Chapter 5

MATHEMATICS OF FINANCE

5.1 Simple and Compound Interest

Your Turn 1

Use the formula for maturity value, with $P = 3000$, $r = 0.058$, and $t = \frac{100}{360}$. We assume a year of 360 days.

$$A = P(1 + rt)$$
$$A = 3000\left[1 + 0.058\left(\frac{100}{360}\right)\right]$$
$$A = 3048.333$$

The maturity value is \$3048.33.

Your Turn 2

Use the formula $A = P(1 + rt)$ with $t = 0.75$ or three quarters of a year.

$$5243.75 = 5000\,[1 + r(0.75)]$$

Solve for r:

$$5243.75 = 5000 + 3750r$$
$$243.75 = 3750r$$
$$r = \frac{243.75}{3750} = 0.065$$

The interest rate is 6.5%.

Your Turn 3

For 7 years compounded quarterly there are $(7)(12) = 84$ periods. The interest rate per month is $\frac{0.042}{12} = 0.0035$. The interest earned on a principal of \$1600 is

$$1600(1 + 0.0035)^{84} - 1600 = 545.75 \text{ or } \$545.75.$$

Your Turn 4

The number of compounding periods is $(8)(12) = 96$. We need to solve the equation

$$6500\left(1 + \frac{r}{12}\right)^{96} = 8665.69$$

$\left(1 + \frac{r}{12}\right)^{96} = \frac{8665.69}{6500} = 1.33318$ Divide both sides by 6500.

$1 + \frac{r}{12} = 1.33318^{1/96} = 1.003$ Raise both sides to the 1/96 power.

$\frac{r}{12} = 0.003$ Subtract 1 from both sides.

$r = 0.036$ Multiply both sides by 12.

The annual interest rate is 3.6%.

Your Turn 5

Use the formula $r_E = \left(1 + \frac{r}{m}\right)^m - 1$ with $r = 0.027$ and $m = 12$.

$$r_E = \left(1 + \frac{0.027}{12}\right)^{12} - 1$$
$$r_E = 1.0273 - 1 = 0.0273$$

The effective rate is 2.73%.

Your Turn 6

The interest rate per quarter is $\frac{0.0425}{4} = 0.010625$. Use the formula for present value with $i = 0.010625$ and $n = (7)(4) = 28$.

$$P = \frac{A}{(1 + i)^n}$$
$$P = \frac{10{,}000}{(1 + 0.010625)^{28}}$$
$$P = 7438.39$$

The present value of the investment is \$7438.39.

Your Turn 7

The semiannual interest rate is $\frac{0.035}{2} = 0.0175$. Let n be the number of compounding periods. Then we want $7000 = 3800(1 + 0.0175)^n$.

Solving, we find

$$(1 + 0.0175)^n = \frac{7000}{3800} = 1.842$$

Using logarithms,

$$n\log(1.0175) = \log(1.842)$$
$$n = \frac{\log(1.842)}{\log(1.0175)} = 35.21$$

Since each period is half a year, this corresponds to $\frac{35.21}{2} = 17.605$ years. Rounding up to the next whole period we get an answer of 18 years.

Your Turn 8

Use the formula for continuous compounding with $P = 5000$, $r = 0.038$ and $t = 9$.

$$A = Pe^{rt}$$
$$A = 5000e^{(0.038)(9)}$$
$$A = 7038.80$$

Subtracting the initial investment we get $7038.80 - 5000 = 2038.80$, or \$2038.80 interest earned.

5.1 Warmup Exercises

W1. $1000[1+0.017(3)] = 1000(1+0.0411)$
$= 1000(1.051) = 1051$

W2. $1500(1+0.05)^6 = 1500(1.06)^6 \approx 2010.14$

W3. $(1+0.04)^8 - 1 = (1.04)^8 - 1 \approx 0.37$

W4. $\dfrac{6000}{(1+0.03)^5} = \dfrac{6000}{(1.03)^5} \approx 5175.65$

5.1 Exercises

5. \$25,000 at 3% for 9 mo

Use the formula for simple interest.

$$I = Prt$$
$$= 25{,}000(0.03)\left(\frac{9}{12}\right)$$
$$= 562.50$$

The simple interest is \$562.50.

6. \$4289 at 4.5% for 9 wk

Use the formula for simple interest.

$$I = Prt$$
$$= 4289(0.045)\left(\frac{35}{52}\right)$$
$$\approx 129.91$$

The simple interest is \$129.91.

7. \$1974 at 6.3% for 25 wk

Use the formula for simple interest.

$$I = Prt$$
$$= 1974(0.063)\left(\frac{25}{52}\right) \approx 59.79$$

The simple interest is \$59.79.

8. \$6125 at 1.25% for 6 mo

Use the formula for simple interest.

$$I = Prt$$
$$= 6125(0.0125)\left(\frac{6}{12}\right)$$
$$\approx 38.28$$

The simple interest is \$38.28.

9. \$8192.17 at 3.1% for 72 days

Use the formula for simple interest.

$$I = Prt$$
$$= 8192.17(0.031)\left(\frac{72}{360}\right)$$
$$\approx 50.79$$

The simple interest is \$50.79.

10. \$7236.15 at 4.25% for 30 days

Use the formula for simple interest.

$$I = Prt$$
$$= 7236.15(0.0425)\left(\frac{30}{360}\right)$$
$$\approx 25.63$$

The simple interest is \$25.63.

11. Use the formula for future value for simple interest.

$$A = P(1 + rt)$$
$$= 3125\left[1 + 0.0285\left(\frac{7}{12}\right)\right]$$
$$\approx 3176.95$$

The maturity value is \$3176.95. The interest earned is $3176.95 - 3125 = \$51.95$.

12. Use the formula for future value for simple interest.

$$A = P(1 + rt)$$
$$= 12{,}000\left[1 + 0.053\left(\frac{11}{12}\right)\right]$$
$$= 12{,}583$$

The maturity value is \$12,583. The interest earned is 12,583 − 12,000 = \$583.

13. Use the formula for simple interest.

$$\begin{aligned} I &= Prt \\ 56.25 &= 1500r\left(\frac{6}{12}\right) \\ r &= 0.075 \end{aligned}$$

The interest rate was 7.5%.

14. Use the formula for simple interest.

$$\begin{aligned} I &= Prt \\ 1057.50 &= 23{,}500r\left(\frac{9}{12}\right) \\ r &= 0.06 \end{aligned}$$

The interest rate was 6%.

19. Use the formula for compound amount with $P = 1000$, $i = 0.06$, and $n = 8$.

$$\begin{aligned} A &= P(1+i)^n \\ &= 1000(1+0.06)^8 \\ &\approx 1593.85 \end{aligned}$$

The compound amount is \$1593.85. The interest earned is 1593.85 − 1000 = \$593.85.

20. Use the formula for compound amount with $P = 1000$, $i = 0.045$, and $n = 6$.

$$\begin{aligned} A &= P(1+i)^n \\ &= 1000(1+0.045)^6 \\ &\approx 1302.26 \end{aligned}$$

The compound amount is \$1302.26. The interest earned is 1302.26 − 1000 = \$302.26.

21. Use the formula for compound amount with $P = 470$, $i = \frac{0.054}{2} = 0.027$, and $n = 12(2) = 24$.

$$\begin{aligned} A &= P(1+i)^n \\ &= 470(1+0.027)^{24} \\ &\approx 890.82 \end{aligned}$$

The compound amount is \$890.82. The interest earned is 890.82 − 470 = \$420.82.

22. Use the formula for compound amount with $P = 15{,}000$, $i = \frac{0.06}{12} = 0.005$, and $n = 10(12) = 120$.

$$\begin{aligned} A &= P(1+i)^n \\ &= 15{,}000(1+0.005)^{120} \\ &\approx 27{,}290.95 \end{aligned}$$

The compound amount is \$27,290.95. The interest earned is 27,290.95 − 15,000 = \$12,290.95.

23. Use the formula for compound amount with $P = 8500$, $i = \frac{0.08}{4} = 0.02$, and $n = 5(4) = 20$.

$$\begin{aligned} A &= P(1+i)^n \\ &= 8500(1+0.02)^{20} \\ &\approx 12{,}630.55 \end{aligned}$$

The compound amount is \$12,630.55. The interest earned is 12,630.55 − 8500 = \$4130.55.

24. Use the formula for compound amount with $P = 9100$, $i = \frac{0.064}{4} = 0.016$, and $n = 9(4) = 36$.

$$\begin{aligned} A &= P(1+i)^n \\ &= 9100(1+0.016)^{36} \\ &\approx 16{,}114.43 \end{aligned}$$

The compound amount is \$16,114.43. The interest earned is 16,114.43 − 9100 = \$7014.43.

25. The number of compounding periods is $(4)(8) = 32$.

$$\begin{aligned} 8000\left(1+\frac{r}{4}\right)^{32} &= 11672.12 \\ \left(1+\frac{r}{4}\right)^{32} &= \frac{11672.12}{8000} = 1.45902 \\ 1+\frac{r}{4} &= 1.45902^{1/32} = 1.011875 \\ \frac{r}{4} &= 0.011875 \\ r &= 0.0475 \end{aligned}$$

The answer is 4.75%.

26. The number of compounding periods is $(4)(9) = 36$.

$$\begin{aligned} 12500\left(1+\frac{r}{4}\right)^{36} &= 20077.43 \\ \left(1+\frac{r}{4}\right)^{36} &= \frac{20077.43}{12500} = 1.606194 \\ 1+\frac{r}{4} &= 1.606194^{1/36} = 1.01325 \\ \frac{r}{4} &= 0.01325 \\ r &= 0.053 \end{aligned}$$

The answer is 5.3%.

27. The number of compounding periods is $(12)(5) = 60$.

$$4500\left(1+\frac{r}{12}\right)^{60} = 5994.79$$
$$\left(1+\frac{r}{12}\right)^{60} = \frac{5994.79}{4500} = 1.332176$$
$$1+\frac{r}{12} = 1.332176^{1/60} = 1.004792$$
$$\frac{r}{12} = 0.004792$$
$$r = 0.0575$$

The answer is 5.75%

28. The number of compounding periods is $(12)(7) = 84$.

$$6725\left(1+\frac{r}{12}\right)^{84} = 10353.47$$
$$\left(1+\frac{r}{12}\right)^{84} = \frac{10353.47}{6725} = 1.539549$$
$$1+\frac{r}{12} = 1.539549^{1/84} = 1.00515$$
$$\frac{r}{12} = 0.00515$$
$$r = 0.0618$$

The answer is 6.18%.

29. 4% compounded quarterly.

Use the formula for effective rate with $r = 0.04$ and $m = 4$.

$$r_E = \left(1+\frac{r}{m}\right)^m - 1 = \left(1+\frac{0.04}{4}\right)^4 - 1 \approx 0.04060$$

The effective rate is about 4.06%.

30. 6% compounded quarterly.

Use the formula for effective rate with $r = 0.06$ and $m = 4$.

$$r_E = \left(1+\frac{r}{m}\right)^m - 1 = \left(1+\frac{0.06}{4}\right)^4 - 1 \approx 0.06136$$

The effective rate is about 6.136%, or rounding to two decimal places, 6.14%.

31. 7.25% compounded semiannually.

Use the formula for effective rate with $r = 0.0725$ and $m = 2$.

$$r_E = \left(1+\frac{r}{m}\right)^m - 1 = \left(1+\frac{0.0725}{2}\right)^2 - 1 \approx 0.07381$$

The effective rate is about 7.381%, or rounding to two decimal places, 7.38%.

32. 6.25% compounded semiannually.

Use the formula for effective rate with $r = 0.0625$ and $m = 2$.

$$r_E = \left(1+\frac{r}{m}\right)^m - 1 = \left(1+\frac{0.0625}{2}\right)^2 - 1 \approx 0.06348$$

The effective rate is about 6.348%, or rounding to two decimal places, 6.35%.

33. Use the formula for present value for compound interest with $A = 12{,}820.77$, $i = 0.048$, and $n = 6$.

$$P = \frac{A}{(1+r)^n} = \frac{12{,}820.77}{(1+0.048)^6} \approx 9677.13$$

The present value is \$9677.13.

34. Use the formula for present value for compound interest with $A = 36{,}527.13$, $i = 0.053$, and $n = 10$.

$$P = \frac{A}{(1+r)^n} = \frac{36{,}527.13}{(1+0.053)^{10}} \approx 21{,}793.74.$$

The present value is \$21,793.74, or \$21,793.75 if we round the cents up.

35. Use the formula for present value for compound interest with $A = 2000$, $i = \frac{0.06}{2} = 0.03$, and $n = 8(2) = 16$.

$$P = \frac{A}{(1+r)^n} = \frac{2000}{(1+0.03)^{16}} \approx 1246.33$$

The present value is $1246.33, or $1246.34 if we round the cents up.

36. Use the formula for present value for compound interest with $A = 2000$, $i = \frac{0.07}{2} = 0.035$, and $n = 8(2) = 16$.

$$P = \frac{A}{(1+r)^n} = \frac{2000}{(1+0.035)^{16}} \approx 1153.41$$

The present value is $1153.41, or $1153.42 if we round the cents up.

37. Use the formula for present value for compound interest with $A = 8800$, $i = \frac{0.05}{4} = 0.0125$, and $n = 5(4) = 20$.

$$P = \frac{A}{(1+r)^n} = \frac{8800}{(1+0.0125)^{20}} \approx 6864.08$$

The present value is $6864.08.

38. Use the formula for present value for compound interest with $A = 7500$, $i = \frac{0.055}{4} = 0.01375$, and $n = 9(4) = 36$.

$$P = \frac{A}{(1+r)^n} = \frac{7500}{(1+0.01375)^{36}} \approx 4587.23$$

The present value is $4587.23.

40. The effective interest rate $\left(1+\frac{i}{n}\right)^n - 1$ equals $1 + n\left(\frac{i}{n}\right) +$ other terms -1. This equals $i+$ other terms, which is greater than the stated interest rate i.

41. The quarterly interest rate is $\frac{0.04}{4} = 0.01$. Let n be the number of compounding periods. Then we want $9000 = 5000(1+0.01)^n$

or

$$(1+0.01)^n = \frac{9000}{5000} = 1.8$$

Using logarithms, we have

$$n\log(1.01) = \log(1.8)$$
$$n = \frac{\log(1.8)}{\log(1.01)}$$
$$n = 59.07$$

Since each period is one quarter of a year, the number of years is $\frac{59.07}{4} = 14.768$. Rounding up to the next whole quarter gives us an answer of 15 years.

42. The quarterly interest rate is $\frac{0.03}{4} = 0.0075$. Let n be the number of compounding periods. Then we want $23000 = 8000(1+0.0075)^n$

or

$$(1+0.0075)^n = \frac{23000}{8000} = 2.875$$

Using logarithms, we have

$$n\log(1.0075) = \log(2.875)$$
$$n = \frac{\log(2.875)}{\log(1.0075)}$$
$$n = 141.33$$

Since each period is one quarter of a year, the number of years is $\frac{141.33}{4} = 35.333$. Rounding up to the next whole quarter gives us an answer of 35.5 years.

43. The monthly interest rate is $\frac{0.036}{12} = 0.003$. Let n be the number of compounding periods. Then we want $11000 = 4500(1+0.003)^n$

or

$$(1+0.003)^n = \frac{11000}{4500} = 2.444$$

Using logarithms, we have

$$\begin{aligned} n\log(1.003) &= \log(2.444) \\ n &= \frac{\log(2.444)}{\log(1.003)} \\ n &= 298.325 \end{aligned}$$

Since each period is one twelfth of a year, the number of years is $\frac{298.325}{12} = 24.86$.

$$\frac{10}{12} = 0.833 \text{ and } \frac{11}{12} = 0.917$$

so rounding up to the next whole month gives us an answer of 24 years and 11 months.

44. The monthly interest rate is $\frac{0.054}{12} = 0.0045$. Let n be the number of compounding periods. Then we want $15000 = 6800(1 + 0.0045)^n$

or

$$(1 + 0.0045)^n = \frac{15000}{6800} = 2.206$$

Using logarithms, we have

$$\begin{aligned} n\log(1.0045) &= \log(2.206) \\ n &= \frac{\log(2.206)}{\log(1.0045)} \\ n &= 176.213 \end{aligned}$$

Since each period is one twelfth of a year, the number of years is $\frac{176.213}{12} = 14.684$.

$$\frac{8}{12} = 0.667 \text{ and } \frac{9}{12} = 0.75$$

so rounding up to the next whole month gives us an answer of 14 years and 9 months.

45. **(a)** The doubling time for an inflation rate of 3.3% is the solution of $2 = (1.033)^n$. Taking logarithms on both sides we have

$$\begin{aligned} \log(2) &= n\log(1.033) \\ n &= \frac{\log(2)}{\log(1.033)} \\ n &= 21.349 \end{aligned}$$

The doubling time is about 21.35 years.

(b) Since $0.001 < 0.033 < 0.05$, this is a small growth rate and we may use the rule of 70 which estimates the doubling time as

$$\frac{70}{(100)(0.033)} = 21.212$$

or about 21.21 years.

46. **(a)** The doubling time for an inflation rate of 6.25% is the solution of $2 = (1.0625)^n$. Taking logarithms on both sides we have

$$\begin{aligned} \log(2) &= n\log(1.0625) \\ n &= \frac{\log(2)}{\log(1.0625)} \\ n &= 11.433 \end{aligned}$$

The doubling time is about 11.43 years.

(b) Since $0.05 < 0.0625 < 0.12$, this is a large growth rate and we may use the rule of 72 which estimates the doubling time as

$$\frac{72}{(100)(0.0625)} = 11.52$$

or about 11.52 years.

47. **(a)** The future value is $5500e^{(0.031)(9)} = 7269.94$, or \$7269.94.

(b) The effective rate is $e^{0.031} - 1 = 0.0315$ or 3.15%.

(c) To find the time to reach \$10,000, we solve

$$\begin{aligned} 10{,}000 &= 5500e^{0.031t} \\ e^{0.031t} &= \frac{10{,}000}{5500} \end{aligned}$$

Using logarithms with base e we have

$$\begin{aligned} 0.031t &= \ln\left(\frac{10{,}000}{5500}\right) \\ t &= \frac{\ln\left(\frac{10{,}000}{5500}\right)}{0.031} \\ t &= 19.285 \end{aligned}$$

The time to reach \$10,000 is 19.29 years.

48. **(a)** The future value is $4700e^{(0.0465)(9)} =$ 7142.50, or \$7142.50.

(b) The effective rate is $e^{0.0465} - 1 = 0.0476$ or 4.76%.

(c) To find the time to reach, \$10,000 we solve

$$\begin{aligned} 10{,}000 &= 4700e^{0.0465t} \\ e^{0.0465t} &= \frac{10{,}000}{4700} \end{aligned}$$

Using logarithms with base e we have

$$0.0465t = \ln\left(\frac{10{,}000}{4700}\right)$$
$$t = \frac{\ln\left(\frac{10{,}000}{4700}\right)}{0.0465}$$
$$t = 16.237$$

The time to reach $10,000 is 16.24 years.

49. Start by finding the total amount repaid. Use the formula for future value for simple interest, with $P = 2700$, $r = 0.062$, and $t = \frac{9}{12}$.

$$A = P(1 + rt)$$
$$= 7200\left[1 + 0.062\left(\frac{9}{12}\right)\right]$$
$$= 7534.80$$

Celeste repaid her father $7534.80. To find the amount of this which was interest, subtract the original loan amount from the repayment amount.

$$7534.80 - 7200 = 334.80$$

Of the amount repaid, $334.80 was interest.

50. To find the total amount paid, use the formula for future value for simple interest, with $P = 321{,}812.85$, $r = 0.134$, and $t = \frac{29}{365}$.

$$A = P(1 + i)^n$$
$$= 321{,}812.85\left[1 + 0.134\left(\frac{29}{365}\right)\right]$$
$$\approx 325{,}239.05$$

The company paid $325,239.05.

51. The interest earned was

$$\$1521.25 - \$1500 = \$21.25$$

Use the formula for simple interest, with $I = 21.25$, $P = 1500$, and $t = \frac{75}{360}$.

$$I = Prt$$
$$21.25 = 1500r\left(\frac{75}{360}\right)$$
$$0.068 = r$$

The interest rate was 6.8%.

52. The interest earned is

$$\$10{,}000 - \$5988.02 = \$4011.98$$

Use the formula for simple interest, with $I = 4011.98$, $P = 5988.02$, and $t = 10$.

$$I = Prt$$
$$4011.98 = 5988.02r(10)$$
$$0.067 \approx r$$

The interest rate was about 6.7%.

53. The interest paid is $378.59 - 300 = 78.59$.

This is for 14 days, so for one year you would pay $78.59\left(\frac{365}{14}\right)$.

To turn this into a percentage of the $300 advance, divide by 300 and multiply by 100:

$$\frac{78.59\left(\frac{365}{14}\right)}{300}(100) \approx 682.98, \text{ or } 682.98\%.$$

54. Start by finding the total interest earned.

$$I = (\$24 - \$22) + \$0.50 = \$2.50$$

Now use the formula for simple interest, with $I = 2.50$, $P = 22$, and $t = 1$.

$$I = Prt$$
$$2.50 = 22r(1)$$
$$0.11364 \approx r$$

The interest rate was about 11.36% or, rounding to one decimal place, 11.4%.

55. Use the formula

$$A = P(1 + i)^n$$

with $P = 10{,}000$ and $r = 0.05$ for 10 years.

(a) If interest is compounding annually,

$$A = 10{,}000(1 + 0.05)^{10}$$
$$\approx 16{,}288.95.$$

The future value is $16,288.95.

(b) If interest is compounding quarterly,

$$A = 10{,}000\left(1 + \frac{0.05}{4}\right)^{40}$$
$$\approx 16{,}436.19.$$

The future value is $16,436.19.

(c) If interest is compounding monthly,

$$A = 10{,}000\left(1 + \frac{0.05}{12}\right)^{120}$$
$$\approx 16{,}470.09.$$

The future value is $16,470.09.

(d) If interest is compounding daily,

$$A = 10{,}000\left(1 + \frac{0.05}{365}\right)^{3650}$$
$$\approx 16{,}486.65.$$

The future value is $16,486.65.

(e) If the interest is compounded continuously for 10 years at 5%, the future value is $10{,}000e^{(0.05)(10)} = 16{,}487.213$ or $16,487.21.

57. **(a)** $A = P(1 + i)^n$

$$= 1000\left(1 + \frac{0.0605}{12}\right)^{12}$$
$$\approx 1062.21$$

The future value is $1062.21.

(b) $A = P(1 - i)^n$

$$= 1000\left(1 + \frac{0.2}{12}\right)^{12}$$
$$\approx 1219.39$$

The future value would be $1219.39.

(c) $A = P(1 - i)^n$

$$= 1000\left(1 + \frac{0.0025}{12}\right)^{12}$$
$$\approx 1002.50$$

The future value would be $1002.50.

(d) The interest rates clearly have a significant effect on the purchasing power of Americans with savins.

58. First use the formula for simple interest where $P = 5200,\ r = 0.03,$ and $t = \frac{10}{12}$.

$$A = P(1 + rt)$$
$$= 5200\left[1 + 0.03\left(\frac{10}{12}\right)\right]$$
$$= 5330.00$$

Now use the formula for compound interest with $P = 5330.00,\ i = \frac{0.033}{4},$ and $n = 5(4) = 20$.

$$A = P(1 + i)^n$$
$$= 5330.00\left(1 + \frac{0.033}{4}\right)^{20}$$
$$= 5330.00(1.00825)^{20}$$
$$\approx 6281.91$$

He will have $6281.91 at the end of the 5 yr.

59. Use the formula for compound amount with

$$P = 40{,}000$$
$$i = \frac{0.0466}{12}$$
$$n = 6$$
$$A = P(1 + i)^n$$
$$A = 40{,}000\left(1 + \frac{0.0466}{12}\right)^6$$
$$\approx 40{,}941.10$$

When Warren begins paying off his loan he will owe $40,941.10.

60. Substitute $P = 10{,}000,\ i = \frac{0.06}{2}$, and $n = 2(3) = 6$ in the formula for compound amount.

$$A = P(1 + i)^n$$
$$= 10{,}000\left(1 + \frac{0.06}{2}\right)^6$$
$$= 10{,}000(1.03)^6$$
$$\approx 11{,}940.52$$

She should contribute about $11,940.52 in 3 yr.

61. Use 5% compounded quarterly for 20 years. Then

$$i = \frac{0.05}{4} = 0.0125$$
$$n = 20(4) = 80$$
$$A = P(1 + 0.0125)^{80} = P(1.0125)^{80}$$

For $10,000

$$A = 10{,}000(1.0125)^{80} \approx 27{,}014.85$$

that is, $27,014.85.

For $149,000

$$A = 149{,}000(1.0125)^{80} \approx 40{,}2521.26$$

that is, $402,521.26.

For $1,000,000

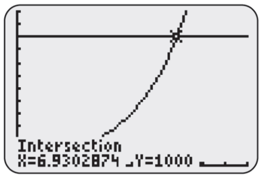
$$A = 1{,}000{,}000(1.0125)^{80} \approx 2{,}701{,}484.94,$$

that is, $2,701,484.94.

62. **(a)** Use the formula for compound amount with $P = 42$ and $i = 0.58$.

$$A = P(1 + i)^n$$
$$A = 42(1 + 0.58)^n$$

Graph $y_1 = 42(1 + 0.58)^x$ and $y_2 = 1000$.

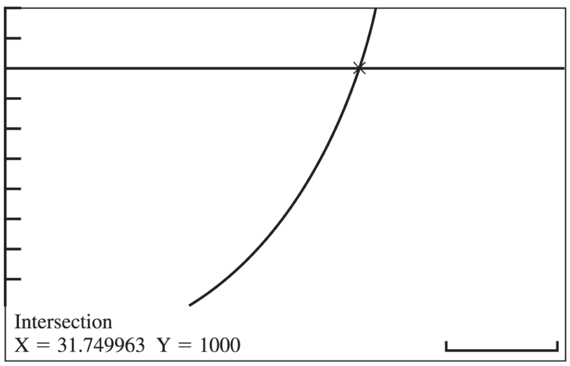


The graphs intersect at (6.9302874, 1000). Thus, 6.93 yr after July 16, 1997, or in June 2004, Bill Gates would be a trillionaire. At that time Mr. Gates would be 48 years old.

(b) Use the formula for compound amount with $P = 42$ and $i = 0.105$.

$$A = P(1 + i)^n$$
$$A = 42(1 + 0.105)^n$$

Graph $y_1 = 42(1 + 0.105)^x$ and $y_2 = 1000$.

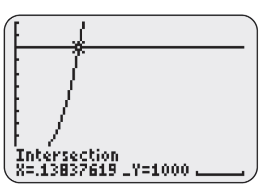


The graphs intersect at (31.749963, 1000). Thus, 31.75 yr after July 16, 1997, or in April 2029, Bill Gates would be a trillionaire. At that time Mr. Gates would be 73 yr old.

(c) January 1, 2022, is 24.46 yr after July 16, 1997. Thus, $t = 24.46$. Since we are compounding once a year, there are 24.46 periods. Thus $n = 24.46$. Use the formula for compound amount with $P = 42$.

$$A = P(1 + i)^n$$
$$A = 42(1 + i)^{24.46}$$

Graph $y_1 = 42(1 + x)^{24.46}$ and $y_2 = 1000$.



The graphs intersect at (0.13837619, 1000). Thus, $i = 0.01384$ and 13.84% growth would be necessary for Bill Gates to be a trillionaire
by January 1, 2022.

(d) We are looking for the annual growth rate which, compounded over 8 years would raise his worth from \$50.0 billion to \$79.1 billion. Solve the equation

$$79.1 = 50(1+i)^8 \text{ for } i.$$

$$\frac{79.1}{50} = (1 + i)^8$$
$$\left(\frac{79.1}{50}\right)^{1/8} - 1 = i$$
$$i \approx 0.05901$$

The annual grwoth rate was about 5.90%.

63. Let $P = 150{,}000$, $i = -2.4\% = -.024$, and $n = 4$.

$$\begin{aligned} A &= P(1 + i)^n \\ &= 150{,}000[1 + (-.024)]^4 \\ &= 150{,}000(.976)^4 \\ &\approx 136{,}110.16 \end{aligned}$$

After 4 yr, the amount on deposit will be \$136,110.16.

64. Let $P = 150{,}000$, $i = -0.024$, and $n = 8$.

$$\begin{aligned} A &= P(1 + i)^n \\ &= 150{,}000(1 - 0.024)^8 \\ &= 150{,}000(0.976)^8 \\ &= 123{,}506.50 \end{aligned}$$

After 8 yr, the amount on deposit will be \$123,506.50.

65. First consider the case of earning interest at a rate of k per annum compounded quarterly for all 8 yr and earning \$2203.76 on the \$1000 investment.

$$2203.76 = 1000\left(1 + \frac{k}{4}\right)^{8(4)}$$
$$2.20376 = \left(1 + \frac{k}{4}\right)^{32}$$

Use a calculator to raise both sides to the power $\frac{1}{32}$.

$$1.025 = 1 + \frac{k}{4}$$
$$0.025 = \frac{k}{4}$$
$$0.1 = k$$

Next consider the actual investments. The \$1000 was invested for the first 5 yr at a rate of j per annum compounded semiannually.

$$A = 1000\left(1 + \frac{j}{2}\right)^{5(2)}$$

$$A = 1000\left(1 + \frac{j}{2}\right)^{10}$$

This amount was then invested for the remaining 3 yr at $k = .1$ per annum compounded quarterly for a final compound amount of \$1990.76.

$$1990.76 = A\left(1 + \frac{0.1}{4}\right)^{3(4)}$$

$$1990.76 = A(1.025)^{12}$$

$$1480.24 \approx A$$

Recall that $A = 1000\left(1 + \frac{j}{2}\right)^{10}$ and substitute this value into the above equation.

$$1480.24 = 1000\left(1 + \frac{j}{2}\right)^{10}$$

$$1.48024 = \left(1 + \frac{j}{2}\right)^{10}$$

Use a calculator to raise both sides to the power $\frac{1}{10}$.

$$1.04 \approx 1 + \frac{j}{2}$$

$$0.04 = \frac{j}{2}$$

$$0.08 = j$$

The ratio of k to j is

$$\frac{k}{j} = \frac{0.1}{0.08} = \frac{10}{8} = \frac{5}{4}.$$

66. Let i be the annual interest rate in question. During the last 6 months of any year Mike earns

$$200\left(\frac{6}{12}i\right) \text{ or } 100i$$

Eric's interest for the last 6 months will be half of the annual interest rate multiplied by his accumulated value after 7.5 years, that is

$$(\text{accumulated value after 7.5 years})(i/2)$$

Eric's interest is compounded semiannualy, and 7.5 years is equal to 15 semiannual periods, so Eric's interest in the last half year is

$$\left(\frac{i}{2}\right)100\left(1 + \frac{i}{2}\right)^{15}$$

Thus we need

$$100i = \left(\frac{i}{2}\right)100\left(1 + \frac{i}{2}\right)^{15}$$

$$\left(1 + \frac{i}{2}\right)^{15} = 2$$

$$i = \left(2^{1/15} - 1\right)(2) = 0.09459 \approx 9.46\%$$

Answer (c) is correct.

67. Let i be the common effective interest rate.

In the 11th year Bruce earns $(i)(100)(1+i)^{10}$.

In the 17th year Bruce earns $(i)(50)(1+i)^{16}$.

Since these are equal,

$$(i)(100)(1+i)^{10} = (i)(50)(1+i)^{16}$$

$$2 = (1+i)^6$$

$$i = 2^{1/6} - 1 \approx 0.122462$$

Use this ratio to compute the amount Eric earns in the 11th year.

$$\left(2^{1/6} - 1\right)(100)\left(2^{10/6}\right) \approx 38.88$$

The correct answer is (e).

68. Use the formula

$$A = P(1 + i)^n$$

with $A = 420{,}000{,}000$, $P = 100$, and $n = 160$.

$$420{,}000{,}000 = 100(1 + r)^{160}$$

$$4{,}200{,}000 = (1 + r)^{160}$$

$$4{,}200{,}000^{1/160} = 1 + r$$

$$r = 4{,}200{,}000^{1/160} - 1$$

$$r \approx 0.1000$$

The rate used was 10.00%.

69. **(a)** Use the formula for effective rate:

$$r_E = \left(1 + \frac{r}{m}\right)^m - 1$$

For the Home Savings Bank

$$r_E = \left(1 + \frac{0.01}{365}\right)^{365} - 1$$

$$\left(1 + \frac{0.01}{365}\right)^{365} - 1 \approx 0.01005,$$

The effective rate is 1.005%.

For the Pallaidan Private Bank

$$r_E = \left(1 + \frac{0.01}{4}\right)^4 - 1$$

$$\left(1 + \frac{0.01}{4}\right)^4 - 1 \approx 0.01004,$$

The effective rate is 1.004%.

(b) For Home Savings the amount is

$$1000\left(1 + \frac{0.01}{365}\right)^{365} \approx 1010.05,$$

or $1010.05.

For the Pallaidan Private the amount is

$$1000\left(1 + \frac{0.01}{4}\right)^4 \approx 1010.04,$$

or $1010.04.

70. $$r_E = \left(1 + \frac{r}{m}\right)^m - 1$$

(a) $$\left(1 + \frac{0.02}{365}\right)^{365} - 1 = 0.02020, \text{ or } 2.020\%$$

$$\left(1 + \frac{0.02}{4}\right)^4 - 1 = 0.02015, \text{ or } 2.015\%$$

(b) $$\left(1 + \frac{0.05}{365}\right)^{365} - 1 = 0.05127, \text{ or } 5.127\%$$

$$\left(1 + \frac{0.05}{4}\right)^4 - 1 = 0.05095, \text{ or } 5.095\%$$

(c)

$$\left(1 + \frac{0.10}{365}\right)^{365} - 1 = 0.105\text{`}6 \text{ or } 10.516\%$$

$$\left(1 + \frac{0.10}{4}\right)^4 - 1 = 0.10381, \text{ or } 10.381\%$$

(d)

$$\left(1 + \frac{0.20}{365}\right)^{365} - 1 \approx 0.22134, \text{ or } 22.134\%$$

$$\left(1 + \frac{0.20}{4}\right)^4 - 1 \approx 0.21551, \text{ or } 21.551\%$$

(e) Answers will vary.

71. Start with the effective-rate formula and solve for thenominal rate r in terms of r_E.

$$r_E = \left(1 + \frac{r}{365}\right)^{365} - 1$$

$$(r_E + 1)^{1/365} = 1 + \frac{r}{365}$$

$$r = (365)\left[(r_E + 1)^{1/365} - 1\right]$$

For the 0.65% APY certificate:

$$r = (365)\ (0.0065 + 1)^{1/365} - 1 \approx 0.00648 \text{ or } 0.65\%$$

For the 1.10% certificate:

$$r = (365)\ (0.0110 + 1)^{1/365} - 1 \approx 0.01094 \text{ or } 1.09\%$$

For the 1.30% certificate:

$$r = (365)\ (0.0130 + 1)^{1/365} - 1 \approx 0.01292 \text{ or } 1.29\%$$

For the 2.30 certificate:

$$r = (365)\ (0.0230 + 1)^{1/365} - 1 \approx 0.02274 \text{ or } 2.27\%$$

72. The total cost of the 8 computers is $\$2309(8) = \$18{,}472$. We want to find the present value of 18,472 dollars compounded at interest rate $i = \frac{0.0479}{12}$ per month for $n = 6$ months.

$$P = \frac{A}{(1+i)^n} = \frac{18{,}472}{\left(1 + \frac{0.0479}{12}\right)^6} \approx 18{,}035.71$$

The department should deposit $18,035.71 now.

73. Use the formula for present value for compound interest with $A = 30{,}000$, $i = \frac{0.055}{4} = 0.01375$, and $n = 5(4) = 20$.

$$P = \frac{A}{(1+r)^n} = \frac{30{,}000}{(1 + 0.01375)^{20}} \approx 22{,}829.89$$

The present value is $22,829.89, or rounding up to the nearest cent (to make sure that the investment really grows to $30,000), $22,829.90. That is how much of the inherited $25,000 Cara should invest in order to have $30,000 for a down payment in 5 years.

74. **(a)** Use the Rule of 72 to find the doubling time.

$$\text{Doubling time} = \frac{72}{4.5} = 16$$

It takes about 16 years for the trust fund to double in size, from $10,000 to $20,000. The grandchild will be 16 years old.

(b) Use the formula for compound amount with $P = 10{,}000$, $i = \frac{0.045}{12}$, and $n = 16(12) = 192$.

$$\begin{aligned} A &= P(1+i)^n \\ &= 10{,}000\left(1+\frac{0.045}{12}\right)^{192} \\ &\approx 20{,}516.69 \end{aligned}$$

The actual amount in the trust fund after 16 years is $20,516.69. Obviously, it actually takes slightly less than 16 years for the fund to reach $20,000.

75. To find the number of years it will take prices to double at 4% annual inflation, find n in the equation

$$2 = (1+0.04)^n,$$

which simplifies to

$$2 = (1.04)^n.$$

By trying various values of n, find that $n = 18$ is approximately correct, because

$$1.04^{18} \approx 2.0258 \approx 2.$$

Prices will double in about 18 yr.

76. $2 = (1+0.05)^n$

$2 = (1.05)^n$

Try various values for n.

$$(1.05)^{14} \approx 1.979932 \approx 2$$

Thus, $n \approx 14$. It would take about 14 yr for the general level of prices in the economy to double at the annual inflation rate of 5%.

77. To find the number of years it will be until the generating capacity will need to be doubled, find n in the equation

$$2 = (1+0.06)^n,$$

which simplifies to

$$2 = (1.06)^n.$$

By trying various values of n, find that $n = 12$ is approximately correct, because

$$1.06^{12} \approx 2.0122 \approx 2.$$

The generating capacity will need to be doubled in about 12 yr.

78. Find n such that

$$2 = (1.02)^n$$

By trying various values of n, we see that $n \approx 35$ is approximately correct because

$$(1.02)^{35} \approx 1.999890 \approx 2.$$

It will take about 35 yr before the utilities will need to double their generating capacity.

79. **(a)** To find this rate of return we must solve $14 = 1(1+r)^{14}$.

$$\begin{aligned} 14 &= 1(1+r)^{14} \\ 1+r &= 14^{1/14} = 1.207 \\ r &= 0.207 \end{aligned}$$

The required rate of return is 20.7%.

(b) With an annual rate of return of 113%, in 14 years an initial investment of $1 million would be worth $1(1+1.13)^{14} = 39{,}565.299$ million dollars or about $39.6 billion.

80. Use the formula

$$A = P(1+i)^n$$

with $P = \frac{2}{8}$ cent $= \$0.0025$ and $r = 0.04$ compounded quarterly for 2000 yr.

$$\begin{aligned} A &= 0.0025\left(1+\frac{0.04}{4}\right)^{4(2000)} \\ &= 0.0025(1.01)^{8000} \\ &\approx 9.31\times 10^{31} \end{aligned}$$

2000 years later, the money would be worth $\$9.31\times 10^{31}$.

5.2 Future Value of an Annuity

Your Turn 1

For the geometric series 4, 12, 36,... the common ratio r is $\frac{12}{4} = 3$. The first term is $a = 4$, and to find the sum of the first nine terms we set $n = 9$ and use the formula for the sum of the first n terms of a geometric series:

$$S_n = \frac{a(r^n - 1)}{r - 1}.$$

$$S_9 = \frac{4(3^9 - 1)}{3 - 1}$$

$$S_9 = 39,364$$

Your Turn 2

Use the formula for the future value of an ordinary annuity,

$$S = R\left[\frac{(1 + i)^n - 1}{i}\right],$$

with $R = 250$, $i = 0.033/12 = 0.00275$, and $n = (11)(12) = 132$.

$$S = 250\left[\frac{(1 + 0.00275)^{132} - 1}{0.00275}\right] = 39719.98$$

The accumulated amount after 11 years is \$39,719.98.

Your Turn 3

Use the formula for a sinking fund payment,

$$R = \frac{Si}{(1 + r)^n - 1},$$

with $S = 13,500$, $i = 0.0375/4 = 0.009375$, and $n = (4)(14) = 56$.

$$R = \frac{(13,500)(0.009375)}{(1 + 0.009375)^{56} - 1}$$

$$R = 184.41$$

The quarterly payment will be \$184.41.

Your Turn 4

Use the formula for the future value of an annuity due,

$$S = R\left[\frac{(1 + i)^{n+1} - 1}{i}\right] - R,$$

with $R = 325$, $i = 0.033/12 = 0.0025$, and $n = (12)(5) = 60$.

$$S = 325\left[\frac{(1 + 0.00275)^{61} - 1}{0.00275}\right] - 325 = 21,227.66$$

The future value of this annuity due is \$21,227.66.

5.2 Warmup Exercises

W1. Use the formula $A = P(1+i)^n$, where

$$A = 1500$$

$$i = \frac{0.032}{4} = 0.008$$

$$n = 6(4) = 24$$

The compound amount is

$$1500(1 + 0.008)^{24} = 1816.12,$$

or \$1816.12. The interest earned is

$$1816.12 - 1500 = 316.12,$$

or \$316.12.

W2. Use the formula $A = P(1+i)^n$, where

$$A = 800$$

$$i = \frac{0.048}{12} = 0.004$$

$$n = 3(12) = 36$$

The compound amount is

$$800(1 + 0.004)^{36} = 923.64,$$

or \$923.64. The interest earned is

$$923.64 - 800 = 123.64,$$

or \$123.64.

5.2 Exercises

1. $a = 3$; $r = 2$

The first five terms are

$$3, 3(2), 3(2)^2, 3(2)^3, 3(2)^4$$

or

$$3, 6, 12, 24, 48.$$

The fifth term is 48.

Or, use the formula $a_n = ar^{n-1}$ with $n = 5$.

$$a_5 = ar^{5-1} = 3(2)^4 = 3(16) = 48$$

2. $a = 7;\ r = 5$

$$a_n = ar^{n-1}$$
$$a_5 = 7(5)^4 = 4375$$

3. $a = -8;\ r = 3;\ n = 5$

$$a_5 = ar^{5-1} = -8(3)^4 = -8(81) = -648$$

The fifth term is -648 .

4. $a = -6;\ r = 2$

$$a_5 = ar^4 = -6(2)^4 = -96$$

5. $a = 1;\ r = -3;\ n = 5$

$$a_5 = ar^{5-1} = 1(-3)^4 = 81$$

The fifth term is 81.

6. $a = 12;\ r = -2$

$$a_5 = ar^4 = 12(-2)^4 = 192.$$

7. $a = 256;\ r = \frac{1}{4};\ n = 5$

$$a_5 = ar^{5-1} = 256\left(\frac{1}{4}\right)^4 = 256\left(\frac{1}{256}\right) = 1$$

The fifth term is 1.

8. $a = 729;\ r = \frac{1}{3}$

$$a_5 = ar^4 = 729\left(\frac{1}{3}\right)^4 = 9$$

9. $a = 1;\ r = 2;\ n = 4$

To find the sum of the first 4 terms, S_4, use the formula for the sum of the first n terms of a geometric sequence.

$$S_n = \frac{a(r^n - 1)}{r - 1}$$
$$S_4 = \frac{1(2^4 - 1)}{2 - 1} = \frac{16 - 1}{1} = 15$$

10. $a = 4;\ r = 4;\ n = 4$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$
$$S_4 = \frac{4(4^4 - 1)}{4 - 1} = 340$$

11. $a = 5;\ r = \frac{1}{5};\ n = 4$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$
$$S_4 = \frac{5\left[\left(\frac{1}{5}\right)^4 - 1\right]}{\frac{1}{5} - 1} = \frac{5\left(-\frac{624}{625}\right)}{-\frac{4}{5}}$$
$$= \frac{-\frac{624}{125}}{-\frac{4}{5}} = \left(-\frac{624}{125}\right)\left(-\frac{5}{4}\right) = \frac{156}{25}$$

12. $a = 6;\ r = \frac{1}{2};\ n = 4$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$
$$S_4 = \frac{6\left[\left(\frac{1}{2}\right)^4 - 1\right]}{\frac{1}{2} - 1} = \frac{6\left(-\frac{15}{16}\right)}{-\frac{1}{2}} = \frac{45}{4}$$

13. $a = 128;\ r = -\frac{3}{2};\ n = 4$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$
$$S_4 = \frac{128\left[\left(-\frac{3}{2}\right)^4 - 1\right]}{-\frac{3}{2} - 1} = \frac{128\left(\frac{65}{16}\right)}{-\frac{5}{2}}$$
$$= -208$$

14. $a = 64;\ r = -\frac{3}{4};\ n = 4$

$$S_4 = \frac{64\left[\left(-\frac{3}{4}\right)^4 - 1\right]}{-\frac{3}{4} - 1} = \frac{64\left(-\frac{175}{256}\right)}{-\frac{7}{4}} = 25$$

17. $R = 100;\ i = 0.06;\ n = 4$

Use the formula for the future value of an ordinary annuity.

$$S = R\left[\frac{(1 + i)^n - 1}{i}\right]$$
$$= 100\left[\frac{(1.06)^4 - 1}{0.06}\right]$$
$$= 100\left(\frac{1.262477 - 1}{0.06}\right)$$
$$\approx 437.46$$

The future value is \$437.46.

18. $R = 1000;\ i = 0.03;\ n = 5$

Use the formula for the future value of an ordinary annuity.

$$S = Rs_{\overline{n}|i} = 1000s_{\overline{5}|0.06}$$
$$= 1000\left[\frac{(1.03)^5 - 1}{0.03}\right]$$
$$\approx 5309.14$$

The future value is $5309.14.

19. $R = 25{,}000;\ i = 0.045;\ n = 36$

$$S = R\left[\frac{(1+i)^n - 1}{i}\right]$$
$$= 25{,}000\left[\frac{(1+0.045)^{36} - 1}{0.045}\right]$$
$$\approx 2{,}154{,}099.15$$

The future value is $2,154,099.15.

20. $R = 29{,}500;\ i = 0.058;\ n = 15$

$$S = R\left[\frac{(1+i)^n - 1}{i}\right]$$
$$= 29{,}500\left[\frac{(1+0.058)^{15} - 1}{0.058}\right]$$
$$\approx 676{,}272.05$$

The future value is $676,272.05.

21. $R = 9200$; 10% interest compounded semiannually for 7 yr

Interest of $\frac{10\%}{2} = 5\%$ is earned semiannually, so $i = 0.05$. In 7 yr, there are $7(2) = 14$ semiannual periods, so $n = 14$.

$$S = R\left[\frac{(1+i)^n - 1}{i}\right]$$
$$= 9200\left[\frac{(1.05)^{14} - 1}{0.05}\right]$$
$$\approx 180{,}307.41$$

The future value is $180,307.41.

$9200 is contributed in each of 14 periods. The total contribution is

$$\$9200(14) = \$128{,}800.$$

The amount from interest is

$$\$180{,}307.41 - 128{,}800 = \$51{,}507.41$$

22. $R = 1250;\ i = \frac{0.05}{2} = 0.025;\ n = 18(2) = 36$

$$S = R\left[\frac{(1+i)^n - 1}{i}\right]$$
$$= 1250\left[\frac{(1+0.025)^{36} - 1}{0.025}\right]$$
$$\approx 71{,}626.77$$

The future value is $71,626.77.

$1250 is contributed in each of 36 periods. The total contribution is

$$\$1250(36) = \$45{,}000.$$

The amount from interest is

$$\$71{,}626.77 - 45{,}000 = \$26{,}626.77$$

23. $R = 800$; 6.51% interest compounded semiannually for 12 yr

Interest of $\frac{6.51\%}{2}$ is earned semiannually, so $i = \frac{0.0651}{2} = 0.03255$. In 12 yr, there are $12(2) = 24$ semiannual periods, so $n = 24$.

$$S = R\left[\frac{(1+i)^n - 1}{i}\right]$$
$$= 800\left[\frac{(1+0.03255)^{24} - 1}{0.03255}\right]$$
$$\approx 28{,}438.21$$

The future value is $28,438.21.

$800 is contributed in each of 24 periods. The total contribution is

$$\$800(24) = \$19{,}200.$$

The amount from interest is

$$\$28{,}438.21 - 19{,}200 = \$9238.21.$$

24. $R = 4600;\ i = \frac{0.0873}{4} = 0.021825;\ n = 9(4) = 36$

$$S = R\left[\frac{(1+i)^n - 1}{i}\right]$$
$$= 4600\left[\frac{(1+0.021825)^{36} - 1}{0.021825}\right]$$
$$\approx 247{,}752.70$$

The future value is $\$247,752.70.$

$\$4600$ is contributed in each of 36 periods. The total contribution is

$$\$4600(36) = \$165,600.$$

The amount from interest is

$$\$247,752.70 - 165,600 = \$82,152.70.$$

25. $R = 12,000;\ i = \frac{0.048}{2} = 0.012;\ n = 16(4) = 64$

$$\begin{aligned} S &= R\left[\frac{(1+i)^n - 1}{i}\right] \\ &= 12,000\left[\frac{(1+0.012)^{64} - 1}{0.012}\right] \\ &\approx 1,145,619.96 \end{aligned}$$

The future value is $\$1,145,619.96.$

$\$12,000$ is contributed in each of 64 periods. The total contribution is

$$\$12,000(64) = \$768,000.$$

The amount from interest is

$$\$1,145,619.96 - 768,000 = \$377,619.96.$$

26. $R = 42,000;\ i = \frac{0.1005}{2} = 0.05025;$ $n = 12(2) = 24$

$$\begin{aligned} S &= R\left[\frac{(1+i)^n - 1}{i}\right] \\ &= 42,000\left[\frac{(1+0.05025)^{24} - 1}{0.05025}\right] \\ &\approx 1,875,230.74 \end{aligned}$$

The future value is $\$1,875,230.74.$

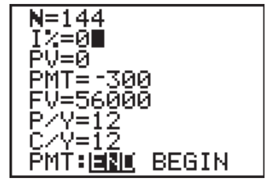
$\$42,000$ is contributed in each of 24 periods. The total contribution is

$$\$42,000(24) = \$1,008,000.$$

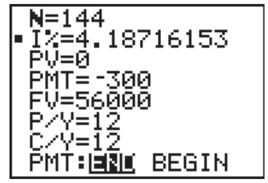
The amount from interest is

$$\$1,875,230.74 - 1,008,000 = \$867,230.74.$$

29. Using the TMV Solver under the FINANCE menu on the TI-84 Plus calculator, set up the following input:

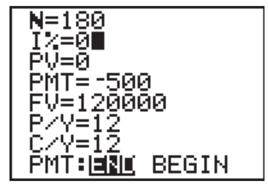


Put the cursor next to I% and press SOLVE to show the solution:

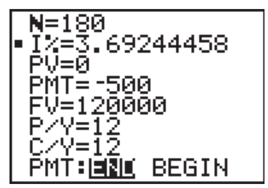


The required interest rate is 4.19%.

30. Using the TMV Solver under the FINANCE menu on the TI-84 Plus calculator, set up the following input:



Put the cursor next to I% and press SOLVE to show the solution:



The required interest rate is 3.69%.

31. $S = \$10,000;$ interest is 5% compounded annually; payments are made at the end of each year for 12 yr.

This is a sinking fund. Use the formula for an ordinary annuity with $S = 10,000,\ i = 0.05,$ and $n = 12$ to find the value of R, the amount of each payment.

$$\begin{aligned} 10,000 &= Rs_{\overline{12}|0.05} \\ R &= \frac{10,000}{s_{\overline{12}|0.05}} \\ &= \frac{10,000}{\frac{(1+0.05)^{12}-1}{0.05}} \\ &\approx 628.25 \end{aligned}$$

The required periodic payment is $\$628.25.$

32. $S = \$150{,}000$; interest is 6% compounded semi-annually; payments are made at the end of each semiannual period for 11 years

This is a sinking fund. Use the formula for an ordinary annuity with $S = 150{,}000$, $i = \frac{0.06}{2} = 0.03$, and $n = 11(2) = 22$ to find the value of R, the amount for each payment.

$$\begin{aligned} 150{,}000 &= Rs \\ &= \frac{150{,}000}{s_{\overline{22}|0.03}} \\ &= \frac{150{,}000}{\frac{(1+0.03)^{22}-1}{0.03}} \\ &\approx 4912.11 \end{aligned}$$

The required periodic payment is \$4912.11.

33. $S = 8500$; $i = 0.08$; $n = 7$

$$\begin{aligned} R &= \frac{S}{s_{\overline{n}|i}} \\ &= \frac{8500}{s_{\overline{7}|0.08}} \\ &= \frac{8500(0.08)}{(1+0.08)^7 - 1} \\ &\approx 952.62 \end{aligned}$$

The payment is \$952.62.

34. \$2750; money earns 5% compounded annually; 5 annual payments

Let R be the amount of each payment.

$$\begin{aligned} 2750 &= Rs_{\overline{5}|0.05} \\ R &= \frac{2750}{s_{\overline{5}|0.05}} \\ &= \frac{2750}{\frac{(1+0.05)^5-1}{0.05}} \\ &\approx 497.68 \end{aligned}$$

The amount of each payment is \$497.68.

35. $S = 75{,}000$; $i = \frac{0.06}{2} = 0.03$; $n = 4\frac{1}{2}(2) = 9$

$$\begin{aligned} R &= \frac{S}{s_{\overline{n}|i}} \\ &= \frac{75{,}000}{s_{\overline{9}|0.03}} \\ &= \frac{75{,}000(0.03)}{(1+0.03)^9 - 1} \\ &\approx 7382.54 \end{aligned}$$

The payment is \$7382.54.

36. \$25,000; money earns 5.7% compounded quarterly for $3\frac{1}{2}$ yr.

Thus, $i = \frac{0.057}{4} = 0.01425$ and $n = \left(3\frac{1}{2}\right)4 = 14$.

$$\begin{aligned} R &= \frac{25{,}000}{s_{\overline{14}|0.01425}} \\ &= \frac{25{,}000}{\frac{(1+0.01425)^{14}-1}{0.01425}} \\ &\approx 1626.16 \end{aligned}$$

The amount of each payment is \$1626.16.

37. \$65,000; money earns 7.5% compounded quarterly for $2\frac{1}{2}$ years

Thus, $i = \frac{0.075}{4} = 0.01875$ and $n = \left(2\frac{1}{2}\right)4 = 10$.

$$\begin{aligned} R &= \frac{65{,}000}{s_{\overline{10}|0.01875}} \\ &= \frac{65{,}000}{\frac{(1+0.01875)^{10}-1}{0.01875}} \\ &\approx 5970.23 \end{aligned}$$

The amount of each payment is \$5970.23.

38. \$9000; money earns 4.8% compounded monthly for $2\frac{1}{2}$ years

Thus, $i = \frac{0.048}{12} = 0.004$ and $n = \left(2\frac{1}{2}\right)12 = 30$.

$$\begin{aligned} R &= \frac{9000}{s_{\overline{30}|0.004}} \\ &= \frac{9000}{\frac{(1+0.004)^{30}-1}{0.004}} \\ &\approx 282.96 \end{aligned}$$

The amount of each payment is \$282.96.

39. $R = 600;\ i = 0.06;\ n = 8$

To find the future value of an annuity due, use the formula for the future value of an ordinary annuity, but include one additional time period and subtract the amount of one payment.

$$\begin{aligned} S &= R\left[\frac{(1+i)^{n+1}-1}{i}\right] - R \\ &= 600\left[\frac{(1+0.06)^{9}-1}{0.06}\right] - 600 \\ &\approx 6294.79 \end{aligned}$$

The future value is $6294.79.

40. $R = 1700;\ i = 0.04;\ n = 15$

To find the future value of an annuity due, use the formula for the future value of an ordinary annuity, but include one additional time period and subtract the amount of one payment.

$$\begin{aligned} S &= R\left[\frac{(1+i)^{n+1}-1}{i}\right] - R \\ &= 1700\left[\frac{(1+0.04)^{16}-1}{0.04}\right] - 1700 \\ &\approx 35{,}401.70 \end{aligned}$$

The future value is $35,401.70.

41. $R = 16{,}000;\ i = 0.05;\ n = 7$

$$\begin{aligned} S &= R\left[\frac{(1+i)^{n+1}-1}{i}\right] - R \\ &= 16{,}000\left[\frac{(1+0.05)^{8}-1}{0.05}\right] - 16{,}000 \\ &\approx 136{,}785.74 \end{aligned}$$

The future value is $136,785.74.

42. $R = 4000;\ i = 0.06;\ n = 11$

$$\begin{aligned} S &= R\left[\frac{(1+i)^{n+1}-1}{i}\right] - R \\ &= 4000\left[\frac{(1+0.06)^{12}-1}{0.06}\right] - 4000 \\ &\approx 63{,}479.76 \end{aligned}$$

The future value is $63,479.76.

43. $R = 1000;\ i = \frac{0.0815}{2} = 0.04075;\ n = 9(2) = 18$

$$\begin{aligned} S &= R\left[\frac{(1+i)^{n+1}-1}{i}\right] - R \\ &= 1000\left[\frac{(1+0.04075)^{19}-1}{0.04075}\right] - 1000 \\ &\approx 26{,}874.97 \end{aligned}$$

The future value is $26,874.97.

$1000 is contributed in each of 18 periods. The total contribution is

$$\$1000(18) = \$18{,}000.$$

The amount from interest is

$$\$26{,}874.97 - 18{,}000 = \$8874.97.$$

44. $R = 750;\ i = \frac{0.059}{12} = 0.00491\overline{6};\ n = 15(12) = 180$

$$\begin{aligned} S &= R\left[\frac{(1+i)^{n+1}-1}{i}\right] - R \\ &= 750\left[\frac{(1+0.00491\overline{6})^{181}-1}{0.00491\overline{6}}\right] - 750 \\ &\approx 217{,}328.08 \end{aligned}$$

The future value is $217,328.08.

$750 is contributed in each of 180 periods. The total contribution is

$$\$750(180) = \$135{,}000.$$

The amount from interest is

$$\$217{,}328.08 - 135{,}000 = \$82{,}328.08.$$

45. $R = 250;\ i = \frac{0.042}{2} = 0.0105;\ n = 12(4) = 48$

$$\begin{aligned} S &= R\left[\frac{(1+i)^{n+1}-1}{i}\right] - R \\ &= 250\left[\frac{(1+0.0105)^{49}-1}{0.0105}\right] - 250 \\ &\approx 15{,}662.40 \end{aligned}$$

The future value is $15,662.40.

$250 is contributed in each of 48 periods. The total contribution is

$$\$250(48) = \$12{,}000.$$

The amount from interest is

$$15{,}662.40 - 12{,}000 = \$3662.40.$$

46. $R = 1500;\ i = \frac{0.056}{2} = 0.028;\ n = 11(2) = 22$

$$\begin{aligned} S &= R\left[\frac{(1+i)^{n+1}-1}{i}\right] - R \\ &= 1500\left[\frac{(1+0.028)^{23}-1}{0.028}\right] - 1500 \\ &\approx 46{,}034.09 \end{aligned}$$

The future value is \$46,034.09.

\$1500 is contributed in each of 22 periods. The total contribution is

$$\$1500(22) = \$33{,}000.$$

The amount from interest is

$$\$46{,}034.09 - 33{,}000 = \$13{,}034.09.$$

47. **(a)** $R = 12{,}000;\ i = 0.08;\ n = 9$

$$\begin{aligned} S &= R\left[\frac{(1+i)^{n}-1}{i}\right] \\ &= 12{,}000\left[\frac{(1+0.08)^{9}-1}{0.08}\right] \\ &\approx 149{,}850.69 \end{aligned}$$

The final amount is \$149,850.69.

(b) $R = 12{,}000;\ i = 0.06;\ n = 9$

$$\begin{aligned} S &= R\left[\frac{(1+i)^{n}-1}{i}\right] \\ &= 12{,}000\left[\frac{(1+0.06)^{9}-1}{0.06}\right] \\ &\approx 137{,}895.79 \end{aligned}$$

She will have \$137,895.79.

(c) The amount that would be lost is the difference between the two amounts in parts (a) and (b), which is

$$\$149{,}850.69 - 137{,}895.79 = \$11{,}954.90.$$

48. Use the formula for the future value of an ordinary annuity with $R = 100$, $i = \frac{0.0225}{12} = 0.001875$, and $n = 12(2) = 24$.

$$\begin{aligned} S &= R\left[\frac{(1+i)^{n+1}-1}{i}\right] \\ &= 100\left[\frac{(1+.001875)^{24}-1}{0.001875}\right] \\ &\approx 2452.47 \end{aligned}$$

The amount in the account after 2 years is \$2452.47. Boyd deposited \$100 in each of 24 periods. The total amount deposited was

$$\$100(24) = \$2400.$$

The amount of interest earned was

$$\$2452.47 - 2400 = \$52.47.$$

49. This is a future value problem with

$R = 179.40$

$i = \dfrac{0.048}{12} = 0.004,$ and

$n = (12)(40) = 480.$

$$\begin{aligned} S &= R\left[\frac{(1+i)^{n}-1}{i}\right], \\ S &= 179.40\left[\frac{(1+0.004)^{480}-1}{0.004}\right] \\ S &= 259{,}900.62 \end{aligned}$$

The account would be worth \$259,900.62.

50. For the first 15 yr, we have an ordinary annuity with $R = 2500$, $i = \frac{0.06}{4} = 0.015$, and $n = 15(4) = 60$. The amount on deposit after 15 yr is

$$\begin{aligned} S &= R\left[\frac{(1+i)^{n}-1}{i}\right] \\ &= 2500\left[\frac{(1+0.015)^{60}-1}{0.015}\right] \\ &\approx 240{,}536.63 \end{aligned}$$

For the remaining 5 yr, this amount earns compound interest at 6% compounded quarterly. To find the final amount on deposit, use the formula for the compound amount with $P = 240{,}536.63$, $i = \frac{0.06}{4} = 0.015$, and $n = 5(4) = 20$.

$$\begin{aligned} A &= P(1+i)^{n} \\ &= 240{,}536.63(1.015)^{20} \\ &\approx 323{,}967.96 \end{aligned}$$

The man will have about $\$323{,}967.96$ in the account when he retires.

51. From ages 50 to 60, we have an ordinary annuity with $R = 3000$, $i = \frac{0.05}{4} = 0.0125$, and $n = 10(4) = 40$. Use the formula for the future value of an ordinary annuity.

$$S = R\left[\frac{(1+i)^n - 1}{i}\right]$$
$$= 3000\left[\frac{(1.0125)^{40} - 1}{0.0125}\right]$$
$$\approx 154,468.67$$

At age 60, the value of the retirement account is \$154,468.67. This amount now earns 6.9% interest compounded monthly for 5 yr. Use the formula for compound amount with $P = 154{,}468.67$, $i = \frac{0.069}{12} = 0.00575$, and $n = 5(12) = 60$ to find the value of this amount after 5 yr.

$$A = P(1+i)^n$$
$$= 154,468.67(1.00575)^{60}$$
$$\approx 217,892.80$$

The value of the amount she withdraws from the retirement account will be \$217,892.80 when she reaches 65.

The deposits of \$300 at the end of each month into the mutual fund form another ordinary annuity. Use the formula for the future value of an ordinary annuity with $R = 300$, $i = \frac{0.069}{12} = 0.00575$, and $n = 12(5) = 60$.

$$S = R\left[\frac{(1+i)^n - 1}{i}\right]$$
$$= 300\left[\frac{(1.00575)^{60} - 1}{0.00575}\right]$$
$$\approx 21,422.37$$

The value of this annuity after 5 yr is \$21,422.37.

The total amount in the mutual fund account when the woman reaches age 65 will be

$$\$217,892.80 + 21,422.37 = \$239,315.17.$$

52. $R = 1000$, $i = \frac{0.06}{4} = 0.015$, and $n = 25(4) = 100$.

$$S = R\left[\frac{(1+i)^n - 1}{i}\right]$$
$$= 1000\left[\frac{(1+0.015)^{100} - 1}{0.015}\right]$$
$$\approx 228,803.04$$

There will be about \$228,803.04 in the IRA.

The total amount deposited was $\$1000(100) = \$100{,}000$.

Thus, the amount of interest earned was

$$\$228,803.04 - 100,000 = \$128,803.04.$$

53. $R = 1000$, $i = \frac{0.08}{4} = 0.02$, and $n = 25(4) = 100$.

$$S = R\left[\frac{(1+i)^n - 1}{i}\right]$$
$$= 1000\left[\frac{(1+0.02)^{100} - 1}{0.02}\right]$$
$$\approx 312,232.31$$

There will be about \$312,232.31 in the IRA.

The total amount deposited was $\$1000(100) = \$100{,}000$.

Thus, the amount of interest earned was

$$\$312,232.31 - 100,000 = \$212,232.31.$$

54. $R = 1000$, $i = \frac{0.04}{4} = 0.01$, and $n = 100$.

$$S = R\left[\frac{(1+i)^n - 1}{i}\right]$$
$$= 1000\left[\frac{(1+0.01)^{100} - 1}{0.01}\right]$$
$$\approx 170,481.38$$

There will be about \$170,481.38 in the IRA. The total amount deposited was \$100,000. Thus, the amount of interest earned was

$$\$170,481.38 - 100,000 = \$70,481.38.$$

55. $R = 1000$, $i = \frac{0.10}{4} = 0.025$, and $n = 100$.

$$S = R\left[\frac{(1+i)^n - 1}{i}\right]$$
$$= 1000\left[\frac{(1+0.025)^{100} - 1}{0.025}\right]$$
$$\approx 432,548.65$$

There will be about \$432,548.65 in the IRA. The total amount deposited was \$100,000. Thus, the amount of interest earned was

$$\$432,548.65 - 100,000 = \$332,548.65.$$

56. **(a)** This is a sinking fund with $S = 10{,}000$, $i = \frac{0.08}{4} = 0.02$, and $n = 8(4) = 32$. Let R represent the amount of each payment.

$$\begin{aligned} S &= Rs_{\overline{n}|i} \\ 10{,}000 &= Rs_{\overline{32}|0.02} \\ R &= \frac{10{,}000}{s_{\overline{32}|0.02}} \\ &= \frac{10{,}000(0.02)}{(1+0.02)^{32}-1} \\ &\approx 226.11 \end{aligned}$$

If the money is deposited at 8% compounded quarterly, Hector's quarterly deposit will need to be about $226.11.

(b) Here $S = 10{,}000$, $i = \frac{0.06}{4} = 0.015$, and $n = 8(4) = 32$. Let R represent the amount of each payment.

$$\begin{aligned} S &= Rs_{\overline{n}|i} \\ 10{,}000 &= Rs_{\overline{32}|0.015} \\ R &= \frac{10{,}000}{s_{\overline{32}|0.015}} \\ &= \frac{10{,}000(0.015)}{(1+0.015)^{32}-1} \\ &\approx 245.77 \end{aligned}$$

If the money is deposited at 6% compounded quarterly, Hector's quarterly deposit will need to be about $245.77, or $245.78 if we round the cents up.

57. This is a sinking fund with $S = 12{,}000$, $i = \frac{0.06}{2} = 0.03$, and $n = 4(2) = 8$.

$$\begin{aligned} R &= \frac{S}{s_{\overline{n}|i}} \\ &= \frac{12{,}000}{s_{\overline{8}|0.03}} \\ &= \frac{12{,}000(0.03)}{(1+0.03)^{8}-1} \\ &\approx 1349.48 \end{aligned}$$

Each payment should be $1349.48.

58. $S = 20{,}000$, $i = \frac{0.032}{4} = 0.008$, $n = 6(4) = 24$

Let R represent the amount of each payment.

$$\begin{aligned} S &= Rs_{\overline{n}|i} \\ 20{,}000 &= Rs_{\overline{24}|0.008} \\ R &= \frac{20{,}000}{s_{\overline{24}|0.008}} \\ &= \frac{20{,}000(0.008)}{(1+0.008)^{24}-1} \\ &\approx 759.21 \end{aligned}$$

She must deposit about $759.21 at the end of each quarter, or $759.22 if we round the cents up.

59. $R = 80$; $i = \frac{0.025}{12}$; $n = 3(12) + 9 = 45$

Because the deposits are made at the beginning of each month, this is an annuity due.

$$\begin{aligned} S &= R\left[\frac{(1+i)^{n+1}-1}{i}\right] - R \\ &= 80\left[\frac{\left(1+\frac{0.025}{12}\right)^{46}-1}{\frac{0.025}{12}}\right] - 80 \\ &\approx 3777.89 \end{aligned}$$

The account will have $3777.89 in it.

60. This may be considered an annuity due, since payments are made at the beginning of each year, starting with the day the daughter is born. However, a payment should not be subtracted at the end, since a twenty-second payment is made on her twenty-first birthday. Thus, the future value is given by

$$S = R\left[\frac{(1+i)^{n+1}-1}{i}\right],$$

where $R = 1000$; $i = 0.0525$; and $n = 21$. Therefore,

$$\begin{aligned} S &= 1000\left[\frac{(1.0525)^{22}-1}{0.0525}\right] \\ &\approx 39{,}664.40 \end{aligned}$$

There will be $39,664.40 in the account at the end of the day on the daughter's twenty-first birthday.

$1000 is contributed in each of the 22 periods. The total contribution is $22,000. Thus

$$\$39{,}664.40 - 22{,}000 = \$17{,}664.40.$$

61. For the first 8 yr, we have an annuity due with $R = 2435$, $i = \frac{0.06}{2} = 0.03$, and $n = 8(2) = 16$.

The amount on deposit after 8 yr is

$$S = R\left[\frac{(1+i)^{n+1}-1}{i}\right] - R$$
$$= 2435\left[\frac{(1+0.03)^{17}-1}{0.03}\right] - 2435$$
$$\approx 50{,}554.47.$$

For the remaining 5 yr, this amount, \$50,554.47, earns compound interest at 6% compounded semiannually. To find the final amount on deposit, use the formula for the compound amount with $P = 50{,}554.47$, $i = \frac{0.06}{2} = 0.03$, and $n = 5(2) = 10$.

$$A = P(1+i)^n$$
$$= 50{,}554.47(1.03)^{10}$$
$$\approx 67{,}940.98$$

The final amount on deposit will be about \$67,940.98.

62. For the first 12 yr, we have an annuity due. To find the amount in this account after 12 yr, use the formula for the future value of an annuity due with $R = 10{,}000$, $i = 0.05$, and $n = 12$.

$$S = R\left[\frac{(1+i)^{n+1}-1}{i}\right] - R$$
$$= 10{,}000\left[\frac{(1.05)^{13}-1}{0.05}\right] - 10{,}000$$
$$\approx 167{,}129.83$$

This amount, \$167,129.83, now earns 6% interest compounded semiannually for another 9 yr, but no new deposits are made. Use the formula for compound amount with $P = 167{,}129.83$, $i = \frac{0.06}{2} = 0.03$, and $n = 9(2) = 18$.

$$A = P(1+i)^n$$
$$= 167{,}129.83(1.03)^{18}$$
$$\approx 284{,}527.35$$

The final amount on deposit after 21 yr is \$284,527.35.

63. Let $x =$ the annual interest rate.

$$n = 20(12) = 240$$

Graph $y_1 = 147{,}126$ and

$$y_2 = 300\left[\frac{\left(1+\frac{x}{12}\right)^{240}-1}{\frac{x}{12}}\right].$$

The x-coordinate of the point of intersection is 0.06499984. Thus, the annual interest rate was about 6.5%.

64. Let $x =$ the annual interest rate.

$n = 30(12) = 360$

Graph $y_1 = 330{,}000$ and

$$y_2 = 250\left[\frac{(1+\frac{x}{12})^{360}-1}{\frac{x}{12}}\right].$$

The x-coordinate of the point of intersection is 0.0739706. Thus, she would need to earn an annual interest rate of about 7.397%.

65. **(a)** Compare the future amounts for an ordinary annuity with $R = 1{,}350{,}000$ and $i = 0.08$ to compound amounts with $P = 7{,}000{,}000$ and $i = .08$ for different values of n, starting with $n = 1$.

n	$S = R\left[\frac{(1+i)^n-1}{i}\right]$	$A = P(1+i)^n$
1	$1{,}350{,}000\left[\frac{1.08-1}{0.08}\right] = \$1{,}350{,}000.00$	\$7,560,000.00
2	$1{,}350{,}000\left[\frac{(1.08)^2-1}{0.08}\right] = \$2{,}808{,}000.00$	\$8,164,800.00
3	$1{,}350{,}000\left[\frac{(1.08)^3-1}{0.08}\right] = \$4{,}382{,}640.00$	\$8,817,984.00
4	$1{,}350{,}000\left[\frac{(1.08)^4-1}{0.08}\right] = \$6{,}083{,}251.20$	\$9,523,422.72
5	$1{,}350{,}000\left[\frac{(1.08)^5-1}{0.08}\right] = \$7{,}919{,}911.30$	\$10,285,296.54
6	$1{,}350{,}000\left[\frac{(1.08)^6-1}{.08}\right] = \$9{,}903{,}504.20$	\$11,108,120.26
7	$1{,}350{,}000\left[\frac{(1.08)^7-1}{0.08}\right] = \$12{,}045{,}784.54$	\$11,996,769.88

After 7 yr, the investors would do better by winning the lottery.

(b) Repeat the calculations from part (a), but change the interest rate to $i = 0.12$.

n	$S = R\left[\frac{(1+i)^n - 1}{i}\right]$	$A = P(1+i)^n$
1	$ 1,350,000.00	$ 7,840,000.00
2	$ 2,862,000.00	$ 8,780,800.00
3	$ 4,555,440.00	$ 9,834,496.00
4	$ 6,452,092.80	$11,014,635.52
5	$ 8,576,343.94	$12,336,391.78
6	$10,955,505.21	$13,816,758.80
7	$13,620,165.83	$15,474,769.85
8	$16,604,585.73	$17,331,742.23
9	$19,947,136.02	$19,411,551.30

After 9 yr, the investors would do better by winning the lottery.

66. This exercise should be solved by graphing calculator or computer methods. The answers, which may vary slightly, are as follows.

(a) The buyer's quarterly interest payment will be

$$\begin{aligned} I &= Prt \\ &= \$60{,}000(0.08)\frac{1}{4} \\ &= \$1200. \end{aligned}$$

(b) The buyer's semiannual payments into the sinking fund will be $3511.58 for each of the first 13 payments and $3511.59 for the last payment. A table showing the amount in the sinking fund after each deposit is as follows.

Payment Number	Amount of Deposit	Interest Earned	Total
1	$3511.58	$ 0	$ 3511.58
2	$3511.58	$ 105.35	$ 7128.51
3	$3511.58	$ 213.86	$10,853.94
4	$3511.58	$ 325.62	$14,691.14
5	$3511.58	$ 440.73	$18,643.46
6	$3511.58	$ 559.30	$22,714.34
7	$3511.58	$ 681.43	$26,907.35
8	$3511.58	$ 807.22	$31,226.15
9	$3511.58	$ 936.78	$35,674.51
10	$3511.58	$1070.24	$40,256.33
11	$3511.58	$1207.69	$44,975.60
12	$3511.58	$1349.27	$49,836.45
13	$3511.58	$1495.09	$54,843.12
14	$3511.59	$1645.29	$60,000.00

67. This exercise should be solved by graphing calculator or computer methods. The answers, which may vary slightly, are as follows.

(a) The amount of each interest payment is $120.

(b) The amount of each payment is $681.83, except the last payment, which is $681.80. A table showing the amount in the sinking fund after each deposit is as follows.

Payment Number	Amount of Deposit	Interest Earned	Total
1	$681.83	$ 0	$ 681.83
2	$681.83	$ 54.55	$1418.21
3	$681.83	$113.46	$2213.49
4	$681.83	$177.08	$3072.40
5	$681.81	$245.79	$4000.00

68. Using the compound amount formula the future value of the down payment, D, is given by

$$A = D(1+i)^n$$

The rest of the payments form an ordinary annuity with future value given by the formula

$$S = R\left[\frac{(1+i)^n - 1}{i}\right]$$

The future value of the loan, including the down payment, is the sum of the future value of the down payment and the future value of the annuity, or

$$S = D(1+i)^n + R\left[\frac{(1+i)^n - 1}{i}\right]$$

5.3 Present Value of an Annuity; Amortization

Your Turn 1

Use the formula for the present value of an annuity,

$$P = R\left[\frac{1-(1+i)^{-n}}{i}\right],$$

with $R = 500$, $i = 0.048/12 = 0.004$, and $n = (12)(5) = 60$.

$$P = 120\left[\frac{1-(1+0.004)^{-60}}{0.004}\right]$$

$$P = 6389.86$$

The present value is $6389.86.

Your Turn 2

Compute the monthly payment using the formula

$$R = \frac{Pi}{1-(1+i)^{-n}}$$

with $P = 17,000,\ i = 0.054/12 = 0.0045,$ and $n = 48.$

$$R = \frac{(17,000)(0.0045)}{1-(1+0.0045)^{-48}}$$
$$R = 394.59$$

The monthly car payment will be $394.59.

Your Turn 3

The monthly payment will be given by

$$R = \frac{Pi}{1-(1+i)^{-n}}$$

with $R = 220,000, i = 0.07/12,$ and $n = (12)(15) = 180.$

$$R = \frac{(220,000)\left(\frac{0.07}{12}\right)}{1-\left(1+\frac{0.07}{12}\right)^{-180}} = 1977.42$$

The monthly payment will be $1977.42. The total of the 180 payments is

$$(1977.42)(180) = 355,935.60$$

To find the interest paid, subtract the principal from this payment total:

$$355,935.60 - 220,000 = 135,935.60$$

The total interest paid is $135,935.60.

Your Turn 4

The only change in the calculation from Example 4 is that number of months remaining is 8 instead of 9, which gives a present value for the remaining balance of

$$88.8488\left[\frac{1-(1.01)^{-8}}{0.01}\right] = 679.84,$$

or $679.84.

5.3 Warmup Exercises

W1. Use this formula for the future value of an annuity with

$$R = 150$$
$$i = \frac{0.03}{12} = 0.0025$$
$$n = (5)(12) = 60$$
$$S = R\left(\frac{(1+i)^n - 1}{i}\right)$$
$$S = 150\left(\frac{(1+0.0025)^{60} - 1}{0.0025}\right) \approx 9697.01$$

The future value of the account is $9697.01 and the amount of the interest earned is

$$9697.01 - (150)(60) = 697.01$$

or $679.01.

W2. Use the formula for the required sinking fund payment R with

$$S = 10,000$$
$$i = \frac{0.048}{12} = 0.004$$
$$n = (5)(12) = 60$$
$$R = \frac{Si}{(1+i)^n - 1}$$
$$R = \frac{(10,000)(0.004)}{(1+0.004)^{60}} \approx 147.80$$

The deposit at the end of each month will be $147.80 and the amount of interest earned is

$$10,000 - (147.80)(60) = 1132,$$

or $1132.

5.3 Exercises

3. Payments of $890 each year for 16 years at 6% compounded annually

Use the formula for present value of an annuity with $R = 890,\ i = 0.06,$ and $n = 16.$

$$P = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$
$$= 890\left[\frac{1-(1+0.06)^{-16}}{0.06}\right]$$
$$\approx 8994.25$$

The present value is $\$8994.25$

4. Payments of \$1400 each year for 8 years at 7% compounded annually

Use the formula for present value of an annuity with $R = 1400,\ i = 0.07,$ and $n = 8.$

$$P = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$
$$= 1400\left[\frac{1-(1+0.07)^{-8}}{0.06}\right]$$
$$\approx 8359.82$$

The present value is $\$8359.82.$

5. Payments of \$10,000 semiannually for 15 years at 5% compounded semiannually

Use the formula for present value of an annuity with $R = 10{,}000,\ i = \frac{0.05}{2} = 0.025,$ and $n = 15(2) = 30.$

$$P = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$
$$= 10{,}000\left[\frac{1-(1+0.025)^{-30}}{0.025}\right]$$
$$\approx 209{,}302.93$$

The present value is \$209,302.93.

6. Payments of \$50,000 quarterly for 10 years at 4% compounded quarterly

Use the formula for present value of an annuity with $R = 50{,}000,\ i = \frac{0.04}{4} = 0.01,$ and $n = 10(4) = 40.$

$$P = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$
$$= 50{,}000\left[\frac{1-(1+0.01)^{-40}}{0.01}\right]$$
$$\approx 1{,}641{,}734.31$$

The present value is \$1,641,734.31.

7. Payments of \$15,806 quarterly for 3 years at 6.8% compounded quarterly

Use the formula for present value of an annuity with $R = 15{,}806,\ i = \frac{0.068}{4} = 0.017,$ and $n = 3(4) = 12.$

$$P = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$
$$= 15{,}806\left[\frac{1-(1+0.017)^{-12}}{0.017}\right]$$
$$\approx 170{,}275.47$$

The present value is $\$170{,}275.47.$

8. Payments of \$18,579 every 6 months (semiannually) for 8 ye ars at 5.4% compounded semiannually Use the formula for present value of an annuity with $R = 18{,}579,$ $i = \frac{0.054}{2} = 0.027.$

$$P = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$
$$= 18{,}579\left[\frac{1-(1+0.027)^{-16}}{0.027}\right]$$
$$\approx 238{,}816.23$$

The present value is \$238,816.23.

9. 4% compounded annually

We want the present value, P, of an annuity with $R = 10{,}000$, $i = 0.04$, and $n = 15$.

$$P = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$
$$= 10{,}000\left[\frac{1-(1.04)^{-15}}{0.04}\right]$$
$$\approx 111{,}183.87$$

The required lump sum is \$111,183.87.

10. We want the present value, P, of an annuity with $R = 10{,}000$, $i = 0.06$, and $n = 15$.

$$P = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$
$$= 10{,}000\left[\frac{1-(1.06)^{-15}}{0.06}\right]$$
$$\approx 97{,}122.49$$

The required lump sum is \$97,122.49.

11. $P = 2500,\ i = \frac{0.06}{4} = 0.015;\ n = 6$

(a) To find the payment amount, use the formula for amortization payments.

$$R = \frac{Pi}{1-(1+i)^{-n}}$$

$$R = \frac{2500(0.015)}{1-(1+0.015)^{-6}}$$

$$\approx 438.81$$

Each payment is \$438.81.

(b) To find the total payments, multiply the amount of one payment by $n = 6$.

$$438.81(6) = 2632.86$$

The total payments come out to \$2632.86.

To find the total amount of interest paid, subtract the original loan amount from the total payments.

$$2632.86 - 2500 = 132.86$$

The total amount of interest paid is \$132.86.

(c) Set $P = 2500$, $i = 0.015$, $n = 6$ and $R = 438.81$ and generate the following amortization table using software.

Payment Number	Amount of Payment	Interest for Period	Portion to Principal	Principal at End of Period
0	0.00	0.00	0.00	2500.00
1	438.81	37.50	401.31	2098.69
2	438.81	31.48	407.33	1691.36
3	438.81	25.37	413.44	1277.92
4	438.81	19.17	419.64	858.28
5	438.81	12.87	425.94	432.34
6	438.83	6.49	432.34	0.00

The sum of the Amount of Payment column gives the total payments, \$2632.88.

The sum of the Interest column gives the total interest paid, \$132.88.

12. $P = 41{,}000$; $i = \frac{0.08}{2} = 0.04$; $n = 10$

(a) To find the payment amount, use the formula for amortization payments.

$$R = \frac{Pi}{1-(1+i)^{-n}}$$

$$R = \frac{41{,}000(0.04)}{1-(1+0.04)^{-10}}$$

$$\approx 5054.93$$

Each payment is \$5054.93.

(b) To find the total payments, multiply the amount of one payment by $n = 10$.

$$5054.93(10) = 50{,}549.30$$

The total payments come out to \$50,549.30.

To find the total amount of interest paid, subtract the original loan amount from the total payments.

$$50{,}549.30 - 41{,}000 = 9549.30$$

The total amount of interest paid is \$9549.30.

(c) Set $P = 41{,}000$; $i = 0.04$, $n = 10$ and $R = 5054.93$ and generate the following amortization table using software.

Payment Number	Amount of Payment	Interest for Period	Portion to Principal	Principal at End of Period
0	0.00	0.00	0.00	41000.00
1	5054.93	1640.00	3414.93	37585.07
2	5054.93	1503.40	3551.53	34033.54
3	5054.93	1361.34	3693.59	30339.95
4	5054.93	1213.60	3841.33	26498.62
5	5054.93	1059.94	3994.99	22503.64
6	5054.93	900.15	4154.78	18348.85
7	5054.93	733.95	4320.98	14027.88
8	5054.93	561.12	4493.81	9534.06
9	5054.93	381.36	4673.57	4860.49
10	5054.91	194.42	4860.49	0.00

The sum of the Amount of Payment column gives the total payments, \$50,549.28.

The sum of the Interest column gives the total interest paid, \$9549.28.

13. $P = 90{,}000$; $i = 0.06$; $n = 12$

(a) To find the payment amount, use the formula for amortization payments.

$$R = \frac{Pi}{1-(1+i)^{-n}}$$

$$R = \frac{90{,}000(0.06)}{1-(1+0.06)^{-12}}$$

$$\approx 10{,}734.93$$

Each payment is \$10,734.93.

(b) To find the total payments, multiply the amount of one payment by $n = 12$.

$$10734.93(12) = 128{,}819.16$$

The total payments come out to \$128,819.16.

To find the total amount of interest paid, subtract the original loan amount from the total payments.

$$128{,}819.16 - 90{,}000 = 38{,}819.16$$

The total amount of interest paid is \$38,819.16.

(c) Set $P = 90{,}000,\ i = 0.06,\ n = 12$ and $R = 10{,}734.93$ and generate the following amortization table using

Payment Number	Amount of Payment	Interest for Period	Portion to Principal	Principal at End of Period
0	0.00	0.00	0.00	90000.00
1	10734.93	5400.00	5334.93	84665.07
2	10734.93	5079.90	5655.03	79010.04
3	10734.93	4740.60	5994.33	73015.72
4	10734.93	4380.94	6353.99	66661.73
5	10734.93	3999.70	6735.23	59926.50
6	10734.93	3595.59	7139.34	52787.16
7	10734.93	3167.23	7567.70	45219.46
8	10734.93	2713.17	8021.76	37197.70
9	10734.93	2231.86	8503.07	28694.63
10	10734.93	1721.68	9013.25	19681.38
11	10734.93	1180.88	9554.05	10127.33
12	10734.97	607.64	10127.33	0.00

The sum of the Amount of Payment column gives the total payments, $128,819.20.

The sum of the Interest column gives the total interest paid, $38,819.20.

14. $P = 140{,}000;\ i = \frac{0.08}{4} = 0.02;\ n = 15$

(a) To find the payment amount, use the formula for amortization payments.

$$R = \frac{Pi}{1-(1+i)^{-n}}$$
$$R = \frac{140{,}000(0.02)}{1-(1+0.02)^{-15}}$$
$$\approx 10{,}895.57$$

Each payment is $\$10{,}895.57$.

(b) To find the total payments, multiply the amount of one payment by $n = 15$.

$$10{,}895.57(15) = 163{,}433.55$$

The total payments come out to $\$163{,}433.55$.

To find the total amount of interest paid, subtract the original loan amount from the total payments

$$163{,}433.55 - 140{,}000 = 23{,}433.55$$

The total amount of interest paid is $\$23{,}433.55$.

(c) Set $P = 140{,}000,\ i = 0.02,\ n = 15$ and $R = 10{,}897.57$ and generate the following amortization table using software.

Payment Number	Amount of Payment	Interest for Period	Portion to Principal	Principal at End of Period
0	0.00	0.00	0.00	140000.00
1	10895.57	2800.00	8095.57	131904.43
2	10895.57	2638.09	8257.48	123646.95
3	10895.57	2472.94	8422.63	115224.32
4	10895.57	2304.49	8591.08	106633.23
5	10895.57	2132.66	8762.91	97870.33
6	10895.57	1957.41	8938.16	88932.17
7	10895.57	1778.64	9116.93	79815.24
8	10895.57	1596.30	9299.27	70515.97
9	10895.57	1410.32	9485.25	61030.72
10	10895.57	1220.61	9674.96	51355.77
11	10895.57	1027.12	9868.45	41487.31
12	10895.57	829.75	10065.82	31421.49
13	10895.57	628.43	10267.14	21154.35
14	10895.57	423.09	10472.48	10681.87
15	10895.50	213.64	10681.86	0.00

The sum of the Amount of Payment column gives the total payments, $163,433.48.

The sum of the Interest column gives the total interest paid, $23,433.48.

15. $P = 7400;\ i = \frac{0.062}{2} = 0.031;\ n = 18$

(a) To find the payment amount, use the formula for amortization payments.

$$R = \frac{Pi}{1-(1+i)^{-n}}$$
$$R = \frac{7400(0.031)}{1-(1+0.031)^{-18}}$$
$$\approx 542.60$$

Each payment is $\$542.60$.

(b) To find the total payments, multiply the amount of one payment by $n = 18$.

$$542.60(18) = 9766.80$$

The total payments come out to $\$9766.80$.

To find the total amount of interest paid, subtract the original loan amount from the total payments

$$9766.80 - 7400 = 2366.80$$

The total amount of interest paid is $\$2366.80$.

(c) Set $P = 7400,\ i = 0.031,\ n = 18$ and $R = 542.60$ and generate the following amortization table using software.

Payment Number	Amount of Payment	Interest for Period	Portion to Principal	Principal at End of Period
0	0.00	0.00	0.00	7400.00
1	542.60	229.40	313.20	7086.80
2	542.60	219.69	322.91	6763.89
3	542.60	209.68	332.92	6430.97
4	542.60	199.36	343.24	6087.73
5	542.60	188.72	353.88	5733.85
6	542.60	177.75	364.85	5369.00
7	542.60	166.44	376.16	4992.84
8	542.60	154.78	387.82	4605.02
9	542.60	142.76	399.84	4205.17
10	542.60	130.36	412.24	3792.93
11	542.60	117.58	425.02	3367.91
12	542.60	104.41	438.19	2929.72
13	542.60	90.82	451.78	2477.94
14	542.60	76.82	465.78	2012.16
15	542.60	62.38	480.22	1531.93
16	542.60	47.49	495.11	1036.82
17	542.60	32.14	510.46	526.37
18	542.68	16.32	526.36	0.00

The sum of the Amount of Payment column gives the total payments, \$9766.88.

The sum of the Interest column gives the total interest paid, \$2366.88.

16. $P = 5500;\ i = \frac{0.10}{12};\ n = 24$

(a) To find the payment amount, use the formula for amortization payments.

$$R = \frac{Pi}{1-(1+i)^{-n}}$$

$$R = \frac{5500\left(\frac{0.10}{12}\right)}{1-\left(1+\frac{0.10}{12}\right)^{-24}} \approx 253.80$$

Each payment is \$253.80.

(b) To find the total payments, multiply the amount of one payment by $n = 24$.

$$253.80(24) = 6091.20$$

The total payments come out to \$6091.20.

To find the total amount of interest paid, subtract the original loan amount from the total payments

$$6091.20 - 5500 = 591.20$$

The total amount of interest paid is \$591.20.

(c) Set $P = 5500$, $i = 0.010/12$, $n = 24$ and $R = 253.80$ and generate the following amortization table using software.

Payment Number	Amount of Payment	Interest for Period	Portion to Principal	Principal at End of Period
0	0.00	0.00	0.00	5500.00
1	253.80	45.83	207.97	5292.03
2	253.80	44.10	209.70	5082.33
3	253.80	42.35	211.45	4870.88
4	253.80	40.59	213.21	4657.67
5	253.80	38.81	214.99	4442.68
6	253.80	37.02	216.78	4225.90
7	253.80	35.22	218.58	4007.32
8	253.80	33.39	220.41	3786.91
9	253.80	31.56	222.24	3564.67
10	253.80	29.71	224.09	3340.58
11	253.80	27.84	225.96	3114.62
12	253.80	25.96	227.84	2886.78
13	253.80	24.06	229.74	2657.04
14	253.80	22.14	231.66	2425.38
15	253.80	20.21	233.59	2191.79
16	253.80	18.26	235.54	1956.26
17	253.80	16.30	237.50	1718.76
18	253.80	14.32	239.48	1479.28
19	253.80	12.33	241.47	1237.81
20	253.80	10.32	243.48	994.32
21	253.80	8.29	245.51	748.81
22	253.80	6.24	247.56	501.25
23	253.80	4.18	249.62	251.63
24	253.72	2.10	251.62	0.00

The sum of the Amount of Payment column gives the total payments, \$6091.12.

The sum of the Interest column gives the total interest paid, \$591.12.

17. Using the first method in Example 4 and carrying more places in the payment amount we have

$$R = \frac{(90{,}000)(0.06)}{1-(1+0.06)^{-12}}$$

$$R = 10{,}734.9326$$

There are $12 - 3 = 9$ payments left, so the amount to pay off the loan is

$$10{,}734.9326\left[\frac{1-(1.06)^{-9}}{0.06}\right] = 73{,}015.71$$

or \$73,015.71.

18. Using the first method in Example 4 and carrying more places in the payment amount we have

$$R = \frac{(140{,}000)(0.02)}{1-(1+0.02)^{-15}}$$

$$R = 10{,}895.5661$$

There are $15-5=10$ payments left, so the amount to pay off the loan is

$$10{,}895.5661\left[\frac{1-(1.02)^{-10}}{0.02}\right]=97{,}870.35$$

or $97,870.35.

19. Using the first method in Example 4 and carrying more places in the payment amount we have

$$R = \frac{(7400)(0.031)}{1-(1+0.031)^{-18}}$$
$$R = 542.6035$$

There are $18-6=12$ payments left, so the amount to pay off the loan is

$$542.6035\left[\frac{1-(1.031)^{-12}}{0.031}\right]=5368.98$$

or $5368.98.

20. Using the first method in Example 4 and carrying more places in the payment amount we have

$$R = \frac{(5500)(0.10/12)}{1-(1+0.10/12)^{-24}}$$
$$R = 253.7971$$

There are $24-7=17$ payments left, so the amount to pay off the loan is

$$253.7971\left[\frac{1-(1+0.10/12)^{-17}}{0.10/12}\right]=4007.35$$

or $4007.35.

21. Look at the entry for payment number 4 under the heading "Interest for Period." The amount of interest included in the fourth payment is $7.61.

22. The "Portion to Principal" column of the table indicates that $87.10 of the 11th payment of $88.85 is used to reduce the debt.

23. To find the amount of interest paid in the first 4 mo of the loan, add the entries for payment 1, 2, 3, and 4 under the heading "Interest for Period."

$$\$10.00 + 9.21 + 8.42 + 7.61 = \$35.24$$

In the first 4 mo of the loan, $35.24 of interest is paid.

24. The amount of interest paid in the last 4 months of the loan is

$$\$3.47 + 2.61 + 1.75 + .88 = \$8.71.$$

25. First, find the value of the annuity at the end of 8 yr. Use the formula for future value of an ordinary annuity.

$$S = R\left[\frac{(1+i)^n - 1}{i}\right]$$
$$= 1000\left[\frac{(1+0.06)^8 - 1}{0.06}\right]$$
$$\approx 9897.47$$

The future value of the annuity is $9897.47.

Now find the present value of $9897.47 at 5% compounded annually for 8 yr. Use the formula for present value for compound interest.

$$P = \frac{A}{(1+i)^n} = \frac{9897.47}{(1.05)^8} \approx 6699.00$$

The required amount is $6699.

26. $4000 deposited every 6 mo for 10 yr at 6% compounded semiannually will be worth

$$4000 \cdot s_{\overline{20}|0.03} \approx \$107{,}481.50.$$

(Note that $i=\frac{0.06}{2}=0.03$ and $n=10(2)=20.$)

For the lump sum investment of x dollars, use $i=\frac{0.08}{4}=0.02$ and $n=10(4)=40$ in the formula $A=P(1+i)^n$. Our unknown amount x will be worth $x(1.02)^{40}$, so

$$x(1.02)^{40} = 107{,}481.50$$
$$x \approx 48{,}677.34.$$

About $48,677.34 should be invested today.

27. $P = 199{,}000;\ i = \frac{0.0701}{12};\ n = 25(12) = 300$

To find the payment amount, use the formula for amortization payments.

$$R = \frac{Pi}{1-(1+i)^{-n}}$$
$$R = \frac{199{,}000\left(\frac{0.0701}{12}\right)}{1-\left(1+\frac{0.0701}{12}\right)^{-300}}$$
$$\approx 1407.76$$

Each payment is $1407.76.

To find the total payment, multiply the amount of one payment by $n = 300$.

$$1407.76(300) = 422,328$$

The total payments come out to \$422,328.

To find the total amount of interest paid, subtract the original loan amount from the total payments.

$$422,328 - 199,000 = 223,328$$

The total amount of interest paid is \$223,328.

28. $P = 175,000, \; i = \frac{0.0624}{12} = 0.0052,$
$n = 30(12) = 360$

To find the payment amount, use the formula for amortization payments.

$$R = \frac{Pi}{1-(1+i)^{-n}}$$

$$R = \frac{175,000(0.0052)}{1-(1+0.0052)^{-360}}$$

$$\approx 1076.37$$

Each payment is \$1076.37.

To find the total payments, multiply the amount of one payment by $n = 360$.

$$1076.37(360) = 387,493.20$$

The total payments come out to \$387,493.20.

To find the total amount of interest paid, subtract the original loan amount from the total payments.

$$387,493.20 - 175,000 = 212,493.20$$

The total amount of interest paid is \$212,493.20.

29. $P = 253,000, \; i = \frac{0.0645}{12}, \; n = 30(12) = 360$

To find the payment amount, use the formula for amortization payments.

$$R = \frac{Pi}{1-(1+i)^{-n}}$$

$$R = \frac{253,000\left(\frac{0.0645}{12}\right)}{1-\left(1+\frac{0.0645}{12}\right)^{-360}}$$

$$\approx 1590.82$$

Each payment is \$1590.82.

To find the total payments, multiply the amount of one payment by $n = 360$.

$$1590.82(360) = 572,695.20$$

The total payments come out to \$572,695.20.

To find the total amount of interest paid, subtract the original loan amount from the total payments.

$$572,695.20 - 253,000 = 319,695.20$$

The total amount of interest paid is \$319,695.20.

30. $P = 310,000, \; i = \frac{0.0596}{12}, \; n = 25(12) = 300$

To find the payment amount, use the formula for amortization payments.

$$R = \frac{Pi}{1-(1+i)^{-n}}$$

$$R = \frac{310,000\left(\frac{0.0596}{12}\right)}{1-\left(1+\frac{0.0596}{12}\right)^{-300}}$$

$$\approx 1989.76$$

Each payment is \$1989.76.

To find the total payments, multiply the amount of one payment by $n = 300$.

$$1989.76(300) = 596,928$$

The total payments come out to \$596,928.

To find the total amount of interest paid, subtract the original loan amount from the total payments.

$$596,928 - 310,000 = 286,928$$

The total amount of interest paid is \$286,928.

31. **(a)** Solve as in Example 6:

$$90,000 = 16,000\left(\frac{1-1.06^{-n}}{0.06}\right)$$

$$\left(\frac{90,000}{16,000}\right)(0.06) = 0.3375$$

$$0.3375 = 1 - 1.06^{-n}$$

$$1.06^{-n} = 1 - 0.3375 = 0.6625$$

$$n = -\frac{\log(0.6625)}{\log(1.06)} = 7.066$$

Rounding to the next whole year, the loan will take 8 years to pay off.

(b) Use software to build an amortization table with $P = 90,000$, $i = 0.06$, $n = 8$, and $R = 16,000$.

Payment Number	Amount of Payment	Interest for Period	Portion to Principal	Principal at End of Period
0	0.00	0.00	0.00	90000.00
1	16000.00	5400.00	10600.00	79400.00
2	16000.00	4764.00	11236.00	68164.00
3	16000.00	4089.84	11910.16	56253.84
4	16000.00	3375.23	12624.77	43629.07
5	16000.00	2617.74	13382.26	30246.81
6	16000.00	1814.81	14185.19	16061.62
7	16000.00	963.70	15036.30	1025.32
8	1086.84	61.52	1025.32	0.00

The amortization table shows that the total of payments is $113,068.84.

(c) Subtracting this value from the answer to Exercise 13(c), we find that the savings in interest is $128,819.20 - 113,086.84 = 15,732.36$ or $15,732.36.

32. (a) Solve as in Example 6:

$$140,000 = 18,000\left(\frac{1-1.02^{-n}}{0.02}\right)$$

$$\left(\frac{140,000}{18,000}\right)(0.02) = \frac{7}{45}$$

$$\frac{7}{45} = 1 - 1.02^{-n}$$

$$1.02^{-n} = 1 - \frac{7}{45} = \frac{38}{45}$$

$$n = -\frac{\log\left(\frac{38}{45}\right)}{\log(1.02)} = 8.538$$

Rounding to the next whole year, the loan will take 9 quarters to pay off.

(b) Use software to build an amortization table with $P = 140,000$, $i = 0.02$, $n = 9$, and $R = 18,000$.

Payment Number	Amount of Payment	Interest for Period	Portion to Principal	Principal at End of Period
0	0.00	0.00	0.00	140000.00
1	18000.00	2800.00	15200.00	124800.00
2	18000.00	2496.00	15504.00	109296.00
3	18000.00	2185.92	15814.08	93481.92
4	18000.00	1869.64	16130.36	77351.56
5	18000.00	1547.03	16452.97	60898.59
6	18000.00	1217.97	16782.03	44116.56
7	18000.00	882.33	17117.67	26998.89
8	18000.00	539.98	17460.02	9538.87
9	9729.65	190.78	9538.87	0.00

The amortization table shows that the total of payments is $153,729.65.

(c) Subtracting this value from the answer to Exercise 14(c), we find that the savings in interest is $163,433.48 - 153,729.65 = 9703.83$ or $9703.83.

33. (a) Solve as in Example 6:

$$7400 = 850\left(\frac{1-1.031^{-n}}{0.031}\right)$$

$$\left(\frac{7400}{850}\right)(0.031) = 0.270$$

$$0.270 = 1 - 1.031^{-n}$$

$$1.031^{-n} = 1 - 0.270 = 0.730$$

$$n = -\frac{\log(0.73)}{\log(1.031)} = 10.309$$

Rounding to the next half year, the loan will take 11 semiannual periods to pay off.

(b) Use software to build an amortization table with $P = 7400$, $i = 0.031$, $n = 11$, and $R = 850$.

Payment Number	Amount of Payment	Interest for Period	Portion to Principal	Principal at End of Period
0	0.00	0.00	0.00	7400.00
1	850.00	229.40	620.60	6779.40
2	850.00	210.16	639.84	6139.56
3	850.00	190.33	659.67	5479.89
4	850.00	169.88	680.12	4799.76
5	850.00	148.79	701.21	4098.56
6	850.00	127.06	722.94	3375.61
7	850.00	104.64	745.36	2630.26
8	850.00	81.54	768.46	1861.79
9	850.00	57.72	792.28	1069.51
10	850.00	33.15	816.85	252.66
11	260.50	7.83	252.67	0.00

The amortization table shows that the total of payments is $8760.50.

(c) Subtracting this value from the answer to Exercise 15(c), we find that the savings in interest is $9766.88 - 8760.50 = 1006.38$ or $1006.38.

34. (a) Solve as in Example 6:

$$5500 = 400\left(\frac{1-(1+0.10/12)^{-n}}{0.10/12}\right)$$

$$\left(\frac{5500}{400}\right)\left(\frac{0.10}{12}\right) = 0.115$$

$$0.115 = 1-(1+0.10/12)^{-n}$$

$$(1+0.10/12)^{-n} = 1-0.115 = 0.885$$

$$n = -\frac{\log(0.885)}{\log(1+0.10/12)} = 14.721$$

Rounding to the next month, the loan will take 15 months to pay off.

(b) Use software to build an amortization table with $P = 5500$, $i = 0.10/12$, $n = 15$, and $R = 400$.

Payment Number	Amount of Payment	Interest for Period	Portion to Principal	Principal at End of Period
0	0.00	0.00	0.00	5500.00
1	400.00	45.83	354.17	5145.83
2	400.00	42.88	357.12	4788.72
3	400.00	39.91	360.09	4428.62
4	400.00	36.91	363.09	4065.53
5	400.00	33.88	366.12	3699.41
6	400.00	30.83	369.17	3330.23
7	400.00	27.75	372.25	2957.99
8	400.00	24.65	375.35	2582.64
9	400.00	21.52	378.48	2204.16
10	400.00	18.37	381.63	1822.53
11	400.00	15.19	384.81	1437.71
12	400.00	11.98	388.02	1049.69
13	400.00	8.75	391.25	658.44
14	400.00	5.49	394.51	263.93
15	266.13	2.20	263.93	0.00

The amortization table shows that the total of payments is $5866.13.

(c) Subtracting this value from the answer to Exercise 16(c), we find that the savings in interest is $6091.12 - 5866.13 = 224.99$ or $224.99.

35. From Example 3, $P = 220{,}000$ and $i = \frac{0.06}{12} = 0.005$. For a 15-year loan, use $n = 15(12) = 180$.

$$R = \frac{Pi}{1-(1+i)^{-n}}$$
$$= \frac{220{,}000(0.005)}{1-(1+0.005)^{-180}}$$
$$\approx 1856.49$$

The monthly payments would be $1856.49. The family makes 180 payments of $1856.49 each, for a total of $334,168.20. Since the amount of the loan was $220,000, the total interest paid is

$$334{,}168.20 - 220{,}000 = 114{,}168.20.$$

The total amount of interest paid is $114,168.20.

The payments for the 15-year loan are

$$1856.49 - \$1319.01 = \$537.48$$

more than those for the 30-year loan in Example 3. However, the total interest paid is

$$254{,}843.60 - \$114{,}168.20 = \$140{,}675.40$$

less than for the 30-year loan in Example 3.

36. (a) $R = 30,\ i = 0.0125,\ n = 12(3) = 36$

Use the formula for the present value of an annuity

$$P = 30\left[\frac{1-(1+0.0125)^{-36}}{0.0125}\right] \approx 865.42$$

Since there was a down payment of $600, the cost of the stereo system will be

$$\$865.45 + 600 = \$1465.42$$

(b) Since a payment of $30 was paid each month for 36 months, the total of the payments will be 36($30) = $1080. From part (a), the cost of the Smart HDTV will be $865.42. Therefore, the total amount of interest paid will be

$$\$1080 - 865.42 = \$214.58.$$

37. (a) $P = 14{,}000,\ i = \frac{0.07}{12},\ n = 4(12) = 48$

$$R = \frac{Pi}{1-(1+i)^{-n}}$$
$$= \frac{14{,}000\left(\frac{0.07}{12}\right)}{1-\left(1+\frac{0.07}{12}\right)^{-48}}$$
$$\approx 335.25$$

The amount of each payment is $335.25.

(b) 48 payments of $335.25 are made, and 48($335.25) = $16,092. The total amount of interest David will pay is $16,092 − $14,000 = $2092.

38. (a) $P = 8430,\ i = \frac{0.27}{12},\ n = 3(12) = 36$

$$R = \frac{Pi}{1-(1+i)^{-n}} = \frac{8430\left(\frac{0.27}{12}\right)}{1-\left(1+\frac{0.27}{12}\right)^{-36}} \approx 344.16$$

The amount of each payment is $344.16.

(b) 36 payments of $344.16 are made, and 36($344.16) = $12,389.76. The total amount of interest Tom will have paid is $12,389.76 − $8430 = $3959.76.

39. **(a)** With 0% financing the monthly payments will be

$$\frac{20,000}{60} \approx 333.33$$

or $333.33. The buyer oaud $20,000 or, if we use the rounded payments,

$$(60)(333.33) = 19,999.80$$

or $19,999.80.

(b) Use the formula for the present value of an annuity, with

$$S = 18,000$$
$$i = \frac{0.0269}{12}$$
$$n = 60$$
$$18,000 = R\left(\frac{1-\left(1+\frac{0.0269}{12}\right)^{-60}}{\frac{0.0269}{12}}\right)$$
$$R = \frac{18,000}{\left(\frac{1-\left(1+\frac{0.0269}{12}\right)^{-60}}{\frac{0.0269}{12}}\right)} \approx 320.96$$

The monthly payment is $320.96. The total payments is

$$(60)(320.96) = 19,257.60,$$

or $19,257.60.

(c) The second option looks better since the total payments are lower and the cash back amount is $2000 more.

40. **(a)** Use the formula for the present value of an annuity with

$$S = 15,000$$
$$i = \frac{0.009}{12} = 0.00075$$
$$n = 36$$
$$15,000 = R\left(\frac{1-(1+0.00075)^{-36}}{(0.00075)}\right)$$
$$R = \frac{15,000}{\left(\frac{1-(1+0.00075)^{-36}}{(0.00075)}\right)} \approx 422.47$$

The monthly payment is $422.47. The total of payments is

$$(36)(422.47) = 15,208.92,$$

Or $15,208.92.

(b) Use the formula for the present value of annuity with

$$S = 15,000$$
$$i = \frac{0.019}{12}$$
$$n = 60$$
$$15,000 = R\left(\frac{1-\left(1+\frac{0.019}{12}\right)^{-60}}{\left(\frac{0.019}{12}\right)}\right)$$
$$R = \frac{15,000}{\left(\frac{1-\left(1+\frac{0.019}{12}\right)^{-60}}{\left(\frac{0.019}{12}\right)}\right)} \approx 262.26$$

The monthly payment is $262.26. The total of payments is

$$(60)(262.26) = 15,735.60,$$

or $15,735.60.

(c) The second option has lower monthly payments but a slightly higher total cost.

41. For parts (a) and (b), if $1 million is divided into 20 equal payments, each payment is $50,000.

(a) $i = 0.05, n = 20$

$$P = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$
$$= 50,000\left[\frac{1-(1+0.05)^{-20}}{0.05}\right]$$
$$\approx 623,110.52$$

The present value is $623,110.52.

(b) $i = 0.09, n = 20$

$$P = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$
$$= 50,000\left[\frac{1-(1+0.09)^{-20}}{0.09}\right]$$
$$\approx 456,427.28$$

The present value is $456,427.28.

For parts (c) and (d), if $1 million is divided into 25 equal payments, each payment is $40,000.

(c) $i = 0.05, n = 25$

$$P = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$
$$= 40,000\left[\frac{1-(1+0.05)^{-25}}{0.05}\right]$$
$$\approx 563,757.78$$

The present value is $563,757.78.

(d) $i = 0.09, n = 25$

$$P = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$
$$= 40,000\left[\frac{1-(1+0.09)^{-25}}{0.09}\right]$$
$$\approx 392,903.18$$

The present value is $392,903.18.

42. Compute the monthly payment using the formula

$$R = \frac{Pi}{1-(1+i)^{-n}}$$

with $P = 55,000,\ i = 0.0466\ /\ 12$ and $n = 120$.

$$R = \frac{(55,000)\left(\frac{0.0466}{12}\right)}{1-\left(1+\frac{0.0466}{12}\right)^{-120}}$$
$$R = 574.26$$

The monthly payment will be $574.26 and the total paid will be $(574.26)(120) = 68,911.20$ or $\$68,911.20$ The interest paid will be $\$68,911.20 - \$55,000 = \$13,911.20$.

43. Compute the monthly payment using the formula

$$R = \frac{Pi}{1-(1+i)^{-n}}$$

with $P = 55,000,\ i = 0.0466/12,$ and $n = 300$.

$$R = \frac{(55,000)\left(\frac{0.0466}{12}\right)}{1-\left(1+\frac{0.0466}{12}\right)^{-300}}$$
$$R = 310.72$$

The monthly payment will be $310.72 and the total paid will be $(310.72)(300) = 93,216.00$ or $\$93,216$. The interest paid will be $\$93,216 - \$55,000 = \$38,216$.

44. The amount of each annual payment is

$$R = \frac{4000}{a_{\overline{4}|0.08}} \approx \$1207.68.$$

On the first payment, the firm owes interest of

$$I = Prt = 4000(0.08)(1) = \$320.$$

Therefore, from the first payment, $320 goes to interest and the balance,

$$\$1207.68 - 320 = \$887.68,$$

goes to principal. The principal at the end of one year is

$$\$4000 - 887.68 = \$3112.32.$$

The interest for the second year is

$$I = Prt = 3112.32(0.08)(1) = \$248.99.$$

Of the second payment, $248.99 goes to interest and

$$\$1207.68 - 248.99 = \$958.69$$

goes to principal. Continuing in this way, we obtain the following amortization schedule.

Payment Number	Amount of Payment	Interest for Period	Portion to Principal	Principal at End of Period
0	———	———	———	$4000.00
1	$1207.68	$320.00	$ 887.68	$3112.32
2	$1207.68	$248.99	$ 958.69	$2153.63
3	$1207.68	$172.29	$1035.39	$1118.24
4	$1207.70	$ 89.46	$1118.24	$ 0.00

45. $P = 110{,}000,\ i = \frac{0.08}{2} = 0.04,\ n = 9$

$$R = \frac{110{,}000}{a_{\overline{9}|0.04}} \approx \$14{,}794.23$$

is the amount of each payment.

Of the first payment, the company owes interest of

$$I = Prt = 110{,}000(0.08)\left(\frac{1}{2}\right) = \$4400.$$

Therefore, from the first payment, $4400 goes to interest, and the balance.

$$\$14{,}794.23 - 4400 = \$10{,}394.23,$$

goes to principal. The principal at the end of this period is

$$\$110{,}000 - 10{,}394.23 = \$99{,}605.77.$$

The interest for the second payment is

$$I = Prt = 99{,}605.77(0.08)\left(\frac{1}{2}\right) \approx \$3984.23$$

Of the second payment, $3984.23 goes to interest and

$$\$14{,}794.23 - 3984.23 = \$10{,}810.00$$

goes to principal. Continue in this fashion to complete the amortization schedule for the first four payments.

Payment Number	Amount of Payment	Interest for Period	Portion to Principal	Principal at End of Period
0	———	———	———	$110,000.00
1	$14,794.23	$4400.00	$10,394.23	$ 99,605.77
2	$14,794.23	$3984.23	$10,810.00	$ 88,795.77
3	$14,794.23	$3551.83	$11,242.40	$ 77,553.37
4	$14,794.23	$3102.13	$11,692.10	$ 65,861.27

46. $P = 1048(8) - 1200 = 7184,\ \ i = \frac{0.06}{12} = 0.005,$

$n = 4(12) = 48$

$$R = \frac{7184}{a_{\overline{48}|0.005}} \approx \$168.72$$

is the amount of each payment.

Of the first payment, the company owes interest of

$$I = Prt = 7184(0.005)(1) = \$35.92.$$

Therefore, from the first payment, $35.92 goes to interest and the balance,

$$\$168.72 - 35.92 = \$132.80,$$

goes to principal. The principal at the end of this period is

$$\$7184 - 132.80 = \$7051.20.$$

The interest for the second payment is

$$I = Prt = 7051.20(0.005)(1) \approx \$35.26.$$

Of the second payment, $35.26 goes to interest and

$$\$168.72 - 35.26 = \$133.46$$

goes to principal. Continue in this fashion to complete the amortization schedule for the first four payments.

Payment Number	Amount of Payment	Interest for Period	Portion to Principal	Principal at End of Period
0	———	———	———	$7184.00
1	$168.72	$35.92	$132.80	$7051.20
2	$168.72	$35.26	$133.46	$6917.74
3	$168.72	$34.59	$134.13	$6783.61
4	$168.72	$33.92	$134.80	$6648.81

47. $150,000 is the future value of an annuity over 79 yr compounded quarterly. So, there are 79(4) = 316 payment periods.

(a) The interest per quarter is $\frac{5.25\%}{4} = 1.3125\%$. Thus, $S = 150{,}000$, $n = 316$, $i = 0.013125$, and we must find the quarterly payment R in the formula

$$S = R\left[\frac{(1+i)^n - 1}{i}\right]$$

$$150{,}000 = R\left[\frac{(1.013125)^{316} - 1}{0.013125}\right]$$

$$R \approx 32.4923796$$

She would have to put $32.49 into her savings at the end of every three months.

(b) For a 2% interest rate, the interest per quarter is $\frac{2\%}{4} = 0.5\%$. Thus, $S = 150{,}000$, $n = 316$, $i = 0.005$, and we must find the quarterly payment R in the formula

$$S = R\left[\frac{(1+i)^n - 1}{i}\right]$$

$$150,000 = R\left[\frac{(1.005)^{316} - 1}{0.005}\right]$$

$$R \approx 195.5222794$$

She would have to put \$195.52 into her savings at the end of every three months.

For a 7% interest rate, the interest per quarter is $\frac{7\%}{4} = 1.75\%$. Thus, $S = 150{,}000$, $n = 316$, $i = 0.0175$, and we must find the quarterly payment R in the formula

$$S = R\left[\frac{(1+i)^n - 1}{i}\right]$$

$$150,000 = R\left[\frac{(1.0175)^{316} - 1}{0.0175}\right]$$

$$R \approx 10.9663932$$

She would have to put \$10.97 into her savings at the end of every three months.

48. The total amount of the loan is

$$14,000 + 7200 - 1200 = 20,000.$$

We have \$20,000 at 12% compounded semiannually for 5 yr.

(a) $i = \frac{0.08}{2} = 0.04,\ n = 5(2) = 10$

$$R = \frac{Pi}{1-(1+i)^{-n}} = \frac{20,000(0.04)}{1-(1+0.04)^{-10}} \approx 2465.82$$

The amount of each payment is \$2465.82.

(b) Graph

$$y_1 = 2465.82\left[\frac{1-1.04^{-(10-x)}}{0.04}\right] \text{ and } y_2 = 5000.$$

The x-coordinate of the point of intersection is 7.8432905, or a little less than 8. Therefore, the loan is reduced below \$5000 after 8 payments, at which time there will be 2 payments left.

49. Throughout this exercise, $i = \frac{0.065}{12}$ and $P =$ the total amount financed, which is

$$\$285,000 - 60,000 = \$225,000.$$

(a) $n = 15(12) = 180$

$$R = \frac{Pi}{1-(1+i)^{-n}} = \frac{225,000\left(\frac{0.065}{12}\right)}{1-\left(1+\frac{0.065}{12}\right)^{-180}} \approx 1959.99$$

The monthly payment is \$1959.99.

$$\text{Total payments} = 180(\$1959.99) = \$352,798.20$$
$$\text{Total interest} = \$352,798.20 - 225,000 = \$127,798.20$$

(b) $n = 20(12) = 240$

$$R = \frac{Pi}{1-(1+i)^{-n}} = \frac{225,000\left(\frac{0.065}{12}\right)}{1-\left(1+\frac{0.065}{12}\right)^{-240}} \approx 1677.54$$

The monthly payment is \$1677.54.

$$\text{Total payments} = 240(\$1677.54) = \$402,609.60$$
$$\text{Total interest} = \$402,609.60 - 225,000 = \$177,609.60$$

(c) $n = 25(12) = 300$

$$R = \frac{Pi}{1-(1+i)^{-n}} = \frac{225,000\left(\frac{0.065}{12}\right)}{1-\left(1+\frac{0.065}{12}\right)^{-300}} \approx 1519.22$$

The monthly payment is \$1519.22.

$$\text{Total payments} = 300(\$1519.22) = \$455,766$$
$$\text{Total interest} = \$455,766 - 225,000 = \$230,766$$

(d) Graph

$$y_1 = 1677.54\left[\frac{1-\left(1+\frac{0.065}{12}\right)^{-(240-x)}}{\frac{0.065}{12}}\right] \text{ and}$$

$$y_2 = \frac{285,000 - 60,000}{2}.$$

The x-coordinate of the point of intersection is 156.44167, which rounds up to 157. Half the loan will be paid after 157 payments.

50. Throughout this exercise, $n = 30(12) = 360$ and $P =$ the total amount financed, which is

$$\$225,000 - 50,000 = \$175,000.$$

(a) $i = \frac{0.06}{12} = 0.005$

$$\begin{aligned} R &= \frac{Pi}{1-(1+i)^{-n}} \\ &= \frac{175,000(0.005)}{1-(1+0.005)^{-360}} \\ &\approx 1049.21 \end{aligned}$$

The monthly payment is \$1049.21.

$$\begin{aligned} \text{Total payments} &= 360(\$1049.21) = \$377,715.60 \\ \text{Total interest} &= \$377,715.60 - 175,000 \\ &= \$202,715.60 \end{aligned}$$

(b) $i = \frac{0.065}{12}$

$$\begin{aligned} R &= \frac{Pi}{1-(1+i)^{-n}} \\ &= \frac{175,000\left(\frac{0.065}{12}\right)}{1-\left(1+\frac{0.065}{12}\right)^{-360}} \\ &\approx 1106.12 \end{aligned}$$

The monthly payment is \$1106.12.

$$\begin{aligned} \text{Total payments} &= 360(\$1106.12) = \$398,203.20 \\ \text{Total interest} &= \$398,203.20 - 175,000 \\ &= \$223,203.20 \end{aligned}$$

(c) $i = \frac{0.07}{12}$

$$\begin{aligned} R &= \frac{Pi}{1-(1+i)^{-n}} \\ &= \frac{175,000\left(\frac{0.07}{12}\right)}{1-\left(1+\frac{0.07}{12}\right)^{-360}} \\ &\approx 1164.28 \end{aligned}$$

The monthly payment is \$1164.28.

$$\begin{aligned} \text{Total payments} &= 360(\$1164.28) = \$419,140.80 \\ \text{Total interest} &= \$419,140.80 - 175,000 \\ &= \$244,140.80 \end{aligned}$$

(d) Graph

$$y_1 = 1164.28\left[\frac{1-\left(1+\frac{0.07}{12}\right)^{-(360-x)}}{\frac{0.07}{12}}\right] \text{ and}$$

$$y_2 = \frac{225,000 - 50,000}{2}.$$

The x-coordinate of the point of intersection is 260.80441, which rounds up to 261. Half the loan will be paid off after 261 payments.

51. $P = 150,000$, $i = \frac{0.082}{12}$, and $n = 30(12) = 360$.

$$\begin{aligned} R &= \frac{Pi}{1-(1+i)^{-n}} \\ &= \frac{150,000\left(\frac{0.082}{12}\right)}{1-\left(1+\frac{0.082}{12}\right)^{-360}} \\ &\approx 1121.63 \end{aligned}$$

The monthly payment is \$1121.63.

$$\begin{aligned} \text{Total payments} &= 360(\$1121.63) = \$403,786.80 \\ \text{Total interest} &= \$403,786.80 - 150,000 \\ &= \$253,786.80 \end{aligned}$$

(b) 15 years of payments means $15(12) = 180$ payments.

$$\begin{aligned} y_{15} &= 1121.63\left[\frac{1-\left(1+\frac{0.082}{12}\right)^{-(360-180)}}{\frac{0.082}{12}}\right] \\ &\approx 115,962.66 \end{aligned}$$

The unpaid balance after 15 years is approximately \$115,962.66.

The total of the remaining 180 payments is

$$180(\$1121.63) = \$201,893.40.$$

(c) The unpaid balance from part (b) is the new loan amount. Now $P = 115,962.66$, $i = \frac{0.065}{12}$, and again $n = 30(12) = 360$.

$$\begin{aligned} R &= \frac{Pi}{1-(1+i)^{-n}} \\ &= \frac{115,962.66\left(\frac{0.065}{12}\right)}{1-\left(1+\frac{0.065}{12}\right)^{-360}} \\ &\approx 732.96 \end{aligned}$$

The new monthly payment would be \$732.96.

$$\begin{aligned} \text{Total payments} &= 360(\$732.96) + \$3400 \\ &= \$267,265.60 \end{aligned}$$

(d) Again the unpaid balance from part (b) is the new loan amount. Again $P = 115,962.66$ and $i = \frac{0.065}{12}$, and this time $n = 15(12) = 180$.

$$R = \frac{Pi}{1-(1+i)^{-n}} = \frac{115{,}962.66\left(\frac{0.065}{12}\right)}{1-\left(1+\frac{0.065}{12}\right)^{-180}} \approx 1010.16$$

The new monthly payment would be $1010.16.

$$\text{Total payments} = 180(\$1010.16) + \$4500 = \$186{,}328.80$$

52. This is an amortization problem with $P = 25{,}000$. R represents the amount of each annual withdrawal.

(a) $i = 0.06, n = 8$

$$R = \frac{Pi}{1-(1+i)^{-n}} = \frac{25{,}000(0.06)}{1-(1+0.06)^{-8}} \approx 4025.90$$

She will be able to withdraw about $4025.90/yr for the 8 yr.

(b) $i = 0.06, n = 12$

$$R = \frac{Pi}{1-(1+i)^{-n}} = \frac{25{,}000(0.06)}{1-(1+0.06)^{-12}} \approx 2981.93$$

She will be able to withdraw about $2981.93/yr for the 12 yr.

53. This is just like a sinking fund in reverse.

(a) $P = 150{,}000,\ i = \frac{0.06}{2} = 0.03,\ n = 2(5) = 10$

$$R = \frac{Pi}{1-(1+i)^{-n}} = \frac{150{,}000(.03)}{1-(1+0.03)^{-10}} \approx 17{,}584.58$$

The amount of each withdrawal is $17,584.58.

(b) $P = 150{,}000,\ i = \frac{0.06}{2} = 0.03,\ n = 2(6) = 12$

$$R = \frac{Pi}{1-(1+i)^{-n}} = \frac{150{,}000(0.03)}{1-(1+0.03)^{-12}} \approx 15{,}069.31$$

If the money must last 6 yr, the amount of each withdrawal is $15,069.31.

54. This exercise should be solved by graphing calculator or computer methods. The amortization schedule, which may vary slightly, is as follows.

Payment Numbar	Amount of Payment	Interest for Period	Portion to Principal	Principal at End of Period
0	———	———	———	$37,948.00
1	$5278.74	$2466.62	$2812.12	$35,135.88
2	$5278.74	$2283.83	$2994.91	$32,140.97
3	$5278.74	$2089.16	$3189.58	$28,951.40
4	$5278.74	$1881.84	$3396.90	$25,554.50
5	$5278.74	$1661.04	$3617.70	$21,936.80
6	$5278.74	$1425.89	$3852.85	$18,083.95
7	$5278.74	$1175.46	$4103.28	$13,980.67
8	$5278.74	$ 908.74	$4370.00	$ 9610.67
9	$5278.74	$ 624.69	$4654.05	$ 4956.62
10	$5278.80	$ 322.18	$4956.62	$ 0.00

55. This exercise should be solved by graphing calculator or computer methods. The amortization schedule, which may vary slightly, is as follows.

Payment Number	Amount of Payment	Interest for Period	Portion to Principal	Principal at End of Period
0	———	———	———	$4836.00
1	$585.16	$175.31	$409.85	$4426.15
2	$585.16	$160.45	$424.71	$4001.43
3	$585.16	$145.05	$440.11	$3561.32
4	$585.16	$129.10	$456.06	$3105.26
5	$585.16	$112.57	$472.59	$2632.67
6	$585.16	$ 95.43	$489.73	$2142.94
7	$585.16	$ 77.68	$507.48	$1635.46
8	$585.16	$ 59.29	$525.87	$1109.59
9	$585.16	$ 40.22	$544.94	$ 564.65
10	$585.12	$ 20.47	$564.65	$ 0.00

57. **(a)** Here $R = 1000$ and $i = 0.04$ and we have

$$P = \frac{R}{i} = \frac{100}{0.04} = 25{,}000$$

Therefore, the present value of the perpetuity is $25,000.

(b) Here $R = 600$ and $i = \frac{0.06}{4} = 0.015$ and we have

$$P = \frac{R}{i} = \frac{600}{0.015} = 40,000$$

Therefore, the present value of the perpetuity is $40,000.

Chapter 5 Review Exercises

1. True

2. False: The ratios of successive pairs of terms are not constant: For example, $\frac{4}{2} = 2$ but $\frac{6}{4} = 1.5$.

3. True

4. False: Both payments and interest on the accumulated value are added to a sinking fund at the end of each time period, so the value increases over time.

5. True

6. True

7. True

8. False: The effective rate formula gives an interest rate, not a present value.

9. False: The correct expression is

$$25,000\left[\frac{0.05/12}{1-(1+0.05/12)^{-72}}\right].$$

10. True

11. $I = Prt$

$$= 15,903(0.06)\left(\frac{8}{12}\right)$$
$$= 636.12$$

The simple interest is $636.12.

12. $I = Prt$

$$= 4902(0.054)\left(\frac{11}{12}\right)$$
$$\approx 242.65$$

The simple interest is $242.65.

13. $I = Prt$

$$= 42,368(0.0522)\left(\frac{7}{12}\right)$$
$$\approx 1290.11$$

The simple interest is $1290.11.

14. $I = Prt$

$$= 3478(0.068)\left(\frac{88}{360}\right)$$
$$\approx 57.81$$

The simple interest is $57.81.

15. For a given amount of money at a given interest rate for a given time period greater than 1, compound interest produces more interest than simple interest.

17. $19,456.11 at 8% compounded semiannually for 7 yr

Use the formula for compound amount with $P = 19{,}456.11$, $i = \frac{0.08}{2} = 0.04$, and $n = 7(2) = 14$.

$$A = P(1+i)^n$$
$$= 19,456.11(1.04)^{14}$$
$$\approx 33,691.69$$

The compound amount is $33,691.69.

18. $2800 at 7% compounded annually for 10 yr

Use the formula for compound amount with $P = 2800$, $i = 0.07$, and $n = 10(1) = 10$.

$$A = P(1+i)^n$$
$$= 2800(1.07)^{10}$$
$$\approx 5508.02$$

The compound amount is $5508.02.

19. $57,809.34 at 6% compounded quarterly for 5 yr

Use the formula for compound amount with $P = 57{,}809.34$, $i = \frac{0.06}{4} = 0.015$, and $n = 5(4) = 20$.

$$A = P(1+i)^n$$
$$= 57,809.34(1.015)^{20}$$
$$\approx 77,860.80$$

The compound amount is $77,860.80.

20. $312.45 at 5.6% compounded semiannually for 16 yr

Use the formula for compound amount with $P = 312.45$, $i = \frac{0.056}{2} = 0.028$, and $n = 16(2) = 32$.

$$A = P(1+i)^n$$
$$= 312.45(1.028)^{32}$$
$$\approx 756.07$$

The compound amount is $756.07.

21. $12,699.36 at 5% compounded semiannually for 7 yr

Here $P = 12{,}699.36$, $i = \frac{0.05}{2} = 0.025$, and $n = 7(2) = 14$. First find the compound amount.

$$\begin{aligned} A &= P(1+i)^n \\ &= 12{,}699.36(1.025)^{14} \\ &\approx 17{,}943.86 \end{aligned}$$

The compound amount is \$17,943.86.

To find the amount of interest earned, subtract the initial deposit from the compound amount. The interest earned is

$$\$17{,}943.86 - 12{,}699.36 = \$5244.50.$$

22. \$3954 at 8% compounded annually for 10 yr

Here $P = 3954$, $i = 0.08$, and $n = 10(1) = 10$. First, find the compound amount.

$$\begin{aligned} A &= P(1+i)^n \\ &= 3954(1.08)^{10} \\ &\approx 8536.39 \end{aligned}$$

The compound amount is \$8536.39. To find the amount of interest earned, subtract the initial deposit from the compound amount.

$$\begin{aligned} \text{Amount of interest} &= A - P \\ &= \$8536.39 - 3954 \\ &= \$4582.39 \end{aligned}$$

23. \$34,677.23 at 4.8% compounded monthly for 32 mo

Here $P = 34{,}677.23$, $i = \frac{0.048}{12} = 0.004$, and $n = 32$.

$$\begin{aligned} A &= P(1+i)^n \\ &= 34{,}677.23(1.004)^{32} \\ &\approx 39{,}402.45 \end{aligned}$$

The compound amount is \$39,402.45

The interest earned is

$$\$39{,}402.45 - 34{,}677.23 = \$4725.22.$$

24. \$12,903.45 at 6.4% compounded quarterly for 29 quarters

Here $P = 12{,}903.45$, $i = \frac{0.064}{4} = 0.016$, and $n = 29$.

$$\begin{aligned} A &= P(1+i)^n \\ &= 12{,}903.45(1.016)^{29} \\ &\approx 20{,}446.71 \end{aligned}$$

The compound amount is \$20,446.71.

$$\begin{aligned} \text{Amount of interest} &= A - P \\ &= \$20{,}446.71 - 12{,}903.45 \\ &= \$7543.26 \end{aligned}$$

25. \$42,000 in 7 yr, 6% compounded monthly

Use the formula for present value for compound interest with $A = 42{,}000$, $i = \frac{0.06}{12} = 0.005$, and $n = 7(12) = 84$.

$$P = \frac{A}{(1+i)^n} = \frac{42{,}000}{(1.005)^{84}} \approx 27{,}624.86$$

The present value is \$27,624.86.

26. \$17,650 in 4 yr, 4% compounded quarterly

Use the formula for present value for compound interest with $A = 17{,}650$, $i = \frac{0.04}{4} = 0.01$, and $n = 4(4) = 16$.

$$\begin{aligned} P &= \frac{A}{(1+i)^n} \\ &= \frac{17{,}650}{(1.01)^{16}} \\ &\approx 15{,}052.30 \end{aligned}$$

The present value is \$15,052.30.

27. \$1347.89 in 3.5 yr, 6.77% compounded semiannually

Use the formula for present value for compound interest with $A = 1347.89$, $i = \frac{0.0677}{2} = 0.03385$, and $n = 3.5(2) = 7$.

$$P = \frac{A}{(1+i)^n} = \frac{1347.89}{(1.03385)^7} \approx 1067.71$$

The present value is \$1067.71.

28. \$2388.90 in 44 mo, 5.93% compounded monthly

$A = 2388.90$, $i = \frac{5.93\%}{12} = \frac{0.0593}{12}$, $n = 44$

$$\begin{aligned} P &= \frac{A}{(1+i)^n} \\ &= \frac{2388.90}{\left(1+\frac{0.0593}{12}\right)^{44}} \\ &\approx 1923.09 \end{aligned}$$

The present value is \$1923.09.

29. $a = 2;\ r = 3$

The first five terms are

$$2,\ 2(3),\ 2(3)^2,\ 2(3)^3, \text{ and } 2(3)^4,$$

or

$$2,\ 6,\ 18,\ 54, \text{ and } 162.$$

30. $a = 4,\ r = \frac{1}{2}$

The first four terms are

$$4,\ 4\left(\frac{1}{2}\right),\ 4\left(\frac{1}{2}\right)^2,\ 4\left(\frac{1}{2}\right)^3$$

or

$$4,\ 2,\ 1,\ \frac{1}{2}.$$

31. $a = -3;\ r = 2$

To find the sixth term, use the formula $a_n = ar^{n-1}$ with $a = -3,\ r = 2$, and $n = 6$.

$$a_6 = ar^{6-1} = -3(2)^5 = -3(32) = -96$$

32. $a = -2,\ r = -2$

For the fifth term, $n = 5$, so

$$a_5 = ar^{5-1} = -2(-2)^4 = -2(16) = -32.$$

The fifth term is -32.

33. $a = -3;\ r = 3$

To find the sum of the first 4 terms of this geometric sequence, use the formula $S_n = \frac{a(r^n - 1)}{r-1}$ with $n = 4$.

$$S_4 = \frac{-3(3^4 - 1)}{3 - 1} = \frac{-3(80)}{2} = \frac{-240}{2} = -120$$

34. $a = 8000,\ r = -\frac{1}{2},\ n = 5$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_5 = \frac{8000\left[\left(-\frac{1}{2}\right)^5 - 1\right]}{-\frac{1}{2} - 1}$$

$$= \frac{8000\left(-\frac{33}{32}\right)}{-\frac{3}{2}} = \frac{-8250}{-\frac{3}{2}}$$

$$= (-8250)\left(-\frac{2}{3}\right) = 5500$$

35. $s_{\overline{n}|i} = \frac{(1+i)^n - 1}{i}$

$$s_{\overline{30}|0.02} = \frac{(1.02)^{30} - 1}{0.02} \approx 40.56808$$

36. $s_{\overline{n}|i} = \frac{(1+i)^n - 1}{i}$

$$s_{\overline{20}|0.06} = \frac{(1.06)^{20} - 1}{0.06} \approx 36.78559$$

38. $R = 500,\ i = \frac{0.06}{2} = 0.03,\ n = 10(2) = 20$

This is an ordinary annuity.

$$S = Rs_{\overline{n}|i}$$
$$S = 500s_{\overline{20}|0.03}$$
$$= 500\left[\frac{(1 + 0.03)^{20} - 1}{0.03}\right]$$
$$\approx 13{,}435.19$$

The future value is \$13,435.19.

The total amount deposited is \$500(20) = \$10,000.

Thus, the amount of interest is

$$\$13{,}435.19 - 10{,}000 = \$3435.19.$$

39. $R = 1288,\ i = 0.04,\ n = 14$

This is an ordinary annuity.

$$S = Rs_{\overline{n}|i}$$
$$S = 1288s_{\overline{14}|0.04}$$
$$= 1288\left[\frac{(1 + 0.04)^{14} - 1}{0.04}\right]$$
$$\approx 23{,}559.98$$

The future value is \$23,559.98.

The total amount deposited is \$1288(14) = \$18,032.

Thus, the amount of interest is

$$\$23{,}559.98 - 18{,}032 = \$5527.98.$$

40. $R = 4000,\ i = \frac{0.05}{4} = 0.0125,\ n = 7(4) = 28$

This is an ordinary annuity.

$$S = R\left[\frac{(1+i)^n - 1}{i}\right]$$
$$S = 4000\left[\frac{(1.0125)^{28} - 1}{0.0125}\right]$$
$$\approx 133{,}117.54$$

The future value is \$133,117.54.

The total amount deposited is $\$4000(28) =$ \$112,000.

Thus, the amount of interest is

$$\$133{,}117.54 - 112{,}000 = \$21{,}117.54.$$

41. $R = 233,\ i = \frac{0.048}{12} = 0.004,\ n = 4(12) = 48$

This is an ordinary annuity.

$$S = R\left[\frac{(1+i)^n - 1}{i}\right]$$
$$S = 233\left[\frac{(1.004)^{48} - 1}{0.004}\right]$$
$$\approx 12{,}302.78$$

The future value is \$12,302.78.

The total amount deposited is $\$233(48) = \$11{,}184$.

Thus, the amount of interest is

$$\$12{,}302.78 - 11{,}184 = \$1118.78.$$

42. $R = 672,\ i = \frac{0.044}{4} = 0.011,\ n = 7(4) = 28$

This is an annuity due, so we use the formula for future value of an ordinary annuity, but include one additional time period and subtract the amount of one payment.

$$S = R\left[\frac{(1+i)^{n+1} - 1}{i}\right] - R$$
$$= 672\left[\frac{(1.011)^{29} - 1}{0.011}\right] - 672$$
$$\approx 22{,}136.73$$

The future value is \$22,136.73.

The total amount deposited is $\$672(28) = \$18{,}816$.

Thus, the amount of interest is

$$\$22{,}136.73 - 18{,}816 = \$3320.73.$$

43. $R = 11{,}900,\ i = \frac{0.06}{12} = 0.005,\ n = 13$

This is an annuity due, so we use the formula for future value of an ordinary annuity, but include one additional time period and subtract the amount of one payment.

$$S = R\left[\frac{(1+i)^{n+1} - 1}{i}\right] - R$$
$$= 11{,}900\left[\frac{(1.005)^{14} - 1}{0.005}\right] - 11{,}900$$
$$\approx 160{,}224.29$$

The future value is \$160,224.29.

The total amount deposited is $\$11{,}900(13) =$ \$154,700.

Thus, the amount of interest is

$$\$160{,}224.29 - 154{,}700 = \$5524.29.$$

45. \$6500; money earns 5% compounded annually; 6 annual payments

$$S = 6500,\ i = 0.05,\ n = 6$$

Let R be the amount of each payment.

$$S = Rs_{\overline{n}|i}$$
$$R = \frac{6500}{s_{\overline{6}|0.05}}$$
$$= \frac{6500(0.05)}{(1.05)^6 - 1}$$
$$\approx 955.61$$

The amount of each payment is \$955.61.

46. 57,000; money earns 4% compounded semiannually for $8\frac{1}{2}$ years

$$S = 57{,}000,\ i = \frac{0.04}{2} = 0.02,$$
$$n = \left(8\tfrac{1}{2}\right)(2) = 17$$

Let R be the amount of each payment.

$$S = Rs_{\overline{n}|i}$$
$$R = \frac{57{,}000}{s_{\overline{17}|0.02}}$$
$$= \frac{57{,}000(0.02)}{(1.02)^{17} - 1}$$
$$\approx 2848.28$$

The amount of each payment is \$2848.28.

47. \$233,188; money earns 5.2% compounded quarterly for $7\frac{3}{4}$ years.

$S = 233{,}188,\ i = \frac{0.052}{4} = 0.013, n = \left(7\frac{3}{4}\right)(4) = 31$

Let R be the amount of each payment.

$$\begin{aligned} S &= Rs_{\overline{n}|i} \\ R &= \frac{233{,}188}{s_{\overline{31}|0.013}} \\ &= \frac{233{,}188(0.013)}{(1.013)^{31} - 1} \\ &\approx 6156.14 \end{aligned}$$

The amount of each payment is $6156.14.

48. $1,056,788; money earns 7.2% compounded monthly for $4\frac{1}{2}$ years

$$S = 1{,}056{,}788,\ i = \frac{0.072}{12} = 0.006,$$
$$n = \left(4\frac{1}{2}\right)(12) = 54$$

Let R represent the amount of each payment.

$$\begin{aligned} S &= Rs_{\overline{n}|i} \\ R &= \frac{1{,}056{,}788}{s_{\overline{54}|0.006}} \\ &= \frac{1{,}056{,}788(0.006)}{(1.006)^{54} - 1} \\ &\approx 16{,}628.83 \end{aligned}$$

The amount of each payment is $16,628.83.

49. Deposits of $850 annually for 4 years at 6% compounded annually

Use the formula for the present value of an annuity with $R = 850$, $i = 0.06$, and $n = 4$.

$$\begin{aligned} P &= R\left[\frac{1 - (1 + i)^{-n}}{i}\right] \\ &= 850\left[\frac{1 - (1 + 0.06)^{-4}}{0.06}\right] \\ &\approx 2945.34 \end{aligned}$$

The present value is $2945.34.

50. Deposits of $1500 quarterly for 7 years at 5% compounded annually

Use the formula for the present value of an annuity with $R = 1500,\ i = \frac{0.05}{4} = 0.0125,$ $n = 7(4) = 28.$

$$\begin{aligned} P &= R\left[\frac{1 - (1 + i)^{-n}}{i}\right] \\ &= 1500\left[\frac{1 - (1 + 0.0125)^{-28}}{0.0125}\right] \\ &\approx 35{,}253.78 \end{aligned}$$

The present value is $35,253.78.

51. Deposits of $4210 semiannually for 8 years at 4.2% compounded annually

Use the formula for the present value of an annuity with $R = 4210,\ i = \frac{0.042}{2} = 0.021,$ $n = 8(2) = 16.$

$$\begin{aligned} P &= R\left[\frac{1 - (1 + i)^{-n}}{i}\right] \\ &= 4210\left[\frac{1 - (1.021)^{-16}}{0.021}\right] \\ &\approx 56{,}711.93 \end{aligned}$$

The present value is $56,711.93.

52. Payments of $877.34 monthly for 17 months at an annuity at 6.4% compounded monthly

Use the formula for the present value of an annuity with $R = 877.34,\ i = \frac{0.064}{12} = 0.005\overline{3},\ n = 17.$

$$P = R\left[\frac{1 - (1 + i)^{-n}}{i}\right]$$
$$P = 877.34\left[\frac{1 - (1 + 0.005\overline{3})^{-17}}{0.05\overline{3}}\right]$$
$$\approx 14{,}222.42$$

The present value is $14,222.42.

53. Two types of loans that are commonly amortized are home loans and auto loans.

54. $P = 80{,}000,\ i = 0.05,\ n = 9$

$$\begin{aligned} R &= \frac{Pi}{1 - (1 + i)^{-n}} \\ &= \frac{80{,}000(0.05)}{1 - (1.05)^{-9}} \\ &\approx 11{,}255.21 \end{aligned}$$

The amount of each payment is $11,255.21.

The total amount paid is $11,255.21(9) = $101,296.89. Thus, the total interest paid is

$$\$101{,}296.89 - 80{,}000 = \$21{,}296.89.$$

55. $P = 3200,\ i = \frac{0.08}{4} = 0.02,\ n = 12$

$$\begin{aligned} R &= \frac{Pi}{1-(1+i)^{-n}} \\ &= \frac{3200(0.02)}{1-(1.02)^{-12}} \\ &\approx 302.59 \end{aligned}$$

The amount of each payment is \$302.59.

The total amount paid is \$302.59(12) = \$3631.08. Thus, the total interest paid is

$$\$3631.08 - 3200 = \$431.08.$$

56. $P = 32{,}000,\ i = \frac{0.064}{4} = 0.016,\ n = 17$

$$\begin{aligned} R &= \frac{Pi}{1-(1+i)^{-n}} \\ &= \frac{32{,}000(0.016)}{1-(1.016)^{-17}} \\ &\approx 2164.87 \end{aligned}$$

The amount of each payment is \$2164.87.

The total amount paid is \$2164.87(17) = \$36,802.79. Thus, the total interest paid is

$$\$36{,}802.79 - 32{,}000 = \$4802.79.$$

57. $P = 51{,}607,\ i = \frac{0.08}{12} = 0.00\overline{6},\ n = 32$

$$\begin{aligned} R &= \frac{Pi}{1-(1+i)^{-n}} \\ &= \frac{51{,}607(0.00\overline{6})}{1-(1.00\overline{6})^{-32}} \\ &\approx 1796.20 \end{aligned}$$

The amount of each payment is \$1796.20.

The total amount paid is \$1796.20(32) = \$57,478.40. Thus, the total interest paid is

$$\$57{,}478.40 - 51{,}607 = \$5871.40.$$

58. $P = 256{,}890,\ i = \frac{0.0596}{12} = 0.0049\overline{6},$
$n = 25(12) = 300.$

$$\begin{aligned} R &= \frac{Pi}{1-(1+i)^{-n}} \\ &= \frac{256{,}890(0.0049\overline{6})}{1-(1.0049\overline{6})^{-300}} \\ &\approx 1648.87 \end{aligned}$$

The amount of each payment is \$1648.87.

The total amount paid is \$1648.87(300) = \$494,661. Thus, the total interest paid is

$$\$494{,}661 - 256{,}890 = \$237{,}771.$$

59. $P = 177{,}110,\ i = \frac{0.0668}{12} = 0.0055\overline{6},$
$n = 30(12) = 360$

$$\begin{aligned} R &= \frac{Pi}{1-(1+i)^{-n}} \\ &= \frac{177{,}110(0.0055\overline{6})}{1-(1.0055\overline{6})^{-360}} \\ &\approx 1140.50 \end{aligned}$$

The amount of each payment is \$1140.50.

The total amount paid is \$1140.50(360) = \$410,580. Thus, the total interest paid is

$$\$410{,}580 - 177{,}110 = \$233{,}470.$$

60. Look in the column titled "Interest for Period." \$896.06 of the fifth payment of \$1022.64 is interest.

61. The answer can be found in the table under payment number 12 in the column labeled "Portion to Principal." The amount of principal repayment included in the fifth payment is \$132.99.

62. In the first 3 months of the loan, the total amount of interest paid is

$$\$899.58 + 898.71 + 897.83 = \$2696.12.$$

63. The last entry in the column "Principal at End of Period," \$125,464.39, shows the debt remaining at the end of the first year (after 12 payments). Since the original debt (loan principal) was \$127,000, the amount by which the debt has been reduced at the end of the first year is

$$\$127{,}000 - 125{,}464.39 = \$1535.61.$$

64. Here $P = 5800$, $r = 5.3\% = 0.053$, and $t = \frac{10}{12}$.

$$\begin{aligned} I &= Prt \\ &= 5800(0.053)\left(\frac{10}{12}\right) \\ &\approx 256.17 \end{aligned}$$

The interest she will pay is $256.17. The total amount she will owe is

$$\$5800 + 256.17 = \$6056.17.$$

65. Here $P = 9820$, $r = 6.7\% = 0.067$, and $t = \frac{7}{12}$.

$$\begin{aligned} I &= Prt \\ &= 9820(0.067)\left(\frac{7}{12}\right) \\ &\approx 383.80 \end{aligned}$$

The interest she will pay is $383.80. The total amount she will owe in 7 mo is

$$\$9820 + 383.80 = \$10{,}203.80.$$

66. Here $P = 28{,}000$, $r = 6.5\% = 0.065$, and $I = 1365$.

Use the formula for simple interest.

$$\begin{aligned} I &= Prt \\ t &= \frac{I}{Pr} \\ &= \frac{1365}{28{,}000(0.065)} \\ &= 0.75 \end{aligned}$$

The loan is for 0.75 yr; convert this to months.

$$0.75\text{yr}\left(\frac{12 \text{ mo}}{1 \text{ yr}}\right) = 9 \text{mo}$$

The loan is for 9 mo.

67. $P = 84{,}720$, $t = \frac{7}{12}$, $I = 4055.46$

Substitute these values into the formula for simple interest to find the value of r.

$$\begin{aligned} I &= Prt \\ 4055.46 &= 84{,}720r\left(\frac{7}{12}\right) \\ 4055.46 &= 49{,}420r \\ 0.0821 &\approx r \end{aligned}$$

The interest rate is 8.21%.

68. $A = 7500$, $i = \frac{0.05}{2} = 0.025$, $n = 3(2) = 6$

Let P represent the lump sum.

$$\begin{aligned} A &= P(1 + i)^n \\ P &= \frac{A}{(1 + i)^n} \\ &= \frac{7500}{(1.025)^6} \approx 6467.23 \end{aligned}$$

She should deposit about $6467.23 today.

69. In both cases use the formula for compound amount with $P = 500$ and $i = \frac{0.05}{4} = 0.0125$. For the investment at age 23 use $n = 42(4) = 168$.

$$\begin{aligned} A &= P(1 + i)^n \\ &= 500(1 + 0.0125)^{168} \\ &\approx 4030.28 \end{aligned}$$

For the investment at age 40 use $n = 25(4) = 100$.

$$\begin{aligned} A &= P(1 + i)^n \\ &= 500(1 + 0.0125)^{100} \\ &\approx 1731.70 \end{aligned}$$

The increased amount of money Ali will have if he invests now is

$$\$4030.28 - 1731.70 = \$2298.58.$$

70. Suppose you receive $6000/yr at age 55 until age 75. Then $R = 6000$, $i = 0.08$, and $n = 20$.

$$S = 6000\left[\frac{(1 + 0.08)^{20} - 1}{0.08}\right] \approx 274{,}571.79$$

You would receive a total of $274,571.79.

Suppose you receive $12,000/yr at age 65 until age 75. Then $R = 12{,}000$, $i = 0.08$, and $n = 10$.

$$S = 12{,}000\left[\frac{(1 + 0.08)^{10} - 1}{0.08}\right] \approx 173{,}838.75$$

You would receive a total of $173,838.75.

Receiving half the pension at 55 would produce the larger amount.

71. $R = 5000$, $i = \frac{0.10}{2} = 0.05$, $n = 7\frac{1}{2}(2) = 15$

This is an ordinary annuity.

$$S = R\left[\frac{(1+i)^n - 1}{i}\right]$$

$$S = 5000\left[\frac{(1+0.05)^{15} - 1}{0.05}\right]$$

$$\approx 107{,}892.82$$

The future value is \$107,892.82. The amount of interest earned is

$$\$107{,}892.82 - 15(5000) = \$32{,}892.82.$$

72. Use the formula for amortization payments with $P = 28{,}000$, $i = \frac{0.08}{4} = 0.02$, $n = 6\frac{1}{2}(4) = 26$.

$$R = \frac{Pi}{1-(1+i)^{-n}} = \frac{28{,}000(0.02)}{1-(1.02)^{-26}} \approx 1391.58$$

The amount of each payment is \$1391.58.

73. Use the formula for amortization payments with $P = 48{,}000$, $i = 0.065$, and $n = 7$.

$$R = \frac{Pi}{1-(1+i)^{-n}}$$

$$= \frac{48{,}000(0.065)}{1-(1.065)^{-7}}$$

$$\approx 8751.91$$

The owner should deposit \$8751.91 at the end of each year.

The total amount deposited is \$8751.91(7) = \$61,263.37. Thus, the total interest paid is

$$\$61{,}263.37 - 48{,}000 = \$13{,}263.37.$$

74. Use the formula for compound amount with $P = 3250$, $i = 0.042$, and $n = 4$.

$$A = P(1+i)^n = 3250(1.042)^4 \approx 3831.37$$

David must pay back \$3831.37.

75. The effective rate paid by First Internet Bank of Indiana would be

$$\left(1+\frac{0.021}{12}\right)^{12} - 1 = 0.0212$$

or 2.12%.

The effective rate paid by Discover Bank would be

$$\left(1+\frac{0.0208}{360}\right)^{360} - 1 = 0.0210$$

or 2.10%. First Internet Bank of Indiana has the higher effective rate.

76. Use the formula for amortization payments with $P = 315{,}700$, $i = \frac{0.075}{12} = 0.00625$, and $n = 300$.

$$R = \frac{Pi}{1-(1+i)^{-n}}$$

$$= \frac{315{,}700(0.00625)}{1-(1.00625)^{-300}}$$

$$\approx 2333.00$$

Each monthly payment will be about \$2333.00. The total amount of interest will be

$$\$2333.00(300) - 315{,}700 = \$384{,}200.$$

77. Compute the monthly payments using the formula

$$R = \frac{Pi}{1-(1+i)^{-n}}$$

(a) $P = 15{,}000$

$i = \dfrac{0.0299}{12}$

$n = 60$

$$R = \frac{(15{,}000)\left(\frac{0.0299}{12}\right)}{1-\left(1+\frac{0.0299}{12}\right)^{-60}} \approx 269.46$$

The monthly payment is \$269.46. The total payment is

$$(60)(269.46) = 16{,}167.60$$

or \$16,167.60.

(b) $P = 15{,}000$

$i = \dfrac{0.0349}{12}$

$n = 84$

$$R = \frac{(15{,}000)\left(\frac{0.0349}{12}\right)}{1-\left(1+\frac{0.0349}{12}\right)^{-84}} \approx 201.53$$

The monthly payment is \$201.53. The total payment is

$$(84)(201.53) = 16{,}928.52$$

or \$16,928.52.

(c) The first deal has higher monthly payments but a lower total cost to the borrower.

78. **(a)** For the 0% financing, the payments are simply 1/72 of the financed amount.

$$\frac{30{,}000}{72} = 416.67$$

Thus the rounded payments are \$416.67, and the total payments are equal to the financed amount of \$30,000. (In fact because of rounding the total payments are 24 cents more than this amount, but the final payment would be reduced by 24 cents to compensate.)

(b) For the 3.18% financing the monthly payment will be

$$\frac{(26{,}000)\left(\frac{0.0318}{12}\right)}{1-\left(1+\frac{0.0318}{12}\right)^{-48}} = 577.56$$

or \$577.56. The total payments will be 48 times this amount, or \$27,722.88.

(c) There is the usual tradeoff between higher payments and lower total cost.

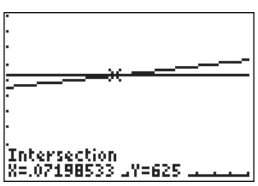
(d) The total payments will be equal when the payment under the second option is

$$\frac{30{,}000}{48} = 625.00,$$

or \$625. We need to solve

$$\frac{(26{,}000)\left(\frac{r}{12}\right)}{1-\left(1+\frac{r}{12}\right)^{-48}} = 625$$

Graphing the two sides of this equality for r from 0.01 to 0.10 and using the Intersect feature gives an r value of 0.0720 or 7.20%.



79. Amount of loan $= \$191{,}000 - 40{,}000$
$= \$151{,}000$

(a) Use the formula for amortization payments with $P = 151{,}000$, $i = \frac{0.065}{12} = 0.00541\overline{6}$, and $n = 30(12) = 360$.

$$R = \frac{Pi}{1-(1+i)^{-n}}$$
$$= \frac{151{,}000(0.00541\overline{6})}{1-(1.00541\overline{6})^{-360}}$$
$$\approx 954.42$$

The monthly payment for this mortgage is \$954.42.

(b) To find the amount of the first payment that goes to interest, use $I = Prt$ with

$P = 151{,}000$, $i = 0.00541\overline{6}$, and $t = 1$.

$$I = 151{,}000(0.065)\left(\frac{1}{12}\right) = 817.92$$

Of the first payment, \$817.92 is interest.

(c) Using method 1, since 180 of 360 payments were made, there are 180 remaining payments. The present value is

$$954.42\left[\frac{1-(1.00541\overline{6})^{-180}}{0.00541\overline{6}}\right] \approx 109{,}563.99,$$

so the remaining balance is \$109,563.99.

Using method 2, since 180 payments were already made, we have

$$954.42\left[\frac{1-(1.00541\overline{6})^{-180}}{0.00541\overline{6}}\right] \approx 109{,}563.99.$$

She still owes

$$\$151{,}000 - 109{,}563.99 = \$41{,}436.01.$$

Furthermore, she owes the interest on this amount for 180 mo, for a total remaining balance of

$$41{,}436.01(1.00541\overline{6})^{180} = 109{,}565.13.$$

(d) Closing costs $= 3700 + 0.025(238{,}000)$

$= 3700 + 5950$

$= 9650$

Closing costs are \$9650.

(e) Amount of money received

= Selling price − Closing costs − Current mortgage balance

Using method 1, the amount received is

$$\$238{,}000 - 9650 - 109{,}563.99 = \$118{,}786.01.$$

Using method 2, the amount received is

$$\$238{,}000 - 9650 - 109{,}565.13 = \$118{,}784.87.$$

80. The death benefit grows to

$$10{,}000(1.05)^7 \approx 14{,}071.$$

This 14,071 is the present value of an annuity due with $P = 14{,}071$, $i = \frac{0.03}{12} = 0.0025$, and $n = 120$.

Let X represent the amount of each monthly payment.

$$\begin{aligned} P &= R \cdot a_{\overline{n+1}|i} - R \\ 14{,}071 &= X \cdot a_{\overline{121}|0.0025} - X \\ 14{,}071 &= (a_{\overline{121}|0.0025} - 1)X \\ 14{,}071 &\approx (104.301 - 1)X \\ 14{,}071 &\approx 103.301X \\ 136 &\approx X \end{aligned}$$

Each payment is about \$136, which corresponds to choice (d).

81. **(a)** Use the formula for effective rate with $r_E = 0.10$ and $m = 12$.

$$\begin{aligned} r_E &= \left(1 + \frac{r}{m}\right)^m - 1 \\ 0.10 &= \left(1 + \frac{r}{12}\right)^{12} - 1 \\ 1.10 &= \left(1 + \frac{r}{12}\right)^{12} \\ (1.10)^{1/12} &= 1 + \frac{r}{12} \\ 1.007974 &\approx 1 + \frac{r}{12} \\ 0.007974 &\approx \frac{r}{12} \\ 0.095688 &\approx r \end{aligned}$$

The annual interest rate is 9.569%.

(b) Use the formula for amortization payments with $P = 140{,}000$, $i = \frac{0.06625}{12}$, and $n = 30(12) = 360$.

$$\begin{aligned} R &= \frac{Pi}{1 - (1+i)^{-n}} \\ &= \frac{140{,}000\left(\frac{0.06625}{12}\right)}{1 - \left(1 + \frac{0.06625}{12}\right)^{-360}} \\ &\approx 896.44 \end{aligned}$$

Her monthly payment is \$896.44.

(c) This investment is an annuity with $R = 1200 - 896.44 = 303.56$, $i = \frac{0.09569}{12}$, and $n = 30(12) = 360$. The future value is

$$\begin{aligned} S &= R\left[\frac{(1+i)^n - 1}{i}\right] \\ &= 303.56\left[\frac{\left(1 + \frac{0.09569}{12}\right)^{360} - 1}{\frac{0.09569}{12}}\right] \\ &\approx 626{,}200.88 \end{aligned}$$

In 30 yr she will have \$626,200.88 in the fund.

(d) Use the formula for amortization payments with $P = 14{,}000$, $i = \frac{0.0625}{12}$, and $n = 15(12) = 180$.

$$\begin{aligned} R &= \frac{Pi}{1 - (1+i)^{-n}} \\ &= \frac{140{,}000\left(\frac{0.0625}{12}\right)}{1 - \left(1 + \frac{0.0625}{12}\right)^{-180}} \\ &\approx 1200.39 \end{aligned}$$

His monthly payment is \$1200.39.

(e) This investment is an annuity with $R = 1200$, $i = \frac{0.09569}{12}$, and $n = 15(12) = 180$. The future value is

$$\begin{aligned} S &= R\left[\frac{(1+i)^n - 1}{i}\right] \\ &= 1200\left[\frac{\left(1 + \frac{0.09569}{12}\right)^{180} - 1}{\frac{0.09569}{12}}\right] \\ &\approx 478{,}134.14 \end{aligned}$$

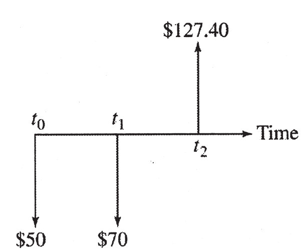
In 30 yr he will have \$478,134.14.

(f) Sue is ahead by

\$626,200.88 − 478,134.14 = \$148,066.74.

Extended Application: Time, Money, and Polynomials

1.



The polynomial equation is

$$50(1+i)^2 + 70(1+i) - 127.40 = 0.$$

Let $x = 1 + i$. The equation becomes

$$50x^2 + 70x - 127.40 = 0.$$

Use the quadratic formula to solve the equation for x.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-70 \pm \sqrt{70^2 - 4(50)(-127.40)}}{2(50)}$$

Reject $x = \frac{-70-\sqrt{30{,}380}}{100}$ because it is negative.

Thus,

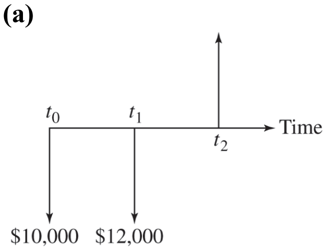
$$x = \frac{-70 + \sqrt{30{,}380}}{100} \approx 1.04298.$$

Since $x = 1 + i$,

$$1 + i = 1.04298$$
$$i \approx 0.043.$$

Thus, the YTM is 4.3%.

2. **(a)**



(b) $A = 1.05(10{,}000) + 0.045(1.05)(10{,}000)$
$+ 1.045(12{,}000)$
$= 23{,}512.5$

At the end of the second year, $23,512.50 was in the account.

(c) The polynomial equation is

$$10{,}000(1+i)^2 + 12{,}000(1+i) = 23{,}512.50 = 0.$$

Let $x = 1 + i$. The equation becomes

$$10{,}000x^2 + 12{,}000x - 23{,}512.50 = 0.$$

Use the quadratic formula to solve for x.

$$x = \frac{-12{,}000 \pm \sqrt{12{,}000^2 - 4(10{,}000)(-23{,}512.50)}}{2(10{,}000)}$$

Reject

$$x = \frac{-12{,}000 - \sqrt{12{,}000^2 - 4(10{,}000)(-23{,}512.50)}}{20{,}000}$$

because it is negative. Thus,

$$x = \frac{-12{,}000 + \sqrt{12{,}000^2 - 4(10{,}000)(-23{,}512.50)}}{20{,}000}$$

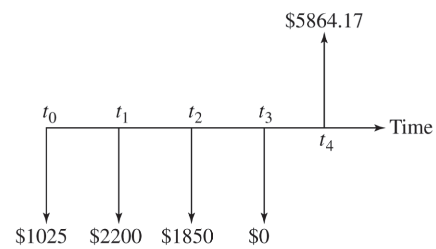
$\approx 1.04659.$

Since $x = 1 + i$,

$$1 + i = 1.04659$$
$$i = 0.04659.$$

Thus, the YTM is 4.7%. As might be expected, the YTM is between 4.5% and 5%.

3. **(a)**



(b) The polynomial equation is

$$1025(1+i)^4 + 2200(1+i)^3$$
$$+1850(1+i)^2 - 5864.17 = 0.$$

Let $x = 1 + i$. The equation becomes

$$1025x^4 + 2200x^3 + 1850x^2 - 5864.17 = 0.$$

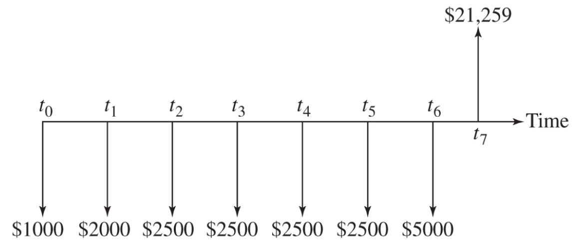
Let $f(x) = 1025x^4 + 2200x^3 + 1850x^2 - 5864.17$.

Since $0 < i < 1$, then $1 < x < 2$ and

$$f(1) = -789.17;$$
$$f(1.1) = 803.2325;$$
$$f(1.05) = -31.8761;$$
$$f(1.052) = 0.00172.$$

The YTM is approximately 5.2%.

4. (a)



(b) The polynomial equation is

$$1000(1+i)^7 + 2000(1+i)^6 + 2500(1+i)^5$$
$$+2500(1+i)^4 + 2500(1+i)^3 + 2500(1+i)^2$$
$$+5000(1+i) - 21{,}259 = 0.$$

(c) Let $x = 1 + i$ and

$$f(x) = 1000(x^7 + 2x^6 + 2.5x^5 + 2.5x^4 + 2.5x^3 + 2.5x^2 + 5x - 21.259).$$

Then

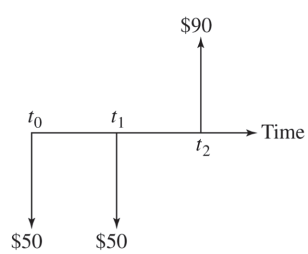
$$f(1.0507) = 7.1216;$$
$$f(1.0505) = -6.9040.$$

Since $f(1.0507)$ is positive and $f(1.0505)$ is negative, the value of x that makes $f(x)$ zero is between 1.0507 and 1.0505.

$$1.0505 < l + i < 1.0507$$
$$0.0505 < i < 0.0507$$
$$5.05\% < i < 5.07\%$$

(d) Graph $y_1 = f(x)$. The graph intersects the x-axis at $x = 1.0505985$. Therefore, $i = 0.0505985$ or 5.06%.

5. (a)



The polynomial equation is

$$50(1+i)^2 + 50(1+i) - 90 = 0.$$

Let $x = 1 + i$. The equation becomes

$$50x^2 + 50x - 90 = 0$$
$$5x^2 + 5x - 9 = 0.$$

Solve for x using the quadratic formula.

$$x = \frac{-5 \pm \sqrt{5^2 - 4(5)(-9)}}{2(5)}$$
$$x = \frac{-5 \pm \sqrt{205}}{10}$$
$$x = \frac{-5 + \sqrt{205}}{10} \text{ or } x = \frac{-5 - \sqrt{205}}{10}$$
$$\approx 0.93178 \qquad \approx -1.93178$$

Then

$$1 + i = 0.93178 \quad \text{or} \quad 1 + i = -1.93178$$
$$i = -0.06822 \qquad i = -2.93178$$
$$i \approx -6.8\% \quad \text{or} \quad i \approx -293.2\%.$$

(b) For $i = -6.8\%$ use the formula for compound amount for each \$50 investment. For the first investment $P = 50$ and $n = 2$.

$$A = P(1+i)^n$$
$$= 50(1 - 0.068)^2$$
$$= 43.4312$$

For the second investment $P = 50$ and $n = 1$.

$$A = P(1+i)^n$$
$$= 50(1 - 0.068)^1$$
$$= 46.6$$

Each value of A seems to be a reasonable future value for the investment considered. Also, note that

$$43.4312 + 46.6 = 90.0312.$$

For $i = -293.2\%$ use the formula for compound amount for each \$50 investment. For the first investment $P = 50$ and $n = 2$.

$$A = P(1+i)^n$$
$$= 50(1 - 2.932)^2$$
$$= 186.6312$$

For the second investment $P = 50$ and $n = 1$.

$$A = P(1+i)^n$$
$$= 50(1 - 2.932)^1$$
$$= -96.6$$

Although $186.6312 - 96.6 = 90.0312$, neither value seems like a reasonable future value for the investment considered. Only $i = -6.8\%$ seems a reasonable interpretation in the context to the problem.