pick 5

$$= 2 \cdot \frac{C(20,3)C(60,2)}{24{,}040{,}016} + 20 \cdot \frac{C(20,4)C(60,1)}{24{,}040{,}016} + 300 \cdot \frac{C(20,5)C(60,0)}{24{,}040{,}016} - 1(1)$$

$\approx -\$0.3968.$

79. (a) (i) When 5 socks are selected, we could get 1 matching pair and 3 odd socks or 2 matching pairs and 1 odd sock.

First consider 1 matching pair and 3 odd socks. The number of ways this could be done is $C(10,1)\left[C(18,3) - C(9,1)C(16,1)\right] = 6720.$

$C(10,1)$ gives the number of ways for 1 pair, while $\left[C(18,3) - C(9,1)C(16,1)\right]$ gives the number of ways for choosing the remaining 3 socks from the 18 socks left. We must subtract the number of ways the last 3 socks could contain a pair from the 9 pairs remaining.

Next consider 2 matching pairs and 1 odd sock. The number of ways this could be done is

$$C(10,2)C(16,1) = 720.$$

$C(10,2)$ gives the number of ways for choosing 2 pairs, while $C(16,1)$ gives the number of ways for choosing the 1 odd sock.

The total number of ways is

$$6720 + 720 = 7440.$$

Then

$$P(\text{matching pair}) = \frac{7440}{C(20,5)} \approx 0.4799.$$

(ii) When 6 socks are selected, we could get 3 matching pairs and no odd socks or 2 matching pairs and 2 odd socks or 1 matching pair and 4 odd socks. The number of ways of obtaining 3 matching pairs is $C(10,3) = 120$. The number of ways of obtaining 2 matching pairs and 2 odd socks is

$$C(10,2)\left[C(16,2) - C(8,1)\right] = 5040.$$

The 2 odd socks must come from the 16 socks remaining but cannot be one of the 8 remaining pairs.

The number of ways of obtaining 1 matching pair and 4 odd socks is

$$C(10,1)\left[C(18,4) - C(9,2) - C(9\,1)[C(16,2) - 8]\right]$$
$$= 20{,}160.$$

The 4 odd socks must come from the 18 socks remaining but cannot be 2 pairs and cannot be 1 pair and 2 odd socks.

The total number of ways is

$$120 + 5040 + 20{,}160 - 25{,}320.$$

Thus,

$$P(\text{matching pair}) = \frac{25{,}320}{C(20,6)} \approx 0.6533.$$

(c) Suppose 6 socks are lost at random. The worst case is they are 6 odd socks. The best case is they are 3 matching pairs.

First find the number of ways of selecting 6 odd socks. This is

$$C(10,6)C(2,1)C(2,1)C(2,1)C(2,1)C(2,1)C(2,1) = 13{,}440.$$

The $C(10,6)$ gives the number of ways of choosing 6 different socks from the 10 pairs. But with each pair, $C(2,1)$ gives the number of ways of selecting 1 sock. Then

$$P(6\,\text{odd socks}) = \frac{13{,}440}{C(20,6)} \approx 0.3467.$$

Next find the number of ways of selecting three matching pairs. This is $C(10,3) = 120.$ Then

$$P(3\,\text{matching pairs}) = \frac{120}{C(20,6)} \approx 0.003096.$$

80. **(a)** $P(5\text{ or more})$

$$= P(5) + P(6) + P(7) + P(8) + P(9)$$
$$= 0.0040 + 0.0018 + 0.0007 + 0.0003 + 0.0001$$
$$= 0.0069$$

(b)
$$P(\text{less than }2) = P(0) + P(1) = 0.7345 + 0.1489 = 0.8834$$

(c) The expected number of runs is

$$E(x) = 0(0.7345) + 1(0.1489) + 2(0.0653) + 3(0.0306) + 4(0.0137) + 5(0.0040) + 6(0.0018) + 7(0.0007) + 8(0.0003) + 9(0.0001) = 0.4651$$

82. There are $C(92,9)$ possible hands.

(a) There are $C(10,9) = 10$ ways to choose 9 of the 10 suits. There are $C(9,1) = 9$ ways to choose a card from each suit. The probability is

$$\frac{10 \cdot 9^9}{C(92,9)} \approx 0.004459.$$

(b) There are $C(10,1) = 10$ ways to choose to choose one suit and $C(9,7) = 36$ ways to choose 7 of the 9 remaining suits. There are $C(9,2) = 36$ ways to choose a pair from one suit and $C(9,1) = 9$ ways to choose a card from each of the 7 suits. The probability is

$$\frac{10 \cdot 36 \cdot 36 \cdot 9^7}{C(92,9)} \approx 0.07135.$$

(c) There are $C(10,2) = 45$ ways to choose two suits and $C(8,5) = 56$ ways to choose 5 of the 8 remaining suits. There are $C(9,2) = 36$ ways to choose a pair from each of the two suits and $C(9,1) = 9$ ways to choose a card from each of the 5 other suits. The probability is

$$\frac{45 \cdot 56 \cdot 36^2 \cdot 9^5}{C(92,9)} \approx 0.2220.$$

Extended Application: Optimal Inventory for a Service Truck

1. **(a)** $C(M_0)$

$$= NL[1-(1-p_1)(1-p_2)(1-p_3)]$$
$$= 3(54)[1-(0.91)(0.76)(0.83)]$$
$$= \$69.01$$

(b)
$$C(M_2) = H_2 + NL[1-(1-p_1)(1-p_3)]$$
$$= 40 + 3(54)[1-(0.91)(0.83)]$$
$$= \$79.64$$

(c)
$$C(M_3) = H_3 + NL[1-(1-p_1)(1-p_2)]$$
$$= 9 + 3(54)[1-(0.91)(0.76)]$$
$$= \$58.96$$

(d)
$$C(M_{12}) = H_1 + H_2 + NL[1-(1-p_3)]$$
$$= 15 + 40 + 3(54)[1-0.83]$$
$$= \$82.54$$

(e) $C(M_{13}) = H_1 + H_3 + NL[1-(1-p_2)]$
$= 15 + 9 + 3(54)[1 - 0.76]$
$= \$62.88$

(f) $C(M_{123}) = H_1 + H_2 + H_3 + NL[1-1]$
$= 15 + 40 + 9$
$= \$64.00$

2. Policy M_3, stocking only part 3 on the truck, leads to the lowest expected cost.

3. It is not necessary for the probabilities to add up to 1 because it is possible that no parts will be needed. That is, the events of needing parts 1, 2, and 3 are not the only events in the sample space.

4. For 3 different parts we have 8 different policies: 1 with no parts, 3 with 1 part, 3 with 2 parts, and 1 with 3 parts. The number of different policies, $8 = 2^3$, is the number of subsets of a set containing 3 distinct elements. If there are *n* different parts, the number of policies is the number of subsets of a set containing *n* distinct elements which is 2^n.