Chapter 6 Radicals and Complex Numbers

Section 6.1 Practice Exercises

1. **a.** b; a

b. principal

c. b^n; a

d. index; radicand

e. cube

f. is not; is

g. even; odd

h. Pythagorean; c^2

i. $\left[0, \infty\right)$; $\left(-\infty, \infty\right)$

j. -5 and -4

2. $\sqrt[3]{8} = \sqrt[3]{2^3} = 2$ because $2^3 = 8$.

3. **a.** 8 is a square root of 64 because $8^2 = 64$.
-8 is a square root of 64 because $\left(-8\right)^2 = 64$.

b. $\sqrt{64} = 8$

c. There are two square roots for every positive number. $\sqrt{64}$ identifies the positive square root.

4. **a.** 11 is a square root of 121 because $11^2 = 121$.
-11 is a square root of 121 because $\left(-11\right)^2 = 121$.

b. $\sqrt{121} = 11$

c. There are two square roots for every positive number. $\sqrt{121}$ identifies the positive square root.

5. **a.** $\sqrt{81} = 9$

b. $-\sqrt{81} = -9$

6. **a.** $\sqrt{100} = 10$

b. $-\sqrt{100} = -10$

7. There is no real number b such that $b^2 = -36$.

8. $\sqrt{25} = 5$

9. $\sqrt{49} = 7$

10. $-\sqrt{25} = -5$

11. $-\sqrt{49} = -7$

12. $\sqrt{-25}$ is not a real number.

13. $\sqrt{-49}$ is not a real number.

14. $\sqrt{\frac{100}{121}} = \frac{10}{11}$

15. $\sqrt{\frac{64}{9}} = \frac{8}{3}$

16. $\sqrt{0.64} = 0.8$

17. $\sqrt{0.81} = 0.9$

18. $-\sqrt{0.0144} = -0.12$

19. $-\sqrt{0.16} = -0.4$

20. There is no real number b such that $b^4 = -16$.

21. **a.** $\sqrt{64} = 8$
b. $\sqrt[3]{64} = 4$
c. $-\sqrt{64} = -8$
d. $-\sqrt[3]{64} = -4$
e. $\sqrt{-64}$ is not a real number.
f. $\sqrt[3]{-64} = -4$

22. **a.** $\sqrt{16} = 4$
b. $\sqrt[4]{16} = 2$
c. $-\sqrt{16} = -4$
d. $-\sqrt[4]{16} = -2$
e. $\sqrt{-16}$ is not a real number.
f. $\sqrt[4]{-16}$ is not a real number.

23. $\sqrt[3]{-27} = -3$

24. $\sqrt[3]{-125} = -5$

25. $\sqrt[3]{\frac{1}{8}} = \frac{1}{2}$

26. $\sqrt[5]{\frac{1}{32}} = \frac{1}{2}$

27. $\sqrt[5]{32} = 2$

28. $\sqrt[4]{1} = 1$

29. $\sqrt[3]{-\frac{125}{64}} = -\frac{5}{4}$

30. $\sqrt[3]{-\frac{8}{27}} = -\frac{2}{3}$

31. $\sqrt[4]{-1}$ is not a real number.

32. $\sqrt[6]{-1}$ is not a real number.

33. $\sqrt[6]{1,000,000} = 10$

34. $\sqrt[4]{10,000} = 10$

35. $-\sqrt[3]{0.008} = -0.2$

36. $-\sqrt[4]{0.0016} = -0.2$

37. $\sqrt[4]{0.0625} = 0.5$

38. $\sqrt[3]{0.064} = 0.4$

39. $\sqrt{a^2} = |a|$

40. $\sqrt[4]{a} = |a|$

41. $\sqrt[3]{a^3} = a$

42. $\sqrt[5]{a^5} = a$

43. $\sqrt[6]{a^6} = |a|$

44. $\sqrt[7]{a^7} = a$

45. $\sqrt{(x+1)^2} = |x+1|$

46. $\sqrt[3]{(y+3)^3} = y+3$

47. $\sqrt{x^2y^4} = \sqrt{x^2\left(y^2\right)^2} = |x|y^2$

48. $\sqrt[3]{(u+v)^3} = u+v$

49. $-\sqrt[3]{\dfrac{x^3}{y^3}} = -\dfrac{x}{y}, y \neq 0$

50. $\sqrt[4]{\dfrac{a^4}{b^8}} = \sqrt[4]{\dfrac{a^4}{\left(b^2\right)^4}} = \dfrac{|a|}{b^2}, b \neq 0$

51. $\dfrac{2}{\sqrt[4]{x^4}} = \dfrac{2}{|x|}, x \neq 0$

52. $\sqrt{(-5)^2} = \sqrt{25} = 5$

53. $\sqrt[3]{(-92)^3} = -92$

54. $\sqrt[6]{(50)^6} = 50$

55. $\sqrt[10]{(-2)^{10}} = |-2| = 2$

56. $\sqrt[5]{(-2)^5} = -2$

57. $\sqrt[7]{(-923)^7} = -923$

58. $\sqrt[6]{(-417)^6} = |-417| = 417$

59. $\sqrt{y^8} = \sqrt{\left(y^4\right)^2} = y^4$

60. $\sqrt{x^4} = \sqrt{\left(x^2\right)^2} = x^2$

61. $\sqrt{\frac{a^6}{b^2}} = \sqrt{\frac{\left(a^3\right)^2}{b^2}} = \frac{a^3}{b}$

62. $\sqrt{\frac{w^2}{z^4}} = \sqrt{\frac{w^2}{\left(z^2\right)^2}} = \frac{w}{z^2}$

63. $-\sqrt{\frac{25}{q^2}} = -\frac{5}{q}$

64. $-\sqrt{\frac{p^6}{81}} = -\sqrt{\frac{\left(p^3\right)^2}{81}} = -\frac{p^3}{9}$

65. $\sqrt{9x^2y^4z^2} = \sqrt{9x^2\left(y^2\right)^2z^2} = 3xy^2z$

66. $\sqrt{4a^4b^2c^6} = \sqrt{4\left(a^2\right)^2b^2\left(c^3\right)^2} = 2a^2bc^3$

67. $\sqrt{\frac{h^2k^4}{16}} = \sqrt{\frac{h^2\left(k^2\right)^2}{16}} = \frac{hk^2}{4}$

68. $\sqrt{\frac{4x^2}{y^8}} = \sqrt{\frac{4x^2}{\left(y^4\right)^2}} = \frac{2x}{y^4}$

69. $-\sqrt[3]{\frac{t^3}{27}} = -\frac{t}{3}$

70. $\sqrt[4]{\frac{16}{w^4}} = \frac{2}{w}$

71. $\sqrt[5]{32y^{10}} = \sqrt[5]{32\left(y^2\right)^5} = 2y^2$

72. $\sqrt[3]{64x^6y^3} = \sqrt[3]{64\left(x^2\right)^3y^3} = 4x^2y$

73. $\sqrt[6]{64p^{12}q^{18}} = \sqrt[6]{64\left(p^2\right)^6\left(q^3\right)^6} = 2p^2q^3$

74. $\sqrt[4]{16r^{12}s^8} = \sqrt[4]{16\left(r^3\right)^4\left(s^2\right)^4} = 2r^3s^2$

75.
$$a^2 + b^2 = c^2$$
$$12^2 + b^2 = 15^2$$
$$144 + b^2 = 225$$
$$b^2 = 81$$
$$b = 9 \text{ cm}$$

76.
$$a^2 + b^2 = c^2$$
$$6^2 + 8^2 = c^2$$
$$36 + 64 = c^2$$
$$100 = c^2$$
$$c = 10 \text{ in.}$$

77.
$$a^2 + b^2 = c^2$$
$$12^2 + 5^2 = c^2$$
$$144 + 25 = c^2$$
$$169 = c^2$$
$$c = 13 \text{ ft}$$

78.
$$a^2 + b^2 = c^2$$
$$10^2 + b^2 = 26^2$$
$$100 + b^2 = 676$$
$$b^2 = 576$$
$$b = 24 \text{ m}$$

79. $a^2+b^2=c^2$

$4^2+3^2=c^2$

$16+9=c^2$

$25=c^2$

$c=5$ mi

They were 5 mi apart.

80. $a^2+b^2=c^2$

$12^2+5^2=c^2$

$144+25=c^2$

$169=c^2$

$c=13$ mi

They were 13 mi apart.

81. $a^2+b^2=c^2$

$20^2+15^2=c^2$

$400+225=c^2$

$625=c^2$

$c=25$ mi

They are 25 mi apart.

82. $a^2+b^2=c^2$

$480^2+200^2=c^2$

$230,400+40,000=c^2$

$270,400=c^2$

$c=520$ ft

They are 520 ft apart.

83. $h(x)=\sqrt{x-2}$

a. $h(0)=\sqrt{0-2}=\sqrt{-2}$ not a real number

b. $h(1)=\sqrt{1-2}=\sqrt{-1}$ not a real numbe

c. $h(2)=\sqrt{2-2}=\sqrt{0}=0$

d. $h(3)=\sqrt{3-2}=\sqrt{1}=1$

e. $h(6)=\sqrt{6-2}=\sqrt{4}=2$

$x-2\ge 0$

$x\ge 2$

Domain: $[2,\infty)$

84. $k(x)=\sqrt{x+1}$

a. $k(-3)=\sqrt{-3+1}=\sqrt{-2}$ not a real number

b. $k(-2)=\sqrt{-2+1}=\sqrt{-1}$ not a real number

c. $k(-1)=\sqrt{-1+1}=\sqrt{0}=0$

d. $k(0)=\sqrt{0+1}=\sqrt{1}=1$

e. $k(3)=\sqrt{3+1}=\sqrt{4}=2$

$x+1\ge 0$

$x\ge -1$

Domain: $[-1,\infty)$

85. $g(x)=\sqrt[3]{x-2}$

a. $g(-6)=\sqrt[3]{-6-2}=\sqrt[3]{-8}=-2$

b. $g(1)=\sqrt[3]{1-2}=\sqrt[3]{-1}=-1$

c. $g(2)=\sqrt[3]{2-2}=\sqrt[3]{0}=0$

d. $g(3)=\sqrt[3]{3-2}=\sqrt[3]{1}=1$

Domain: $(-\infty,\infty)$

86. $f(x)=\sqrt[3]{x+1}$

a. $f(-9)=\sqrt[3]{-9+1}=\sqrt[3]{-8}=-2$

b. $f(-2)=\sqrt[3]{-2+1}=\sqrt[3]{-1}=-1$

c. $f(0)=\sqrt[3]{0+1}=\sqrt[3]{1}=1$

d. $f(7)=\sqrt[3]{7+1}=\sqrt[3]{8}=2$

Domain: $(-\infty,\infty)$

87. $f(x)=\sqrt{5-2x}$

$5-2x\geq 0$

$-2x\geq -5$

$x\leq \frac{5}{2}$

Domain: $\left(-\infty,\frac{5}{2}\right]$

88. $g(x)=\sqrt{3-4x}$

$3-4x\geq 0$

$-4x\geq -3$

$x\leq \frac{3}{4}$

Domain: $\left(-\infty,\frac{3}{4}\right]$

89. $k(x)=\sqrt[3]{4x-7}$

Domain: $(-\infty,\infty)$

90. $R(x)=\sqrt[3]{x+1}$

Domain: $(-\infty,\infty)$

91. $M(x)=\sqrt{x-5}+3$

$x-5\geq 0$

$x\geq 5$

Domain: $[5,\infty)$

92. $N(x)=\sqrt{x+3}-1$

$x+3\geq 0$

$x\geq -3$

Domain: $[-3,\infty)$

93. $F(x)=\sqrt[3]{x+7}-2$

Domain: $(-\infty,\infty)$

94. $G(x)=\sqrt[3]{x-10}+4$

Domain: $(-\infty,\infty)$

95. **a.** $f(x)=\sqrt{1-x}$

$1-x\geq 0$

$-x\geq -1$

$x\leq 1$

$(-\infty,1]$

b. Create a table of ordered pairs where x values are taken to be less than or equal to 1.

96. **a.** $f(x)=\sqrt{2-x}$

$2-x\geq 0$

$-x\geq -2$

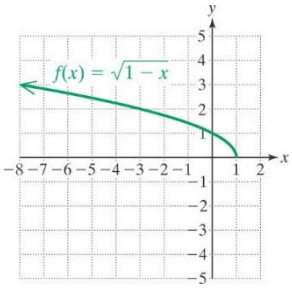
$x\leq 2$

$(-\infty,2]$

b. Create a table of ordered pairs where x values are taken to be less than or equal to 2.

x	$f(x)$
1	0
0	1
−3	2
−8	3
−15	4

$f(1)=\sqrt{1-1}=\sqrt{0}=0$
$f(0)=\sqrt{1-0}=\sqrt{1}=1$
$f(-3)=\sqrt{1-(-3)}=\sqrt{4}=2$
$f(-8)=\sqrt{1-(-8)}=\sqrt{9}=3$
$f(-15)=\sqrt{1-(-15)}=\sqrt{16}=4$



97. **a.** $f(x)=\sqrt{x+3}$

$x+3\geq 0$

$x\geq -3$

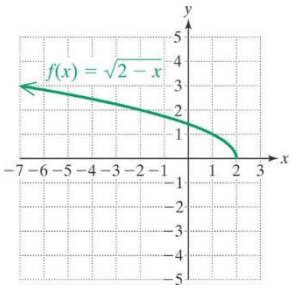
$[-3,\infty)$

b. Create a table of ordered pairs where x values are taken to be greater than or equal to -3.

x	$f(x)$
−3	0
−2	1
1	2
6	3
13	4

x	$f(x)$
2	0
1	1
−2	2
−7	3
−14	4

$f(2)=\sqrt{2-2}=\sqrt{0}=0$
$f(1)=\sqrt{2-1}=\sqrt{1}=1$
$f(-2)=\sqrt{2-(-2)}=\sqrt{4}=2$
$f(-7)=\sqrt{2-(-7)}=\sqrt{9}=3$
$f(-14)=\sqrt{2-(-14)}=\sqrt{16}=4$



98. **a.** $f(x)=\sqrt{x+1}$

$x+1\geq 0$

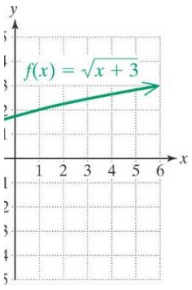
$x\geq -1$

$[-1,\infty)$

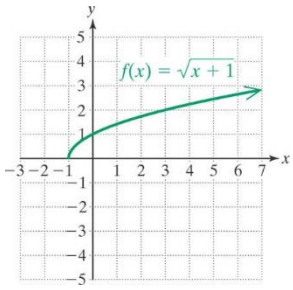
b. Create a table of ordered pairs where x values are taken to be greater than or equal to -1.

x	$f(x)$
1	0
0	1
3	2
8	3
15	4

$f(-3)=\sqrt{-3+3}=\sqrt{0}=0$
$f(-2)=\sqrt{-2+3}=\sqrt{1}=1$
$f(1)=\sqrt{1+3}=\sqrt{4}=2$
$f(6)=\sqrt{6+3}=\sqrt{9}=3$
$f(13)=\sqrt{13+3}=\sqrt{16}=4$



$f(1)=\sqrt{-1+1}=\sqrt{0}=0$
$f(0)=\sqrt{0+1}=\sqrt{1}=1$
$f(3)=\sqrt{3+1}=\sqrt{4}=2$
$f(8)=\sqrt{8+1}=\sqrt{9}=3$
$f(15)=\sqrt{15+1}=\sqrt{16}=4$



99. **a.** $f(x)=\sqrt{x}+2$

$x \geq 0$

$[0,\infty)$

b. Create a table of ordered pairs where x values are taken to be greater than or equal to 0.

x	$f(x)$
0	2
1	3
4	4
9	5
16	6

$f(0)=\sqrt{0}+2=0+2=2$
$f(1)=\sqrt{1}+2=1+2=3$
$f(4)=\sqrt{4}+2=2+2=4$
$f(9)=\sqrt{9}+2=3+2=5$
$f(16)=\sqrt{16}+2=4+2=6$

100. **a.** $f(x)=\sqrt{x}-1$

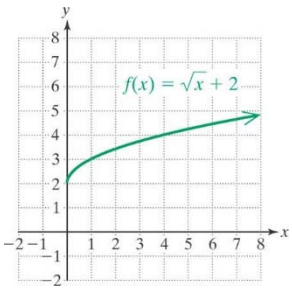
$x \geq 0$

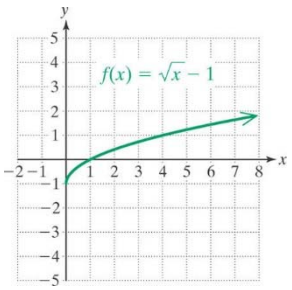
$[0,\infty)$

b. Create a table of ordered pairs where x values are taken to be greater than or equal to 0.

x	$f(x)$
0	−1
1	0
4	1
9	2
16	3

$f(0)=\sqrt{0}-1=0-1=-1$
$f(1)=\sqrt{1}-1=1-1=0$
$f(4)=\sqrt{4}-1=2-1=1$
$f(9)=\sqrt{9}-1=3-1=2$
$f(16)=\sqrt{16}-1=4-1=3$





101. **a.** $f(x)=\sqrt[3]{x-1}$

The index is odd; therefore the domain is all real numbers. $(-\infty,\infty)$

b. Create a table of ordered pairs where x values are taken to be all real numbers.

x	$f(x)$
-7	-2
0	-1
1	0
2	1
9	2

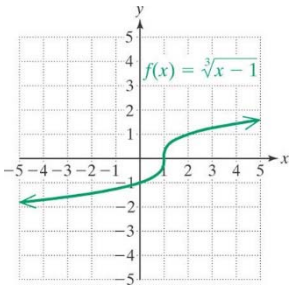
$f(-7)=\sqrt[3]{-7-1}=\sqrt[3]{-8}=-2$

$f(0)=\sqrt[3]{0-1}=\sqrt[3]{-1}=-1$

$f(1)=\sqrt[3]{1-1}=\sqrt[3]{0}=0$

$f(2)=\sqrt[3]{2-1}=\sqrt[3]{1}=1$

$f(9)=\sqrt[3]{9-1}=\sqrt[3]{8}=2$



102. **a.** $f(x)=\sqrt[3]{x+2}$

The index is odd; therefore the domain is all real numbers. $(-\infty,\infty)$

b. Create a table of ordered pairs where x values are taken to be all real numbers.

x	$f(x)$
-10	-2
-3	-1
-2	0
-1	1
6	2

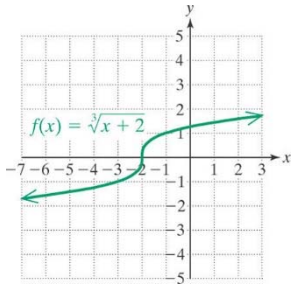
$f(-10)=\sqrt[3]{-10+2}=\sqrt[3]{-8}=-2$

$f(-3)=\sqrt[3]{-3+2}=\sqrt[3]{-1}=-1$

$f(-2)=\sqrt[3]{-2+2}=\sqrt[3]{0}=0$

$f(-1)=\sqrt[3]{-1+2}=\sqrt[3]{1}=1$

$f(6)=\sqrt[3]{6+2}=\sqrt[3]{8}=2$



103. $q+p^2$

104. $11\sqrt[3]{x}$

105. $\dfrac{6}{\sqrt[3]{x}}$

106. $y-\sqrt{x}$

107. $s^2 = 64$

$s = \sqrt{64}$

$= 8$ in.

108. $s^2 = 121$

$s = \sqrt{121}$

$= 11$ m

109. $\sqrt{69} \approx 8.3066$

110. $\sqrt{5798} \approx 76.1446$

111. $2+\sqrt[3]{5} \approx 2+1.7100$

$= 3.7100$

112. $3-2\sqrt[4]{10} \approx -0.5566$

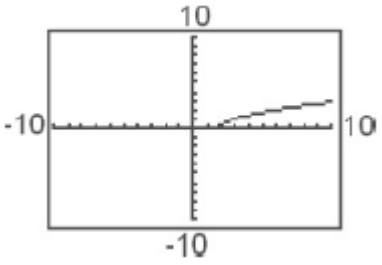
113. $7\sqrt[4]{25} \approx 15.6525$

114. $-3\sqrt[3]{9} \approx -6.2403$

115. $\dfrac{3-\sqrt{19}}{11} \approx -0.1235$

116. $\dfrac{5+2\sqrt{15}}{12} \approx 1.0622$

117.



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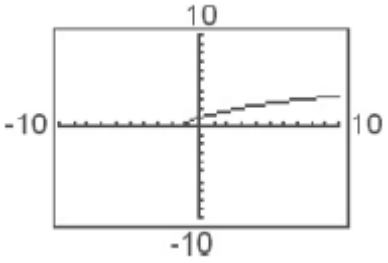

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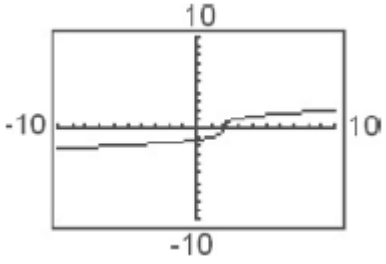

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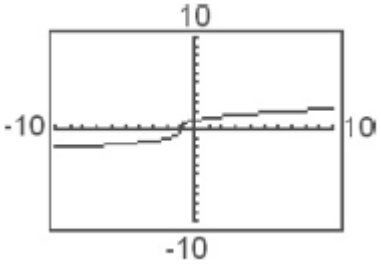


Section 6.2 Practice Exercises

1. **a.** $\sqrt[n]{a}$

b. $\left(\sqrt[n]{a}\right)^m$ or $\sqrt[n]{a^m}$

c. $\dfrac{1}{\sqrt{x}}$

d. 2; $\dfrac{1}{2}$

2. Index: 3; radicand: 27

3. $\sqrt{25} = 5$

4. $\sqrt[3]{8} = 2$

5. $\sqrt[4]{81} = 3$

6. $\left(\sqrt[4]{16}\right)^3 = 2^3 = 8$

7. $144^{1/2} = \sqrt{144} = 12$

8. $16^{1/4} = \sqrt[4]{16} = 2$

9. $-144^{1/2} = -\sqrt{144} = -12$

10. $-16^{1/4} = -\sqrt[4]{16} = -2$

11. $\left(-144\right)^{1/2} = \sqrt{-144}$ is not a real number.

12. $\left(-16\right)^{1/4} = \sqrt[4]{-16}$ is not a real number.

13. $\left(-64\right)^{1/3} = \sqrt[3]{-64} = -4$

14. $\left(-32\right)^{1/5} = \sqrt[5]{-32} = -2$

15. $25^{-1/2} = \dfrac{1}{25^{1/2}} = \dfrac{1}{\sqrt{25}} = \dfrac{1}{5}$

16. $27^{-1/3} = \dfrac{1}{27^{1/3}} = \dfrac{1}{\sqrt[3]{27}} = \dfrac{1}{3}$

17. $-49^{-1/2} = -\dfrac{1}{49^{1/2}} = -\dfrac{1}{\sqrt{49}} = -\dfrac{1}{7}$

18. $-64^{-1/2} = -\dfrac{1}{64^{1/2}} = -\dfrac{1}{\sqrt{64}} = -\dfrac{1}{8}$

19. $a^{m/n} = \sqrt[n]{a^m}$; The numerator of the exponent represents the power of the base. The denominator of the exponent represents the index of the radical.

20. $\left(\sqrt[3]{8}\right)^4$ works within the parentheses first. Taking the root first results in a smaller number that must be raised to the fourth power. $\sqrt[3]{8^4}$ takes the fourth power of 8

first. This results in a larger number whose cube root may be difficult to identify.

21. **a.** $16^{3/4} = \left(\sqrt[4]{16}\right)^3 = 2^3 = 8$

b. $-16^{3/4} = -\left(\sqrt[4]{16}\right)^3 = -\left(2^3\right) = -8$

c. $(-16)^{3/4} = \left(\sqrt[4]{-16}\right)^3$ is not a real number.

d. $16^{-3/4} = \dfrac{1}{16^{3/4}} = \dfrac{1}{\left(\sqrt[4]{16}\right)^3} = \dfrac{1}{2^3} = \dfrac{1}{8}$

e. $-16^{-3/4} = \dfrac{1}{-16^{3/4}} = \dfrac{1}{-\left(\sqrt[4]{16}\right)^3} = \dfrac{1}{-\left(2^3\right)}$

$= \dfrac{1}{8}$

f. $(-16)^{-3/4} = \dfrac{1}{(-16)^{3/4}}$

$= \dfrac{1}{\left(\sqrt[4]{-16}\right)^3}$ is not a real number.

22. **a.** $81^{3/4} = \left(\sqrt[4]{81}\right)^3 = 3^3 = 27$

b. $-81^{3/4} = -\left(\sqrt[4]{81}\right)^3 = -\left(3^3\right) = -27$

c. $(-81)^{3/4} = \left(\sqrt[4]{-81}\right)^3$ is not a real number.

d. $81^{-3/4} = \dfrac{1}{81^{3/4}} = \dfrac{1}{\left(\sqrt[4]{81}\right)^3} = \dfrac{1}{3^3} = \dfrac{1}{27}$

e. $-81^{-3/4} = \dfrac{1}{-81^{3/4}} = \dfrac{1}{-\left(\sqrt[4]{81}\right)^3} = \dfrac{1}{-\left(3^3\right)}$

$= -\dfrac{1}{27}$

f. $(-81)^{-3/4} = \dfrac{1}{(-81)^{3/4}}$

$= \dfrac{1}{\left(\sqrt[4]{-81}\right)^3}$ is not a real number.

23. **a.** $25^{3/2} = \left(\sqrt{25}\right)^3 = 5^3 = 125$

b. $-25^{3/2} = -\left(\sqrt{25}\right)^3 = -\left(5^3\right) = -125$

c. $(-25)^{3/2} = \left(\sqrt{-25}\right)^3$ is not a real number.

d. $25^{-3/2} = \dfrac{1}{25^{3/2}} = \dfrac{1}{\left(\sqrt{25}\right)^3} = \dfrac{1}{5^3} = \dfrac{1}{125}$

e. $-25^{-3/2} = \dfrac{1}{-25^{3/2}} = \dfrac{1}{-\left(\sqrt{25}\right)^3} = \dfrac{1}{-\left(5^3\right)} = -\dfrac{1}{125}$

24. **a.** $4^{3/2} = \left(\sqrt{4}\right)^3 = 2^3 = 8$

b. $-4^{3/2} = -\left(\sqrt{4}\right)^3 = -\left(2^3\right) = -8$

c. $(-4)^{3/2} = \left(\sqrt{-4}\right)^3$ is not a real number.

d. $4^{-3/2} = \dfrac{1}{4^{3/2}} = \dfrac{1}{\left(\sqrt{4}\right)^3} = \dfrac{1}{2^3} = \dfrac{1}{8}$

e. $-4^{-3/2} = \dfrac{1}{-4^{3/2}} = \dfrac{1}{-\left(\sqrt{4}\right)^3} = \dfrac{1}{-\left(2^3\right)} = -\dfrac{1}{8}$

f. $(-25)^{-3/2} = \dfrac{1}{(-25)^{3/2}}$

$= \dfrac{1}{\left(\sqrt{-25}\right)^3}$ is not a real number.

f. $(-4)^{-3/2} = \dfrac{1}{(-4)^{3/2}}$

$= \dfrac{1}{\left(\sqrt{-4}\right)^3}$ is not a real number.

25. $64^{-3/2} = \dfrac{1}{64^{3/2}} = \dfrac{1}{\left(\sqrt{64}\right)^3} = \dfrac{1}{8^3} = \dfrac{1}{512}$

26. $81^{-3/2} = \dfrac{1}{81^{3/2}} = \dfrac{1}{\left(\sqrt{81}\right)^3} = \dfrac{1}{9^3} = \dfrac{1}{729}$

27. $243^{3/5} = \left(\sqrt[5]{243}\right)^3 = 3^3 = 27$

28. $1^{5/3} = \left(\sqrt[3]{1}\right)^5 = 1^5 = 1$

29. $-27^{-4/3} = \dfrac{1}{-27^{4/3}} = \dfrac{1}{-\left(\sqrt[3]{27}\right)^4} = \dfrac{1}{-(3)^4} = -\dfrac{1}{81}$

30. $-16^{-5/4} = \dfrac{1}{-16^{5/4}} = \dfrac{1}{-\left(\sqrt[4]{16}\right)^5} = \dfrac{1}{-(2)^5} = -\dfrac{1}{32}$

31. $\left(\dfrac{100}{9}\right)^{-3/2} = \dfrac{1}{\left(\dfrac{100}{9}\right)^{3/2}} = \dfrac{1}{\left(\sqrt{\dfrac{100}{9}}\right)^3}$

$= \dfrac{1}{\left(\dfrac{10}{3}\right)^3} = \dfrac{1}{\dfrac{1000}{27}}$

$= \dfrac{27}{1000}$

32. $\left(\dfrac{49}{100}\right)^{-1/2} = \dfrac{1}{\left(\dfrac{49}{100}\right)^{1/2}}$

$= \dfrac{1}{\sqrt{\dfrac{49}{100}}} = \dfrac{1}{\dfrac{7}{10}}$

$= \dfrac{10}{7}$

33. $(-4)^{-3/2} = \dfrac{1}{(-4)^{3/2}}$

$= \dfrac{1}{\left(\sqrt{-4}\right)^3}$ is not a real number

34. $(-49)^{-3/2} = \dfrac{1}{(-49)^{3/2}}$

$= \dfrac{1}{\left(\sqrt{-49}\right)^3}$ is not a real number

35. $(-8)^{1/3} = \sqrt[3]{-8} = -2$

36. $(-9)^{1/2} = \sqrt{-9}$ is not a real number

37. $-8^{1/3} = -\sqrt[3]{8} = -2$

38. $-9^{1/2} = -\sqrt{9} = -3$

39. $\dfrac{1}{36^{-1/2}} = 36^{1/2} = \sqrt{36} = 6$

40. $\dfrac{1}{16^{-1/2}} = 16^{1/2} = \sqrt{16} = 4$

41. $\dfrac{1}{1000^{-1/3}} = 1000^{1/3} = \sqrt[3]{1000} = 10$

42. $\dfrac{1}{81^{-3/4}} = 81^{3/4} = \left(\sqrt[4]{81}\right)^3 = 3^3 = 27$

43. $\left(\dfrac{1}{8}\right)^{2/3} + \left(\dfrac{1}{4}\right)^{1/2} = \left(\sqrt[3]{\dfrac{1}{8}}\right)^2 + \sqrt{\dfrac{1}{4}} = \left(\dfrac{1}{2}\right)^2 + \dfrac{1}{2}$
$= \dfrac{1}{4} + \dfrac{1}{2} = \dfrac{3}{4}$

44. $\left(\dfrac{1}{8}\right)^{-2/3} + \left(\dfrac{1}{4}\right)^{-1/2} = 8^{2/3} + 4^{1/2} = \left(\sqrt[3]{8}\right)^2 + \sqrt{4}$
$= (2)^2 + 2 = 4 + 2 = 6$

45. $\left(\dfrac{1}{16}\right)^{-3/4} - \left(\dfrac{1}{49}\right)^{-1/2} = 16^{3/4} - 49^{1/2}$
$= \left(\sqrt[4]{16}\right)^3 - \sqrt{49} = (2)^3 - 7 = 8 - 7 = 1$

46. $\left(\dfrac{1}{16}\right)^{1/4} - \left(\dfrac{1}{49}\right)^{1/2} = \sqrt[4]{\dfrac{1}{16}} - \sqrt{\dfrac{1}{49}} = \dfrac{1}{2} - \dfrac{1}{7}$
$= \dfrac{1}{2}\cdot\dfrac{7}{7} - \dfrac{1}{7}\cdot\dfrac{2}{2} = \dfrac{7-2}{14} = \dfrac{5}{14}$

47. $\left(\dfrac{1}{4}\right)^{1/2} + \left(\dfrac{1}{64}\right)^{-1/3} = \left(\dfrac{1}{4}\right)^{1/2} + (64)^{1/3}$
$= \sqrt{\dfrac{1}{4}} + \sqrt[3]{64} = \dfrac{1}{2} + 4 = \dfrac{9}{2}$

48. $\left(\dfrac{1}{36}\right)^{1/2} + \left(\dfrac{1}{64}\right)^{-5/6} = \left(\dfrac{1}{36}\right)^{1/2} + (64)^{5/6}$
$= \sqrt{\dfrac{1}{36}} + \left(\sqrt[6]{64}\right)^5 = \dfrac{1}{6} + 2^5 = \dfrac{1}{6} + 32 = \dfrac{193}{6}$

49. $q^{2/3} = \sqrt[3]{q^2}$

50. $t^{3/5} = \sqrt[5]{t^3}$

51. $6y^{3/4} = 6\sqrt[4]{y^3}$

52. $8b^{4/9} = 8\sqrt[9]{b^4}$

53. $x^{2/3}y^{1/3} = \left(x^2y\right)^{1/3} = \sqrt[3]{x^2y}$

54. $c^{2/5}d^{3/5} = \left(c^2d^3\right)^{1/5} = \sqrt[5]{c^2d^3}$

55. $6r^{-2/5} = 6\cdot\dfrac{1}{r^{2/5}} = \dfrac{6}{\sqrt[5]{r^2}}$

56. $7x^{-3/4} = 7\cdot\dfrac{1}{x^{3/4}} = \dfrac{7}{\sqrt[4]{x^3}}$

57. $\sqrt[3]{x} = x^{1/3}$

58. $\sqrt[4]{a} = a^{1/4}$

59. $10\sqrt{b} = 10b^{1/2}$

60. $-2\sqrt[3]{t} = -2t^{1/3}$

61. $\sqrt[3]{y^2} = y^{2/3}$

62. $\sqrt[6]{z^5} = z^{5/6}$

63. $\sqrt[4]{a^2b^3} = \left(a^2b^3\right)^{1/4}$

64. $\sqrt{abc} = \left(abc\right)^{1/2}$

65. $x^{1/4}x^{-5/4} = x^{1/4+(-5/4)} = x^{-1} = \dfrac{1}{x}$

66. $2^{2/3}2^{-5/3} = 2^{(2/3)+(-5/3)} = 2^{-1} = \dfrac{1}{2}$

67. $\dfrac{p^{5/3}}{p^{2/3}} = p^{(5/3)-(2/3)} = p^1 = p$

68. $\dfrac{q^{5/4}}{q^{1/4}} = q^{(5/4)-(1/4)} = q^1 = q$

69. $\left(y^{1/5}\right)^{10} = y^{(1/5)(10)} = y^2$

70. $\left(x^{1/2}\right)^8 = x^{(1/2)(8)} = x^4$

71. $6^{-1/5}6^{3/5} = 6^{(-1/5)+(3/5)} = 6^{2/5}$

72. $a^{-1/3}a^{2/3} = a^{(-1/3)+(2/3)} = a^{1/3}$

73. $\dfrac{4t^{-1/3}}{t^{4/3}} = 4t^{(-1/3)-(4/3)} = 4t^{-5/3} = \dfrac{4}{t^{5/3}}$

74. $\dfrac{5s^{-1/3}}{s^{5/3}} = 5s^{(-1/3)-(5/3)} = 5s^{-6/3} = \dfrac{5}{s^2}$

75. $\left(a^{1/3}a^{1/4}\right)^{12} = \left(a^{1/3}\right)^{12}\left(a^{1/4}\right)^{12} = a^{12/3}a^{12/4}$
$= a^4a^3 = a^{3+4} = a^7$

76. $\left(x^{2/3}x^{1/2}\right)^6 = \left(x^{2/3}\right)^6\left(x^{1/2}\right)^6 = x^{12/3}x^{6/2}$
$= x^4x^3 = x^{3+4} = x^7$

77. $\left(5a^2c^{-1/2}d^{1/2}\right)^2 = 5^2\left(a^2\right)^2\left(c^{-1/2}\right)^2\left(d^{1/2}\right)^2$
$= 5^2a^4c^{-2/2}d^{2/2} = 25a^4c^{-1}d^1$
$= \dfrac{25a^4d}{c}$

78. $\left(2x^{-1/3}y^2z^{5/3}\right)^3 = 2^3\left(x^{-1/3}\right)^3\left(y^2\right)^3\left(z^{5/3}\right)^3$
$= 2^3x^{-3/3}y^6z^{15/3} = 8x^{-1}y^6z^5$
$= \dfrac{8y^6z^5}{x}$

79. $\left(\dfrac{x^{-2/3}}{y^{-3/4}}\right)^{12} = \dfrac{\left(x^{-2/3}\right)^{12}}{\left(y^{-3/4}\right)^{12}} = \dfrac{x^{-24/3}}{y^{-36/4}} = \dfrac{x^{-8}}{y^{-9}} = \dfrac{y^9}{x^8}$

80. $\left(\dfrac{m^{-1/4}}{n^{-1/2}}\right)^{-4} = \dfrac{\left(m^{-1/4}\right)^{-4}}{\left(n^{-1/2}\right)^{-4}} = \dfrac{m^{4/4}}{n^{4/2}} = \dfrac{m^1}{n^2} = \dfrac{m}{n^2}$

81. $\left(\dfrac{16w^{-2}z}{2wz^{-8}}\right)^{1/3} = \left(8w^{-2-1}z^{1-(-8)}\right)^{1/3} = \left(8w^{-3}z^{9}\right)^{1/3}$

$= 8^{1/3}\left(w^{-3}\right)^{1/3}\left(z^{9}\right)^{1/3} = \sqrt[3]{8}w^{-3/3}z^{9/3}$

$= 2w^{-1}z^{3} = \dfrac{2z^3}{w}$

82. $\left(\dfrac{50p^{-1}q}{2pq^{-3}}\right)^{1/2} = \left(25p^{-1-1}q^{1-(-3)}\right)^{1/2} = \left(25p^{-2}q^{4}\right)^{1/2}$

$= 25^{1/2}\left(p^{-2}\right)^{1/2}\left(q^{4}\right)^{1/2} = \sqrt{25}p^{-2/2}q^{4/2}$

$= 5p^{-1}q^{2} = \dfrac{5q^2}{p}$

83. $\left(25x^2y^4z^6\right)^{1/2} = 25^{1/2}\left(x^2\right)^{1/2}\left(y^4\right)^{1/2}\left(z^6\right)^{1/2}$

$= \sqrt{25}x^{2/2}y^{4/2}z^{6/2}$

$= 5xy^2z^3$

84. $\left(8a^6b^3c^9\right)^{2/3} = 8^{2/3}\left(a^6\right)^{2/3}\left(b^3\right)^{2/3}\left(c^9\right)^{2/3}$

$= \left(\sqrt[3]{8}\right)^2 a^{12/3}b^{6/3}c^{18/3} = 2^2a^4b^2c^6$

$= 4a^4b^2c^6$

85. $\left(x^2y^{-1/3}\right)^6\left(x^{1/2}yz^{2/3}\right)^2$

$= \left(x^2\right)^6\left(y^{-1/3}\right)^6\left(x^{1/2}\right)^2y^2\left(z^{2/3}\right)^2$

$= x^{12}y^{-2}x^{1}y^{2}z^{4/3} = x^{12+1}y^{-2+2}z^{4/3}$

$= x^{13}y^{0}z^{4/3} = x^{13}z^{4/3}$

86. $\left(a^{-1/3}b^{1/2}\right)^4\left(a^{-1/2}b^{3/5}\right)^{10}$

$= \left(a^{-1/3}\right)^4\left(b^{1/2}\right)^4\left(a^{-1/2}\right)^{10}\left(b^{3/5}\right)^{10}$

$= a^{-4/3}b^{4/2}a^{-10/2}b^{30/5} = a^{-4/3}b^2a^{-5}b^6$

$= a^{(-4/3)+(-5)}b^{2+6} = a^{-19/3}b^8 = \dfrac{b^8}{a^{19/3}}$

87. $\left(\dfrac{x^{3m}y^{2m}}{z^{5m}}\right)^{1/m} = \dfrac{\left(x^{3m}\right)^{1/m}\left(y^{2m}\right)^{1/m}}{\left(z^{5m}\right)^{1/m}}$

$= \dfrac{x^{3m/m}y^{2m/m}}{z^{5m/m}} = \dfrac{x^3y^2}{z^5}$

88. $\left(\dfrac{a^{4n}b^{3n}}{c^n}\right)^{1/n} = \dfrac{\left(a^{4n}\right)^{1/n}\left(b^{3n}\right)^{1/n}}{\left(c^n\right)^{1/n}}$

$= \dfrac{a^{4n/n}b^{3n/n}}{c^{n/n}} = \dfrac{a^4b^3}{c^1} = \dfrac{a^4b^3}{c}$

89. **a.** $r = \left(\dfrac{A}{P}\right)^{1/t} - 1$

$r = \left(\dfrac{16{,}802}{10{,}000}\right)^{1/5} - 1 \approx 0.109 = 10.9\%$

b. $r = \left(\dfrac{18{,}000}{10{,}000}\right)^{1/7} - 1 \approx 0.088 = 8.8\%$

c. The account in part (a).

90. **a.** $s = A^{1/2}$

b. $s = 100^{1/2} = \sqrt{100} = 10$ in

$s = A^{1/2}$

$s = 72^{1/2}$

$= \sqrt{72}$

≈ 8.5 in

91. $r = \left(\frac{3V}{4\pi}\right)^{1/3}$

$r = \left(\frac{3(85)}{4\pi}\right)^{1/3} = \sqrt[3]{\frac{3(85)}{4\pi}} \approx 2.7 \text{ in}$

92. They are not the same. For example:

$(9+16)^{1/2} = 25^{1/2} = \sqrt{25} = 5$

$9^{1/2} + 16^{1/2} = \sqrt{9} + \sqrt{16} = 3 + 4 = 7$

93. $\sqrt[6]{y^3} = y^{3/6} = y^{1/2} = \sqrt{y}$

94. $\sqrt[4]{w^2} = w^{2/4} = w^{1/2} = \sqrt{w}$

95. $\sqrt[12]{z^3} = z^{3/12} = z^{1/4} = \sqrt[4]{z}$

96. $\sqrt[18]{t^3} = t^{3/18} = t^{1/6} = \sqrt[6]{t}$

97. $\sqrt[9]{x^6} = x^{6/9} = x^{2/3} = \sqrt[3]{x^2}$

98. $\sqrt[12]{p^9} = p^{9/12} = p^{3/4} = \sqrt[4]{p^3}$

99. $\sqrt[6]{x^3y^6} = \left(x^3y^6\right)^{1/6} = x^{3/6}y^{6/6} = x^{1/2}y = y\sqrt{x}$

100. $\sqrt[8]{m^2p^8} = m^{2/8}p^{8/8} = m^{1/4}p = p\sqrt[4]{m}$

101. $\sqrt{16x^8y^6} = 16^{1/2}x^{8/2}y^{6/2} = 4x^4y^3$

102. $\sqrt{81a^{12}b^{20}} = 81^{1/2}a^{12/2}b^{20/2} = 9a^6b^{10}$

103. $\sqrt[3]{8x^3y^2z} = 2x\sqrt[3]{y^2z}$

104. $\sqrt[3]{64m^2n^3p} = 4n\sqrt[3]{m^2p}$

105. $\sqrt{\sqrt[3]{x}} = \sqrt{x^{1/3}} = \left(x^{1/3}\right)^{1/2} = x^{(1/3)(1/2)}$

$= x^{1/6} = \sqrt[6]{x}$

106. $\sqrt[3]{\sqrt{x}} = \sqrt[3]{x^{1/2}} = \left(x^{1/2}\right)^{1/3} = x^{(1/2)(1/3)}$

$= x^{1/6} = \sqrt[6]{x}$

107. $\sqrt[5]{\sqrt[3]{w}} = \sqrt[5]{w^{1/3}} = \left(w^{1/3}\right)^{1/5} = w^{(1/3)(1/5)}$

$= w^{1/15} = \sqrt[15]{w}$

108. $\sqrt[3]{\sqrt[4]{w}} = \sqrt[3]{w^{1/4}} = \left(w^{1/4}\right)^{1/3} = w^{(1/4)(1/3)}$

$= w^{1/12} = \sqrt[12]{w}$

109. $9^{1/2} = 3$

110. $125^{-1/3} = 0.2$

111. $50^{-1/4} \approx 0.3761$

112. $172^{3/5} \approx 21.9441$

113. $\sqrt[3]{5^2} \approx 2.9240$

114. $\sqrt[4]{6^3} \approx 3.8337$

115. $\sqrt{10^3} \approx 31.6228$

116. $\sqrt[3]{16} \approx 2.5198$

Section 6.3 Practice Exercises

1. **a.** $\sqrt[n]{a}$; $\sqrt[n]{b}$

b. The exponent within the radicand is greater than the index.

c. is not

d. 3

e. t^{12}

f. No. $\sqrt{2}$ is an irrational number and the decimal form is a nonterminating, nonrepeating decimal.

2.
$$\begin{aligned}\left(a^2b^{-4}\right)^{1/2}\left(\frac{a}{b^{-3}}\right) &= \left(a^2\right)^{1/2}\left(b^{-4}\right)^{1/2}\left(\frac{a}{b^{-3}}\right)\\ &= a^{2/2}b^{-4/2}\left(\frac{a}{b^{-3}}\right)\\ &= ab^{-2}\frac{a}{b^{-3}}\\ &= a^{1+1}b^{(-2)-(-3)}\\ &= a^2b^1\\ &= a^2b\end{aligned}$$

3.
$$\begin{aligned}\left(\frac{p^4}{q^{-6}}\right)^{-1/2}\left(p^3q^{-2}\right) &= \frac{\left(p^4\right)^{-1/2}}{\left(q^{-6}\right)^{-1/2}}\left(p^3q^{-2}\right)\\ &= \frac{p^{-4/2}}{q^{6/2}}\left(p^3q^{-2}\right)\\ &= \frac{p^{-2}}{q^3}\cdot p^3q^{-2} = p^{-2+3}q^{-2-3}\\ &= pq^{-5} = \frac{p}{q^5}\end{aligned}$$

4.
$$\begin{aligned}\left(x^{1/3}y^{5/6}\right)^{-6} &= \left(x^{1/3}\right)^{-6}\left(y^{5/6}\right)^{-6}\\ &= x^{-6/3}y^{-30/6}\\ &= x^{-2}y^{-5}\\ &= \frac{1}{x^2y^5}\end{aligned}$$

5. $x^{4/7} = \sqrt[7]{x^4}$

6. $y^{2/5} = \sqrt[5]{y^2}$

7. $\sqrt{y^9} = y^{9/2}$

8. $\sqrt[3]{x^2} = x^{2/3}$

9. $\sqrt{x^5} = \sqrt{x^4\cdot x} = \sqrt{x^4}\cdot\sqrt{x} = x^2\sqrt{x}$

10. $\sqrt{p^{15}} = \sqrt{p^{14}\cdot p} = \sqrt{p^{14}}\cdot\sqrt{p} = p^7\sqrt{p}$

11. $\sqrt[3]{q^7} = \sqrt[3]{q^6\cdot q} = \sqrt[3]{q^6}\cdot\sqrt[3]{q} = q^2\sqrt[3]{q}$

12. $\sqrt[3]{r^{17}} = \sqrt[3]{r^{15}\cdot r^2} = \sqrt[3]{r^{15}}\cdot\sqrt[3]{r^2} = r^5\sqrt[3]{r^2}$

13.
$$\begin{aligned}\sqrt{a^5b^4} &= \sqrt{a^4b^4\cdot a} = \sqrt{a^4b^4}\cdot\sqrt{a}\\ &= a^2b^2\sqrt{a}\end{aligned}$$

14.
$$\begin{aligned}\sqrt{c^9d^6} &= \sqrt{c^8d^6\cdot c} = \sqrt{c^8d^6}\cdot\sqrt{c}\\ &= c^4d^3\sqrt{c}\end{aligned}$$

15. $-\sqrt[4]{x^8y^{13}} = -\sqrt[4]{x^8y^{12}\cdot y} = -\sqrt[4]{x^8y^{12}}\cdot\sqrt[4]{y}$
$= -x^2y^3\sqrt[4]{y}$

16. $-\sqrt[4]{p^{16}q^{17}} = -\sqrt[4]{p^{16}q^{16}\cdot q} = -\sqrt[4]{p^{16}q^{16}}\cdot\sqrt[4]{q}$
$= -p^4q^4\sqrt[4]{q}$

17. $\sqrt{28} = \sqrt{4\cdot 7} = \sqrt{4}\cdot\sqrt{7} = 2\sqrt{7}$

18. $\sqrt{63} = \sqrt{9\cdot 7} = \sqrt{9}\cdot\sqrt{7} = 3\sqrt{7}$

19. $\sqrt{20} = \sqrt{4\cdot 5} = \sqrt{4}\cdot\sqrt{5} = 2\sqrt{5}$

20. $\sqrt{50} = \sqrt{25\cdot 2} = \sqrt{25}\cdot\sqrt{2} = 5\sqrt{2}$

21. $5\sqrt{18} = 5\sqrt{9\cdot 2} = 5\sqrt{9}\cdot\sqrt{2} = 5\cdot 3\sqrt{2}$
$= 15\sqrt{2}$

22. $2\sqrt{24} = 2\sqrt{4\cdot 6} = 2\sqrt{4}\cdot\sqrt{6} = 2\cdot 2\sqrt{6}$
$= 4\sqrt{6}$

23. $\sqrt[3]{54} = \sqrt[3]{27\cdot 2} = \sqrt[3]{27}\cdot\sqrt[3]{2} = 3\sqrt[3]{2}$

24. $\sqrt[3]{250} = \sqrt[3]{125\cdot 2} = \sqrt[3]{125}\cdot\sqrt[3]{2} = 5\sqrt[3]{2}$

25. $\sqrt{25ab^3} = \sqrt{25b^2\cdot ab}$
$= \sqrt{25b^2}\cdot\sqrt{ab} = 5b\sqrt{ab}$

26. $\sqrt{64m^5n^{20}} = \sqrt{64m^4n^{20}\cdot m}$
$= \sqrt{64m^4n^{20}}\cdot\sqrt{m} = 8m^2n^{10}\sqrt{m}$

27. $\sqrt[3]{40x^7} = \sqrt[3]{8\cdot 5\cdot x^6\cdot x} = \sqrt[3]{8x^6}\cdot\sqrt[3]{5x}$
$= 2x^2\sqrt[3]{5x}$

28. $\sqrt[3]{81y^{17}} = \sqrt[3]{27\cdot 3\cdot y^{15}\cdot y^2} = \sqrt[3]{27y^{15}}\cdot\sqrt[3]{3y^2}$
$= 3y^5\sqrt[3]{3y^2}$

29. $\sqrt[3]{-16x^6yz^3} = \sqrt[3]{-8x^6z^3\cdot 2y}$
$= \sqrt[3]{-8x^6z^3}\cdot\sqrt[3]{2y}$
$= -2x^2z\sqrt[3]{2y}$

30. $\sqrt[3]{-192a^6bc^2} = \sqrt[3]{-64a^6\cdot 3bc^2}$
$= \sqrt[3]{-64a^6}\cdot\sqrt[3]{3bc^2}$
$= -4a^2\sqrt[3]{3bc^2}$

31. $\sqrt[4]{80w^4z^7} = \sqrt[4]{16w^4z^4\cdot 5z^3}$
$= \sqrt[4]{16w^4z^4}\cdot\sqrt[4]{5z^3}$
$= 2wz\sqrt[4]{5z^3}$

32. $\sqrt[4]{32p^8qr^5} = \sqrt[4]{16p^8r^4\cdot 2qr}$
$= \sqrt[4]{16p^8r^4}\cdot\sqrt[4]{2qr}$
$= 2p^2r\sqrt[4]{2qr}$

33. $\sqrt{\frac{x^3}{x}} = \sqrt{x^2} = x$

34. $\sqrt{\frac{y^5}{y}} = \sqrt{y^4} = y^2$

35. $\sqrt{\frac{p^7}{p^3}} = \sqrt{p^4} = p^2$

36. $\sqrt{\frac{q^{11}}{q^5}} = \sqrt{q^6} = q^3$

37. $\sqrt{\frac{50}{2}} = \sqrt{25} = 5$

38. $\sqrt{\frac{98}{2}} = \sqrt{49} = 7$

39. $\sqrt[3]{\frac{3}{24}} = \sqrt[3]{\frac{1}{8}} = \frac{1}{2}$

40. $\sqrt[3]{\frac{2}{250}} = \sqrt[3]{\frac{1}{125}} = \frac{1}{5}$

41. $\frac{5\sqrt[3]{16}}{6} = \frac{5\sqrt[3]{8\cdot 2}}{6} = \frac{5\sqrt[3]{8}\cdot\sqrt[3]{2}}{6}$
$= \frac{5\cdot 2\sqrt[3]{2}}{6} = \frac{5\sqrt[3]{2}}{3}$

42. $\frac{7\sqrt{18}}{9} = \frac{7\sqrt{9\cdot 2}}{9} = \frac{7\sqrt{9}\cdot\sqrt{2}}{9}$
$= \frac{7\cdot 3\sqrt{2}}{9} = \frac{7\sqrt{2}}{3}$

43. $\frac{5\sqrt[3]{72}}{12} = \frac{5\sqrt[3]{8\cdot 9}}{12} = \frac{5\sqrt[3]{8}\cdot\sqrt[3]{9}}{12}$
$= \frac{5\cdot 2\sqrt[3]{9}}{12} = \frac{5\sqrt[3]{9}}{6}$

44. $\frac{3\sqrt[3]{250}}{10} = \frac{3\sqrt[3]{125\cdot 2}}{10} = \frac{3\sqrt[3]{125}\cdot\sqrt[3]{2}}{10}$
$= \frac{3\cdot 5\sqrt[3]{2}}{10} = \frac{3\sqrt[3]{2}}{2}$

45. $\sqrt{80} = \sqrt{16\cdot 5} = \sqrt{16}\cdot\sqrt{5} = 4\sqrt{5}$

46. $\sqrt{108} = \sqrt{36\cdot 3} = \sqrt{36}\cdot\sqrt{3} = 6\sqrt{3}$

47. $-6\sqrt{75} = -6\sqrt{25\cdot 3} = -6\sqrt{25}\cdot\sqrt{3}$
$= -6\cdot 5\sqrt{3} = -30\sqrt{3}$

48. $-8\sqrt{8} = -8\sqrt{4\cdot 2} = -8\sqrt{4}\cdot\sqrt{2}$
$= -8\cdot 2\sqrt{2} = -16\sqrt{2}$

49. $\sqrt{25x^4y^3} = \sqrt{25x^4y^2\cdot y}$
$= \sqrt{25x^4y^2}\cdot\sqrt{y}$
$= 5x^2y\sqrt{y}$

50. $\sqrt{125p^3q^2} = \sqrt{25p^2q^2\cdot 5p}$
$= \sqrt{25p^2q^2}\cdot\sqrt{5p}$
$= 5pq\sqrt{5p}$

51. $\sqrt[3]{27x^2y^3z^4} = \sqrt[3]{27y^3z^3\cdot x^2z}$
$= \sqrt[3]{27y^3z^3}\cdot\sqrt[3]{x^2z}$
$= 3yz\sqrt[3]{x^2z}$

52. $\sqrt[3]{108a^3bc^2} = \sqrt[3]{27a^3\cdot 4bc^2}$
$= \sqrt[3]{27a^3}\cdot\sqrt[3]{4bc^2}$
$= 3a\sqrt[3]{4bc^2}$

53. $\sqrt{\frac{12w^5}{3w}} = \sqrt{4w^4} = 2w^2$

54. $\sqrt{\frac{64x^9}{4x^3}} = \sqrt{16x^6} = 4x^3$

55. $\sqrt{\frac{3y^3}{300y^{15}}} = \sqrt{\frac{1}{100y^{12}}} = \frac{1}{10y^6}$

56. $\sqrt{\frac{4h}{100h^5}} = \sqrt{\frac{1}{25h^4}} = \frac{1}{5h^2}$

57. $\sqrt[3]{\dfrac{16a^2b}{2a^2b^4}} = \sqrt[3]{\dfrac{8}{b^3}} = \dfrac{2}{b}$

58. $\sqrt[3]{\dfrac{-27a^4}{8a}} = \sqrt[3]{\dfrac{-27a^3}{8}} = -\dfrac{3a}{2}$

59. $\begin{aligned}\sqrt{2^3a^{14}b^8c^{31}d^{22}} &= \sqrt{2^2a^{14}b^8c^{30}d^{22}\cdot 2c}\\ &= \sqrt{2^2a^{14}b^8c^{30}d^{22}}\cdot\sqrt{2c}\\ &= 2a^7b^4c^{15}d^{11}\sqrt{2c}\end{aligned}$

60. $\begin{aligned}\sqrt{7^5u^{12}v^{20}w^{65}x^{80}} &= \sqrt{7^4u^{12}v^{20}w^{64}x^{80}\cdot 7w}\\ &= \sqrt{7^4u^{12}v^{20}w^{64}x^{80}}\cdot\sqrt{7w}\\ &= 7^2u^6v^{10}w^{32}x^{40}\sqrt{7w}\\ &= 49u^6v^{10}w^{32}x^{40}\sqrt{7w}\end{aligned}$

61. $\begin{aligned}\sqrt[3]{54a^6b^4} &= \sqrt[3]{27a^6b^3\cdot 2b}\\ &= \sqrt[3]{27a^6b^3}\cdot\sqrt[3]{2b}\\ &= 3a^2b\sqrt[3]{2b}\end{aligned}$

62. $\begin{aligned}\sqrt[3]{72m^5n^3} &= \sqrt[3]{8m^3n^3\cdot 9m^2}\\ &= \sqrt[3]{8m^3n^3}\cdot\sqrt[3]{9m^2}\\ &= 2mn\sqrt[3]{9m^2}\end{aligned}$

63. $\begin{aligned}-5a\sqrt{12a^3b^4c} &= -5a\sqrt{4\cdot 3\cdot a^2\cdot a\cdot b^4\cdot c}\\ &= -5a\cdot 2ab^2\sqrt{3ac}\\ &= -10a^2b^2\sqrt{3ac}\end{aligned}$

64. $\begin{aligned}-7y\sqrt{75xy^5z^6} &= -7y\sqrt{25\cdot 3x\cdot y^4yz^6}\\ &= -7y\cdot 5y^2z^3\sqrt{3xy}\\ &= -35y^3z^3\sqrt{3xy}\end{aligned}$

65. $\begin{aligned}\sqrt[4]{7x^5y} &= \sqrt[4]{x^4\cdot 7xy}\\ &= \sqrt[4]{x^4}\cdot\sqrt[4]{7xy}\\ &= x\sqrt[4]{7xy}\end{aligned}$

66. $\begin{aligned}\sqrt[4]{10cd^7} &= \sqrt[4]{d^4\cdot 10cd^3}\\ &= \sqrt[4]{d^4}\cdot\sqrt[4]{10cd^3}\\ &= d\sqrt[4]{10cd^3}\end{aligned}$

67. $\sqrt{54a^4b^2} = \sqrt{6\cdot 9a^4b^2} = 3a^2b\sqrt{6}$

68. $\sqrt{48r^6s^2} = \sqrt{3\cdot 16r^6s^2} = 4r^3s\sqrt{3}$

69. $\dfrac{2\sqrt{27}}{3} = \dfrac{2\sqrt{9\cdot 3}}{3} = \dfrac{2\cdot 3\sqrt{3}}{3} = 2\sqrt{3}$

70. $\dfrac{7\sqrt{24}}{2} = \dfrac{7\sqrt{4\cdot 6}}{2} = \dfrac{7\cdot 2\sqrt{6}}{2} = 7\sqrt{6}$

71. $\dfrac{3\sqrt{125}}{20} = \dfrac{3\sqrt{25\cdot 5}}{20} = \dfrac{3\cdot 5\sqrt{5}}{20} = \dfrac{3\sqrt{5}}{4}$

72. $\dfrac{10\sqrt{63}}{12} = \dfrac{10\sqrt{9\cdot 7}}{12} = \dfrac{10\cdot 3\sqrt{7}}{12} = \dfrac{30\sqrt{7}}{12} = \dfrac{5\sqrt{7}}{2}$

73. $\dfrac{1}{\sqrt[3]{w^6}} = \dfrac{1}{w^2}$

74. $\sqrt{\dfrac{h^2}{49}} = \dfrac{\sqrt{h^2}}{\sqrt{49}} = \dfrac{h}{7}$

75. $\sqrt{k^3} = \sqrt{k^2\cdot k} = k\sqrt{k}$

76. $\sqrt[3]{2x^4} = \sqrt[3]{x^3\cdot 2x} = x\sqrt[3]{2x}$

77. $a^2+b^2=c^2$
$8^2+10^2=c^2$
$64+100=c^2$
$164=c^2$
$c=\sqrt{164}$
$=\sqrt{4\cdot 41}$
$=2\sqrt{41}$ ft

78. $a^2+b^2=c^2$
$4^2+12^2=c^2$
$16+144=c^2$
$160=c^2$
$c=\sqrt{160}$
$=\sqrt{16\cdot 10}$
$=4\sqrt{10}$ in.

79. $a^2+b^2=c^2$
$a^2+12^2=18^2$
$a^2+144=324$
$a^2=180$
$a=\sqrt{180}=\sqrt{36\cdot 5}$
$=6\sqrt{5}$ m

80. $a^2+b^2=c^2$
$3^2+b^2=7^2$
$9+b^2=49$
$b^2=40$
$b=\sqrt{40}=\sqrt{4\cdot 10}$
$=2\sqrt{10}$ cm

81. $a^2+b^2=c^2$
$90^2+90^2=c^2$
$8100+8100=c^2$
$16200=c^2$
$c=\sqrt{16200}$
$=\sqrt{8100\cdot 2}$
$=90\sqrt{2}$ ft ≈ 127.3 ft

The distance is $90\sqrt{2}$ ft or approximately 127.3 ft.

82. $a^2+b^2=c^2$
$60^2+b^2=100^2$
$3600+b^2=10{,}000$
$b^2=6400$
$b=\sqrt{6400}$
$=80$ft

Since Linda is 5 ft tall, the kite is 85 ft above the ground.

83. Let b = the distance from B to C
$40^2+b^2=50^2$
$1600+b^2=2500$
$b^2=900$
$b=\sqrt{900}=30$ mi

The distance along the four lane highway is 40 + 30 = 70 mi. The time from A to C via B is

84. Let x = the width of the pasture
$80^2+x^2=330^2$
$6400+x^2=108{,}900$
$x^2=102{,}500$
$x=\sqrt{102{,}500}=320.156$ ft

The perimeter of the pasture is:
$P=2(80)+2(320.156)$
$=160+640.4$
$=800.312$ ft

$t = \dfrac{70}{55} = \dfrac{14}{11} \approx 1.27$ hr.

The time from A to C along the direct route is $t = \dfrac{50}{35} = \dfrac{10}{7} \approx 1.43$ hr.

The route form A to C via B is the faster.

The cost of the fence is:

$C = 3.29(800.312)$

$\approx \$2633$

Section 6.4 Practice Exercises

1. **a.** index; radicand

b. $2\sqrt{3x}$

c. cannot; can

d. $4\sqrt{2}$

2. $\sqrt[3]{-16s^4t^9} = \sqrt[3]{-8s^3t^9 \cdot 2s}$

$= \sqrt[3]{-8s^3t^9} \cdot \sqrt[3]{2s}$

$= -2st^3\sqrt[3]{2s}$

3. $-\sqrt[4]{x^7y^4} = -\sqrt[4]{x^4y^4 \cdot x^3}$

$= -\sqrt[4]{x^4y^4} \cdot \sqrt[4]{x^3}$

$= -xy\sqrt[4]{x^3}$

4. $\sqrt{36a^2b^3} = \sqrt{36a^2b^2 \cdot b}$

$= \sqrt{36a^2b^2} \cdot \sqrt{b} = 6ab\sqrt{b}$

5. $\dfrac{\sqrt[3]{7b^8}}{\sqrt[3]{56b^2}} = \sqrt[3]{\dfrac{7b^8}{56b^2}} = \sqrt[3]{\dfrac{b^6}{8}} = \dfrac{b^2}{2}$

6. $\left(4x^2\right)^{1/3} = \sqrt[3]{4x^2}$

7. $\sqrt[4]{x^3y} = \left(x^3y\right)^{1/4} = \left(x^3\right)^{1/4} y^{1/4} = x^{3/4}y^{1/4}$

8. $32^{-1/5} = \dfrac{1}{32^{1/5}} = \dfrac{1}{\sqrt[5]{32}} = \dfrac{1}{2}$

9. $y^{2/3}y^{1/4} = y^{(2/3)+(1/4)} = y^{11/12}$

10. $\left(x^{1/2}y^{-3/4}\right)^{-4} = \left(x^{1/2}\right)^{-4}\left(y^{-3/4}\right)^{-4} = x^{-4/2}y^{12/4} = x^{-2}y^3 = \dfrac{y^3}{x^2}$

11. **a.** $\sqrt{2}$ and $\sqrt[3]{2}$ are not like radicals. The indices are different.

b. $\sqrt{2}$ and $3\sqrt{2}$ are like radicals.

c. $\sqrt{2}$ and $\sqrt{5}$ are not like radicals. The radicands are different.

12. **a.** $7\sqrt[3]{x}$ and $\sqrt[3]{x}$ are like radicals.

b. $\sqrt[3]{x}$ and $\sqrt[4]{x}$ are not like radicals. The indices are different.

c. $2\sqrt[4]{x}$ and $x\sqrt[4]{2}$ are not like radicals. The radicands are different.

13. **a.** $7\sqrt{5}+4\sqrt{5}$ and $7x+4x$
Both expressions can be simplified by using the distributive property.

b. $-2\sqrt{6}-9\sqrt{3}$ and $-2x-9y$
Neither expression can be simplified because they do not contain like radicals or like terms.

14. **a.** $-4\sqrt{3}+5\sqrt{3}$ and $-4z+5z$
Both expressions can be simplified by using the distributive property.

b. $13\sqrt{7}-18$ and $13a-18$
Neither expression can be simplified because they do not contain like radicals or like terms.

15. $3\sqrt{5}+6\sqrt{5}=(3+6)\sqrt{5}$
$=9\sqrt{5}$

16. $5\sqrt{a}+3\sqrt{a}=(5+3)\sqrt{a}$
$=8\sqrt{a}$

17. $3\sqrt[3]{tw}-2\sqrt[3]{tw}+\sqrt[3]{tw}=(3-2+1)\sqrt[3]{tw}$
$=2\sqrt[3]{tw}$

18. $6\sqrt[3]{7}-2\sqrt[3]{7}+\sqrt[3]{7}=(6-2+1)\sqrt[3]{7}$
$=5\sqrt[3]{7}$

19. $6\sqrt{10}-\sqrt{10}=(6-1)\sqrt{10}=5\sqrt{10}$

20. $13\sqrt{11}-\sqrt{11}=(13-1)\sqrt{11}=12\sqrt{11}$

21. $\sqrt[4]{3}+7\sqrt[4]{3}-\sqrt[4]{14}=(1+7)\sqrt[4]{3}-\sqrt[4]{14}$
$=8\sqrt[4]{3}-\sqrt[4]{14}$

22. $2\sqrt{11}+3\sqrt{13}+5\sqrt{11}=(2+5)\sqrt{11}+3\sqrt{13}$
$=7\sqrt{11}+3\sqrt{13}$

23. $8\sqrt{x}+2\sqrt{y}-6\sqrt{x}=(8-6)\sqrt{x}+2\sqrt{y}$
$=2\sqrt{x}+2\sqrt{y}$

24. $10\sqrt{10}-8\sqrt{10}+\sqrt{2}=(10-8)\sqrt{10}+\sqrt{2}$
$=2\sqrt{10}+\sqrt{2}$

25. $\sqrt[3]{ab}+a\sqrt[3]{b}$ cannot be simplified further.

26. $x\sqrt[4]{y}-y\sqrt[4]{x}$ cannot be simplified further.

27. $\sqrt{2t}+\sqrt[3]{2t}$ cannot be simplified further.

28. $\sqrt[4]{5c}+\sqrt[3]{5c}$ cannot be simplified further.

29. $\frac{5}{6}z\sqrt[3]{6}+\frac{7}{9}z\sqrt[3]{6}=\left(\frac{5}{6}+\frac{7}{9}\right)z\sqrt[3]{6}$
$=\left(\frac{15}{18}+\frac{14}{18}\right)z\sqrt[3]{6}$
$=\frac{29}{18}z\sqrt[3]{6}$

30. $\frac{3}{4}a\sqrt[4]{b}+\frac{1}{6}a\sqrt[4]{b}=\left(\frac{3}{4}+\frac{1}{6}\right)a\sqrt[4]{b}$
$=\left(\frac{9}{12}+\frac{2}{12}\right)a\sqrt[4]{b}$
$=\frac{11}{12}a\sqrt[4]{b}$

31. $0.81x\sqrt{y}-0.11x\sqrt{y}=\left(0.81-0.11\right)x\sqrt{y}$
$=0.70x\sqrt{y}$

32. $7.5\sqrt{pq}-6.3\sqrt{pq}=\left(7.5-6.3\right)\sqrt{pq}$
$=1.2\sqrt{pq}$

33. Simplify each radical. Then add like radicals.

$3\sqrt{2}+7\sqrt{50}=3\sqrt{2}+7\sqrt{25\cdot 2}$
$=3\sqrt{2}+7\cdot 5\sqrt{2}$
$=3\sqrt{2}+35\sqrt{2}$
$=\left(3+35\right)\sqrt{2}$
$=38\sqrt{2}$

34. Simplify each radical. Then subtract *like* radicals.

$\sqrt{12x}-\sqrt{75x}=\sqrt{4\cdot 3x}-\sqrt{25\cdot 3x}$
$=2\sqrt{3x}-5\sqrt{3x}$
$=-3\sqrt{3x}$

35. $\sqrt{36}+\sqrt{81}=6+9$
$=15$

36. $3\sqrt{80}-5\sqrt{45}=3\sqrt{16\cdot 5}-5\sqrt{9\cdot 5}$
$=3\cdot 4\sqrt{5}-5\cdot 3\sqrt{5}$
$=12\sqrt{5}-15\sqrt{5}$
$=\left(12-15\right)\sqrt{5}$
$=-3\sqrt{5}$

37. $2\sqrt{12}+\sqrt{48}=2\sqrt{4\cdot 3}+\sqrt{16\cdot 3}$
$=2\cdot 2\sqrt{3}+4\sqrt{3}$
$=4\sqrt{3}+4\sqrt{3}$
$=\left(4+4\right)\sqrt{3}$
$=8\sqrt{3}$

38. $5\sqrt{32}+2\sqrt{50}=5\sqrt{16\cdot 2}+2\sqrt{25\cdot 2}$
$=5\cdot 4\sqrt{2}+2\cdot 5\sqrt{2}$
$=20\sqrt{2}+10\sqrt{2}$
$=30\sqrt{2}$

39. $4\sqrt{7}+\sqrt{63}-2\sqrt{28}=4\sqrt{7}+\sqrt{9\cdot 7}-2\sqrt{4\cdot 7}$
$=4\sqrt{7}+3\sqrt{7}-2\cdot 2\sqrt{7}$
$=4\sqrt{7}+3\sqrt{7}-4\sqrt{7}$
$=\left(4+3-4\right)\sqrt{7}$
$=3\sqrt{7}$

40. $8\sqrt{3}-2\sqrt{27}+\sqrt{75}=8\sqrt{3}-2\sqrt{9\cdot 3}+\sqrt{25\cdot 3}$
$=8\sqrt{3}-2\cdot 3\sqrt{3}+5\sqrt{3}$
$=8\sqrt{3}-6\sqrt{3}+5\sqrt{3}$
$=\left(8-6+5\right)\sqrt{3}$
$=7\sqrt{3}$

41. $5\sqrt{18}+\sqrt{27}-4\sqrt{50}$

$$\begin{aligned}&=5\sqrt{9\cdot 2}+\sqrt{9\cdot 3}-4\sqrt{25\cdot 2}\\&=5\cdot 3\sqrt{2}+3\sqrt{3}-4\cdot 5\sqrt{2}\\&=15\sqrt{2}+3\sqrt{3}-20\sqrt{2}\\&=(15-20)\sqrt{2}+3\sqrt{3}\\&=-5\sqrt{2}+3\sqrt{3}\end{aligned}$$

42. $7\sqrt{40}-\sqrt{8}+4\sqrt{50}$

$$\begin{aligned}&=7\sqrt{4\cdot 10}-\sqrt{4\cdot 2}+4\sqrt{25\cdot 2}\\&=7\cdot 2\sqrt{10}-2\sqrt{2}+4\cdot 5\sqrt{2}\\&=14\sqrt{10}-2\sqrt{2}+20\sqrt{2}\\&=14\sqrt{10}+(-2+20)\sqrt{2}\\&=14\sqrt{10}+18\sqrt{2}\end{aligned}$$

43. $\sqrt[3]{81}-\sqrt[3]{24}=\sqrt[3]{27\cdot 3}-\sqrt[3]{8\cdot 3}$

$$\begin{aligned}&=3\sqrt[3]{3}-2\sqrt[3]{3}\\&=(3-2)\sqrt[3]{3}\\&=\sqrt[3]{3}\end{aligned}$$

44. $17\sqrt[3]{81}-2\sqrt[3]{24}=17\sqrt[3]{27\cdot 3}-2\sqrt[3]{8\cdot 3}$

$$\begin{aligned}&=17\cdot 3\sqrt[3]{3}-2\cdot 2\sqrt[3]{3}\\&=51\sqrt[3]{3}-4\sqrt[3]{3}\\&=(51-4)\sqrt[3]{3}\\&=47\sqrt[3]{3}\end{aligned}$$

45. $3\sqrt{2a}-\sqrt{8a}-\sqrt{72a}=3\sqrt{2a}-\sqrt{4\cdot 2a}-\sqrt{36\cdot 2a}$

$$\begin{aligned}&=3\sqrt{2a}-2\sqrt{2a}-6\sqrt{2a}\\&=(3-2-6)\sqrt{2a}\\&=-5\sqrt{2a}\end{aligned}$$

46. $\sqrt{12t}-\sqrt{27t}+5\sqrt{3t}=\sqrt{4\cdot 3t}-\sqrt{9\cdot 3t}+5\sqrt{3t}$

$$\begin{aligned}&=2\sqrt{3t}-3\sqrt{3t}+5\sqrt{3t}\\&=(2-3+5)\sqrt{3t}\\&=4\sqrt{3t}\end{aligned}$$

47. $2s^2\sqrt[3]{s^2t^6}+3t^2\sqrt[3]{8s^8}$

$$\begin{aligned}&=2s^2\sqrt[3]{t^6\cdot s^2}+3t^2\sqrt[3]{8s^6\cdot s^2}\\&=2s^2\cdot t^2\sqrt[3]{s^2}+3t^2\cdot 2s^2\sqrt[3]{s^2}\\&=2s^2t^2\sqrt[3]{s^2}+6s^2t^2\sqrt[3]{s^2}\\&=\left(2s^2t^2+6s^2t^2\right)\sqrt[3]{s^2}\\&=8s^2t^2\sqrt[3]{s^2}\end{aligned}$$

48. $4\sqrt[3]{x^4}-2x\sqrt[3]{x}=4\sqrt[3]{x^3\cdot x}-2x\sqrt[3]{x}$

$$\begin{aligned}&=4x\sqrt[3]{x}-2x\sqrt[3]{x}\\&=(4x-2x)\sqrt[3]{x}\\&=2x\sqrt[3]{x}\end{aligned}$$

49. $7\sqrt[3]{x^4}-x\sqrt[3]{x}=7\sqrt[3]{x^3\cdot x}-x\sqrt[3]{x}$

$$\begin{aligned}&=7x\sqrt[3]{x}-x\sqrt[3]{x}\\&=(7x-x)\sqrt[3]{x}\\&=6x\sqrt[3]{x}\end{aligned}$$

50. $6\sqrt[3]{y^{10}}-3y^2\sqrt[3]{y^4}=6\sqrt[3]{y^9\cdot y}-3y^2\sqrt[3]{y^3\cdot y}$

$$\begin{aligned}&=6y^3\sqrt[3]{y}-3y^2\cdot y\sqrt[3]{y}\\&=6y^3\sqrt[3]{y}-3y^3\sqrt[3]{y}\\&=(6y^3-3y^3)\sqrt[3]{y}\\&=3y^3\sqrt[3]{y}\end{aligned}$$

51. $5p\sqrt{20p^2}+p^2\sqrt{80}=5p\sqrt{4p^2\cdot 5}+p^2\sqrt{16\cdot 5}$
$=5p\cdot 2p\sqrt{5}+p^2\cdot 4\sqrt{5}$
$=10p^2\sqrt{5}+4p^2\sqrt{5}$
$=\left(10p^2+4p^2\right)\sqrt{5}$
$=14p^2\sqrt{5}$

52. $2q\sqrt{48q^2}-\sqrt{27q^4}=2q\sqrt{16q^2\cdot 3}-\sqrt{9q^4\cdot 3}$
$=2q\cdot 4q\sqrt{3}-3q^2\sqrt{3}$
$=8q^2\sqrt{3}-3q^2\sqrt{3}$
$=\left(8q^2-3q^2\right)\sqrt{3}$
$=5q^2\sqrt{3}$

53. $\sqrt[3]{a^2b}-\sqrt[3]{8a^2b}=\sqrt[3]{a^2b}-2\sqrt[3]{a^2b}$
$=\left(1-2\right)\sqrt[3]{a^2b}$
$=-\sqrt[3]{a^2b}$

54. $w\sqrt{80}-3\sqrt{125w^2}=w\sqrt{16\cdot 5}-3\sqrt{25w^2\cdot 5}$
$=4w\sqrt{5}-3\cdot 5w\sqrt{5}$
$=4w\sqrt{5}-15w\sqrt{5}$
$=\left(4w-15w\right)\sqrt{5}$
$=-11w\sqrt{5}$

55. $5x\sqrt{x}+6\sqrt{x}=\left(5x+6\right)\sqrt{x}$

56. $9y^2\sqrt{2}+4\sqrt{2}=\left(9y^2+4\right)\sqrt{2}$

57. $\sqrt{50x^2}-3\sqrt{8}=\sqrt{25x^2\cdot 2}-3\sqrt{4\cdot 2}$
$=5x\sqrt{2}-3\cdot 2\sqrt{2}$
$=5x\sqrt{2}-6\sqrt{2}$
$=\left(5x-6\right)\sqrt{2}$

58. $\sqrt{9x^3}-\sqrt{25x}=\sqrt{9x^2\cdot x}-\sqrt{25x}$
$=3x\sqrt{x}-5\sqrt{x}$
$=\left(3x-5\right)\sqrt{x}$

59. $11\sqrt[3]{54cd^3}-2\sqrt[3]{2cd^3}+d\sqrt[3]{16c}$
$=11\sqrt[3]{27\cdot 2cd^3}-2\sqrt[3]{2cd^3}+d\sqrt[3]{8\cdot 2c}$
$=11\cdot 3d\sqrt[3]{2c}-2\cdot d\sqrt[3]{2c}+d\cdot 2\sqrt[3]{2c}$
$=33d\sqrt[3]{2c}-2d\sqrt[3]{2c}+2d\sqrt[3]{2c}$
$=\left(33-2+2\right)d\sqrt[3]{2c}$
$=33d\sqrt[3]{2c}$

60. $x\sqrt[3]{64x^5y^2}-x^2\sqrt[3]{x^2y^2}+5\sqrt[3]{x^8y^2}$
$=x\sqrt[3]{64x^3\cdot x^2y^2}-x^2\sqrt[3]{x^2y^2}+5\sqrt[3]{x^6\cdot x^2y^2}$
$=x\cdot 4x\sqrt[3]{x^2y^2}-x^2\sqrt[3]{x^2y^2}+5x^2\sqrt[3]{x^2y^2}$
$=4x^2\sqrt[3]{x^2y^2}-x^2\sqrt[3]{x^2y^2}+5x^2\sqrt[3]{x^2y^2}$
$=\left(4-1+5\right)x^2\sqrt[3]{x^2y^2}$
$=8x^2\sqrt[3]{x^2y^2}$

61. $\frac{3}{2}ab\sqrt{24a^3}+\frac{4}{3}\sqrt{54a^5b^2}-a^2b\sqrt{150a}=\frac{3}{2}ab\sqrt{4a^2\cdot 6a}+\frac{4}{3}\sqrt{9a^4b^2\cdot 6a}-a^2b\sqrt{25\cdot 6a}$

$=\frac{3}{2}ab\cdot 2a\sqrt{6a}+\frac{4}{3}\cdot 3a^2b\sqrt{6a}-a^2b\cdot 5\sqrt{6a}=3a^2b\sqrt{6a}+4a^2b\sqrt{6a}-5a^2b\sqrt{6a}$

$=\left(3+4-5\right)a^2b\sqrt{6a}$

$=2a^2b\sqrt{6a}$

62. $mn\sqrt{72n}+\frac{2}{3}n\sqrt{8m^2n}-\frac{5}{6}\sqrt{50m^2n^3}=mn\sqrt{36\cdot 2n}+\frac{2}{3}n\sqrt{4m^2\cdot 2n}-\frac{5}{6}\sqrt{25m^2n^2\cdot 2n}$

$=6mn\sqrt{2n}+\frac{2}{3}n\cdot 2m\sqrt{2n}-\frac{5}{6}\cdot 5mn\sqrt{2n}=6mn\sqrt{2n}+\frac{4}{3}mn\sqrt{2n}-\frac{25}{6}mn\sqrt{2n}$

$=\left(6+\frac{4}{3}-\frac{25}{6}\right)mn\sqrt{2n}=\left(\frac{36}{6}+\frac{8}{6}-\frac{25}{6}\right)mn\sqrt{2n}$

$=\frac{19}{6}mn\sqrt{2n}$

63. $x\sqrt[3]{16}-2\sqrt[3]{27x}+\sqrt[3]{54x^3}$

$=x\sqrt[3]{8\cdot 2}-2\sqrt[3]{27x}+\sqrt[3]{27x^3\cdot 2}$

$=x\cdot 2\sqrt[3]{2}-2\cdot 3\sqrt[3]{x}+3x\sqrt[3]{2}$

$=2x\sqrt[3]{2}-6\sqrt[3]{x}+3x\sqrt[3]{2}$

$=\left(2+3\right)x\sqrt[3]{2}-6\sqrt[3]{x}$

$=5x\sqrt[3]{2}-6\sqrt[3]{x}$

64. $5\sqrt[4]{y^5}-2y\sqrt[4]{y}+\sqrt[4]{16y^7}$

$=5\sqrt[4]{y^4\cdot y}-2y\sqrt[4]{y}+\sqrt[4]{16y^4\cdot y^3}$

$=5y\sqrt[4]{y}-2y\sqrt[4]{y}+2y\sqrt[4]{y^3}$

$=\left(5-2\right)y\sqrt[4]{y}+2y\sqrt[4]{y^3}$

$=3y\sqrt[4]{y}+2y\sqrt[4]{y^3}$

65. $\sqrt{x}+\sqrt{y}=\sqrt{x+y}$ False.

$\sqrt{9}+\sqrt{16}\neq\sqrt{9+16}$

$3+4\neq\sqrt{25}$

$7\neq 5$

66. $\sqrt{x}+\sqrt{x}=2\sqrt{x}$ True.

67. $5\sqrt[3]{x}+2\sqrt[3]{x}=7\sqrt[3]{x}$ True.

68. $6\sqrt{x}+5\sqrt[3]{x}=11\sqrt{x}$ False.

Radicals must have the same index if they are to be added.

69. $\sqrt{y}+\sqrt{y}=\sqrt{2y}$ False.

$\sqrt{y}+\sqrt{y}=2\sqrt{y}\neq\sqrt{2y}$

70. $\sqrt{c^2+d^2}=c+d$ False.

$$\sqrt{3^2+4^2} \neq 3+4$$
$$\sqrt{9+16} \neq 7$$
$$\sqrt{25} \neq 7$$
$$5 \neq 7$$

71. $2w\sqrt{5}+4w\sqrt{5}=6w^2\sqrt{5}$

False: $2w\sqrt{5}+4w\sqrt{5}=6w\sqrt{5}\neq 6w^2\sqrt{5}$

72. $7x\sqrt{3}-2\sqrt{3}=(7x-2)\sqrt{3}$

True

73. $\sqrt{48}+\sqrt{12}=\sqrt{16\cdot 3}+\sqrt{4\cdot 3}$
$=4\sqrt{3}+2\sqrt{3}$
$=(4+2)\sqrt{3}$
$=6\sqrt{3}$

74. $\sqrt[3]{16}+\sqrt[3]{2}=\sqrt[3]{8\cdot 2}+\sqrt[3]{2}$
$=2\sqrt[3]{2}+\sqrt[3]{2}$
$=(2+1)\sqrt[3]{2}$
$=3\sqrt[3]{2}$

75. $5\sqrt[3]{x^6}-x^2=5x^2-x^2$
$=4x^2$

76. $y^3+\sqrt[4]{y^{12}}=y^3+y^3$
$=2y^3$

77. $\sqrt{18}-5^2$

The difference of the principal square root of 18 and the square of 5.

78. $4^3-\sqrt[3]{4}$

The difference of the cube of 4 and the cube root of 4.

79. $\sqrt[4]{x}+y^3$

The sum of the principal fourth root of x and the cube of y.

80. $a^4+\sqrt{a}$

The sum of the fourth power of a and the principal square root of a.

81. $P=2\sqrt{6}+2\sqrt{24}+\sqrt{54}$
$=2\sqrt{6}+2\sqrt{4\cdot 6}+\sqrt{9\cdot 6}$
$=2\sqrt{6}+2\cdot 2\sqrt{6}+3\sqrt{6}$
$=2\sqrt{6}+4\sqrt{6}+3\sqrt{6}$
$=(2+4+3)\sqrt{6}$
$=9\sqrt{6}$ cm
≈ 22.0 cm

82. $P=\sqrt{3}+\sqrt{75}+\sqrt{27}$
$=\sqrt{3}+\sqrt{25\cdot 3}+\sqrt{9\cdot 3}$
$=\sqrt{3}+5\sqrt{3}+3\sqrt{3}$
$=(1+5+3)\sqrt{3}$
$=9\sqrt{3}$ ft
≈ 15.6 ft

83. $2\sqrt{50}+2x=14\sqrt{2}$

$2\sqrt{25\cdot 2}+2x=14\sqrt{2}$

$2\cdot 5\sqrt{2}+2x=14\sqrt{2}$

$10\sqrt{2}+2x=14\sqrt{2}$

$2x=4\sqrt{2}$

$x=2\sqrt{2}$ ft

84. $2\sqrt{112}+2x=12\sqrt{7}$

$2\sqrt{16\cdot 7}+2x=12\sqrt{7}$

$2\cdot 4\sqrt{7}+2x=12\sqrt{7}$

$8\sqrt{7}+2x=12\sqrt{7}$

$2x=4\sqrt{7}$

$x=2\sqrt{7}$ cm

85. **a.** Side from (0, 6) to (6, 9):

$c^2=3^2+6^2=9+36=45$

$c=\sqrt{45}=\sqrt{9\cdot 5}=3\sqrt{5}$

Side from (6, 9) to (7, 7):

$c^2=1^2+2^2=1+4=5$

$c=\sqrt{5}$

Side from (7, 7) to (4, 1):

$c^2=3^2+6^2=9+36=45$

$c=\sqrt{45}=\sqrt{9\cdot 5}=3\sqrt{5}$

Side from (4, 1) to (2, 2):

$c^2=1^2+2^2=1+4=5$

$c=\sqrt{5}$

Side from (2, 2) to (0, 6):

$c^2=2^2+4^2=4+16=20$

$c=\sqrt{20}=\sqrt{4\cdot 5}=2\sqrt{5}$

$P=3\sqrt{5}+\sqrt{5}+3\sqrt{5}+\sqrt{5}+2\sqrt{5}$

$=\left(3+1+3+1+2\right)\sqrt{5}=10\sqrt{5}$ yd

b. $10\sqrt{5}\approx 22.36$ yd

c. $C=1.49\left(22.36\right)\left(3\right)+0.06\left(1.49\left(22.36\right)\left(3\right)\right)$

$=99.95+6.00=\$105.95$

Section 6.5 Practice Exercises

1. **a.** $\sqrt[n]{ab}$

b. x

c. a

d. conjugates

e. $m-n$

f. $c+8\sqrt{c}+16$

2. $\sqrt[3]{-16x^5y^6z^7}=\sqrt[3]{-8x^3y^6z^6\cdot 2x^2z}$

$=-2xy^2z^2\sqrt[3]{2x^2z}$

3 $-\sqrt{20a^2b^3c}=-\sqrt{4a^2b^2\cdot 5bc}$

$=-2ab\sqrt{5bc}$

4. $\sqrt{\dfrac{8y^3z^5}{y}}=\sqrt{8y^2z^5}=\sqrt{4\cdot 2\cdot y^2\cdot z^4\cdot z}$

$=2yz^2\sqrt{2z}$

5. $x^{1/3}y^{1/4}x^{-1/6}y^{1/3}=x^{(1/3)+(-1/6)}y^{(1/4)+(1/3)}$

$=x^{1/6}y^{7/12}$

6. $\dfrac{b^{1/4}}{b^{3/2}}=b^{(1/4)-(3/2)}=b^{-5/4}=\dfrac{1}{b^{5/4}}$

7. $-2\sqrt[3]{7}+4\sqrt[3]{7}=\left(-2+4\right)\sqrt[3]{7}$
$=2\sqrt[3]{7}$

8. $4\sqrt{8x^3}-x\sqrt{50x}=4\sqrt{4x^2\cdot 2x}-x\sqrt{25\cdot 2x}$
$=4\cdot 2x\sqrt{2x}-x\cdot 5\sqrt{2x}$
$=8x\sqrt{2x}-5x\sqrt{2x}$
$=\left(8-5\right)x\sqrt{2x}=3x\sqrt{2x}$

9. $\sqrt[3]{7}\cdot\sqrt[3]{3}=\sqrt[3]{21}$

10. $\sqrt[4]{6}\cdot\sqrt[4]{2}=\sqrt[4]{12}$

11. $\sqrt{2}\cdot\sqrt{10}=\sqrt{20}=\sqrt{4\cdot 5}=2\sqrt{5}$

12. $\sqrt[3]{4}\cdot\sqrt[3]{12}=\sqrt[3]{48}=\sqrt[3]{8\cdot 6}=2\sqrt[3]{6}$

13. $\sqrt[4]{16}\cdot\sqrt[4]{64}=\sqrt[4]{2^4}\cdot\sqrt[4]{2^6}=2\sqrt[4]{2^4\cdot 2^2}$
$=2\cdot 2\sqrt[4]{2^2}=4\sqrt[4]{4}$

14. $\sqrt{5x^3}\cdot\sqrt{10x^4}=\sqrt{50x^7}$
$=\sqrt{25x^6\cdot 2x}=5x^3\sqrt{2x}$

15. $\left(4\sqrt[3]{4}\right)\left(2\sqrt[3]{5}\right)=\left(4\cdot 2\right)\left(\sqrt[3]{4}\cdot\sqrt[3]{5}\right)$
$=8\sqrt[3]{20}$

16. $\left(2\sqrt{5}\right)\left(3\sqrt{7}\right)=\left(2\cdot 3\right)\left(\sqrt{5}\cdot\sqrt{7}\right)$
$=6\sqrt{35}$

17. $\left(8a\sqrt{b}\right)\left(-3\sqrt{ab}\right)=\left(8a\right)\left(-3\right)\left(\sqrt{b}\cdot\sqrt{ab}\right)$
$=-24a\sqrt{ab^2}$
$=-24ab\sqrt{a}$

18. $\left(p\sqrt[4]{q^3}\right)\left(\sqrt[4]{pq}\right)=p\left(\sqrt[4]{q^3}\cdot\sqrt[4]{pq}\right)$
$=p\sqrt[4]{pq^4}$
$=pq\sqrt[4]{p}$

19. $\sqrt{30}\cdot\sqrt{12}=\sqrt{360}=\sqrt{36\cdot 10}$
$=6\sqrt{10}$

20. $\sqrt{20}\cdot\sqrt{54}=\sqrt{4\cdot 5}\cdot\sqrt{9\cdot 6}=2\sqrt{5}\cdot 3\sqrt{6}$
$=2\cdot 3\left(\sqrt{5}\cdot\sqrt{6}\right)=6\sqrt{30}$

21. $\sqrt{6x}\sqrt{12x}=\sqrt{72x^2}$
$=\sqrt{36x^2\cdot 2}$
$=6x\sqrt{2}$

22. $\left(\sqrt{3ab^2}\right)\left(\sqrt{21a^2b}\right)=b\sqrt{3a}\cdot a\sqrt{21b}$
$=ab\sqrt{63ab}$
$=ab\sqrt{9\cdot 7ab}=3ab\sqrt{7ab}$

23. $\sqrt{5a^3b^2}\sqrt{20a^3b^3}=\sqrt{100a^6b^5}$
$=\sqrt{100a^6b^4\cdot b}$
$=10a^3b^2\sqrt{b}$

24. $\sqrt[3]{m^2n^2}\cdot\sqrt[3]{48m^4n^2}=\sqrt[3]{48m^6n^4}$
$=\sqrt[3]{8m^6n^3\cdot 6n}=2m^2n\sqrt[3]{6n}$

25. $\left(4\sqrt{3xy^3}\right)\left(-2\sqrt{6x^3y^2}\right) = -8\sqrt{18x^4y^5}$
$= -8\sqrt{9x^4y^4 \cdot 2y}$
$= -8 \cdot 3x^2y^2\sqrt{2y}$
$= -24x^2y^2\sqrt{2y}$

26. $\left(2\sqrt[4]{3x}\right)\left(4\sqrt[4]{27x^6}\right) = 8\sqrt[4]{81x^7}$
$= 8\sqrt[4]{81x^4 \cdot x^3}$
$= 8 \cdot 3x\sqrt[4]{x^3}$
$= 24x\sqrt[4]{x^3}$

27. $\left(\sqrt[3]{4a^2b}\right)\left(\sqrt[3]{2ab^3}\right)\left(\sqrt[3]{54a^2b}\right)$
$= \sqrt[3]{8a^3b^3 \cdot b} \cdot \sqrt[3]{27 \cdot 2a^2b}$
$= 2ab\sqrt[3]{b} \cdot 3\sqrt[3]{2a^2b}$
$= 6ab\sqrt[3]{2a^2b^2}$

28. $\left(\sqrt[3]{9x^3y}\right)\left(\sqrt[3]{6xy}\right)\left(\sqrt[3]{8x^2y^5}\right) = \sqrt[3]{54 \cdot 8x^6y^7}$
$= \sqrt[3]{27 \cdot 2 \cdot 8x^6y^6 \cdot y}$
$= 3 \cdot 2x^2y^2\sqrt[3]{2y}$
$= 6x^2y^2\sqrt[3]{2y}$

29. $\sqrt{3}\left(4\sqrt{3} - 6\right) = \sqrt{3} \cdot 4\sqrt{3} - \sqrt{3} \cdot (6)$
$= 4\sqrt{9} - 6\sqrt{3}$
$= 4 \cdot 3 - 6\sqrt{3} = 12 - 6\sqrt{3}$

30. $3\sqrt{5}\left(2\sqrt{5} + 4\right) = 3\sqrt{5} \cdot 2\sqrt{5} + 3\sqrt{5}(4)$
$= 6\sqrt{25} + 12\sqrt{5} = 6 \cdot 5 + 12\sqrt{5}$
$= 30 + 12\sqrt{5}$

31. $\sqrt{2}\left(\sqrt{6} - \sqrt{3}\right) = \sqrt{2} \cdot \sqrt{6} - \sqrt{2} \cdot \sqrt{3}$
$= \sqrt{12} - \sqrt{6} = \sqrt{4 \cdot 3} - \sqrt{6}$
$= 2\sqrt{3} - \sqrt{6}$

32. $\sqrt{5}\left(\sqrt{3} + \sqrt{7}\right) = \sqrt{5} \cdot \sqrt{3} + \sqrt{5} \cdot \sqrt{7}$
$= \sqrt{15} + \sqrt{35}$

33. $-\frac{1}{3}\sqrt{x}\left(6\sqrt{x} + 7\right) = -\frac{1}{3}\sqrt{x} \cdot 6\sqrt{x} - \frac{1}{3}\sqrt{x} \cdot (7)$
$= -2\sqrt{x^2} - \frac{7}{3}\sqrt{x}$
$= -2x - \frac{7}{3}\sqrt{x}$

34. $-\frac{1}{2}\sqrt{y}\left(8 - 3\sqrt{y}\right) = -\frac{1}{2}\sqrt{y}(8) + \frac{1}{2}\sqrt{y} \cdot 3\sqrt{y}$
$= -4\sqrt{y} + \frac{3}{2}\sqrt{y^2}$
$= -4\sqrt{y} + \frac{3}{2}y$

35. $\left(\sqrt{3} + 2\sqrt{10}\right)\left(4\sqrt{3} - \sqrt{10}\right) = \sqrt{3} \cdot 4\sqrt{3} - \sqrt{3} \cdot \sqrt{10} + 2\sqrt{10} \cdot 4\sqrt{3} - 2\sqrt{10} \cdot \sqrt{10}$
$= 4\sqrt{9} - \sqrt{30} + 8\sqrt{30} - 2\sqrt{100} = 4 \cdot 3 + 7\sqrt{30} - 2 \cdot 10 = 12 + 7\sqrt{30} - 20 = -8 + 7\sqrt{30}$

36. $\left(8\sqrt{7} - \sqrt{5}\right)\left(\sqrt{7} + 3\sqrt{5}\right) = 8\sqrt{7} \cdot \sqrt{7} + 8\sqrt{7} \cdot 3\sqrt{5} - \sqrt{5} \cdot \sqrt{7} - \sqrt{5} \cdot 3\sqrt{5}$
$= 8\sqrt{49} + 24\sqrt{35} - \sqrt{35} - 3\sqrt{25} = 8 \cdot 7 + 23\sqrt{35} - 3 \cdot 5 = 56 + 23\sqrt{35} - 15 = 41 + 23\sqrt{35}$

37. $\left(\sqrt{x} + 4\right)\left(\sqrt{x} - 9\right) = \sqrt{x} \cdot \sqrt{x} - \sqrt{x} \cdot 9 + 4 \cdot \sqrt{x} - 4 \cdot 9 = \sqrt{x^2} - 9\sqrt{x} + 4\sqrt{x} - 36 = x - 5\sqrt{x} - 36$

38. $\left(\sqrt{w}-2\right)\left(\sqrt{w}-9\right)=\sqrt{w}\cdot\sqrt{w}-\sqrt{w}\cdot 9-2\cdot\sqrt{w}+2\cdot 9=\sqrt{w^2}-9\sqrt{w}-2\sqrt{w}+18=w-11\sqrt{w}+18$

39. $\left(\sqrt[3]{y}+2\right)\left(\sqrt[3]{y}-3\right)=\sqrt[3]{y}\cdot\sqrt[3]{y}-\sqrt[3]{y}\cdot 3+2\cdot\sqrt[3]{y}-2\cdot 3=\sqrt[3]{y^2}-3\sqrt[3]{y}+2\sqrt[3]{y}-6=\sqrt[3]{y^2}-\sqrt[3]{y}-6$

40. $\left(4+\sqrt[5]{p}\right)\left(5+\sqrt[5]{p}\right)=4\cdot 5+4\cdot\sqrt[5]{p}+\sqrt[5]{p}\cdot 5+\sqrt[5]{p}\cdot\sqrt[5]{p}=20+4\sqrt[5]{p}+5\sqrt[5]{p}+\sqrt[5]{p^2}=20+9\sqrt[5]{p}+\sqrt[5]{p^2}$

41. $\left(\sqrt{a}-3\sqrt{b}\right)\left(9\sqrt{a}-\sqrt{b}\right)=\sqrt{a}\cdot 9\sqrt{a}-\sqrt{a}\cdot\sqrt{b}-3\sqrt{b}\cdot 9\sqrt{a}+3\sqrt{b}\cdot\sqrt{b}$
$=9\sqrt{a^2}-\sqrt{ab}-27\sqrt{ab}+3\sqrt{b^2}=9a-28\sqrt{ab}+3b$

42. $\left(11\sqrt{m}+4\sqrt{n}\right)\left(\sqrt{m}+\sqrt{n}\right)=11\sqrt{m}\cdot\sqrt{m}+11\sqrt{m}\cdot\sqrt{n}+4\sqrt{n}\cdot\sqrt{m}+4\sqrt{n}\cdot\sqrt{n}$
$=11\sqrt{m^2}+11\sqrt{mn}+4\sqrt{mn}+4\sqrt{n^2}=11m+15\sqrt{mn}+4n$

43. $\left(\sqrt{p}+2\sqrt{q}\right)\left(8+3\sqrt{p}-\sqrt{q}\right)=\sqrt{p}\cdot 8+\sqrt{p}\cdot 3\sqrt{p}-\sqrt{p}\cdot\sqrt{q}+2\sqrt{q}\cdot 8+2\sqrt{q}\cdot 3\sqrt{p}-2\sqrt{q}\cdot\sqrt{q}$
$=8\sqrt{p}+3\sqrt{p^2}-\sqrt{pq}+16\sqrt{q}+6\sqrt{pq}-2\sqrt{q^2}=8\sqrt{p}+3p+5\sqrt{pq}+16\sqrt{q}-2q$

44. $\left(5\sqrt{s}-\sqrt{t}\right)\left(\sqrt{s}+5+6\sqrt{t}\right)=5\sqrt{s}\cdot\sqrt{s}+5\sqrt{s}\cdot 5+5\sqrt{s}\cdot 6\sqrt{t}-\sqrt{t}\cdot\sqrt{s}-\sqrt{t}\cdot 5-\sqrt{t}\cdot 6\sqrt{t}$
$=5\sqrt{s^2}+25\sqrt{s}+30\sqrt{st}-\sqrt{st}-5\sqrt{t}-6\sqrt{t^2}=5s+25\sqrt{s}+29\sqrt{st}-5\sqrt{t}-6t$

45. $\left(\sqrt{15}\right)^2=15$

46. $\left(\sqrt{58}\right)^2=58$

47. $\left(\sqrt{3y}\right)^2=3y$

48. $\left(\sqrt{19yz}\right)^2=19yz$

49. $\left(\sqrt[3]{6}\right)^3=6$

50. $\left(\sqrt[5]{24}\right)^5=24$

51. $\sqrt{709}\cdot\sqrt{709}=\left(\sqrt{709}\right)^2=709$

52. $\sqrt{401}\cdot\sqrt{401}=\left(\sqrt{401}\right)^2=401$

53. **(a)** $\left(x+y\right)\left(x-y\right)=x^2-y^2$

54. **(a)** $\left(x+y\right)^2=x^2+2xy+y^2$

(b) $(x+5)(x-5) = x^2 - 5^2$
$= x^2 - 25$

(b) $(x+5)^2 = x^2 + 2x(5) + 5^2$
$= x^2 + 10x + 25$

55. $\left(\sqrt{13}+4\right)^2 = \left(\sqrt{13}\right)^2 + 2\left(\sqrt{13}\right)(4) + 4^2$
$= 13 + 8\sqrt{13} + 16$
$= 29 + 8\sqrt{13}$

56. $\left(6-\sqrt{11}\right)^2 = (6)^2 - 2(6)\left(\sqrt{11}\right) + \left(\sqrt{11}\right)^2$
$= 36 - 12\sqrt{11} + 11$
$= 47 - 12\sqrt{11}$

57. $\left(\sqrt{p}-\sqrt{7}\right)^2 = \left(\sqrt{p}\right)^2 - 2\left(\sqrt{p}\right)\left(\sqrt{7}\right) + \left(\sqrt{7}\right)^2$
$= p - 2\sqrt{7p} + 7$

58. $\left(\sqrt{q}+\sqrt{2}\right)^2 = \left(\sqrt{q}\right)^2 + 2\left(\sqrt{q}\right)\left(\sqrt{2}\right) + \left(\sqrt{2}\right)^2$
$= q + 2\sqrt{2q} + 2$

59. $\left(\sqrt{2a}-3\sqrt{b}\right)^2$
$= \left(\sqrt{2a}\right)^2 - 2\left(\sqrt{2a}\right)\left(3\sqrt{b}\right) + \left(3\sqrt{b}\right)^2$
$= 2a - 6\sqrt{2ab} + 9b$

60. $\left(\sqrt{3w}+4\sqrt{z}\right)^2$
$= \left(\sqrt{3w}\right)^2 + 2\left(\sqrt{3w}\right)\left(4\sqrt{z}\right) + \left(4\sqrt{z}\right)^2$
$= 3w + 8\sqrt{3wz} + 16z$

61. $\left(\sqrt{3}+x\right)\left(\sqrt{3}-x\right) = \left(\sqrt{3}\right)^2 - x^2 = 3 - x^2$

62. $\left(y+\sqrt{6}\right)\left(y-\sqrt{6}\right) = y^2 - \left(\sqrt{6}\right)^2 = y^2 - 6$

63. $\left(\sqrt{6}+\sqrt{2}\right)\left(\sqrt{6}-\sqrt{2}\right) = \left(\sqrt{6}\right)^2 - \left(\sqrt{2}\right)^2$
$= 6 - 2 = 4$

64. $\left(\sqrt{15}+\sqrt{5}\right)\left(\sqrt{15}-\sqrt{5}\right) = \left(\sqrt{15}\right)^2 - \left(\sqrt{5}\right)^2$
$= 15 - 5 = 10$

65. $\left(\frac{2}{3}\sqrt{x}+\frac{1}{2}\sqrt{y}\right)\left(\frac{2}{3}\sqrt{x}-\frac{1}{2}\sqrt{y}\right)$
$= \left(\frac{2}{3}\sqrt{x}\right)^2 - \left(\frac{1}{2}\sqrt{y}\right)^2$
$= \frac{4}{9}x - \frac{1}{4}y$

66. $\left(\frac{1}{4}\sqrt{s}+\frac{1}{5}\sqrt{t}\right)\left(\frac{1}{4}\sqrt{s}-\frac{1}{5}\sqrt{t}\right)$
$= \left(\frac{1}{4}\sqrt{s}\right)^2 - \left(\frac{1}{5}\sqrt{t}\right)^2$
$= \frac{1}{16}s - \frac{1}{25}t$

67. a. $\left(\sqrt{3}+\sqrt{x}\right)\left(\sqrt{3}-\sqrt{x}\right) = \sqrt{3}^2 - \sqrt{x}^2$
$= 3 - x$

b. $\left(\sqrt{3}+\sqrt{x}\right)\left(\sqrt{3}+\sqrt{x}\right)$
$= \sqrt{3}^2 + 2\sqrt{3}\sqrt{x} + \sqrt{x}^2$
$= 3 + 2\sqrt{3x} + x$

68. a. $\left(\sqrt{5}+\sqrt{y}\right)\left(\sqrt{5}-\sqrt{y}\right) = \sqrt{5}^2 - \sqrt{y}^2$
$= 5 - y$

b. $\left(\sqrt{5}+\sqrt{y}\right)\left(\sqrt{5}+\sqrt{y}\right)$
$= \sqrt{5}^2 + 2\sqrt{5}\sqrt{y} + \sqrt{y}^2$
$= 5 + 2\sqrt{5y} + y$

c. $\left(\sqrt{3}-\sqrt{x}\right)\left(\sqrt{3}-\sqrt{x}\right)$

$= \sqrt{3}^2 - 2\sqrt{3}\sqrt{x} + \sqrt{x}^2$

$= 3 - 2\sqrt{3x} + x$

c. $\left(\sqrt{5}-\sqrt{y}\right)\left(\sqrt{5}-\sqrt{y}\right)$

$= \sqrt{5}^2 - 2\sqrt{5}\sqrt{y} + \sqrt{y}^2$

$= 5 - 2\sqrt{5y} + y$

69. $\sqrt{3}\cdot\sqrt{2} = \sqrt{6}$ True.

70. $\sqrt{5}\cdot\sqrt[3]{2} = \sqrt{10}$ False.

The radicals have different indices.

71. $\left(x-\sqrt{5}\right)^2 = x-5$ False.

$\left(x-\sqrt{5}\right)^2 = x^2 - 2x\sqrt{5} + \left(\sqrt{5}\right)^2$

$= x^2 - 2x\sqrt{5} + 5$

72. $3\left(2\sqrt{5x}\right) = 6\sqrt{5x}$ True.

73. $5\left(3\sqrt{4x}\right) = 15\sqrt{20x}$ False.

5 is multiplied by 3 only.

74. $\dfrac{\sqrt{5x}}{5} = \sqrt{x}$ False. $\dfrac{\sqrt{5}}{5} \neq 1$

75. $\dfrac{3\sqrt{x}}{3} = \sqrt{x}$ True.

76. $\left(\sqrt{t}-1\right)\left(\sqrt{t}+1\right) = t-1$ True.

77. $\left(-\sqrt{6x}\right)^2 = 6x$

78. $\left(-\sqrt{8a}\right)^2 = 8a$

79. $\left(\sqrt{3x+1}\right)^2 = 3x+1$

80. $\left(\sqrt{x-1}\right)^2 = x-1$

81. $\left(\sqrt{x+3}-4\right)^2$

$= \left(\sqrt{x+3}\right)^2 - 2\left(\sqrt{x+3}\right)(4) + (4)^2$

$= x+3-8\sqrt{x+3}+16$

$= x+19-8\sqrt{x+3}$

82. $\left(\sqrt{x+1}+3\right)^2$

$= \left(\sqrt{x+1}\right)^2 + 2\left(\sqrt{x+1}\right)(3) + (3)^2$

$= x+1+6\sqrt{x+1}+9$

$= x+10+6\sqrt{x+1}$

83. $\left(\sqrt{2t-3}+5\right)^2$

$=\left(\sqrt{2t-3}\right)^2+2\left(\sqrt{2t-3}\right)(5)+(5)^2$

$=2t-3+10\sqrt{2t-3}+25$

$=2t+22+10\sqrt{2t-3}$

84. $\left(\sqrt{3w-2}-4\right)^2$

$=\left(\sqrt{3w-2}\right)^2-2\left(\sqrt{3w-2}\right)(4)+(4)^2$

$=3w-2-8\sqrt{3w-2}+16$

$=3w+14-8\sqrt{3w-2}$

85. $A=\sqrt{40}\cdot 3\sqrt{2}=3\sqrt{80}=3\sqrt{16\cdot 5}$

$=3\cdot 4\sqrt{5}=12\sqrt{5}\text{ ft}^2$

86. $A=\frac{1}{2}\cdot 6\sqrt{2}\cdot 10\sqrt{12}=30\sqrt{24}=30\sqrt{4\cdot 6}$

$=30\cdot 2\sqrt{6}=60\sqrt{6}\text{ m}^2$

87. $A=\frac{1}{2}\cdot 3\sqrt{5}\cdot 6\sqrt{12}=9\sqrt{60}=9\sqrt{4\cdot 15}$

$=9\cdot 2\sqrt{15}=18\sqrt{15}\text{ in.}^2$

88. $A=2\sqrt{18}\cdot 7\sqrt{6}=14\sqrt{108}=14\sqrt{36\cdot 3}$

$=14\cdot 6\sqrt{3}=84\sqrt{3}\text{ yd}^2$

89. $\sqrt{x}\cdot\sqrt[4]{x}=x^{1/2}\cdot x^{1/4}=x^{(1/2)+(1/4)}$

$=x^{3/4}=\sqrt[4]{x^3}$

90. $\sqrt[3]{y}\cdot\sqrt{y}=y^{1/3}\cdot y^{1/2}=y^{(1/3)+(1/2)}$

$=y^{5/6}=\sqrt[6]{y^5}$

91. $\sqrt[5]{2z}\cdot\sqrt[3]{2z}=(2z)^{1/5}\cdot(2z)^{1/3}$

$=(2z)^{(1/5)+(1/3)}$

$=(2z)^{8/15}=\sqrt[15]{(2z)^8}$

92. $\sqrt[3]{5w}\cdot\sqrt[4]{5w}=(5w)^{1/3}\cdot(5w)^{1/4}$

$=(5w)^{(1/3)+(1/4)}$

$=(5w)^{7/12}=\sqrt[12]{(5w)^7}$

93. $\sqrt[3]{p^2}\cdot\sqrt{p^3}=p^{2/3}\cdot p^{3/2}$

$=p^{(2/3)+(3/2)}=p^{13/6}$

$=\sqrt[6]{p^{13}}=\sqrt[6]{p^{12}\cdot p}=p^2\sqrt[6]{p}$

94. $\sqrt[4]{q^3}\cdot\sqrt[3]{q^2}=q^{3/4}\cdot q^{2/3}$

$=q^{(3/4)+(2/3)}=q^{17/12}$

$=\sqrt[12]{q^{17}}=\sqrt[12]{q^{12}\cdot q^5}=q\sqrt[12]{q^5}$

95. $\frac{\sqrt{u^3}}{\sqrt[3]{u}}=\frac{u^{3/2}}{u^{1/3}}=u^{(3/2)-(1/3)}$

$=u^{7/6}=\sqrt[6]{u^7}$

$=\sqrt[6]{u^6\cdot u}=u\sqrt[6]{u}$

96. $\frac{\sqrt{v^5}}{\sqrt[4]{v}}=\frac{v^{5/2}}{v^{1/4}}=v^{(5/2)-(1/4)}$

$=v^{9/4}=\sqrt[4]{v^9}$

$=\sqrt[4]{v^8\cdot v}=v^2\sqrt[4]{v}$

97. $\sqrt[3]{x}\cdot\sqrt[6]{y}=x^{1/3}\cdot y^{1/6}=x^{2/6}\cdot y^{1/6}$

$=\left(x^2y\right)^{1/6}=\sqrt[6]{x^2y}$

98. $\sqrt{a}\cdot\sqrt[6]{b}=a^{1/2}\cdot b^{1/6}=a^{3/6}b^{1/6}$

$=\left(a^3b\right)^{1/6}=\sqrt[6]{a^3b}$

99. $\sqrt[4]{8}\cdot\sqrt{3}=\sqrt[4]{2^3}\cdot\sqrt{3}$
$=2^{3/4}\cdot 3^{1/2}=2^{3/4}\cdot 3^{2/4}$
$=\left(2^3\cdot 3^2\right)^{1/4}=\sqrt[4]{2^3\cdot 3^2}$
$=\sqrt[4]{8\cdot 9}$
$=\sqrt[4]{72}$

100. $\sqrt{11}\cdot\sqrt[6]{2}=11^{1/2}\cdot 2^{1/6}$
$=11^{3/6}2^{1/6}$
$=\left(11^3\cdot 2\right)^{1/6}$
$=\sqrt[6]{11^3\cdot 2}$
$=\sqrt[6]{2662}$

101. $\sqrt[3]{2xy}\cdot\sqrt[4]{5xy}=\left(2xy\right)^{1/3}\left(5xy\right)^{1/4}=2^{1/3}x^{1/3}y^{1/3}5^{1/4}x^{1/4}y^{1/4}=2^{4/12}x^{4/12}y^{4/12}5^{3/12}x^{3/12}y^{3/12}$
$=2^{4/12}\cdot 5^{3/12}x^{7/12}y^{7/12}=\left(2^4\cdot 5^3x^7y^7\right)^{1/12}$
$=\sqrt[12]{2^4 5^3 x^7 y^7}$

102. $\sqrt{6ab}\cdot\sqrt[3]{7ab}=\left(6ab\right)^{1/2}\left(7ab\right)^{1/3}=6^{1/2}a^{1/2}b^{1/2}7^{1/3}a^{1/3}b^{1/3}=6^{3/6}a^{3/6}b^{3/6}7^{2/6}a^{2/6}b^{2/6}=6^{3/6}\cdot 7^{2/6}a^{5/6}b^{5/6}$
$=\left(6^3\cdot 7^2a^5b^5\right)^{1/6}=\sqrt[6]{6^3 7^2 a^5 b^5}$

103. $\sqrt[3]{4m^2n}\cdot\sqrt{6mn}=\left(4m^2n\right)^{1/3}\left(6mn\right)^{1/2}=4^{1/3}m^{2/3}n^{1/3}6^{1/2}m^{1/2}n^{1/2}=4^{2/6}m^{4/6}n^{2/6}6^{3/6}m^{3/6}n^{3/6}$
$=4^{2/6}\cdot 6^{3/6}m^{7/6}n^{5/6}=\left(4^2\cdot 6^3m^7n^5\right)^{1/6}=\sqrt[6]{2^4 2^3 3^3 m^7 n^5}=\sqrt[6]{2^6\cdot 2\cdot 3^3 m^6 mn^5}$
$=2m\sqrt[6]{2\cdot 3^3 mn^5}$

104. $\sqrt[4]{5xy^3}\cdot\sqrt[3]{10x^2y}=\left(5xy^3\right)^{1/4}\left(10x^2y\right)^{1/3}=5^{1/4}x^{1/4}y^{3/4}2^{1/3}\cdot 5^{1/3}x^{2/3}y^{1/3}=5^{3/12}x^{3/12}y^{9/12}2^{4/12}5^{4/12}x^{8/12}y^{4/12}$
$=2^{4/12}\cdot 5^{7/12}x^{11/12}y^{13/12}=\left(2^4\cdot 5^7x^{11}y^{13}\right)^{1/12}=\sqrt[12]{2^4 5^7 x^{11} y^{13}}=\sqrt[12]{2^4 5^7 x^{11} y^{12} y}$
$=y\sqrt[12]{2^4 5^7 x^{11} y}$

105. $\left(\sqrt[3]{a}+\sqrt[3]{b}\right)\left(\sqrt[3]{a^2}-\sqrt[3]{ab}+\sqrt[3]{b^2}\right)=\sqrt[3]{a}\cdot\sqrt[3]{a^2}-\sqrt[3]{a}\cdot\sqrt[3]{ab}+\sqrt[3]{a}\cdot\sqrt[3]{b^2}+\sqrt[3]{b}\cdot\sqrt[3]{a^2}-\sqrt[3]{b}\cdot\sqrt[3]{ab}+\sqrt[3]{b}\cdot\sqrt[3]{b^2}$
$=\sqrt[3]{a^3}-\sqrt[3]{a^2b}+\sqrt[3]{ab^2}+\sqrt[3]{a^2b}-\sqrt[3]{ab^2}+\sqrt[3]{b^3}$
$=a+b$

106. $\left(\sqrt[3]{a}-\sqrt[3]{b}\right)\left(\sqrt[3]{a^2}+\sqrt[3]{ab}+\sqrt[3]{b^2}\right)=\sqrt[3]{a}\cdot\sqrt[3]{a^2}+\sqrt[3]{a}\cdot\sqrt[3]{ab}+\sqrt[3]{a}\cdot\sqrt[3]{b^2}-\sqrt[3]{b}\cdot\sqrt[3]{a^2}-\sqrt[3]{b}\cdot\sqrt[3]{ab}-\sqrt[3]{b}\cdot\sqrt[3]{b^2}$
$=\sqrt[3]{a^3}+\sqrt[3]{a^2b}+\sqrt[3]{ab^2}-\sqrt[3]{a^2b}-\sqrt[3]{ab^2}-\sqrt[3]{b^3}$
$=a-b$

Problem Recognition Exercises

1. **a.** $\sqrt{24} = \sqrt{4 \cdot 6} = 2\sqrt{6}$

b. $\sqrt[3]{24} = \sqrt[3]{8 \cdot 3} = 2\sqrt[3]{3}$

2. **a.** $\sqrt{54} = \sqrt{9 \cdot 6} = 3\sqrt{6}$

b. $\sqrt[3]{54} = \sqrt[3]{27 \cdot 2} = 3\sqrt[3]{2}$

3. **a.** $\sqrt{200y^6} = \sqrt{100 \cdot 2y^6} = 10y^3\sqrt{2}$

b. $\sqrt[3]{200y^6} = \sqrt[3]{8 \cdot 25y^6} = 2y^2\sqrt[3]{25}$

4. **a.** $\sqrt{32z^{15}} = \sqrt{16 \cdot 2 \cdot z^{14} \cdot z} = 4z^7\sqrt{2z}$

b. $\sqrt[3]{32z^{15}} = \sqrt[3]{8 \cdot 4z^{15}} = 2z^5\sqrt[3]{4}$

5. **a.** $\sqrt{80} = \sqrt{16 \cdot 5} = 4\sqrt{5}$

b. $\sqrt[3]{80} = \sqrt[3]{8 \cdot 10} = 2\sqrt[3]{10}$

c. $\sqrt[4]{80} = \sqrt[4]{16 \cdot 5} = 2\sqrt[4]{5}$

6. **a.** $\sqrt{48} = \sqrt{16 \cdot 3} = 4\sqrt{3}$

b. $\sqrt[3]{48} = \sqrt[3]{8 \cdot 6} = 2\sqrt[3]{6}$

c. $\sqrt[4]{48} = \sqrt[4]{16 \cdot 3} = 2\sqrt[4]{3}$

7. **a.** $\sqrt{x^5y^6} = \sqrt{x^4 \cdot x \cdot y^6} = x^2y^3\sqrt{x}$

b. $\sqrt[3]{x^5y^6} = \sqrt[3]{x^3 \cdot x^2 \cdot y^6} = xy^2\sqrt[3]{x^2}$

c. $\sqrt[4]{x^5y^6} = \sqrt[4]{x^4 \cdot x \cdot y^4 \cdot y^2} = xy\sqrt[4]{xy^2}$

8. **a.** $\sqrt{a^{10}b^{12}} = a^5b^6$

b. $\sqrt[3]{a^{10}b^{12}} = \sqrt[3]{a^9 \cdot a \cdot b^{12}} = a^3b^4\sqrt[3]{a}$

c. $\sqrt[4]{a^{10}b^{12}} = \sqrt[4]{a^8 \cdot a^2 \cdot b^{12}} = a^2b^3\sqrt[4]{a^2}$

9. **a.** $\sqrt[3]{32s^5t^6} = \sqrt[3]{8 \cdot 4 \cdot s^3 \cdot s^2 \cdot t^6}$
$= 2st^2\sqrt[3]{4s^2}$

b. $\sqrt[4]{32s^5t^6} = \sqrt[4]{16 \cdot 2 \cdot s^4 \cdot s \cdot t^4 \cdot t^2}$
$= 2st\sqrt[4]{2st^2}$

c. $\sqrt[5]{32s^5t^6} = \sqrt[5]{32 \cdot s^5 \cdot t^5 \cdot t}$
$= 2st\sqrt[5]{t}$

10. **a.** $\sqrt[3]{96v^7w^{20}} = \sqrt[3]{8 \cdot 12 \cdot v^6 \cdot v \cdot w^{18} \cdot w^2}$
$= 2v^2w^6\sqrt[3]{12vw^2}$

b. $\sqrt[4]{96v^7w^{20}} = \sqrt[4]{16 \cdot 6 \cdot v^4 \cdot v^3 \cdot w^{20}}$
$= 2vw^5\sqrt[4]{6v^3}$

c. $\sqrt[5]{96v^7w^{20}} = \sqrt[5]{32 \cdot 3 \cdot v^5 \cdot v^2 \cdot w^{20}}$
$= 2vw^4\sqrt[5]{3v^2}$

11. **a.** $\sqrt{5} + \sqrt{5} = 2\sqrt{5}$

b. $\sqrt{5} \cdot \sqrt{5} = \sqrt{25} = 5$

12. **a.** $\sqrt{10} + \sqrt{10} = 2\sqrt{10}$

b. $\sqrt{10} \cdot \sqrt{10} = \sqrt{100} = 10$

13. **a.** $2\sqrt{6} - 5\sqrt{6} = -3\sqrt{6}$

b. $2\sqrt{6} \cdot 5\sqrt{6} = 10\sqrt{36} = 10 \cdot 6 = 60$

14. **a.** $3\sqrt{7} - 10\sqrt{7} = -7\sqrt{7}$

b. $3\sqrt{7} \cdot 10\sqrt{7} = 30\sqrt{49} = 30 \cdot 7 = 210$

15. **a.** $\sqrt{8}+\sqrt{2}=\sqrt{4\cdot 2}+\sqrt{2}=2\sqrt{2}+\sqrt{2}$
$=3\sqrt{2}$

b. $\sqrt{8}\cdot\sqrt{2}=\sqrt{16}=4$

16. **a.** $\sqrt{12}+\sqrt{3}=\sqrt{4\cdot 3}+\sqrt{3}=2\sqrt{3}+\sqrt{3}$
$=3\sqrt{3}$

b. $\sqrt{12}\cdot\sqrt{3}=\sqrt{36}=6$

17. **a.** $5\sqrt{18}-4\sqrt{8}=5\sqrt{9\cdot 2}-4\sqrt{4\cdot 2}$
$=5\cdot 3\sqrt{2}-4\cdot 2\sqrt{2}$
$=15\sqrt{2}-8\sqrt{2}=7\sqrt{2}$

b. $5\sqrt{18}\cdot 4\sqrt{8}=20\sqrt{144}=20\cdot 12=240$

18. **a.** $\sqrt{50}-\sqrt{72}=\sqrt{25\cdot 2}-\sqrt{36\cdot 2}$
$=5\sqrt{2}-6\sqrt{2}$
$=-\sqrt{2}$

b. $\sqrt{50}\cdot\sqrt{72}=\sqrt{3600}=\sqrt{36\cdot 100}$
$=6\cdot 10=60$

19. **a.** $4\sqrt[3]{24}+6\sqrt[3]{3}=4\sqrt[3]{8\cdot 3}+6\sqrt[3]{3}$
$=4\cdot 2\sqrt[3]{3}+6\sqrt[3]{3}$
$=8\sqrt[3]{3}+6\sqrt[3]{3}=14\sqrt[3]{3}$

b. $4\sqrt[3]{24}\cdot 6\sqrt[3]{3}=24\sqrt[3]{72}$
$=24\sqrt[3]{8\cdot 9}$
$=24\cdot 2\sqrt[3]{9}=48\sqrt[3]{9}$

20. **a.** $2\sqrt[3]{54}-5\sqrt[3]{2}=2\sqrt[3]{27\cdot 2}-5\sqrt[3]{2}$
$=2\cdot 3\sqrt[3]{2}-5\sqrt[3]{2}$
$=6\sqrt[3]{2}-5\sqrt[3]{2}=\sqrt[3]{2}$

b. $2\sqrt[3]{54}\cdot 5\sqrt[3]{2}=10\sqrt[3]{108}$
$=10\sqrt[3]{27\cdot 4}$
$=10\cdot 3\sqrt[3]{4}=30\sqrt[3]{4}$

Section 6.6 Practice Exercises

1. **a.** radical

b. $\dfrac{\sqrt[n]{a}}{\sqrt[b]{b}}$

c. $\dfrac{4}{x^2}$

d. rationalizing

e. is; is not

f. denominator

2. $\left(4x^2y^4\right)^{1/2}\left(64y^{-3}\right)^{1/3}=\left(2^2x^2y^4\right)^{1/2}\left(4^3y^{-3}\right)^{1/3}=2xy^2\cdot 4y^{-1}=8xy$

3. $2y\sqrt{45}+3\sqrt{20y^2}=2y\sqrt{9\cdot 5}+3\sqrt{4y^2\cdot 5}$
$=2y\cdot 3\sqrt{5}+3\cdot 2y\sqrt{5}$
$=6y\sqrt{5}+6y\sqrt{5}$
$=12y\sqrt{5}$

4. $3x\sqrt{72x}-9\sqrt{50x^3}$
$=3x\sqrt{36\cdot 2x}-9\sqrt{25x^2\cdot 2x}$
$=3x\cdot 6\sqrt{2x}-9\cdot 5x\sqrt{2x}$
$=18x\sqrt{2x}-45x\sqrt{2x}=-27x\sqrt{2x}$

5. $\left(-6\sqrt{y}+3\right)\left(3\sqrt{y}+1\right)$

$=-6\sqrt{y}\cdot 3\sqrt{y}-6\sqrt{y}\cdot(1)+3\cdot 3\sqrt{y}+3\cdot 1$

$=-18\sqrt{y^2}-6\sqrt{y}+9\sqrt{y}+3$

$=-18y+3\sqrt{y}+3$

6. $\left(\sqrt{w}+12\right)\left(2\sqrt{w}-4\right)$

$=\sqrt{w}\cdot 2\sqrt{w}-\sqrt{w}\cdot 4+12\cdot 2\sqrt{w}-12\cdot 4$

$=2\sqrt{w^2}-4\sqrt{w}+24\sqrt{w}-48$

$=2w+20\sqrt{w}-48$

7. $\left(8-\sqrt{t}\right)^2=8^2-2\cdot 8\cdot\sqrt{t}+\left(\sqrt{t}\right)^2$

$=64-16\sqrt{t}+t$

8. $\left(\sqrt{p}+4\right)^2=\left(\sqrt{p}\right)^2+2\cdot\sqrt{p}\cdot(4)+4^2$

$=p+8\sqrt{p}+16$

9. $\left(\sqrt{2}+\sqrt{7}\right)\left(\sqrt{2}-\sqrt{7}\right)=\left(\sqrt{2}\right)^2-\left(\sqrt{7}\right)^2$

$=2-7$

$=-5$

10. $\left(\sqrt{3}+5\right)\left(\sqrt{3}-5\right)=\left(\sqrt{3}\right)^2-5^2$

$=3-25$

$=-22$

11. $\sqrt{\dfrac{49x^4}{y^6}}=\dfrac{\sqrt{49x^4}}{\sqrt{y^6}}=\dfrac{7x^2}{y^3}$

12. $\sqrt{\dfrac{100p^2}{q^8}}=\dfrac{\sqrt{100p^2}}{\sqrt{q^8}}=\dfrac{10p}{q^4}$

13. $\sqrt{\dfrac{8a^2}{x^6}}=\dfrac{\sqrt{2\cdot 4a^2}}{\sqrt{x^6}}=\dfrac{2a\sqrt{2}}{x^3}$

14. $\sqrt{\dfrac{4w^3}{25y^4}}=\dfrac{\sqrt{4w^2\cdot w}}{\sqrt{25y^4}}=\dfrac{2w\sqrt{w}}{5y^2}$

15. $\sqrt[3]{\dfrac{-16j^3}{k^3}}=\dfrac{\sqrt[3]{-8j^3\cdot 2}}{\sqrt[3]{k^3}}=\dfrac{-2j\sqrt[3]{2}}{k}$

16. $\sqrt[5]{\dfrac{32x}{y^{10}}}=\dfrac{\sqrt[5]{32x}}{\sqrt[5]{y^{10}}}=\dfrac{2\sqrt[5]{x}}{y^2}$

17. $\dfrac{\sqrt{72ab^5}}{\sqrt{8ab}}=\sqrt{\dfrac{72ab^5}{8ab}}=\sqrt{9b^4}=3b^2$

18. $\dfrac{\sqrt{6x^3}}{\sqrt{24x}}=\sqrt{\dfrac{6x^3}{24x}}=\sqrt{\dfrac{x^2}{4}}=\dfrac{\sqrt{x^2}}{\sqrt{4}}=\dfrac{x}{2}$

19. $\dfrac{\sqrt[4]{3b^3}}{\sqrt[4]{48b^{11}}}=\sqrt[4]{\dfrac{3b^3}{48b^{11}}}=\sqrt[4]{\dfrac{1}{16b^8}}=\dfrac{\sqrt[4]{1}}{\sqrt[4]{16b^8}}$

$=\dfrac{1}{2b^2}$

20. $\dfrac{\sqrt[3]{128wz^8}}{\sqrt[3]{2wz^2}}=\sqrt[3]{\dfrac{128wz^8}{2wz^2}}=\sqrt[3]{64z^6}$

$=4z^2$

21. $\dfrac{\sqrt{3yz^2}}{\sqrt{w^4}}=\dfrac{z\sqrt{3y}}{w^2}$

22. $\dfrac{\sqrt{50x^3z}}{\sqrt{9y^4}}=\dfrac{\sqrt{2xz\cdot 25x^2}}{\sqrt{9y^4}}=\dfrac{5x\sqrt{2xz}}{3y^2}$

23. $\dfrac{x}{\sqrt{5}} = \dfrac{x}{\sqrt{5}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}}$

24. $\dfrac{2}{\sqrt{x}} = \dfrac{2}{\sqrt{x}} \cdot \dfrac{\sqrt{x}}{\sqrt{x}}$

25. $\dfrac{7}{\sqrt[3]{x}} = \dfrac{7}{\sqrt[3]{x}} \cdot \dfrac{\sqrt[3]{x^2}}{\sqrt[3]{x^2}}$

26. $\dfrac{5}{\sqrt[4]{y}} = \dfrac{5}{\sqrt[4]{y}} \cdot \dfrac{\sqrt[4]{y^3}}{\sqrt[4]{y^3}}$

27. $\dfrac{8}{\sqrt{3z}} = \dfrac{8}{\sqrt{3z}} \cdot \dfrac{\sqrt{3z}}{\sqrt{3z}}$

28. $\dfrac{10}{\sqrt{7w}} = \dfrac{10}{\sqrt{7w}} \cdot \dfrac{\sqrt{7w}}{\sqrt{7w}}$

29. $\dfrac{1}{\sqrt[4]{8a^2}} = \dfrac{1}{\sqrt[4]{8a^2}} \cdot \dfrac{\sqrt[4]{2a^2}}{\sqrt[4]{2a^2}}$

30. $\dfrac{1}{\sqrt[3]{9b^2}} = \dfrac{1}{\sqrt[3]{3^2 b^2}} \cdot \dfrac{\sqrt[3]{3b}}{\sqrt[3]{3b}}$

31. $\dfrac{1}{\sqrt{3}} = \dfrac{1}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{1\sqrt{3}}{\sqrt{3^2}} = \dfrac{\sqrt{3}}{3}$

32. $\dfrac{1}{\sqrt{7}} = \dfrac{1}{\sqrt{7}} \cdot \dfrac{\sqrt{7}}{\sqrt{7}} = \dfrac{1\sqrt{7}}{\sqrt{7^2}} = \dfrac{\sqrt{7}}{7}$

33. $\sqrt{\dfrac{1}{x}} = \dfrac{\sqrt{1}}{\sqrt{x}} = \dfrac{1}{\sqrt{x}} \cdot \dfrac{\sqrt{x}}{\sqrt{x}} = \dfrac{1\sqrt{x}}{\sqrt{x^2}} = \dfrac{\sqrt{x}}{x}$

34. $\sqrt{\dfrac{1}{z}} = \dfrac{\sqrt{1}}{\sqrt{z}} = \dfrac{1}{\sqrt{z}} \cdot \dfrac{\sqrt{z}}{\sqrt{z}} = \dfrac{1\sqrt{z}}{\sqrt{z^2}} = \dfrac{\sqrt{z}}{z}$

35. $\dfrac{6}{\sqrt{2y}} = \dfrac{6}{\sqrt{2y}} \cdot \dfrac{\sqrt{2y}}{\sqrt{2y}} = \dfrac{6\sqrt{2y}}{\sqrt{(2y)^2}}$

$= \dfrac{6\sqrt{2y}}{2y} = \dfrac{3\sqrt{2y}}{y}$

36. $\dfrac{9}{\sqrt{3t}} = \dfrac{9}{\sqrt{3t}} \cdot \dfrac{\sqrt{3t}}{\sqrt{3t}} = \dfrac{9\sqrt{3t}}{\sqrt{(3t)^2}}$

$= \dfrac{9\sqrt{3t}}{3t} = \dfrac{3\sqrt{3t}}{t}$

37. $\sqrt{\dfrac{a^3}{2}} = \dfrac{\sqrt{a^3}}{\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{\sqrt{2a \cdot a^2}}{\sqrt{4}} = \dfrac{a\sqrt{2a}}{2}$

38. $\sqrt{\dfrac{b^3}{3}} = \dfrac{\sqrt{b^3}}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{\sqrt{3b \cdot b^2}}{\sqrt{9}} = \dfrac{b\sqrt{3b}}{3}$

39. $\dfrac{6}{\sqrt{8}} = \dfrac{6}{\sqrt{4 \cdot 2}} = \dfrac{6}{2\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}}$

$= \dfrac{3\sqrt{2}}{\sqrt{(2)^2}} = \dfrac{3\sqrt{2}}{2}$

40. $\dfrac{2}{\sqrt{48}} = \dfrac{2}{\sqrt{16 \cdot 3}} = \dfrac{2}{4\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{\sqrt{3}}{2\sqrt{(3)^2}}$

$= \dfrac{\sqrt{3}}{2 \cdot 3} = \dfrac{\sqrt{3}}{6}$

41. $\dfrac{3}{\sqrt[3]{2}} = \dfrac{3}{\sqrt[3]{2}} \cdot \dfrac{\sqrt[3]{2^2}}{\sqrt[3]{2^2}} = \dfrac{3\sqrt[3]{2^2}}{\sqrt[3]{2^3}} = \dfrac{3\sqrt[3]{4}}{2}$

42. $\dfrac{2}{\sqrt[3]{7}} = \dfrac{2}{\sqrt[3]{7}} \cdot \dfrac{\sqrt[3]{7^2}}{\sqrt[3]{7^2}} = \dfrac{2\sqrt[3]{7^2}}{\sqrt[3]{7^3}} = \dfrac{2\sqrt[3]{49}}{7}$

43. $\dfrac{-6}{\sqrt[4]{x}} = \dfrac{-6}{\sqrt[4]{x}} \cdot \dfrac{\sqrt[4]{x^3}}{\sqrt[4]{x^3}} = \dfrac{-6\sqrt[4]{x^3}}{\sqrt[4]{x^4}} = \dfrac{-6\sqrt[4]{x^3}}{x}$

44. $\dfrac{-2}{\sqrt[5]{y}} = \dfrac{-2}{\sqrt[5]{y}} \cdot \dfrac{\sqrt[5]{y^4}}{\sqrt[5]{y^4}} = \dfrac{-2\sqrt[5]{y^4}}{\sqrt[5]{y^5}} = \dfrac{-2\sqrt[5]{y^4}}{y}$

45. $\dfrac{7}{\sqrt[3]{4}} = \dfrac{7}{\sqrt[3]{2^2}} = \dfrac{7}{\sqrt[3]{2^2}} \cdot \dfrac{\sqrt[3]{2}}{\sqrt[3]{2}}$

$= \dfrac{7\sqrt[3]{2}}{\sqrt[3]{2^3}} = \dfrac{7\sqrt[3]{2}}{2}$

46. $\dfrac{1}{\sqrt[3]{9}} = \dfrac{1}{\sqrt[3]{3^2}} = \dfrac{1}{\sqrt[3]{3^2}} \cdot \dfrac{\sqrt[3]{3}}{\sqrt[3]{3}}$

$= \dfrac{\sqrt[3]{3}}{\sqrt[3]{3^3}} = \dfrac{\sqrt[3]{3}}{3}$

47. $\sqrt[3]{\dfrac{4}{w^2}} = \dfrac{\sqrt[3]{4}}{\sqrt[3]{w^2}} = \dfrac{\sqrt[3]{4}}{\sqrt[3]{w^2}} \cdot \dfrac{\sqrt[3]{w}}{\sqrt[3]{w}}$

$= \dfrac{\sqrt[3]{4}\sqrt[3]{w}}{\sqrt[3]{w^3}} = \dfrac{\sqrt[3]{4w}}{w}$

48. $\sqrt[3]{\dfrac{5}{z^2}} = \dfrac{\sqrt[3]{5}}{\sqrt[3]{z^2}} = \dfrac{\sqrt[3]{5}}{\sqrt[3]{z^2}} \cdot \dfrac{\sqrt[3]{z}}{\sqrt[3]{z}}$

$= \dfrac{\sqrt[3]{5}\sqrt[3]{z}}{\sqrt[3]{z^3}} = \dfrac{\sqrt[3]{5z}}{z}$

49. $\sqrt[4]{\dfrac{16}{3}} = \dfrac{\sqrt[4]{16}}{\sqrt[4]{3}} = \dfrac{2}{\sqrt[4]{3}} \cdot \dfrac{\sqrt[4]{3^3}}{\sqrt[4]{3^3}} = \dfrac{2\sqrt[4]{3^3}}{\sqrt[4]{3^4}} = \dfrac{2\sqrt[4]{27}}{3}$

50. $\sqrt[4]{\dfrac{81}{8}} = \dfrac{\sqrt[4]{3^4}}{\sqrt[4]{2^3}} = \dfrac{3}{\sqrt[4]{2^3}} \cdot \dfrac{\sqrt[4]{2}}{\sqrt[4]{2}} = \dfrac{3\sqrt[4]{2}}{\sqrt[4]{2^4}} = \dfrac{3\sqrt[4]{2}}{2}$

51. $\dfrac{2}{\sqrt[3]{4x^2}} = \dfrac{2}{\sqrt[3]{2^2x^2}} \cdot \dfrac{\sqrt[3]{2x}}{\sqrt[3]{2x}} = \dfrac{2\sqrt[3]{2x}}{\sqrt[3]{2^3x^3}}$

$= \dfrac{2\sqrt[3]{2x}}{2x} = \dfrac{\sqrt[3]{2x}}{x}$

52. $\dfrac{6}{\sqrt[3]{3y^2}} = \dfrac{6}{\sqrt[3]{3y^2}} \cdot \dfrac{\sqrt[3]{3^2y}}{\sqrt[3]{3^2y}} = \dfrac{6\sqrt[3]{9y}}{\sqrt[3]{3^3y^3}}$

$= \dfrac{6\sqrt[3]{9y}}{3y} = \dfrac{2\sqrt[3]{9y}}{y}$

53. $\dfrac{8}{7\sqrt{24}} = \dfrac{8}{7\sqrt{4\cdot 6}} = \dfrac{8}{7\cdot 2\sqrt{6}} = \dfrac{4}{7\sqrt{6}} \cdot \dfrac{\sqrt{6}}{\sqrt{6}}$

$= \dfrac{4\sqrt{6}}{7\sqrt{6^2}} = \dfrac{4\sqrt{6}}{7\cdot 6} = \dfrac{2\sqrt{6}}{21}$

54. $\dfrac{5}{3\sqrt{50}} = \dfrac{5}{3\sqrt{25\cdot 2}} = \dfrac{5}{3\cdot 5\sqrt{2}} = \dfrac{1}{3\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}}$

$= \dfrac{\sqrt{2}}{3\sqrt{2^2}} = \dfrac{\sqrt{2}}{3\cdot 2} = \dfrac{\sqrt{2}}{6}$

55. $\dfrac{1}{\sqrt{x^7}} = \dfrac{1}{\sqrt{x^6 \cdot x}} = \dfrac{1}{x^3\sqrt{x}}$

$= \dfrac{1}{x^3\sqrt{x}} \cdot \dfrac{\sqrt{x}}{\sqrt{x}}$

$= \dfrac{\sqrt{x}}{x^3\sqrt{x^2}} = \dfrac{\sqrt{x}}{x^3 \cdot x} = \dfrac{\sqrt{x}}{x^4}$

56. $\dfrac{1}{\sqrt{y^5}} = \dfrac{1}{\sqrt{y^4 \cdot y}} = \dfrac{1}{y^2\sqrt{y}}$

$= \dfrac{1}{y^2\sqrt{y}} \cdot \dfrac{\sqrt{y}}{\sqrt{y}}$

$= \dfrac{\sqrt{y}}{y^2\sqrt{y^2}} = \dfrac{\sqrt{y}}{y^2 \cdot y} = \dfrac{\sqrt{y}}{y^3}$

57. $\dfrac{2}{\sqrt{8x^5}} = \dfrac{2}{\sqrt{4x^4 \cdot 2x}} = \dfrac{2}{2x^2\sqrt{2x}}$

$= \dfrac{1}{x^2\sqrt{2x}} \cdot \dfrac{\sqrt{2x}}{\sqrt{2x}} = \dfrac{\sqrt{2x}}{x^2\sqrt{2^2x^2}}$

$= \dfrac{\sqrt{2x}}{x^2 \cdot 2x}$

$= \dfrac{\sqrt{2x}}{2x^3}$

58. $\dfrac{6}{\sqrt{27t^7}} = \dfrac{6}{\sqrt{9t^6 \cdot 3t}} = \dfrac{6}{3t^3\sqrt{3t}}$

$= \dfrac{2}{t^3\sqrt{3t}} \cdot \dfrac{\sqrt{3t}}{\sqrt{3t}} = \dfrac{2\sqrt{3t}}{t^3\sqrt{3^2t^2}}$

$= \dfrac{2\sqrt{3t}}{t^3 \cdot 3t}$

$= \dfrac{2\sqrt{3t}}{3t^4}$

59. $\sqrt{2}+\sqrt{6}$

60. $\sqrt{11}-\sqrt{5}$

61. $\sqrt{x}-23$

62. $17+\sqrt{y}$

63. $\dfrac{4}{\sqrt{2}+3} = \dfrac{4}{\sqrt{2}+3} \cdot \dfrac{\sqrt{2}-3}{\sqrt{2}-3}$

$= \dfrac{4\left(\sqrt{2}-3\right)}{\left(\sqrt{2}\right)^2 - 3^2}$

$= \dfrac{4\sqrt{2}-12}{2-9} = \dfrac{4\sqrt{2}-12}{-7}$

64. $\dfrac{6}{4-\sqrt{3}} = \dfrac{6}{4-\sqrt{3}} \cdot \dfrac{4+\sqrt{3}}{4+\sqrt{3}}$

$= \dfrac{6\left(4+\sqrt{3}\right)}{4^2 - \left(\sqrt{3}\right)^2}$

$= \dfrac{24+6\sqrt{3}}{16-3} = \dfrac{24+6\sqrt{3}}{13}$

65. $\dfrac{8}{\sqrt{6}-2}=\dfrac{8}{\sqrt{6}-2}\cdot\dfrac{\sqrt{6}+2}{\sqrt{6}+2}$

$=\dfrac{8\left(\sqrt{6}+2\right)}{\left(\sqrt{6}\right)^2-2^2}$

$=\dfrac{8\left(\sqrt{6}+2\right)}{6-4}=\dfrac{8\left(\sqrt{6}+2\right)}{2}$

$=4\left(\sqrt{6}+2\right)=4\sqrt{6}+8$

66. $\dfrac{-12}{\sqrt{5}-3}=\dfrac{-12}{\sqrt{5}-3}\cdot\dfrac{\sqrt{5}+3}{\sqrt{5}+3}$

$=\dfrac{-12\left(\sqrt{5}+3\right)}{\left(\sqrt{5}\right)^2-3^2}$

$=\dfrac{-12\left(\sqrt{5}+3\right)}{5-9}=\dfrac{-12\left(\sqrt{5}+3\right)}{-4}$

$=3\left(\sqrt{5}+3\right)=3\sqrt{5}+9$

67. $\dfrac{\sqrt{7}}{\sqrt{3}+2}=\dfrac{\sqrt{7}}{\sqrt{3}+2}\cdot\dfrac{\sqrt{3}-2}{\sqrt{3}-2}$

$=\dfrac{\sqrt{7}\left(\sqrt{3}-2\right)}{\left(\sqrt{3}\right)^2-2^2}$

$=\dfrac{\sqrt{7}\cdot\sqrt{3}-\sqrt{7}\cdot(2)}{3-4}$

$=\dfrac{\sqrt{21}-2\sqrt{7}}{-1}$

$=-\sqrt{21}+2\sqrt{7}$

68. $\dfrac{\sqrt{8}}{\sqrt{3}+1}=\dfrac{\sqrt{4\cdot 2}}{\sqrt{3}+1}\cdot\dfrac{\sqrt{3}-1}{\sqrt{3}-1}$

$=\dfrac{2\sqrt{2}\left(\sqrt{3}-1\right)}{\left(\sqrt{3}\right)^2-1^2}$

$=\dfrac{2\sqrt{2}\left(\sqrt{3}-1\right)}{3-1}=\dfrac{2\sqrt{2}\left(\sqrt{3}-1\right)}{2}$

$=\sqrt{2}\left(\sqrt{3}-1\right)$

$=\sqrt{6}-\sqrt{2}$

69. $\dfrac{-1}{\sqrt{p}+\sqrt{q}}=\dfrac{-1}{\sqrt{p}+\sqrt{q}}\cdot\dfrac{\sqrt{p}-\sqrt{q}}{\sqrt{p}-\sqrt{q}}$

$=\dfrac{-1\left(\sqrt{p}-\sqrt{q}\right)}{\left(\sqrt{p}\right)^2-\left(\sqrt{q}\right)^2}$

$=\dfrac{-\sqrt{p}+\sqrt{q}}{p-q}$

70. $\dfrac{6}{\sqrt{a}-\sqrt{b}}=\dfrac{6}{\sqrt{a}-\sqrt{b}}\cdot\dfrac{\sqrt{a}+\sqrt{b}}{\sqrt{a}+\sqrt{b}}$

$=\dfrac{6\left(\sqrt{a}+\sqrt{b}\right)}{\left(\sqrt{a}\right)^2-\left(\sqrt{b}\right)^2}$

$=\dfrac{6\sqrt{a}+6\sqrt{b}}{a-b}$

71. $\dfrac{x-5}{\sqrt{x}+\sqrt{5}} = \dfrac{x-5}{\sqrt{x}+\sqrt{5}} \cdot \dfrac{\sqrt{x}-\sqrt{5}}{\sqrt{x}-\sqrt{5}}$

$= \dfrac{(x-5)\left(\sqrt{x}-\sqrt{5}\right)}{\left(\sqrt{x}\right)^2-\left(\sqrt{5}\right)^2}$

$= \dfrac{\cancel{(x-5)}\left(\sqrt{x}-\sqrt{5}\right)}{\cancel{x-5}} = \sqrt{x}-\sqrt{5}$

72. $\dfrac{y-2}{\sqrt{y}-\sqrt{2}} = \dfrac{y-2}{\sqrt{y}-\sqrt{2}} \cdot \dfrac{\sqrt{y}+\sqrt{2}}{\sqrt{y}+\sqrt{2}}$

$= \dfrac{(y-2)\left(\sqrt{y}+\sqrt{2}\right)}{\left(\sqrt{y}\right)^2-\left(\sqrt{2}\right)^2}$

$= \dfrac{\cancel{(y-2)}\left(\sqrt{y}+\sqrt{2}\right)}{\cancel{y-2}} = \sqrt{y}+\sqrt{2}$

73. $\dfrac{\sqrt{w}+2}{9-\sqrt{w}} = \dfrac{\sqrt{w}+2}{9-\sqrt{w}} \cdot \dfrac{9+\sqrt{w}}{9+\sqrt{w}}$

$= \dfrac{\left(\sqrt{w}+2\right)\left(9+\sqrt{w}\right)}{(9)^2-\left(\sqrt{w}\right)^2}$

$= \dfrac{w+9\sqrt{w}+2\sqrt{w}+18}{81-w}$

$= \dfrac{w+11\sqrt{w}+18}{81-w}$

74. $\dfrac{10-\sqrt{t}}{\sqrt{t}-6} = \dfrac{10-\sqrt{t}}{\sqrt{t}-6} \cdot \dfrac{\sqrt{t}+6}{\sqrt{t}+6}$

$= \dfrac{\left(10-\sqrt{t}\right)\left(\sqrt{t}+6\right)}{\left(\sqrt{t}\right)^2-(6)^2}$

$= \dfrac{-t-6\sqrt{t}+10\sqrt{t}+60}{t-36}$

$= \dfrac{-t+4\sqrt{t}+60}{t-36}$

75. $\dfrac{3\sqrt{x}-\sqrt{y}}{\sqrt{y}+\sqrt{x}} = \dfrac{3\sqrt{x}-\sqrt{y}}{\sqrt{y}+\sqrt{x}} \cdot \dfrac{\sqrt{y}-\sqrt{x}}{\sqrt{y}-\sqrt{x}}$

$= \dfrac{\left(3\sqrt{x}-\sqrt{y}\right)\left(\sqrt{y}-\sqrt{x}\right)}{\left(\sqrt{y}\right)^2-\left(\sqrt{x}\right)^2}$

$= \dfrac{3\sqrt{xy}-3\sqrt{x^2}-\sqrt{y^2}+\sqrt{xy}}{y-x}$

$= \dfrac{4\sqrt{xy}-3x-y}{y-x}$

76. $\dfrac{2\sqrt{a}+\sqrt{b}}{\sqrt{b}-\sqrt{a}} = \dfrac{2\sqrt{a}+\sqrt{b}}{\sqrt{b}-\sqrt{a}} \cdot \dfrac{\sqrt{b}+\sqrt{a}}{\sqrt{b}+\sqrt{a}}$

$= \dfrac{\left(2\sqrt{a}+\sqrt{b}\right)\left(\sqrt{b}+\sqrt{a}\right)}{\left(\sqrt{b}\right)^2-\left(\sqrt{a}\right)^2}$

$= \dfrac{2\sqrt{ab}+2\sqrt{a^2}+\sqrt{b^2}+\sqrt{ab}}{b-a}$

$= \dfrac{3\sqrt{ab}+2a+b}{b-a}$

77. $\dfrac{3\sqrt{10}}{2+\sqrt{10}} = \dfrac{3\sqrt{10}}{2+\sqrt{10}} \cdot \dfrac{2-\sqrt{10}}{2-\sqrt{10}}$

$= \dfrac{3\sqrt{10}\left(2-\sqrt{10}\right)}{(2)^2-\left(\sqrt{10}\right)^2} = \dfrac{6\sqrt{10}-3\sqrt{100}}{4-10}$

$= \dfrac{6\sqrt{10}-3\cdot 10}{-6} = \dfrac{6\sqrt{10}-30}{-6}$

$= \dfrac{\cancel{-6}\left(5-\sqrt{10}\right)}{\cancel{-6}} = 5-\sqrt{10}$

78. $\dfrac{4\sqrt{7}}{3+\sqrt{7}} = \dfrac{4\sqrt{7}}{3+\sqrt{7}} \cdot \dfrac{3-\sqrt{7}}{3-\sqrt{7}} = \dfrac{4\sqrt{7}\left(3-\sqrt{7}\right)}{(3)^2-\left(\sqrt{7}\right)^2}$

$= \dfrac{12\sqrt{7}-4\sqrt{49}}{9-7} = \dfrac{12\sqrt{7}-4\cdot 7}{2}$

$= \dfrac{\cancel{2}\left(6\sqrt{7}-14\right)}{\cancel{2}}$

$= 6\sqrt{7}-14$

79. $\dfrac{2\sqrt{3}+\sqrt{7}}{3\sqrt{3}-\sqrt{7}} = \dfrac{2\sqrt{3}+\sqrt{7}}{3\sqrt{3}-\sqrt{7}} \cdot \dfrac{3\sqrt{3}+\sqrt{7}}{3\sqrt{3}+\sqrt{7}}$

$= \dfrac{\left(2\sqrt{3}+\sqrt{7}\right)\left(3\sqrt{3}+\sqrt{7}\right)}{\left(3\sqrt{3}\right)^2-\left(\sqrt{7}\right)^2}$

$= \dfrac{6\sqrt{9}+2\sqrt{21}+3\sqrt{21}+\sqrt{49}}{9\cdot 3-7}$

$= \dfrac{6\cdot 3+5\sqrt{21}+7}{27-7}$

$= \dfrac{18+5\sqrt{21}+7}{20}$

$= \dfrac{25+5\sqrt{21}}{20} = \dfrac{\cancel{5}\left(5+\sqrt{21}\right)}{\cancel{5}\cdot 4}$

$= \dfrac{5+\sqrt{21}}{4}$

80. $\dfrac{5\sqrt{2}-\sqrt{5}}{5\sqrt{2}+\sqrt{5}} = \dfrac{5\sqrt{2}-\sqrt{5}}{5\sqrt{2}+\sqrt{5}} \cdot \dfrac{5\sqrt{2}-\sqrt{5}}{5\sqrt{2}-\sqrt{5}}$

$= \dfrac{\left(5\sqrt{2}-\sqrt{5}\right)^2}{\left(5\sqrt{2}\right)^2-\left(\sqrt{5}\right)^2}$

$= \dfrac{\left(5\sqrt{2}\right)^2-2\cdot 5\sqrt{2}\cdot\sqrt{5}+\left(\sqrt{5}\right)^2}{25\cdot 2-5}$

$= \dfrac{25\cdot 2-10\sqrt{10}+5}{50-5}$

$= \dfrac{50-10\sqrt{10}+5}{45}$

$= \dfrac{55-10\sqrt{10}}{45} = \dfrac{\cancel{5}\left(11-2\sqrt{10}\right)}{\cancel{5}\cdot 9}$

$= \dfrac{11-2\sqrt{10}}{9}$

81. $\dfrac{\sqrt{5}+4}{2-\sqrt{5}} = \dfrac{\sqrt{5}+4}{2-\sqrt{5}} \cdot \dfrac{2+\sqrt{5}}{2+\sqrt{5}}$

$= \dfrac{\left(\sqrt{5}+4\right)\left(2+\sqrt{5}\right)}{(2)^2-\left(\sqrt{5}\right)^2}$

$= \dfrac{2\sqrt{5}+\sqrt{25}+8+4\sqrt{5}}{4-5}$

$= \dfrac{6\sqrt{5}+5+8}{-1} = \dfrac{6\sqrt{5}+13}{-1}$

$= -6\sqrt{5}-13$

82. $\dfrac{3+\sqrt{2}}{\sqrt{2}-5} = \dfrac{3+\sqrt{2}}{\sqrt{2}-5} \cdot \dfrac{\sqrt{2}+5}{\sqrt{2}+5}$

$= \dfrac{\left(3+\sqrt{2}\right)\left(\sqrt{2}+5\right)}{\left(\sqrt{2}\right)^2-(5)^2}$

$= \dfrac{3\sqrt{2}+15+\sqrt{4}+5\sqrt{2}}{2-25}$

$= \dfrac{8\sqrt{2}+15+2}{-23}$

$= \dfrac{8\sqrt{2}+17}{-23}$

83. $\dfrac{16}{\sqrt[3]{4}} = \dfrac{16}{\sqrt[3]{2^2}} \cdot \dfrac{\sqrt[3]{2}}{\sqrt[3]{2}} = \dfrac{16\sqrt[3]{2}}{\sqrt[3]{2^3}} = \dfrac{16\sqrt[3]{2}}{2} = 8\sqrt[3]{2}$

84. $\dfrac{21}{\sqrt[4]{27}} = \dfrac{21}{\sqrt[4]{3^3}} \cdot \dfrac{\sqrt[4]{3}}{\sqrt[4]{3}} = \dfrac{21\sqrt[4]{3}}{\sqrt[4]{3^4}} = \dfrac{21\sqrt[4]{3}}{3} = 7\sqrt[4]{3}$

85. $\dfrac{4}{x-\sqrt{2}} = \dfrac{4}{x-\sqrt{2}} \cdot \dfrac{x+\sqrt{2}}{x+\sqrt{2}}$

$= \dfrac{4\left(x+\sqrt{2}\right)}{x^2-\left(\sqrt{2}\right)^2} = \dfrac{4x+4\sqrt{2}}{x^2-2}$

86. $\dfrac{8}{y+\sqrt{3}} = \dfrac{8}{y+\sqrt{3}} \cdot \dfrac{y-\sqrt{3}}{y-\sqrt{3}}$

$= \dfrac{8\left(y-\sqrt{3}\right)}{y^2-\left(\sqrt{3}\right)^2} = \dfrac{8y-8\sqrt{3}}{y^2-3}$

87. $T(x) = 2\pi\sqrt{\dfrac{x}{32}}$

$T(1) = 2\pi\sqrt{\dfrac{1}{32}} = 2\pi\dfrac{1}{\sqrt{16\cdot 2}} = \dfrac{2\pi}{4\sqrt{2}}$

$= \dfrac{2\pi}{4\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{2\pi\sqrt{2}}{4\cdot 2} = \dfrac{\pi\sqrt{2}}{4}$ sec

≈ 1.11 sec

88. $T(x) = \sqrt{\dfrac{10x}{49}}$

$T(230) = \sqrt{\dfrac{10(230)}{49}} = \sqrt{\dfrac{2300}{49}} = \dfrac{\sqrt{100\cdot 23}}{\sqrt{49}}$

$= \dfrac{10\sqrt{23}}{7}$ sec

≈ 6.85 sec

89. **a.** $\dfrac{1}{\sqrt{2}} = \dfrac{1}{\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{\sqrt{2}}{\sqrt{2^2}} = \dfrac{\sqrt{2}}{2}$

b. $\dfrac{1}{\sqrt[3]{2}} = \dfrac{1}{\sqrt[3]{2}} \cdot \dfrac{\sqrt[3]{2^2}}{\sqrt[3]{2^2}} = \dfrac{\sqrt[3]{2^2}}{\sqrt[3]{2^3}} = \dfrac{\sqrt[3]{4}}{2}$

90. **a.** $\dfrac{1}{\sqrt[3]{x}} = \dfrac{1}{\sqrt[3]{x}} \cdot \dfrac{\sqrt[3]{x^2}}{\sqrt[3]{x^2}} = \dfrac{\sqrt[3]{x^2}}{\sqrt[3]{x^3}} = \dfrac{\sqrt[3]{x^2}}{x}$

b. $\dfrac{1}{\sqrt[3]{x^2}} = \dfrac{1}{\sqrt[3]{x^2}} \cdot \dfrac{\sqrt[3]{x}}{\sqrt[3]{x}} = \dfrac{\sqrt[3]{x}}{\sqrt[3]{x^3}} = \dfrac{\sqrt[3]{x}}{x}$

91. **a.** $\dfrac{1}{\sqrt{5a}} = \dfrac{1}{\sqrt{5a}} \cdot \dfrac{\sqrt{5a}}{\sqrt{5a}} = \dfrac{\sqrt{5a}}{\sqrt{(5a)^2}} = \dfrac{\sqrt{5a}}{5a}$

b. $\dfrac{1}{\sqrt{5}+a} = \dfrac{1}{\sqrt{5}+a} \cdot \dfrac{\sqrt{5}-a}{\sqrt{5}-a} = \dfrac{\sqrt{5}-a}{\left(\sqrt{5}\right)^2 - a^2}$

$= \dfrac{\sqrt{5}-a}{5-a^2}$

92. $\dfrac{1}{\sqrt{7}} + \sqrt{7} = \dfrac{1}{\sqrt{7}} \cdot \dfrac{\sqrt{7}}{\sqrt{7}} + \sqrt{7} = \dfrac{\sqrt{7}}{7} + \dfrac{\sqrt{7}}{1}$

$= \dfrac{\sqrt{7}}{7} + \dfrac{\sqrt{7}}{1} \cdot \dfrac{7}{7} = \dfrac{\sqrt{7}+7\sqrt{7}}{7}$

$= \dfrac{8\sqrt{7}}{7}$

93. $\dfrac{\sqrt{6}}{2} + \dfrac{1}{\sqrt{6}} = \dfrac{\sqrt{6}}{2} + \dfrac{1}{\sqrt{6}} \cdot \dfrac{\sqrt{6}}{\sqrt{6}}$

$= \dfrac{\sqrt{6}}{2} + \dfrac{\sqrt{6}}{6}$

$= \dfrac{\sqrt{6}}{2} \cdot \dfrac{3}{3} + \dfrac{\sqrt{6}}{6}$

$= \dfrac{3\sqrt{6}+\sqrt{6}}{6}$

$= \dfrac{4\sqrt{6}}{6}$

$= \dfrac{2\sqrt{6}}{3}$

94. **a.** $\dfrac{4}{\sqrt{2x}} = \dfrac{4}{\sqrt{2x}} \cdot \dfrac{\sqrt{2x}}{\sqrt{2x}}$

$= \dfrac{4\sqrt{2x}}{\sqrt{(2x)^2}} = \dfrac{4\sqrt{2x}}{2x}$

$= \dfrac{2\sqrt{2x}}{x}$

b. $\dfrac{4}{2-\sqrt{x}} = \dfrac{4}{2-\sqrt{x}} \cdot \dfrac{2+\sqrt{x}}{2+\sqrt{x}}$

$= \dfrac{4\left(2+\sqrt{x}\right)}{(2)^2 - \left(\sqrt{x}\right)^2}$

$= \dfrac{8+4\sqrt{x}}{4-x}$

95. $\sqrt{15}-\sqrt{\frac{3}{5}}+\sqrt{\frac{5}{3}}=\sqrt{15}-\frac{\sqrt{3}}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}}+\frac{\sqrt{5}}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}$

$=\frac{\sqrt{15}}{1}-\frac{\sqrt{15}}{5}+\frac{\sqrt{15}}{3}$

$=\frac{\sqrt{15}}{1}\cdot\frac{15}{15}-\frac{\sqrt{15}}{5}\cdot\frac{3}{3}+\frac{\sqrt{15}}{3}\cdot\frac{5}{5}$

$=\frac{15\sqrt{15}-3\sqrt{15}+5\sqrt{15}}{15}=\frac{17\sqrt{15}}{15}$

96. $\sqrt{\frac{6}{2}}-\sqrt{12}+\sqrt{\frac{2}{6}}=\sqrt{3}-\sqrt{4\cdot 3}+\sqrt{\frac{1}{3}}$

$=\sqrt{3}-2\sqrt{3}+\frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=-\sqrt{3}+\frac{\sqrt{3}}{3}$

$=-\sqrt{3}\cdot\frac{3}{3}+\frac{\sqrt{3}}{3}=\frac{-3\sqrt{3}+\sqrt{3}}{3}$

$=\frac{-2\sqrt{3}}{3}$

97. $\sqrt[3]{25}+\frac{3}{\sqrt[3]{5}}=\sqrt[3]{5^2}+\frac{3}{\sqrt[3]{5}}\cdot\frac{\sqrt[3]{5^2}}{\sqrt[3]{5^2}}=\frac{\sqrt[3]{5^2}}{1}+\frac{3\sqrt[3]{5^2}}{\sqrt[3]{5^3}}$

$=\frac{\sqrt[3]{5^2}}{1}+\frac{3\sqrt[3]{5^2}}{5}=\frac{\sqrt[3]{5^2}}{1}\cdot\frac{5}{5}+\frac{3\sqrt[3]{5^2}}{5}$

$=\frac{5\sqrt[3]{5^2}+3\sqrt[3]{5^2}}{5}=\frac{8\sqrt[3]{5^2}}{5}=\frac{8\sqrt[3]{25}}{5}$

98. $\frac{1}{\sqrt[3]{4}}+\sqrt[3]{54}=\frac{1}{\sqrt[3]{2^2}}+\sqrt[3]{27\cdot 2}$

$=\frac{1}{\sqrt[3]{2^2}}\cdot\frac{\sqrt[3]{2}}{\sqrt[3]{2}}+3\sqrt[3]{2}=\frac{\sqrt[3]{2}}{\sqrt[3]{2^3}}+\frac{3\sqrt[3]{2}}{1}$

$=\frac{\sqrt[3]{2}}{2}+\frac{3\sqrt[3]{2}}{1}\cdot\frac{2}{2}=\frac{\sqrt[3]{2}+6\sqrt[3]{2}}{2}=\frac{7\sqrt[3]{2}}{2}$

99. $\frac{\sqrt{3}+6}{2}=\frac{\sqrt{3}+6}{2}\cdot\frac{\sqrt{3}-6}{\sqrt{3}-6}=\frac{\left(\sqrt{3}\right)^2-6^2}{2\left(\sqrt{3}-6\right)}$

$=\frac{3-36}{2\sqrt{3}-12}=\frac{-33}{2\sqrt{3}-12}$

100. $\frac{\sqrt{7}-2}{5}=\frac{\sqrt{7}-2}{5}\cdot\frac{\sqrt{7}+2}{\sqrt{7}+2}=\frac{\left(\sqrt{7}\right)^2-2^2}{5\left(\sqrt{7}+2\right)}$

$=\frac{7-4}{5\sqrt{7}+10}=\frac{3}{5\sqrt{7}+10}$

101. $\frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}}=\frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}}\cdot\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}+\sqrt{b}}$

$=\frac{\left(\sqrt{a}\right)^2-\left(\sqrt{b}\right)^2}{\left(\sqrt{a}+\sqrt{b}\right)^2}$

$=\frac{a-b}{\left(\sqrt{a}\right)^2+2\sqrt{a}\cdot\sqrt{b}+\left(\sqrt{b}\right)^2}$

$=\frac{a-b}{a+2\sqrt{ab}+b}$

102. $\frac{\sqrt{p}+\sqrt{q}}{\sqrt{p}-\sqrt{q}}=\frac{\sqrt{p}+\sqrt{q}}{\sqrt{p}-\sqrt{q}}\cdot\frac{\sqrt{p}-\sqrt{q}}{\sqrt{p}-\sqrt{q}}$

$=\frac{\left(\sqrt{p}\right)^2-\left(\sqrt{q}\right)^2}{\left(\sqrt{p}-\sqrt{q}\right)^2}$

$=\frac{p-q}{\left(\sqrt{p}\right)^2-2\sqrt{p}\cdot\sqrt{q}+\left(\sqrt{q}\right)^2}$

$=\frac{p-q}{p-2\sqrt{pq}+q}$

103. $\dfrac{\sqrt{5+3h}-\sqrt{5}}{h}=\dfrac{\sqrt{5+3h}-\sqrt{5}}{h}\cdot\dfrac{\sqrt{5+3h}+\sqrt{5}}{\sqrt{5+3h}+\sqrt{5}}$

$$=\frac{\left(\sqrt{5+3h}\right)^2-\left(\sqrt{5}\right)^2}{h\left(\sqrt{5+3h}+\sqrt{5}\right)}$$

$$=\frac{5+3h-5}{h\left(\sqrt{5+3h}+\sqrt{5}\right)}$$

$$=\frac{3h}{h\left(\sqrt{5+3h}+\sqrt{5}\right)}=\frac{3}{\sqrt{5+3h}+\sqrt{5}}$$

104. $\dfrac{\sqrt{7+2h}-\sqrt{7}}{h}=\dfrac{\sqrt{7+2h}-\sqrt{7}}{h}\cdot\dfrac{\sqrt{7+2h}+\sqrt{7}}{\sqrt{7+2h}+\sqrt{7}}$

$$=\frac{\left(\sqrt{7+2h}\right)^2-\left(\sqrt{7}\right)^2}{h\left(\sqrt{7+2h}+\sqrt{7}\right)}$$

$$=\frac{7+2h-7}{h\left(\sqrt{7+2h}+\sqrt{7}\right)}$$

$$=\frac{2h}{h\left(\sqrt{7+2h}+\sqrt{7}\right)}=\frac{2}{\sqrt{7+2h}+\sqrt{7}}$$

105. $\dfrac{\sqrt{4+5h}-2}{h}=\dfrac{\sqrt{4+5h}-2}{h}\cdot\dfrac{\sqrt{4+5h}+2}{\sqrt{4+5h}+2}$

$$=\frac{\left(\sqrt{4+5h}\right)^2-(2)^2}{h\left(\sqrt{4+5h}+2\right)}$$

$$=\frac{4+5h-4}{h\left(\sqrt{4+5h}+2\right)}$$

$$=\frac{5h}{h\left(\sqrt{4+5h}+2\right)}$$

$$=\frac{5}{\sqrt{4+5h}+2}$$

106. $\dfrac{\sqrt{9+4h}-3}{h}=\dfrac{\sqrt{9+4h}-3}{h}\cdot\dfrac{\sqrt{9+4h}+3}{\sqrt{9+4h}+3}$

$$=\frac{\left(\sqrt{9+4h}\right)^2-(3)^2}{h\left(\sqrt{9+4h}+3\right)}$$

$$=\frac{9+4h-9}{h\left(\sqrt{9+4h}+3\right)}$$

$$=\frac{4h}{h\left(\sqrt{9+4h}+3\right)}$$

$$=\frac{4}{\sqrt{9+4h}+3}$$

Section 6.7 Practice Exercises

1. **a.** radical

b. isolate; 7

c. extraneous

d. third

2. **a.** $2x+3=23$ Linear

$2x=20$

$x=10$ $\{10\}$

b. $2x^2-9x=5$ Quadratic

$2x^2-9x-5=0$

$(2x+1)(x-5)=0$

$2x+1=0$ or $x-5=0$

$2x=-1$ or $x=5$

$x=-\frac{1}{2}$ or $x=5$ $\left\{-\frac{1}{2},5\right\}$

3. $\sqrt{\frac{9w^3}{16}}=\sqrt{\frac{9w^2\cdot w}{16}}=\frac{3w\sqrt{w}}{4}$

4. $\sqrt{\frac{a^2}{3}}=\frac{a}{\sqrt{3}}=\frac{a}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{a\sqrt{3}}{3}$

5. $\sqrt[3]{54c^4}=\sqrt[3]{27c^3\cdot 2c}$

$=3c\sqrt[3]{2c}$

6. $\sqrt{\frac{49}{5t^3}}=\frac{\sqrt{49}}{\sqrt{t^2\cdot 5t}}=\frac{7}{t\sqrt{5t}}=\frac{7}{t\sqrt{5t}}\cdot\frac{\sqrt{5t}}{\sqrt{5t}}$

$=\frac{7\sqrt{5t}}{t\sqrt{25t^2}}=\frac{7\sqrt{5t}}{t\cdot 5t}=\frac{7\sqrt{5t}}{5t^2}$

7. $\left(\sqrt{4x-6}\right)^2=4x-6$

8. $\left(\sqrt{5y+2}\right)^2=5y+2$

9. $\left(\sqrt[3]{9p+7}\right)^3=9p+7$

10. $\left(\sqrt[3]{4t+13}\right)^3=4t+13$

11. $\sqrt{x}=10$

$\left(\sqrt{x}\right)^2=(10)^2$

$x=100$

Check: $\sqrt{100}=10$, $10=10$

The solution is $\{100\}$.

12. $\sqrt{y}=7$

$\left(\sqrt{y}\right)^2=(7)^2$

$y=49$

Check: $\sqrt{49}=7$, $7=7$

The solution is $\{49\}$.

13. $\sqrt{x}+4=6$

$\sqrt{x}=2$

$\left(\sqrt{x}\right)^2=2^2$

$x=4$

Check: $\sqrt{4}+4=6$, $2+4=6$, $6=6$

The solution is $\{4\}$.

14. $\sqrt{x}+2=8$

$\sqrt{x}=6$

$\left(\sqrt{x}\right)^2=6^2$

$x=36$

Check: $\sqrt{36}+2=8$, $6+2=8$, $8=8$

The solution is $\{36\}$.

15. $\sqrt{5y+1}=4$

$\left(\sqrt{5y+1}\right)^2=4^2$

$5y+1=16$

$5y=15$

$y=3$

Check:

$\sqrt{5(3)+1}=4$

$\sqrt{15+1}=4$

$\sqrt{16}=4$

$4=4$

The solution is $\{3\}$.

16. $\sqrt{9z-5}-2=9$

$\sqrt{9z-5}=11$

$\left(\sqrt{9z-5}\right)^2=11^2$

$9z-5=121$

$9z=126$

$z=14$

Check:

$\sqrt{9(14)-5}-2=9$

$\sqrt{126-5}-2=9$

$\sqrt{121}-2=9$

$11-2=9$

$9=9$

The solution is $\{14\}$.

17. $6=\sqrt{2z-3}-3$

$9=\sqrt{2z-3}$

$9^2=\left(\sqrt{2z-3}\right)^2$

$81=2z-3$

$2z=84$

$z=42$

Check:

$6=\left(2(42)-3\right)^{1/2}-3$

$6=\left(84-3\right)^{1/2}-3$

$6=81^{1/2}-3$

$6=9-3$

$6=6$

The solution is $\{42\}$.

18. $4=\sqrt{8+3a}-1$

$5=\sqrt{8+3a}$

$5^2=\left(\sqrt{8+3a}\right)^2$

$25=8+3a$

$3a=17$

$a=\dfrac{17}{3}$

Check:

$4=\left(8+3\left(\dfrac{17}{3}\right)\right)^{1/2}-1$

$4=\left(8+17\right)^{1/2}-1$

$4=25^{1/2}-1$

$4=5-1$

$4=4$

The solution is $\left\{\frac{17}{3}\right\}$.

19. $\sqrt{x^2+5}=x+1$

$\left(\sqrt{x^2+5}\right)^2=\left(x+1\right)^2$

$x^2+5=x^2+2x+1$

$-2x=-4$

$x=2$

Check:

$\sqrt{(2)^2+5}=2+1$

$\sqrt{4+5}=3$

20. $\sqrt{y^2-8}=y-2$

$\left(\sqrt{y^2-8}\right)^2=\left(y-2\right)^2$

$y^2-8=y^2-4y+4$

$4y=12$

$y=3$

Check:

$\sqrt{(3)^2-8}=3-2$

$\sqrt{9-8}=1$

$$\sqrt{9} = 43$$
$$3 = 3$$

The solution is $\{2\}$.

$$\sqrt{1} = 1$$
$$1 = 1$$

The solution is $\{3\}$.

21. $\sqrt[3]{x-2} - 1 = 2$

$$\sqrt[3]{x-2} = 3$$
$$\left(\sqrt[3]{x-2}\right)^3 = 3^3$$
$$x - 2 = 27$$
$$x = 29$$

Check:

$$\sqrt[3]{29-2} - 1 = 2$$
$$\sqrt[3]{27} - 1 = 2$$
$$3 - 1 = 2$$
$$2 = 2$$

The solution is $\{29\}$.

22. $\sqrt[3]{2x-5} - 1 = 1$

$$\sqrt[3]{2x-5} = 2$$
$$\left(\sqrt[3]{2x-5}\right)^3 = 2^3$$
$$2x - 5 = 8$$
$$2x = 13$$
$$x = \frac{13}{2}$$

Check:

$$\sqrt[3]{2\left(\frac{13}{2}\right) - 5} - 1 = 1$$
$$\sqrt[3]{13-5} - 1 = 1$$
$$\sqrt[3]{8} - 1 = 1$$
$$2 - 1 = 1$$
$$1 = 1$$

The solution is $\left\{\frac{13}{2}\right\}$.

23. $(15-w)^{1/3} + 7 = 2$

$$(15-w)^{1/3} = -5$$
$$\left(\sqrt[3]{15-w}\right)^3 = (-5)^3$$
$$15 - w = -125$$
$$140 = w$$

Check:

$$(15-w)^{1/3} + 7 = 2$$
$$(15-140)^{1/3} + 7 = 2$$
$$(-125)^{1/3} + 7 = 2$$
$$-5 + 7 = 2$$
$$2 = 2$$

The solution is $\{140\}$.

24. $(k+18)^{1/3} + 5 = 3$

$$(k+18)^{1/3} = -2$$
$$\left(\sqrt[3]{k+18}\right)^3 = (-2)^3$$
$$k + 18 = -8$$
$$k = -26$$

Check:

$$(-26+18)^{1/3} + 5 = 3$$
$$(-8)^{1/3} + 5 = 3$$
$$-2 + 5 = 3$$
$$3 = 3$$

The solution is $\{-26\}$.

25. $3+\sqrt{x-16}=0$

$\sqrt{x-16}=-3$

$\left(\sqrt{x-16}\right)^2=(-3)^2$

$x-16=9$

$x=25$

Check:

$3+\sqrt{25-16}=0$

$3+\sqrt{9}=0$

$3+3=0$

$6\neq 0$

{ } ($x=25$ does not check).

26. $12+\sqrt{2x+1}=0$

$\sqrt{2x+1}=-12$

$\left(\sqrt{2x+1}\right)^2=(-12)^2$

$2x+1=144$

$2x=143$

$x=\dfrac{143}{2}$

Check:

$12+\sqrt{2\left(\dfrac{143}{2}\right)+1}=0$

$12+\sqrt{143+1}=0$

$12+\sqrt{144}=0$

$12+12=0$

$24\neq 0$

{ } ($x=\frac{143}{2}$ does not check).

27. $2\sqrt{6a+7}-2a=0$

$2\sqrt{6a+7}=2a$

$\sqrt{6a+7}=a$

$\left(\sqrt{6a+7}\right)^2=(a)^2$

$6a+7=a^2$

$a^2-6a-7=0$

$(a-7)(a+1)=0$

$a-7=0$ or $a+1=0$

$a=7$ or $a=-1$

Check $a=7$:

$2\sqrt{6(7)+7}-2(7)=0$

$2\sqrt{42+7}-14=0$

$2\sqrt{49}-14=0$

$2\cdot 7-14=0$

$14-14=0$

$0=0$

Check $a=-1$:

$2\sqrt{6(-1)+7}-2(-1)=0$

$2\sqrt{-6+7}+2=0$

$2\sqrt{1}+2=0$

$2\cdot 1+2=0$

$2+2=0$

$4\neq 0$

The solution is {7}. ($a=-1$ does not check).

28.
$$2\sqrt{3-w}-w=0$$
$$2\sqrt{3-w}=w$$
$$\left(2\sqrt{3-w}\right)^2=(w)^2$$
$$4(3-w)=w^2$$
$$12-4w=w^2$$
$$w^2+4w-12=0$$
$$(w+6)(w-2)=0$$
$$w+6=0 \text{ or } w-2=0$$
$$w=-6 \text{ or } \quad w=2$$

Check $w=-6$:
$$2\sqrt{3-(-6)}-(-6)=0$$
$$2\sqrt{9}+6=0$$
$$2\cdot 3+6=0$$
$$6+6=0$$
$$12\neq 0$$

Check $w=2$:
$$2\sqrt{3-2}-2=0$$
$$2\sqrt{1}-2=0$$
$$2\cdot 1-2=0$$
$$2-2=0$$
$$0=0$$

The solution is $\{2\}$. ($w=-6$ does not check).

29.
$$(2x-5)^{1/4}=-1$$
$$\sqrt[4]{2x-5}=-1$$
$$\left(\sqrt[4]{2x-5}\right)^4=(-1)^4$$
$$2x-5=1$$
$$2x=6$$
$$x=3$$

Check:
$$\sqrt[4]{2(3)-5}=-1$$
$$\sqrt[4]{6-5}=-1$$
$$\sqrt[4]{1}=-1$$
$$1\neq -1$$

$\{\ \}$ ($x=3$ does not check).

30.
$$(x+16)^{1/4}=-4$$
$$\sqrt[4]{x+16}=-4$$
$$\left(\sqrt[4]{x+16}\right)^4=(-4)^4$$
$$x+16=256$$
$$x=240$$

Check:
$$\sqrt[4]{240+16}=-4$$
$$\sqrt[4]{256}=-4$$
$$4\neq -4$$

$\{\ \}$ ($x=240$ does not check).

31.
$$r=\sqrt[3]{\frac{3V}{4\pi}} \quad \text{for } V$$
$$r^3=\left(\sqrt[3]{\frac{3V}{4\pi}}\right)^3$$
$$r^3=\frac{3V}{4\pi}$$
$$4\pi r^3=3V$$
$$\frac{4\pi r^3}{3}=V$$

32.
$$r=\sqrt{\frac{V}{h\pi}} \quad \text{for } V$$
$$r^2=\left(\sqrt{\frac{V}{h\pi}}\right)^2$$
$$r^2=\frac{V}{h\pi}$$
$$\pi r^2 h=V$$

33. $r = \pi\sqrt{r^2 + h^2}$ for h^2

$$\frac{r}{\pi} = \sqrt{r^2 + h^2}$$
$$\left(\frac{r}{\pi}\right)^2 = \left(\sqrt{r^2 + h^2}\right)^2$$
$$\frac{r^2}{\pi^2} = r^2 + h^2$$
$$\frac{r^2}{\pi^2} - r^2 = h^2$$

34. $s = 1.3\sqrt{d}$ for d

$$\frac{s}{1.3} = \sqrt{d}$$
$$\left(\frac{s}{1.3}\right)^2 = \left(\sqrt{d}\right)^2$$
$$\frac{s^2}{1.3^2} = d$$
$$d = \frac{s^2}{1.69}$$

35. $(a+5)^2 = a^2 + 2\cdot a\cdot 5 + 5^2 = a^2 + 10a + 25$

36. $(b+7)^2 = b^2 + 2\cdot b\cdot 7 + 7^2 = b^2 + 14b + 49$

37. $\left(\sqrt{5a} - 3\right)^2 = \left(\sqrt{5a}\right)^2 - 2\cdot\sqrt{5a}\cdot 3 + 3^2$
$= 5a - 6\sqrt{5a} + 9$

38. $\left(2 + \sqrt{b}\right)^2 = 2^2 + 2\cdot 2\cdot\sqrt{b} + \left(\sqrt{b}\right)^2$
$= 4 + 4\sqrt{b} + b$

39. $\left(\sqrt{r-3} + 5\right)^2 = \left(\sqrt{r-3}\right)^2 + 2\cdot\sqrt{r-3}\cdot 5 + 5^2$
$= r - 3 + 10\sqrt{r-3} + 25$
$= r + 22 + 10\sqrt{r-3}$

40. $\left(2 - \sqrt{2t-4}\right)^2 = 2^2 - 2\cdot 2\cdot\sqrt{2t-4} + \left(\sqrt{2t-4}\right)^2$
$= 4 - 4\sqrt{2t-4} + 2t - 4$
$= 2t - 4\sqrt{2t-4}$

41. $\sqrt{a^2 + 2a + 1} = a + 5$

$$\left(\sqrt{a^2 + 2a + 1}\right)^2 = (a+5)^2$$
$$a^2 + 2a + 1 = a^2 + 10a + 25$$
$$-8a = 24$$
$$a = -3$$

The solution is $\{-3\}$.

Check:

$$\sqrt{(-3)^2 + 2(-3) + 1} = -3 + 5$$
$$\sqrt{9 - 6 + 1} = 2$$
$$\sqrt{4} = 2$$
$$2 = 2$$

42. $\sqrt{b^2 - 5b - 8} = b + 7$

$$\left(\sqrt{b^2 - 5b - 8}\right)^2 = (b+7)^2$$
$$b^2 - 5b - 8 = b^2 + 14b + 49$$
$$-19b = 57$$
$$b = -3$$

The solution is $\{-3\}$.

Check:

$$\sqrt{(-3)^2 - 5(-3) - 8} = -3 + 7$$
$$\sqrt{9 + 15 - 8} = 4$$
$$\sqrt{16} = 4$$
$$4 = 4$$

43.

$$\sqrt{25w^2-2w-3}=5w-4$$
$$\left(\sqrt{25w^2-2w-3}\right)^2=(5w-4)^2$$
$$25w^2-2w-3=25w^2-40w+16$$
$$38w=19$$
$$w=\frac{1}{2}$$

Check:

$$\sqrt{25\left(\frac{1}{2}\right)^2-2\left(\frac{1}{2}\right)-3}=5\left(\frac{1}{2}\right)-4$$
$$\sqrt{\frac{25}{4}-1-3}=\frac{5}{2}-4$$
$$\sqrt{\frac{9}{4}}=-\frac{3}{2}$$
$$\frac{3}{2}\neq-\frac{3}{2}$$

{ } ($w=\frac{1}{2}$ does not check.)

44.

$$\sqrt{4p^2-2p+1}=2p-3$$
$$\left(\sqrt{4p^2-2p+1}\right)^2=(2p-3)^2$$
$$4p^2-2p+1=4p^2-12p+9$$
$$10p=8$$
$$p=\frac{4}{5}$$

Check:

$$\sqrt{4\left(\frac{4}{5}\right)^2-2\left(\frac{4}{5}\right)+1}=2\left(\frac{4}{5}\right)-3$$
$$\sqrt{\frac{64}{25}-\frac{8}{5}+1}=\frac{8}{5}-3$$
$$\sqrt{\frac{49}{25}}=-\frac{7}{5}$$
$$\frac{7}{5}\neq-\frac{7}{5}$$

{ } ($p=\frac{4}{5}$ does not check.)

45.

$$4\sqrt{p-2}-2=-p$$
$$4\sqrt{p-2}=-p+2$$
$$\left(4\sqrt{p-2}\right)^2=(-p+2)^2$$
$$16(p-2)=p^2-4p+4$$
$$16p-32=p^2-4p+4$$
$$0=p^2-20p+36$$
$$0=(p-18)(p-2)$$
$$p-18=0 \quad \text{or} \quad p-2=0$$
$$p=18 \text{ or } \quad p=2$$

Check $p=18$:

$$4\sqrt{18-2}-2=-18$$
$$4\sqrt{16}-2=-18$$
$$4(4)-2=-18$$
$$16-2=-18$$
$$14\neq-18$$

Check $p=2$:

$$4\sqrt{2-2}-2=-2$$
$$4\sqrt{0}-2=-2$$
$$4(0)-2=-2$$
$$0-2=-2$$
$$-2=-2$$

The solution is {2}. ($p=18$ does not check.)

46.
$$x-3\sqrt{x-5}=5$$
$$-3\sqrt{x-5}=-x+5$$
$$\left(-3\sqrt{x-5}\right)^2=\left(-x+5\right)^2$$
$$9\left(x-5\right)=x^2-10x+25$$
$$9x-45=x^2-10x+25$$
$$x^2-19x+70=0$$
$$\left(x-14\right)\left(x-5\right)=0$$
$$x-14=0 \quad \text{or} \quad x-5=0$$
$$x=14 \text{ or } \quad x=5$$

Check $x=14$:
$$14-3\sqrt{14-5}=5$$
$$14-3\sqrt{9}=5$$
$$14-3\left(3\right)=5$$
$$14-9=5$$
$$5=5$$

Check $x=5$:
$$5-3\sqrt{5-5}=5$$
$$5-3\sqrt{0}=5$$
$$5-3\left(0\right)=5$$
$$5-0=5$$
$$5=5$$

The solution is $\{14, 5\}$.

47.
$$\sqrt[4]{h+4}=\sqrt[4]{2h-5}$$
$$\left(\sqrt[4]{h+4}\right)^4=\left(\sqrt[4]{2h-5}\right)^4$$
$$h+4=2h-5$$
$$9=h$$

Check:
$$\sqrt[4]{9+4}=\sqrt[4]{2\left(9\right)-5}$$
$$\sqrt[4]{13}=\sqrt[4]{18-5}$$
$$\sqrt[4]{13}=\sqrt[4]{13}$$

The solution is $\{9\}$.

48.
$$\sqrt[4]{3b+6}=\sqrt[4]{7b-6}$$
$$\left(\sqrt[4]{3b+6}\right)^4=\left(\sqrt[4]{7b-6}\right)^4$$
$$3b+6=7b-6$$
$$-4b=-12$$
$$b=3$$

Check:
$$\sqrt[4]{3\left(3\right)+6}=\sqrt[4]{7\left(3\right)-6}$$
$$\sqrt[4]{9+6}=\sqrt[4]{21-6}$$
$$\sqrt[4]{15}=\sqrt[4]{15}$$

The solution is $\{3\}$.

49.
$$\sqrt[3]{5a+3}-\sqrt[3]{a-13}=0$$
$$\sqrt[3]{5a+3}=\sqrt[3]{a-13}$$
$$\left(\sqrt[3]{5a+3}\right)^3=\left(\sqrt[3]{a-13}\right)^3$$
$$5a+3=a-13$$
$$4a=-16$$
$$a=-4$$

Check:
$$\sqrt[3]{5\left(-4\right)+3}-\sqrt[3]{-4-13}=0$$
$$\sqrt[3]{-20+3}-\sqrt[3]{-17}=0$$
$$\sqrt[3]{-17}-\sqrt[3]{-17}=0$$
$$0=0$$

The solution is $\{-4\}$.

50. $\sqrt[3]{k-8}-\sqrt[3]{4k+1}=0$

$$\sqrt[3]{k-8}=\sqrt[3]{4k+1}$$
$$\left(\sqrt[3]{k-8}\right)^3=\left(\sqrt[3]{4k+1}\right)^3$$
$$k-8=4k+1$$
$$-3k=9$$
$$k=-3$$

Check:

$$\sqrt[3]{-3-8}-\sqrt[3]{4(-3)+1}=0$$
$$\sqrt[3]{-11}-\sqrt[3]{-12+1}=0$$
$$\sqrt[3]{-11}-\sqrt[3]{-11}=0$$
$$0=0$$

The solution is $\{-3\}$.

51. $\sqrt{5a-9}=\sqrt{5a}-3$

$$\left(\sqrt{5a-9}\right)^2=\left(\sqrt{5a}-3\right)^2$$
$$5a-9=5a-6\sqrt{5a}+9$$
$$6\sqrt{5a}=18$$
$$\sqrt{5a}=3$$
$$\left(\sqrt{5a}\right)^2=3^2$$
$$5a=9$$
$$a=\frac{9}{5}$$

Check:

$$\sqrt{5\left(\frac{9}{5}\right)-9}=\sqrt{5\left(\frac{9}{5}\right)}-3$$
$$\sqrt{9-9}=\sqrt{9}-3$$
$$\sqrt{0}=3-3$$
$$0=0$$

The solution is $\left\{\frac{9}{5}\right\}$.

52. $\sqrt{8+b}=2+\sqrt{b}$

$$\left(\sqrt{8+b}\right)^2=\left(2+\sqrt{b}\right)^2$$
$$8+b=4+4\sqrt{b}+b$$
$$4=4\sqrt{b}$$
$$\sqrt{b}=1$$
$$\left(\sqrt{b}\right)^2=1^2$$
$$b=1$$

Check:

$$\sqrt{8+1}=2+\sqrt{1}$$
$$\sqrt{9}=2+1$$
$$3=3$$

The solution is $\{1\}$.

53. $\sqrt{2h+5}-\sqrt{2h}=1$

$$\sqrt{2h+5}=\sqrt{2h}+1$$
$$\left(\sqrt{2h+5}\right)^2=\left(\sqrt{2h}+1\right)^2$$
$$2h+5=2h+2\sqrt{2h}+1$$
$$4=2\sqrt{2h}$$
$$\sqrt{2h}=2$$
$$\left(\sqrt{2h}\right)^2=2^2$$
$$2h=4$$
$$h=2$$

Check:

$$\sqrt{2(2)+5}-\sqrt{2(2)}=1$$
$$\sqrt{4+5}-\sqrt{4}=1$$
$$\sqrt{9}-\sqrt{4}=1$$
$$3-2=1$$
$$1=1$$

The solution is $\{2\}$.

54. $\sqrt{3k-5}-\sqrt{3k}=-1$

$$\sqrt{3k-5}=\sqrt{3k}-1$$
$$\left(\sqrt{3k-5}\right)^2=\left(\sqrt{3k}-1\right)^2$$
$$3k-5=3k-2\sqrt{3k}+1$$
$$-6=-2\sqrt{3k}$$
$$\sqrt{3k}=3$$
$$\left(\sqrt{3k}\right)^2=3^2$$
$$3k=9$$
$$k=3$$

Check:

$$\sqrt{3(3)-5}-\sqrt{3(3)}=-1$$
$$\sqrt{9-5}-\sqrt{9}=-1$$
$$\sqrt{4}-\sqrt{9}=-1$$
$$2-3=-1$$
$$-1=-1$$

The solution is $\{3\}$.

55. $(t-9)^{1/2}-t^{1/2}=3$

$$\sqrt{t-9}-\sqrt{t}=3$$
$$\sqrt{t-9}=\sqrt{t}+3$$
$$\left(\sqrt{t-9}\right)^2=\left(\sqrt{t}+3\right)^2$$
$$t-9=t+6\sqrt{t}+9$$
$$-18=6\sqrt{t}$$
$$\sqrt{t}=-3$$

Check:

$$(9-9)^{1/2}-9^{1/2}=3$$
$$\sqrt{9-9}-\sqrt{9}=3$$
$$\sqrt{0}-\sqrt{9}=3$$
$$0-3=3$$
$$-3\neq 3$$

$$\left(\sqrt{t}\right)^2 = (-3)^2$$
$$t = 9$$

{ } (The value 9 does not check.)

56. $\left(y-16\right)^{1/2} - y^{1/2} = 4$

$$\sqrt{y-16} - \sqrt{y} = 4$$
$$\sqrt{y-16} = \sqrt{y} + 4$$
$$\left(\sqrt{y-16}\right)^2 = \left(\sqrt{y}+4\right)^2$$
$$y-16 = y + 8\sqrt{y} + 16$$
$$-32 = 8\sqrt{y}$$
$$\sqrt{y} = -4$$
$$\left(\sqrt{y}\right)^2 = (-4)^2$$
$$y = 16$$

Check:

$$\left(16-16\right)^{1/2} - 16^{1/2} = 4$$
$$\sqrt{16-16} - \sqrt{16} = 4$$
$$\sqrt{0} - \sqrt{16} = 4$$
$$0 - 4 = 4$$
$$-4 \neq 4$$

{ } (The value 16 does not check.)

57.
$$6 = \sqrt{x^2+3} - x \quad \{\ \}$$
$$x + 6 = \sqrt{x^2+3}$$
$$\left(x+6\right)^2 = \left(\sqrt{x^2+3}\right)^2$$
$$x^2 + 12x + 36 = x^2 + 3$$
$$12x = -33$$
$$x = -\frac{33}{12}$$
$$= -\frac{11}{4}$$

Check:

$$6 = \sqrt{\left(-\frac{11}{4}\right)^2 + 3} - \left(-\frac{11}{4}\right)$$
$$6 = \sqrt{\frac{121}{16} + 3} + \frac{11}{4}$$
$$6 = \sqrt{\frac{169}{16}} + \frac{11}{4}$$
$$6 = \frac{13}{4} + \frac{11}{4}$$
$$6 = \frac{24}{4}$$
$$6 = 6$$

The solution is $\left\{-\frac{11}{4}\right\}$.

58.

$$2 = \sqrt{y^2+5} - y$$

$$y + 2 = \sqrt{y^2+5}$$

$$(y+2)^2 = \left(\sqrt{y^2+5}\right)^2$$

$$y^2 + 4y + 4 = y^2 + 5$$

$$4y = 1$$

$$y = \frac{1}{4}$$

Check:

$$2 = \sqrt{\left(\frac{1}{4}\right)^2 + 5} - \frac{1}{4}$$

$$2 = \sqrt{\frac{1}{16} + 5} - \frac{1}{4}$$

$$2 = \sqrt{\frac{81}{16}} - \frac{1}{4}$$

$$2 = \frac{9}{4} - \frac{1}{4}$$

$$2 = \frac{8}{4}$$

$$2 = 2$$

The solution is $\left\{\frac{1}{4}\right\}$.

59.

$$\sqrt{3t-7} = 2 - \sqrt{3t+1}$$

$$\left(\sqrt{3t-7}\right)^2 = \left(2 - \sqrt{3t+1}\right)^2$$

$$3t - 7 = 4 - 4\sqrt{3t+1} + 3t + 1$$

$$-12 = -4\sqrt{3t+1}$$

$$\sqrt{3t+1} = 3$$

$$\left(\sqrt{3t+1}\right)^2 = (3)^2$$

$$3t + 1 = 9$$

$$3t = 8$$

$$t = \frac{8}{3}$$

Check:

$$\sqrt{3\left(\frac{8}{3}\right) - 7} = 2 - \sqrt{3\left(\frac{8}{3}\right) + 1}$$

$$\sqrt{8-7} = 2 - \sqrt{8+1}$$

$$\sqrt{1} = 2 - \sqrt{9}$$

$$1 = 2 - 3$$

$$1 \neq -1$$

{ } ($t = \frac{8}{3}$ does not check.)

60. $\sqrt{p-6} = \sqrt{p+2} - 4$

$$\left(\sqrt{p-6}\right)^2 = \left(\sqrt{p+2} - 4\right)^2$$
$$p - 6 = p + 2 - 8\sqrt{p+2} + 16$$
$$-24 = -8\sqrt{p+2}$$
$$\sqrt{p+2} = 3$$
$$\left(\sqrt{p+2}\right)^2 = (3)^2$$
$$p + 2 = 9$$
$$p = 7$$

Check:

$$\sqrt{7-6} = \sqrt{7+2} - 4$$
$$\sqrt{1} = \sqrt{9} - 4$$
$$1 = 3 - 4$$
$$1 \neq -1$$

{ } ($p = 7$ does not check.)

61. $\sqrt{z+1} + \sqrt{2z+3} = 1$

$$\sqrt{2z+3} = 1 - \sqrt{z+1}$$
$$\left(\sqrt{2z+3}\right)^2 = \left(1 - \sqrt{z+1}\right)^2$$
$$2z + 3 = 1 - 2\sqrt{z+1} + z + 1$$
$$z + 1 = -2\sqrt{z+1}$$
$$(z+1)^2 = \left(-2\sqrt{z+1}\right)^2$$
$$z^2 + 2z + 1 = 4(z+1)$$
$$z^2 + 2z + 1 = 4z + 4$$
$$z^2 - 2z - 3 = 0$$
$$(z-3)(z+1) = 0$$
$$z - 3 = 0 \text{ or } z + 1 = 0$$
$$z = 3 \text{ or } \quad z = -1$$

Check $z = 3$:

$$\sqrt{3+1} + \sqrt{2(3)+3} = 1$$
$$\sqrt{4} + \sqrt{6+3} = 1$$
$$\sqrt{4} + \sqrt{9} = 1$$
$$2 + 3 = 1$$
$$5 \neq 1$$

Check $z = -1$:

$$\sqrt{-1+1} + \sqrt{2(-1)+3} = 1$$
$$\sqrt{0} + \sqrt{-2+3} = 1$$
$$\sqrt{0} + \sqrt{1} = 1$$
$$0 + 1 = 1$$
$$1 = 1$$

The solution is $\{-1\}$. ($z = 3$ does not check.)

62.

$$\sqrt{2y+6}=\sqrt{7-2y}+1$$
$$\left(\sqrt{2y+6}\right)^2=\left(\sqrt{7-2y}+1\right)^2$$
$$2y+6=7-2y+2\sqrt{7-2y}+1$$
$$4y-2=2\sqrt{7-2y}$$
$$2y-1=\sqrt{7-2y}$$
$$\left(2y-1\right)^2=\left(\sqrt{7-2y}\right)^2$$
$$4y^2-4y+1=7-2y$$
$$4y^2-2y-6=0$$
$$2\left(2y^2-y-3\right)=0$$
$$2\left(2y-3\right)\left(y+1\right)=0$$
$$2y-3=0 \text{ or } y+1=0$$
$$2y=3 \text{ or } \quad y=-1$$
$$y=\frac{3}{2} \text{ or } \quad y=-1$$

Check $y=\frac{3}{2}$:

$$\sqrt{2\left(\frac{3}{2}\right)+6}=\sqrt{7-2\left(\frac{3}{2}\right)}+1$$
$$\sqrt{3+6}=\sqrt{7-3}+1$$
$$\sqrt{9}=\sqrt{4}+1$$
$$3=2+1$$
$$3=3$$

Check $y=-1$:

$$\sqrt{2\left(-1\right)+6}=\sqrt{7-2\left(-1\right)}+1$$
$$\sqrt{-2+6}=\sqrt{7+2}+1$$
$$\sqrt{4}=\sqrt{9}+1$$
$$2=3+1$$
$$2\neq 4$$

The solution is $\left\{\frac{3}{2}\right\}$. ($y=-1$ does not check.)

63.

$$\sqrt{6m+7}-\sqrt{3m+3}=1$$
$$\sqrt{6m+7}=1+\sqrt{3m+3}$$
$$\left(\sqrt{6m+7}\right)^2=\left(1+\sqrt{3m+3}\right)^2$$
$$6m+7=1+2\sqrt{3m+3}+3m+3$$
$$3m+3=2\sqrt{3m+3}$$
$$\left(3m+3\right)^2=\left(2\sqrt{3m+3}\right)^2$$
$$9m^2+18m+9=4\left(3m+3\right)$$
$$9m^2+18m+9=12m+12$$
$$9m^2+6m-3=0$$
$$3\left(3m^2+2m-1\right)=0$$
$$3\left(3m-1\right)\left(m+1\right)=0$$
$$3m-1=0 \text{ or } m+1=0$$
$$3m=1 \text{ or } \quad m=-1$$
$$m=\frac{1}{3} \text{ or } \quad m=-1$$

Check $m=\frac{1}{3}$:

$$\sqrt{6\left(\frac{1}{3}\right)+7}-\sqrt{3\left(\frac{1}{3}\right)+3}=1$$
$$\sqrt{2+7}-\sqrt{1+3}=1$$
$$\sqrt{9}-\sqrt{4}=1$$
$$3-2=1$$
$$1=1$$

Check $m=-1$:

$$\sqrt{6\left(-1\right)+7}-\sqrt{3\left(-1\right)+3}=1$$
$$\sqrt{-6+7}-\sqrt{-3+3}=1$$
$$\sqrt{1}-\sqrt{0}=1$$
$$1-0=1$$
$$1=1$$

The solution is $\left\{\frac{1}{3},-1\right\}$.

64. $\sqrt{5w+1}-\sqrt{3w}=1$

$$\sqrt{5w+1}=1+\sqrt{3w}$$
$$\left(\sqrt{5w+1}\right)^2=\left(1+\sqrt{3w}\right)^2$$
$$5w+1=1+2\sqrt{3w}+3w$$
$$2w=2\sqrt{3w}$$
$$w=\sqrt{3w}$$
$$\left(w\right)^2=\left(\sqrt{3w}\right)^2$$
$$w^2=3w$$
$$w^2-3w=0$$
$$w\left(w-3\right)=0$$
$$w=0 \text{ or } w-3=0$$
$$w=0 \text{ or } \quad w=3$$

The solution is $\left\{0.3\right\}$.

Check $w=0$:

$$\sqrt{5\left(0\right)+1}-\sqrt{3\left(0\right)}=1$$
$$\sqrt{0+1}-\sqrt{0}=1$$
$$\sqrt{1}-\sqrt{0}=1$$
$$1-0=1$$
$$1=1$$

Check $w=3$:

$$\sqrt{5\left(3\right)+1}-\sqrt{3\left(3\right)}=1$$
$$\sqrt{15+1}-\sqrt{9}=1$$
$$\sqrt{16}-\sqrt{9}=1$$
$$4-3=1$$
$$1=1$$

65. $2+2\sqrt{2t+3}+2\sqrt{3t-5}=0$

$$2+2\sqrt{2t+3}=-2\sqrt{3t-5}$$
$$1+\sqrt{2t+3}=-\sqrt{3t-5}$$
$$\left(1+\sqrt{2t+3}\right)^2=\left(-\sqrt{3t-5}\right)^2$$
$$1+2\sqrt{2t+3}+2t+3=3t-5$$
$$2\sqrt{2t+3}=t-9$$
$$\left(2\sqrt{2t+3}\right)^2=\left(t-9\right)^2$$
$$4\left(2t+3\right)=t^2-18t+81$$
$$8t+12=t^2-18t+81$$
$$t^2-26t+69=0$$
$$\left(t-3\right)\left(t-23\right)=0$$
$$t-3=0 \text{ or } t-23=0$$
$$t=3 \text{ or } \quad t=23$$

Check $t=3$:

$$2+2\sqrt{2\left(3\right)+3}+2\sqrt{3\left(3\right)-5}=0$$
$$2+2\sqrt{6+3}+2\sqrt{9-5}=0$$
$$2+2\sqrt{9}+2\sqrt{4}=0$$
$$2+2\cdot3+2\cdot2=0$$
$$2+6+4=0$$
$$12\neq0$$

Check $t=23$:

$$2+2\sqrt{2\left(23\right)+3}+2\sqrt{3\left(23\right)-5}=0$$
$$2+2\sqrt{46+3}+2\sqrt{69-5}=0$$
$$2+2\sqrt{49}+2\sqrt{64}=0$$
$$2+2\cdot7+2\cdot8=0$$
$$2+14+16=0$$
$$32\neq0$$

{ } ($t=3$ and $t=23$ do not check.)

66. $6+3\sqrt{3x+1}+3\sqrt{x-1}=0$

$6+3\sqrt{3x+1}=-3\sqrt{x-1}$

$2+\sqrt{3x+1}=-\sqrt{x-1}$

$\left(2+\sqrt{3x+1}\right)^2=\left(-\sqrt{x-1}\right)^2$

$4+4\sqrt{3x+1}+3x+1=x-1$

$4\sqrt{3x+1}=-2x-6$

$2\sqrt{3x+1}=-x-3$

$\left(2\sqrt{3x+1}\right)^2=\left(-x-3\right)^2$

$4\left(3x+1\right)=x^2+6x+9$

$12x+4=x^2+6x+9$

$x^2-6x+5=0$

$\left(x-5\right)\left(x-1\right)=0$

$x-5=0$ or $x-1=0$

$x=5$ or $x=1$

{ } ($x=5$ and $x=1$ do not check.)

Check $x=5$:

$6+3\sqrt{3\left(5\right)+1}+3\sqrt{5-1}=0$

$6+3\sqrt{15+1}+3\sqrt{4}=0$

$6+3\sqrt{16}+3\sqrt{4}=0$

$6+3\cdot4+3\cdot2=0$

$6+12+6=0$

$24\neq0$

Check $x=1$:

$6+3\sqrt{3\left(1\right)+1}+3\sqrt{1-1}=0$

$6+3\sqrt{3+1}+3\sqrt{0}=0$

$6+3\sqrt{4}+3\sqrt{0}=0$

$6+3\cdot2+3\cdot0=0$

$6+6+0=0$

$12\neq0$

67. $3\sqrt{y-3}=\sqrt{4y+3}$

$\left(3\sqrt{y-3}\right)^2=\left(\sqrt{4y+3}\right)^2$

$9\left(y-3\right)=4y+3$

$9y-27=4y+3$

$5y=30$

$y=6$

The solution is {6}.

Check:

$3\sqrt{6-3}=\sqrt{4\left(6\right)+3}$

$3\sqrt{3}=\sqrt{24+3}$

$3\sqrt{3}=\sqrt{27}$

$3\sqrt{3}=3\sqrt{3}$

68. $\sqrt{5x-8}=2\sqrt{x-1}$

$\left(\sqrt{5x-8}\right)^2=\left(2\sqrt{x-1}\right)^2$

$5x-8=4\left(x-1\right)$

$5x-8=4x-4$

$x=4$

The solution is {4}.

Check:

$\sqrt{5\left(4\right)-8}=2\sqrt{4-1}$

$\sqrt{20-8}=2\sqrt{3}$

$\sqrt{12}=2\sqrt{3}$

$\sqrt{4\cdot3}=2\sqrt{3}$

$2\sqrt{3}=2\sqrt{3}$

69.

$$\sqrt{p+7} = \sqrt{2p}+1$$
$$\left(\sqrt{p+7}\right)^2 = \left(\sqrt{2p}+1\right)^2$$
$$p+7 = 2p+2\sqrt{2p}+1$$
$$-p+6 = 2\sqrt{2p}$$
$$\left(-p+6\right)^2 = \left(2\sqrt{2p}\right)^2$$
$$p^2-12p+36 = 4\left(2p\right)$$
$$p^2-12p+36 = 8p$$
$$p^2-20p+36 = 0$$
$$\left(p-18\right)\left(p-2\right) = 0$$

$$p-18=0 \quad \text{or } p-2=0$$
$$p=18 \text{ or } \quad p=2$$

Check $p=18$:

$$\sqrt{18+7} = \sqrt{2\cdot 18}+1$$
$$\sqrt{25} = \sqrt{36}+1$$
$$5 = 6+1$$
$$5 \neq 7$$

Check $p=2$:

$$\sqrt{2+7} = \sqrt{2\cdot 2}+1$$
$$\sqrt{9} = \sqrt{4}+1$$
$$3 = 2+1$$
$$3 = 3$$

The solution is $\{2\}$. ($p = 18$ does not check.)

70.

$$\sqrt{t} = \sqrt{t-12}+2$$
$$\left(\sqrt{t}\right)^2 = \left(\sqrt{t-12}+2\right)^2$$
$$t = t-12+4\sqrt{t-12}+4$$
$$8 = 4\sqrt{t-12}$$
$$2 = \sqrt{t-12}$$
$$\left(2\right)^2 = \left(\sqrt{t-12}\right)^2$$
$$4 = t-12$$
$$16 = t$$

Check:

$$\sqrt{16} = \sqrt{16-12}+2$$
$$\sqrt{16} = \sqrt{4}+2$$
$$4 = 2+2$$
$$4 = 4$$

The solution is $\{16\}$.

71. $v = \sqrt{2gh}$

a. $44 = \sqrt{2(32)h}$

$44 = \sqrt{64h}$

$44 = 8\sqrt{h}$

$11 = 2\sqrt{h}$

$11^2 = \left(2\sqrt{h}\right)^2$

$121 = 4h$

$h = \dfrac{121}{4} = 30.25$ ft

b. $26 = \sqrt{2(9.8)h}$

$26 = \sqrt{19.6h}$

$26^2 = \left(\sqrt{19.6h}\right)^2$

$676 = 19.6h$

$h = \dfrac{676}{19.6}$

≈ 34.5 m

72. $T = 2\pi\sqrt{\dfrac{L}{g}}$

a. $1.36 = 2\pi\sqrt{\dfrac{L}{32}}$

$(1.36)^2 = \left(2\pi\sqrt{\dfrac{L}{32}}\right)^2$

$1.8496 = 4\pi^2 \cdot \dfrac{L}{32}$

$32(1.8496) = 4\pi^2 L$

$L = \dfrac{32(1.8496)}{4\pi^2}$

≈ 1.5 ft

b. $T = 2\pi\sqrt{\dfrac{4}{32}}$

$= 2\pi\sqrt{\dfrac{1}{8}}$

$= 2\pi \cdot \dfrac{1}{\sqrt{8}}$

$= \dfrac{2\pi}{2\sqrt{2}}$

≈ 2.2 sec

73. $C(x) = \sqrt{0.3x+1}$

a. $C(x) = \sqrt{0.3(10)+1} = \sqrt{3+1} = \sqrt{4}$

$= \$2$ million

b. $P(x) = R(x) - C(x)$

$P(x) = 320(10{,}000) - 2{,}000{,}000$

$= 3{,}200{,}000 - 2{,}000{,}000$

$= \$1.2$ million

74. $t(d) = \sqrt{\dfrac{d}{4.9}}$

a. $7.89 = \sqrt{\dfrac{d}{4.9}}$

$(7.89)^2 = \left(\sqrt{\dfrac{d}{4.9}}\right)^2$

$62.2521 = \dfrac{d}{4.9}$

$d = 4.9(62.2521)$

≈ 305 m

c. $4 = \sqrt{0.3x+1}$

$4^2 = \left(\sqrt{0.3x+1}\right)^2$

$16 = 0.3x + 1$

$15 = 0.3x$

$x = \dfrac{15}{0.3}$

$= 50$ (50,000 passengers)

b. $9.51 = \sqrt{\dfrac{d}{4.9}}$

$\left(9.51\right)^2 = \left(\sqrt{\dfrac{d}{4.9}}\right)^2$

$90.4401 = \dfrac{d}{4.9}$

$d = 4.9\left(90.4401\right)$

≈ 443 m

75. $t\left(x\right) = 0.90\sqrt[5]{x^3}$

a. $4 = 0.90\sqrt[5]{x^3}$

$\dfrac{4}{0.90} = \sqrt[5]{x^3}$

$\left(\dfrac{4}{0.90}\right)^5 = \left(\sqrt[5]{x^3}\right)^5$

$1734.15 = x^3$

$x = \sqrt[3]{1734.15}$

≈ 12 lb

b. $t\left(18\right) = 0.90\sqrt[5]{18^3}$

≈ 5.1 hr

An 18-lb turkey will take about 5.1 hr to cook.

76. $a^2 + b^2 = c^2$

$2^2 + b^2 = y^2$

$b^2 = y^2 - 2^2$

$b = \sqrt{y^2 - 4}$

77. $a^2 + b^2 = c^2$

$h^2 + b^2 = 5^2$

$b^2 = 5^2 - h^2$

$b = \sqrt{25 - h^2}$

78. $a^2 + b^2 = c^2$

$a^2 + x^2 = 8^2$

$a^2 = 8^2 - x^2$

$a = \sqrt{64 - x^2}$

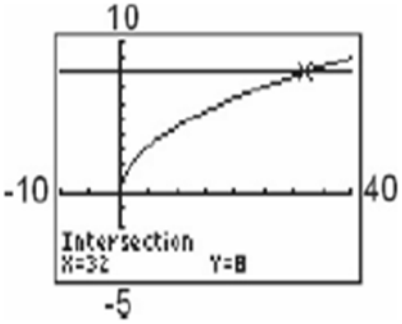
79. $a^2 + b^2 = c^2$

$a^2 + 14^2 = k^2$

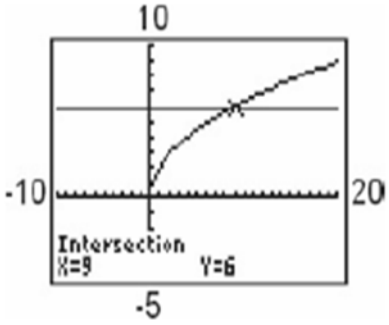
$a^2 = k^2 - 14^2$

$a = \sqrt{k^2 - 196}$

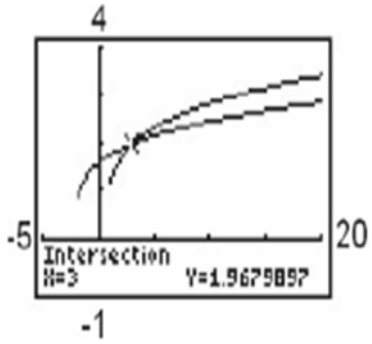
80. The x-coordinate of the point of intersection is the solution to the equation.



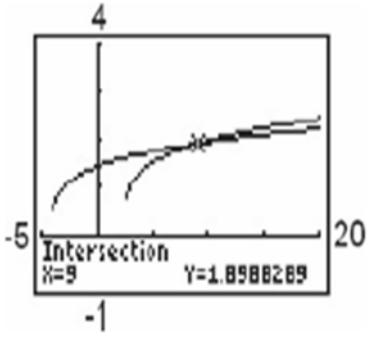
81. The x-coordinate of the point of intersection is the solution to the equation.



82.



83.



Section 6.8 Practice Exercises

1. **a.** imaginary

b. $\sqrt{-1}\,;\ -1$

c. $i\sqrt{b}$

d. $a+bi\,;\ \sqrt{-1}$

e. real; b

f. $a+bi$

g. True

h. True

2. $-2\sqrt{5}-3\sqrt{50}+\sqrt{125}$

$=-2\sqrt{5}-3\sqrt{25\cdot 2}+\sqrt{25\cdot 5}$

$=-2\sqrt{5}-3\cdot 5\sqrt{2}+5\sqrt{5}$

$=3\sqrt{5}-15\sqrt{2}$

3. $\left(3-\sqrt{x}\right)\left(3+\sqrt{x}\right)=3^2-\left(\sqrt{x}\right)^2$

$=9-x$

4. $\left(\sqrt{5}+\sqrt{2}\right)^2=\left(\sqrt{5}\right)^2+2\cdot\sqrt{5}\cdot\sqrt{2}+\left(\sqrt{2}\right)^2$

$=5+2\sqrt{10}+2=7+2\sqrt{10}$

5. $\sqrt[3]{3p+7}-\sqrt[3]{2p-1}=0$

$\sqrt[3]{3p+7}=\sqrt[3]{2p-1}$

$\left(\sqrt[3]{3p+7}\right)^3=\left(\sqrt[3]{2p-1}\right)^3$

$3p+7=2p-1$

$p=-8$

The solution is $\{-8\}$.

Check:

$\sqrt[3]{3(-8)+7}-\sqrt[3]{2(-8)-1}=0$

$\sqrt[3]{-24+7}-\sqrt[3]{-16-1}=0$

$\sqrt[3]{-17}-\sqrt[3]{-17}=0$

$0=0$

6. $\sqrt{9t+10}+18=10$

$\sqrt{9t+10}=-8$

$\left(\sqrt{9t+10}\right)^2=(-8)^2$

$9t+10=64$

$9t=54$

$t=6$

$\{\ \}$ ($t=6$ does not check.)

Check:

$\sqrt{9(6)+10}+18=10$

$\sqrt{54+10}+18=10$

$\sqrt{64}+18=10$

$8+18=10$

$26=10$

7. $\sqrt{4a+29}=2\sqrt{a}+5$

$\left(\sqrt{4a+29}\right)^2=\left(2\sqrt{a}+5\right)^2$

$4a+29=4a+20\sqrt{a}+25$

$4=20\sqrt{a}$

$5\sqrt{a}=1$

$\left(5\sqrt{a}\right)^2=1^2$

$25a=1$

$a=\frac{1}{25}$

The solution is $\left\{\frac{1}{25}\right\}$.

Check:

$\sqrt{4\left(\frac{1}{25}\right)+29}=2\sqrt{\frac{1}{25}}+5$

$\sqrt{\frac{4}{25}+29}=2\cdot\frac{1}{5}+5$

$\sqrt{\frac{729}{25}}=\frac{2}{5}+5$

$\frac{27}{5}=\frac{27}{5}$

8. $\frac{2}{\sqrt{x}-3}\cdot\frac{\sqrt{x}+3}{\sqrt{x}+3}=\frac{2\left(\sqrt{x}+3\right)}{\left(\sqrt{x}\right)^2-3^2}=\frac{2\sqrt{x}+6}{x-9}$

9. $\sqrt{-1}=i$ and $-\sqrt{1}=-1$

10. $i^2=-1$

11. $\sqrt{-144}=i\sqrt{144}=12i$

12. $\sqrt{-81}=i\sqrt{81}=9i$

13. $\sqrt{-3} = i\sqrt{3}$

14. $\sqrt{-17} = i\sqrt{17}$

15. $-\sqrt{-20} = -i\sqrt{20} = -i\sqrt{4\cdot 5}$
$= -2i\sqrt{5}$

16. $-\sqrt{-75} = -i\sqrt{75} = -i\sqrt{25\cdot 3}$
$= -5i\sqrt{3}$

17. $2\sqrt{-25}\cdot 3\sqrt{-4} = 2i\sqrt{25}\cdot 3i\sqrt{4}$
$= 5\cdot 2i\cdot 2\cdot 3i$
$= 10i\cdot 6i = 60i^2$
$= 60(-1) = -60$

18. $\left(-4\sqrt{-9}\right)\left(-3\sqrt{-1}\right) = \left(-4\cdot 3i\right)\left(-3\cdot i\right)$
$= \left(-12i\right)\left(-3i\right)$
$= 36i^2$
$= 36(-1) = -36$

19. $7\sqrt{-63} - 4\sqrt{-28} = 7\sqrt{-9\cdot 7} - 4\sqrt{-4\cdot 7}$
$= 7\cdot 3i\sqrt{7} - 4\cdot 2i\sqrt{7}$
$= 21i\sqrt{7} - 8i\sqrt{7}$
$= 13i\sqrt{7}$

20. $7\sqrt{-3} - 4\sqrt{-27} = 7\sqrt{-3} - 4\sqrt{-9\cdot 3}$
$= 7i\sqrt{3} - 4\cdot 3i\sqrt{3}$
$= 7i\sqrt{3} - 12i\sqrt{3}$
$= -5i\sqrt{3}$

21. $\sqrt{-7}\cdot\sqrt{-7} = i\sqrt{7}\cdot i\sqrt{7} = i^2\sqrt{49}$
$= -1\cdot 7 = -7$

22. $\sqrt{-11}\cdot\sqrt{-11} = i\sqrt{11}\cdot i\sqrt{11} = i^2\sqrt{121}$
$= -1\cdot 11 = -11$

23. $\sqrt{-9}\cdot\sqrt{-16} = 3i\cdot 4i = 12i^2$
$= 12(-1) = -12$

24. $\sqrt{-25}\cdot\sqrt{-36} = 5i\cdot 6i = 30i^2$
$= 30(-1) = -30$

25. $\sqrt{-15}\cdot\sqrt{-6} = i\sqrt{15}\cdot i\sqrt{6} = i^2\sqrt{90}$
$= -1\sqrt{9\cdot 10}$
$= -3\sqrt{10}$

26. $\sqrt{-12}\cdot\sqrt{-50} = \sqrt{-4\cdot 3}\cdot\sqrt{-25\cdot 2}$
$= 2i\sqrt{3}\cdot 5i\sqrt{2} = 10i^2\sqrt{6}$
$= 10(-1)\sqrt{6} = -10\sqrt{6}$

27. $\dfrac{\sqrt{-50}}{\sqrt{25}} = \dfrac{\sqrt{-25\cdot 2}}{5} = \dfrac{5i\sqrt{2}}{5} = i\sqrt{2}$

28. $\dfrac{\sqrt{-27}}{\sqrt{9}} = \dfrac{\sqrt{-9\cdot 3}}{3} = \dfrac{3i\sqrt{3}}{3} = i\sqrt{3}$

29. $\dfrac{\sqrt{-90}}{\sqrt{-10}} = \dfrac{\sqrt{-9\cdot 10}}{\sqrt{-1\cdot 10}} = \dfrac{3i\sqrt{10}}{i\sqrt{10}} = 3$

30. $\dfrac{\sqrt{-125}}{\sqrt{-45}} = \dfrac{\sqrt{-25\cdot 5}}{\sqrt{-9\cdot 5}} = \dfrac{5i\sqrt{5}}{3i\sqrt{5}} = \dfrac{5}{3}$

31. $i^7 = i^4\cdot i^3 = 1(-i) = -i$

32. $i^{38} = i^{36}\cdot i^2 = \left(i^4\right)^9\cdot i^2 = 1^9(-1) = 1(-1) = -1$

33. $i^{64} = \left(i^4\right)^{16} = 1^{64}$
$= 1$

34. $i^{75} = i^{72} \cdot i^3 = \left(i^4\right)^{18} \cdot i^3 = 1^{18}\left(-i\right)$
$= 1\left(-i\right) = -i$

35. $i^{41} = i^{40} \cdot i = \left(i^4\right)^{10} \cdot i = 1^{10} \cdot i$
$= 1 \cdot i = i$

36. $i^{25} = i^{24} \cdot i^1 = \left(i^4\right)^6 \cdot i = 1^6 \cdot i$
$= 1 \cdot i = i$

37. $i^{52} = \left(i^4\right)^{13} = 1^{13} = 1$

38. $i^0 = 1$

39. $i^{23} = i^{20} \cdot i^3 = \left(i^4\right)^5 \cdot i^3 = 1^5\left(-i\right)$
$= 1\left(-i\right) = -i$

40. $i^{103} = i^{100} \cdot i^3 = \left(i^4\right)^{25}\left(-i\right) = 1^{25}\left(-i\right)$
$= 1\left(-i\right) = -i$

41. $i^6 = i^4 \cdot i^2 = 1\left(-1\right) = -1$

42. $i^{82} = i^{80} \cdot i^2 = \left(i^4\right)^{20}\left(-1\right) = 1^{20}\left(-1\right)$
$= 1\left(-1\right) = -1$

43. $a - bi$

44. **a.** True. Every real number can be written in the form $a + 0i$.

b. False. The real numbers do not include the square roots of negative numbers.

45. $-5 + 12i$
Real part: –5; Imaginary part: 12

46. $22 - 16i$
Real part: 22; Imaginary part: –16

47. $-6i = 0 - 6i$
Real part: 0; Imaginary part: –6

48. $10i = 0 + 10i$
Real part: 0; Imaginary part: 10

49. $35 = 35 + 0i$
Real part: 35; Imaginary part: 0

50. $-1 = -1 + 0i$
Real part: –1; Imaginary part: 0

51. $\frac{3}{5} + i$

52. $-\frac{1}{2} - \frac{1}{4}i$

Real part: $\frac{3}{5}$; Imaginary part: 1

Real part: $-\frac{1}{2}$; Imaginary part: $-\frac{1}{4}$

53. $(2-i)+(5+7i)=(2+5)+(-1+7)i$
$=7+6i$

54. $(5-2i)+(3+4i)=(5+3)+(-2+4)i$
$=8+2i$

55. $\left(\frac{1}{2}+\frac{2}{3}i\right)-\left(\frac{1}{5}-\frac{5}{6}i\right)=\frac{1}{2}+\frac{2}{3}i-\frac{1}{5}+\frac{5}{6}i$
$=\left(\frac{1}{2}-\frac{1}{5}\right)+\left(\frac{2}{3}+\frac{5}{6}\right)i$
$=\left(\frac{5}{10}-\frac{2}{10}\right)+\left(\frac{4}{6}+\frac{5}{6}\right)i$
$=\frac{3}{10}+\frac{9}{6}i=\frac{3}{10}+\frac{3}{2}i$

56. $\left(\frac{11}{10}-\frac{7}{5}i\right)-\left(-\frac{2}{5}+\frac{3}{5}i\right)=\frac{11}{10}-\frac{7}{5}i+\frac{2}{5}-\frac{3}{5}i$
$=\left(\frac{11}{10}+\frac{2}{5}\right)+\left(-\frac{7}{5}-\frac{3}{5}\right)i$
$=\left(\frac{11}{10}+\frac{4}{10}\right)+\left(-\frac{7}{5}-\frac{3}{5}\right)i$
$=\frac{15}{10}-\frac{10}{5}i=\frac{3}{2}-2i$

57. $\sqrt{-98}-\sqrt{-8}=i\sqrt{98}-i\sqrt{8}$
$=i\sqrt{7^2\cdot 2}-i\sqrt{2^2\cdot 2}$
$=7i\sqrt{2}-2i\sqrt{2}$
$=5i\sqrt{2}$

58. $\sqrt{-75}+\sqrt{-12}=i\sqrt{75}+i\sqrt{12}$
$=i\sqrt{5^2\cdot 3}+i\sqrt{2^2\cdot 3}$
$=5i\sqrt{3}+2i\sqrt{3}$
$=7i\sqrt{3}$

59. $(2+3i)-(1-4i)+(-2+3i)$
$=2+3i-1+4i-2+3i$
$=(2-1-2)+(3+4+3)i$
$=-1+10i$

60. $(2+5i)-(7-2i)+(-3+4i)$
$=2+5i-7+2i-3+4i$
$=(2-7-3)+(5+2+4)i$
$=-8+11i$

61. $(8i)(3i)=24i^2=24(-1)=-24+0i$

62. $(2i)(4i)=8i^2=8(-1)=-8+0i$

63. $6i(1-3i)=6i(1)-6i(3i)$
$=6i-18i^2$
$=6i-18(-1)$
$=18+6i$

64. $-i(3+4i)=-i(3)-i(4i)$
$=-3i-4i^2$
$=-3i-4(-1)$
$=4-3i$

65. $(2-10i)(3+2i)$
$= 2(3)+2(2i)-10i(3)-10i(2i)$
$= 6+4i-30i-20i^2$
$= 6-26i-20(-1)$
$= 6-26i+20 = 26-26i$

66. $(4+7i)(2-3i) = 4(2)-4(3i)+7i(2)-7i(3i)$
$= 8-12i+14i-21i^2$
$= 8+2i-21(-1)$
$= 8+2i+21$
$= 29+2i$

67. $(-5+2i)(5+2i)$
$= -5(5)-5(2i)+2i(5)+(2i)(2i)$
$= -25-10i+10i+4i^2$
$= -25+4(-1) = -25-4$
$= -29+0i$

68. $(4-11i)(4+11i)$
$= 4(4)+4(11i)-11i(4)-11i(11i)$
$= 16+44i-44i-121i^2$
$= 16-121(-1) = 16+121$
$= 137+0i$

69. $(4+5i)^2 = 4^2+2\cdot 4\cdot 5i+(5i)^2$
$= 16+40i+25i^2 = 16+40i+25(-1)$
$= 16+40i-25 = -9+40i$

70. $(3-2i)^2 = 3^2-2\cdot 3\cdot 2i+(2i)^2$
$= 9-12i+4i^2 = 9-12i+4(-1)$
$= 9-12i-4 = 5-12i$

71. $(2+i)(3-2i)(4+3i)$
$= \left[2\cdot 3-2\cdot 2i+i\cdot 3-i(2i)\right](4+3i)$
$= (6-4i+3i-2i^2)(4+3i)$
$= (6-i-2(-1))(4+3i)$
$= (6-i+2)(4+3i)$
$= (8-i)(4+3i)$
$= 8\cdot 4+8\cdot 3i-i\cdot 4-i(3i)$
$= 32+24i-4i-3i^2$
$= 32+20i-3(-1)$
$= 32+20i+3 = 35+20i$

72. $(3-i)(3+i)(4-i) = \left[3^2-i^2\right](4-i)$
$= (9-(-1))(4-i)$
$= (10)(4-i)$
$= 10(4)-10(i)$
$= 40-10i$

73. $(-4-6i)^2 = (-4)^2+2\cdot 4\cdot 6i+(6i)^2$
$= 16+48i+36i^2$
$= 16+48i+36(-1)$
$= 16+48i-36$
$= -20+48i$

74. $(-3-5i)^2 = (-3)^2+2\cdot 3\cdot 5i+(5i)^2$
$= 9+30i+25i^2$
$= 9+30i+25(-1)$
$= 9+30i-25$
$= -16+30i$

75. $\left(-\frac{1}{2}-\frac{3}{4}i\right)\left(-\frac{1}{2}+\frac{3}{4}i\right)=\left(-\frac{1}{2}\right)^2-\left(\frac{3}{4}i\right)^2$

$=\frac{1}{4}-\frac{9}{16}i^2$

$=\frac{1}{4}-\frac{9}{16}(-1)$

$=\frac{1}{4}+\frac{9}{16}$

$=\frac{4}{16}+\frac{9}{16}$

$=\frac{13}{16}+0i$

76. $\left(-\frac{2}{3}+\frac{1}{6}i\right)\left(-\frac{2}{3}-\frac{1}{6}i\right)=\left(-\frac{2}{3}\right)^2-\left(\frac{1}{6}i\right)^2$

$=\frac{4}{9}-\frac{1}{36}i^2$

$=\frac{4}{9}-\frac{1}{36}(-1)$

$=\frac{4}{9}+\frac{1}{36}$

$=\frac{16}{36}+\frac{1}{36}$

$=\frac{17}{36}+0i$

77. $\frac{2}{1+3i}=\frac{2}{1+3i}\cdot\frac{1-3i}{1-3i}$

$=\frac{2(1-3i)}{1^2-(3i)^2}=\frac{2(1-3i)}{1-9i^2}$

$=\frac{2(1-3i)}{1-9(-1)}=\frac{2(1-3i)}{1+9}$

$=\frac{2(1-3i)}{10}$

$=\frac{1-3i}{5}$

$=\frac{1}{5}-\frac{3}{5}i$

78. $\frac{-2}{3+i}=\frac{-2}{3+i}\cdot\frac{3-i}{3-i}$

$=\frac{-2(3-i)}{3^2-i^2}$

$=\frac{-2(3-i)}{9-(-1)}$

$=\frac{-2(3-i)}{10}$

$=\frac{3-i}{-5}$

$=-\frac{3}{5}+\frac{1}{5}i$

79. $\dfrac{-i}{4-3i} = \dfrac{-i}{4-3i}\cdot\dfrac{4+3i}{4+3i}$

$= \dfrac{-i(4+3i)}{4^2-(3i)^2}$

$= \dfrac{-i\cdot 4 - i(3i)}{16-9i^2} = \dfrac{-4i-3i^2}{16-9(-1)}$

$= \dfrac{-4i-3(-1)}{16+9} = \dfrac{3-4i}{25}$

$= \dfrac{3}{25} - \dfrac{4}{25}i$

80. $\dfrac{3-3i}{1-i} = \dfrac{3-3i}{1-i}\cdot\dfrac{1+i}{1+i}$

$= \dfrac{3\cdot 1+3\cdot i-3i\cdot 1-3i\cdot i}{1^2-i^2}$

$= \dfrac{3+3i-3i-3i^2}{1-(-1)}$

$= \dfrac{3-3(-1)}{2}$

$= \dfrac{3+3}{2} = \dfrac{6}{2} = 3+0i$

81. $\dfrac{5+2i}{5-2i} = \dfrac{5+2i}{5-2i}\cdot\dfrac{5+2i}{5+2i}$

$= \dfrac{5\cdot 5+5\cdot 2i+2i\cdot 5+2i\cdot 2i}{5^2-(2i)^2}$

$= \dfrac{25+10i+10i+4i^2}{25-4i^2}$

$= \dfrac{25+20i+4(-1)}{25-4(-1)}$

$= \dfrac{25+20i-4}{25+4}$

$= \dfrac{21+20i}{29}$

$= \dfrac{21}{29} + \dfrac{20}{29}i$

82. $\dfrac{7+3i}{4-2i} = \dfrac{7+3i}{4-2i}\cdot\dfrac{4+2i}{4+2i}$

$= \dfrac{7\cdot 4+7\cdot 2i+3i\cdot 4+3i\cdot 2i}{4^2-(2i)^2}$

$= \dfrac{28+14i+12i+6i^2}{16-4i^2}$

$= \dfrac{28+26i+6(-1)}{16-4(-1)}$

$= \dfrac{28+26i-6}{16+4}$

$= \dfrac{22+26i}{20} = \dfrac{2(11+13i)}{20}$

$= \dfrac{11+13i}{10} = \dfrac{11}{10} + \dfrac{13}{10}i$

83. $\dfrac{3+7i}{-2-4i} = \dfrac{3+7i}{-2-4i}\cdot\dfrac{-2+4i}{-2+4i}$

$= \dfrac{3(-2)+3\cdot 4i-7i\cdot 2+7i\cdot 4i}{(-2)^2-(4i)^2}$

$= \dfrac{-6+12i-14i+28i^2}{4-16i^2}$

$= \dfrac{-6-2i+28(-1)}{4-16(-1)}$

$= \dfrac{-6-2i-28}{4+16}$

$= \dfrac{-34-2i}{20} = \dfrac{2(-17-i)}{20}$

$= \dfrac{-17-i}{10} = -\dfrac{17}{10}-\dfrac{1}{10}i$

84. $\dfrac{-2+9i}{-1-4i} = \dfrac{-2+9i}{-1-4i}\cdot\dfrac{-1+4i}{-1+4i}$

$= \dfrac{-2(-1)-2\cdot 4i-9i\cdot 1+9i\cdot 4i}{(-1)^2-(4i)^2}$

$= \dfrac{2-8i-9i+36i^2}{1-16i^2}$

$= \dfrac{2-17i+36(-1)}{1-16(-1)}$

$= \dfrac{2-17i-36}{1+16}$

$= \dfrac{-34-17i}{17} = \dfrac{17(-2-i)}{17}$

$= -2-i$

85. $\dfrac{13i}{-5-i} = \dfrac{13i}{-5-i}\cdot\dfrac{-5+i}{-5+i}$

$= \dfrac{13i(-5)+13i\cdot i}{(-5)^2-i^2}$

$= \dfrac{-65i+13i^2}{25-(-1)} = \dfrac{-65i+13(-1)}{25+1}$

$= \dfrac{-13-65i}{26} = \dfrac{13(-1-5i)}{26}$

$= \dfrac{-1-5i}{2} = -\dfrac{1}{2}-\dfrac{5}{2}i$

86. $\dfrac{15i}{-2-i} = \dfrac{15i}{-2-i}\cdot\dfrac{-2+i}{-2+i}$

$= \dfrac{15i(-2)+15i\cdot i}{(-2)^2-i^2}$

$= \dfrac{-30i+15i^2}{4-(-1)} = \dfrac{-30i+15(-1)}{5}$

$= \dfrac{-15-30i}{5} = \dfrac{5(-3-6i)}{5}$

$= -3-6i$

87. $\dfrac{2+3i}{6i}=\dfrac{2+3i}{6i}\cdot\dfrac{-6i}{-6i}$

$=\dfrac{-12i-18i^2}{-36i^2}$

$=\dfrac{-12i-18(-1)}{-36(-1)}=\dfrac{18-12i}{36}$

$=\dfrac{6(3-2i)}{36}=\dfrac{3-2i}{6}$

$=\dfrac{1}{2}-\dfrac{1}{3}i$

88. $\dfrac{4-i}{2i}=\dfrac{4-i}{2i}\cdot\dfrac{-2i}{-2i}=\dfrac{-8i+2i^2}{-4i^2}$

$=\dfrac{-8i+2(-1)}{-4(-1)}$

$=\dfrac{-2-8i}{4}=\dfrac{2(-1-4i)}{4}$

$=\dfrac{-1-4i}{2}$

$=-\dfrac{1}{2}-2i$

89. $\dfrac{-10+i}{i}=\dfrac{-10+i}{i}\cdot\dfrac{-i}{-i}$

$=\dfrac{10i-i^2}{-i^2}$

$=\dfrac{10i-(-1)}{-(-1)}=\dfrac{1+10i}{1}$

$=1+10i$

90. $\dfrac{-6-i}{-i}=\dfrac{-6-i}{-i}\cdot\dfrac{i}{i}$

$=\dfrac{-6i-i^2}{-i^2}$

$=\dfrac{-6i-(-1)}{-(-1)}=\dfrac{1-6i}{1}$

$=1-6i$

91. $\dfrac{2+\sqrt{-16}}{8}=\dfrac{2+4i}{8}=\dfrac{2(1+2i)}{8}$

$=\dfrac{1+2i}{4}$

$=\dfrac{1}{4}+\dfrac{1}{2}i$

92. $\dfrac{6-\sqrt{-4}}{4}=\dfrac{6-2i}{4}=\dfrac{2(3-i)}{4}$

$=\dfrac{3-i}{2}$

$=\dfrac{3}{2}-\dfrac{1}{2}i$

93. $\dfrac{-6+\sqrt{-72}}{6} = \dfrac{-6+\sqrt{-36\cdot 2}}{6}$

$= \dfrac{-6+6i\sqrt{2}}{6}$

$= \dfrac{6\left(-1+i\sqrt{2}\right)}{6}$

$= -1+i\sqrt{2}$

94. $\dfrac{-20+\sqrt{-500}}{10} = \dfrac{-20+\sqrt{-100\cdot 5}}{10}$

$= \dfrac{-20+10i\sqrt{5}}{10}$

$= \dfrac{10\left(-2+i\sqrt{5}\right)}{10}$

$= -2+i\sqrt{5}$

95. $\dfrac{-8-\sqrt{-48}}{4} = \dfrac{-8-\sqrt{-16\cdot 3}}{4}$

$= \dfrac{-8-4i\sqrt{3}}{4}$

$= \dfrac{4\left(-2-i\sqrt{3}\right)}{4}$

$= -2-i\sqrt{3}$

96. $\dfrac{-18-\sqrt{-72}}{3} = \dfrac{-18-\sqrt{-36\cdot 2}}{3}$

$= \dfrac{-18-6i\sqrt{2}}{3}$

$= \dfrac{3\left(-6-2i\sqrt{2}\right)}{3}$

$= -6-2i\sqrt{2}$

97. $\dfrac{-5+\sqrt{-50}}{10} = \dfrac{-5+\sqrt{-25\cdot 2}}{10}$

$= \dfrac{-5+5i\sqrt{2}}{10}$

$= \dfrac{5\left(-1+i\sqrt{2}\right)}{5\cdot 2}$

$= -\dfrac{1}{2}+\dfrac{\sqrt{2}}{2}i$

98. $\dfrac{14+\sqrt{-98}}{7} = \dfrac{14+\sqrt{-49\cdot 2}}{7}$

$= \dfrac{14+7i\sqrt{2}}{7}$

$= \dfrac{7\left(2+i\sqrt{2}\right)}{7}$

$= 2+i\sqrt{2}$

99. $x^2 - 4x + 5 = 0 \qquad x = 2 + i$

$$(2+i)^2 - 4(2+i) + 5 = 0$$
$$4 + 4i + i^2 - 8 - 4i + 5 = 0$$
$$4 + 4i - 1 - 8 - 4i + 5 = 0$$
$$0 = 0$$

$2 + i$ is a solution.

100. $x^2 - 6x + 25 = 0 \qquad x = 3 - 4i$

$$(3-4i)^2 - 6(3-4i) + 25 = 0$$
$$9 - 24i + 16i^2 - 18 + 24i + 25 = 0$$
$$9 - 24i - 16 - 18 + 24i + 25 = 0$$
$$0 = 0$$

$3 - 4i$ is a solution.

101. $x^2 + 12 = 0 \qquad x = -2i\sqrt{3}$

$$\left(-2i\sqrt{3}\right)^2 + 12 = 0$$
$$4i^2 \cdot 3 + 12 = 0$$
$$-12 + 12 = 0$$
$$0 = 0$$

$-2i\sqrt{3}$ is a solution.

102. $x^2 + 18 = 0 \qquad x = 3i\sqrt{2}$

$$\left(3i\sqrt{2}\right)^2 + 18 = 0$$
$$9i^2 \cdot 2 + 18 = 0$$
$$-18 + 18 = 0$$
$$0 = 0$$

$3i\sqrt{2}$ is a solution.

Group Activity

1.–6. Answers will vary.

Chapter 6 Review Exercises

Section 6.1

1. **a.** False; $\sqrt{0} = 0$ is not positive.

b. False; $\sqrt[3]{-8} = -2$

2. $\sqrt{(-3)^2} = \sqrt{9} = 3$

3. **a.** False

b. True

4. $\sqrt{\frac{50}{32}} = \sqrt{\frac{25}{16}} = \frac{5}{4}$

5. $\sqrt[4]{625} = \sqrt[4]{5^4} = 5$

6. $\sqrt{(-6)^2} = \sqrt{36} = 6$

7. $f(x)=\sqrt{x-1}$

a. $f(10)=\sqrt{10-1}=\sqrt{9}=3$

b. $f(1)=\sqrt{1-1}=\sqrt{0}=0$

c. $f(8)=\sqrt{8-1}=\sqrt{7}$

d. $x-1\geq 0$

$x\geq 1 \quad [1,\infty)$

8. $g(t)=\sqrt{5+t}$

a. $g(-5)=\sqrt{5+(-5)}=\sqrt{0}=0$

b. $g(-4)=\sqrt{5+(-4)}=\sqrt{1}=1$

c. $g(4)=\sqrt{5+4}=\sqrt{9}=3$

d. $5+t\geq 0$

$t\geq -5 \quad [-5,\infty)$

9. $\dfrac{\sqrt[3]{2x}}{\sqrt[4]{2x}}+4$

10. **a.** $\sqrt{x^2}=|x|$

b. $\sqrt[3]{x^3}=x$

c. $\sqrt[4]{x^4}=|x|$

d. $\sqrt[5]{(x+1)^5}=x+1$

11. **a.** $\sqrt{4y^2}=2|y|$

b. $\sqrt[3]{27y^3}=3y$

c. $\sqrt[100]{y^{100}}=|y|$

d. $\sqrt[101]{y^{101}}=y$

12. Let x = the third side of the triangle

$x^2+15^2=17^2$

$x^2=17^2-15^2$

$x^2=289-225$

$x^2=64$

$x=\sqrt{64}$

$=8$ cm

Section 6.2

13. Yes, provided the expressions are well defined. For example: $x^5\cdot x^3=x^8$ and $x^{1/5}\cdot x^{2/5}=x^{3/5}$

14. n represents the root.

15. Take the reciprocal of the base and change the exponent to positive.

16. $(-125)^{1/3}=\sqrt[3]{-125}=-5$

17. $16^{-1/4} = \left(\frac{1}{16}\right)^{1/4} = \sqrt[4]{\frac{1}{16}}$
$= \frac{1}{2}$

18. $\left(\frac{1}{16}\right)^{-3/4} - \left(\frac{1}{8}\right)^{-2/3} = 16^{3/4} - 8^{2/3}$
$= \left(\sqrt[4]{16}\right)^3 - \left(\sqrt[3]{8}\right)^2 = 2^3 - 2^2 = 8 - 4 = 4$

19. $\left(b^{1/2} \cdot b^{1/3}\right)^{12} = \left(b^{1/2}\right)^{12}\left(b^{1/3}\right)^{12}$
$= b^6 \cdot b^4$
$= b^{6+4}$
$= b^{10}$

20. $\left(\frac{x^{-1/4}y^{-1/3}z^{3/4}}{2^{1/3}x^{-1/3}y^{2/3}}\right)^{-12} = \left(\frac{2^{1/3}x^{-1/3}y^{2/3}}{x^{-1/4}y^{-1/3}z^{3/4}}\right)^{12}$
$= \frac{\left(2^{1/3}\right)^{12}\left(x^{-1/3}\right)^{12}\left(y^{2/3}\right)^{12}}{\left(x^{-1/4}\right)^{12}\left(y^{-1/3}\right)^{12}\left(z^{3/4}\right)^{12}} = \frac{2^4x^{-4}y^8}{x^{-3}y^{-4}z^9}$
$= \frac{2^4x^{(-4)-(-3)}y^{8-(-4)}}{z^9} = \frac{2^4x^{-1}y^{12}}{z^9} = \frac{16y^{12}}{xz^9}$

21. $\left(\frac{a^{12}b^{-4}c^7}{a^3b^2c^4}\right)^{1/3} = \left(a^{12-3}b^{-4-2}c^{7-4}\right)^{1/3}$
$= \left(a^9b^{-6}c^3\right)^{1/3}$
$= \left(a^9\right)^{1/3}\left(b^{-6}\right)^{1/3}\left(c^3\right)^{1/3}$
$= a^3b^{-2}c^1 = \frac{a^3c}{b^2}$

22. $\sqrt[4]{x^3} = x^{3/4}$

23. $\sqrt[3]{2y^2} = \left(2y^2\right)^{1/3}$

24. $10^{1/3} \approx 2.1544$

25. $17.8^{2/3} \approx 6.8173$

26. $\sqrt[5]{147^4} = 147^{4/5} \approx 54.1819$

Section 6.3

27. 1. The radicand has no factor raised to a power greater than or equal to the index.
2. The radicand does not contain a fraction.
3. There are no radicals in the denominator of a fraction.

28. $\sqrt{108} = \sqrt{36 \cdot 3} = 6\sqrt{3}$

29. $\sqrt[4]{x^5yz^4} = \sqrt[4]{x^4z^4 \cdot xy} = xz\sqrt[4]{xy}$

30. $-2\sqrt[3]{250a^3b^{10}} = -2\sqrt[3]{2b \cdot 125a^3b^9}$
$= -2 \cdot 5ab^3\sqrt[3]{2b}$
$= -10ab^3\sqrt[3]{2b}$

31. $\sqrt[3]{\dfrac{-16a^4}{2ab^3}} = \sqrt[3]{\dfrac{-8a^3}{b^3}}$
$= -\dfrac{2a}{b}$

32. **a.** The principal square root of the quotient of 2 and x.

b. The cube of the sum of x and 1.

33. Let $h =$ the height of the bulge

$\frac{1}{2}$ length of the bridge $= \frac{1}{8}$ mi $= \frac{1}{8}(5280)$
$= 660$ ft

$h^2 + 660^2 = 660.75^2$
$h^2 = 660.75^2 - 660^2$
$h^2 = 436,590.5625 - 435,600$
$h^2 = 990.5625$
$h = \sqrt{990.5625} \approx 31$ ft

Section 6.4

34. Radicals may be added or subtracted if they are like radicals.

35. $\sqrt[3]{2x} - 2\sqrt{2x}$ cannot be combined; the indices are different.

36. $2 + \sqrt{x}$ cannot be combined; one term has a radical but the other does not.

37. $\sqrt[4]{3xy} + 2\sqrt[4]{3xy} = 3\sqrt[4]{3xy}$ can be combined.

38. $-4\sqrt{32} + 7\sqrt{50} = -4\sqrt{16 \cdot 2} + 7\sqrt{25 \cdot 2}$
$= -4 \cdot 4\sqrt{2} + 7 \cdot 5\sqrt{2}$
$= -16\sqrt{2} + 35\sqrt{2} = 19\sqrt{2}$
can be added after simplifying.

39. $4\sqrt{7} - 2\sqrt{7} + 3\sqrt{7} = (4 - 2 + 3)\sqrt{7} = 5\sqrt{7}$

40. $2\sqrt[3]{64} + 3\sqrt[3]{54} - 16 = 2 \cdot 4 + 3\sqrt[3]{27 \cdot 2} - 16$
$= 8 + 3 \cdot 3\sqrt[3]{2} - 16$
$= 9\sqrt[3]{2} - 8$

41. $\sqrt{50} + 7\sqrt{2} - \sqrt{8} = \sqrt{25 \cdot 2} + 7\sqrt{2} - \sqrt{4 \cdot 2}$
$= 5\sqrt{2} + 7\sqrt{2} - 2\sqrt{2}$
$= 10\sqrt{2}$

42. $x\sqrt[3]{16x^2} - 4\sqrt[3]{2x^5} + 5x\sqrt[3]{54x^2}$
$= x\sqrt[3]{8\cdot 2x^2} - 4\sqrt[3]{x^3\cdot 2x^2} + 5x\sqrt[3]{27\cdot 2x^2}$
$= 2x\sqrt[3]{2x^2} - 4x\sqrt[3]{2x^2} + 5x\cdot 3\sqrt[3]{2x^2}$
$= 2x\sqrt[3]{2x^2} - 4x\sqrt[3]{2x^2} + 15x\sqrt[3]{2x^2}$
$= 13x\sqrt[3]{2x^2}$

43. False; 5 and $3\sqrt{x}$ are not like radicals.

44. False; $\sqrt{y} + \sqrt{y} = 2\sqrt{y}$ (add the coefficients)

Section 6.5

45. $\sqrt{3}\cdot\sqrt{12} = \sqrt{36} = 6$

46. $\sqrt[4]{4}\cdot\sqrt[4]{8} = \sqrt[4]{32} = \sqrt[4]{16\cdot 2} = 2\sqrt[4]{2}$

47. $-2\sqrt{3}\left(\sqrt{7} - 3\sqrt{11}\right)$
$= -2\sqrt{3}\cdot\sqrt{7} + 2\sqrt{3}\cdot 3\sqrt{11}$
$= -2\sqrt{21} + 6\sqrt{33}$

48. $-3\sqrt{5}\left(2\sqrt{3} - \sqrt{5}\right) = -3\sqrt{5}\cdot 2\sqrt{3} + 3\sqrt{5}\cdot\sqrt{5}$
$= -6\sqrt{15} + 3\sqrt{25} = -6\sqrt{15} + 3\cdot 5$
$= -6\sqrt{15} + 15$

49. $\left(2\sqrt{x} - 3\right)\left(2\sqrt{x} + 3\right) = \left(2\sqrt{x}\right)^2 - 3^2$
$= 4x - 9$

50. $\left(\sqrt{y} + 4\right)\left(\sqrt{y} - 4\right) = \left(\sqrt{y}\right)^2 - 4^2$
$= y - 16$

51. $\left(\sqrt{7y} - \sqrt{3x}\right)^2$
$= \left(\sqrt{7y}\right)^2 - 2\cdot\sqrt{7y}\cdot\sqrt{3x} + \left(\sqrt{3x}\right)^2$
$= 7y - 2\sqrt{21xy} + 3x$

52. $\left(2\sqrt{3w} + 5\right)^2 = \left(2\sqrt{3w}\right)^2 + 2\cdot 2\sqrt{3w}\cdot 5 + 5^2$
$= 4\cdot 3w + 20\sqrt{3w} + 25$
$= 12w + 20\sqrt{3w} + 25$

53. $\left(-\sqrt{z} - \sqrt{6}\right)\left(2\sqrt{z} + 7\sqrt{6}\right) = \left(-\sqrt{z}\right)\left(2\sqrt{z}\right) - \sqrt{z}\left(7\sqrt{6}\right) - \sqrt{6}\left(2\sqrt{z}\right) - \sqrt{6}\left(7\sqrt{6}\right)$
$= -2\sqrt{z^2} - 7\sqrt{6z} - 2\sqrt{6z} - 7\sqrt{36} = -2z - 9\sqrt{6z} - 7\cdot 6 = -2z - 9\sqrt{6z} - 42$

54. $\left(3\sqrt{a} - \sqrt{5}\right)\left(\sqrt{a} + 2\sqrt{5}\right) = \left(3\sqrt{a}\right)\left(\sqrt{a}\right) + 3\sqrt{a}\left(2\sqrt{5}\right) - \sqrt{5}\left(\sqrt{a}\right) - \sqrt{5}\left(2\sqrt{5}\right)$
$= 3\sqrt{a^2} + 6\sqrt{5a} - \sqrt{5a} - 2\sqrt{25} = 3a + 5\sqrt{5a} - 2\cdot 5 = 3a + 5\sqrt{5a} - 10$

55. $\sqrt[3]{u}\cdot\sqrt{u^5} = u^{1/3}\cdot u^{5/2}$

$= u^{(1/3)+(5/2)} = u^{17/6}$

$= \sqrt[6]{u^{17}} = \sqrt[6]{u^{12}\cdot u^5}$

$= u^2\sqrt[6]{u^5}$

56. $\sqrt{2}\cdot\sqrt[4]{w^3} = 2^{1/2}\cdot w^{3/4}$

$= 2^{2/4}\cdot w^{3/4}$

$= \left(2^2 w^3\right)^{1/4}$

$= \sqrt[4]{4w^3}$

Section 6.6

57. $\sqrt{\dfrac{3y^5}{25x^6}} = \dfrac{\sqrt{3y\cdot y^4}}{\sqrt{25x^6}}$

$= \dfrac{y^2\sqrt{3y}}{5x^3}$

58. $\sqrt[3]{\dfrac{-16x^7y^6}{z^9}} = \dfrac{\sqrt[3]{2x\cdot -8x^6y^6}}{\sqrt[3]{z^9}}$

$= \dfrac{-2x^2y^2\sqrt[3]{2x}}{z^3}$

59. $\dfrac{\sqrt{324w^7}}{\sqrt{4w^3}} = \sqrt{\dfrac{324w^7}{4w^3}}$

$= \sqrt{81w^4}$

$= 9w^2$

60. $\dfrac{\sqrt[3]{3t^{14}}}{\sqrt[3]{192t^2}} = \sqrt[3]{\dfrac{3t^{14}}{192t^2}} = \sqrt[3]{\dfrac{t^{12}}{64}}$

$= \dfrac{\sqrt[3]{t^{12}}}{\sqrt[3]{64}} = \dfrac{t^4}{4}$

61. $\sqrt{\dfrac{7}{2y}} = \dfrac{\sqrt{7}}{\sqrt{2y}} = \dfrac{\sqrt{7}}{\sqrt{2y}}\cdot\dfrac{\sqrt{2y}}{\sqrt{2y}}$

$= \dfrac{\sqrt{14y}}{\sqrt{4y^2}} = \dfrac{\sqrt{14y}}{2y}$

62. $\sqrt{\dfrac{5}{3w}} = \dfrac{\sqrt{5}}{\sqrt{3w}} = \dfrac{\sqrt{5}}{\sqrt{3w}}\cdot\dfrac{\sqrt{3w}}{\sqrt{3w}}$

$= \dfrac{\sqrt{15w}}{\sqrt{9w^2}} = \dfrac{\sqrt{15w}}{3w}$

63. $\dfrac{4}{\sqrt[3]{9p^2}} = \dfrac{4}{\sqrt[3]{3^2\cdot p^2}}\cdot\dfrac{\sqrt[3]{3p}}{\sqrt[3]{3p}}$

$= \dfrac{4\sqrt[3]{3p}}{\sqrt[3]{3^3p^3}} = \dfrac{4\sqrt[3]{3p}}{3p}$

64. $\dfrac{-2}{\sqrt[3]{2x}} = \dfrac{-2}{\sqrt[3]{2x}}\cdot\dfrac{\sqrt[3]{4x^2}}{\sqrt[3]{4x^2}} = \dfrac{-2\sqrt[3]{4x^2}}{\sqrt[3]{8x^3}}$

$= \dfrac{-2\sqrt[3]{4x^2}}{2x} = \dfrac{-\sqrt[3]{4x^2}}{x}$

65. $\dfrac{-5}{\sqrt{15}+\sqrt{10}} = \dfrac{-5}{\sqrt{15}+\sqrt{10}} \cdot \dfrac{\sqrt{15}-\sqrt{10}}{\sqrt{15}-\sqrt{10}}$

$= \dfrac{-5\left(\sqrt{15}-\sqrt{10}\right)}{\left(\sqrt{15}\right)^2-\left(\sqrt{10}\right)^2}$

$= \dfrac{-5\left(\sqrt{15}-\sqrt{10}\right)}{15-10}$

$= \dfrac{-5\left(\sqrt{15}-\sqrt{10}\right)}{5}$

$= -\sqrt{15}+\sqrt{10}$

66. $\dfrac{-6}{\sqrt{7}+\sqrt{5}} = \dfrac{-6}{\sqrt{7}+\sqrt{5}} \cdot \dfrac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}}$

$= \dfrac{-6\left(\sqrt{7}-\sqrt{5}\right)}{\left(\sqrt{7}\right)^2-\left(\sqrt{5}\right)^2}$

$= \dfrac{-6\left(\sqrt{7}-\sqrt{5}\right)}{7-5}$

$= \dfrac{-6\left(\sqrt{7}-\sqrt{5}\right)}{2} = -3\left(\sqrt{7}-\sqrt{5}\right)$

$= -3\sqrt{7}+3\sqrt{5}$

67. $\dfrac{t-3}{\sqrt{t}-\sqrt{3}} = \dfrac{t-3}{\sqrt{t}-\sqrt{3}} \cdot \dfrac{\sqrt{t}+\sqrt{3}}{\sqrt{t}+\sqrt{3}}$

$= \dfrac{(t-3)\left(\sqrt{t}+\sqrt{3}\right)}{\left(\sqrt{t}\right)^2-\left(\sqrt{3}\right)^2} = \dfrac{\cancel{(t-3)}\left(\sqrt{t}+\sqrt{3}\right)}{\cancel{t-3}}$

$= \sqrt{t}+\sqrt{3}$

68. $\dfrac{w-7}{\sqrt{w}-\sqrt{7}} = \dfrac{w-7}{\sqrt{w}-\sqrt{7}} \cdot \dfrac{\sqrt{w}+\sqrt{7}}{\sqrt{w}+\sqrt{7}}$

$= \dfrac{(w-7)\left(\sqrt{w}+\sqrt{7}\right)}{\left(\sqrt{w}\right)^2-\left(\sqrt{7}\right)^2}$

$= \dfrac{\cancel{(w-7)}\left(\sqrt{w}+\sqrt{7}\right)}{\cancel{w-7}} = \sqrt{w}+\sqrt{7}$

69. The quotient of the principal square root of 2 and the square of x.

Section 6.7

70. $\sqrt{2y} = 7$

$\left(\sqrt{2y}\right)^2 = 7^2$

$2y = 49$

$y = \dfrac{49}{2}$

The solution is $\left\{\frac{49}{2}\right\}$.

Check:

$\sqrt{2\left(\dfrac{49}{2}\right)} = 7$

$\sqrt{49} = 7$

$7 = 7$

71. $\sqrt{a-6}-5 = 0$

$\sqrt{a-6} = 5$

$\left(\sqrt{a-6}\right)^2 = 5^2$

$a-6 = 25$

$a = 31$

The solution is $\{31\}$.

Check:

$\sqrt{31-6}-5 = 0$

$\sqrt{25}-5 = 0$

$5-5 = 0$

$0 = 0$

72. $\sqrt[3]{2w-3}+5=2$

$\sqrt[3]{2w-3}=-3$

$\left(\sqrt[3]{2w-3}\right)^3=(-3)^3$

$2w-3=-27$

$2w=-24$

$w=-12$

Check:

$\sqrt[3]{2(-12)-3}+5=2$

$\sqrt[3]{-24-3}+5=2$

$\sqrt[3]{-27}+5=2$

$-3+5=2$

$2=2$

The solution is $\{-12\}$.

73. $\sqrt[4]{p+12}-\sqrt[4]{5p-16}=0$

$\sqrt[4]{p+12}=\sqrt[4]{5p-16}$

$\left(\sqrt[4]{p+12}\right)^4=\left(\sqrt[4]{5p-16}\right)^4$

$p+12=5p-16$

$-4p=-28$

$p=7$

Check:

$\sqrt[4]{7+12}-\sqrt[4]{5(7)-16}=0$

$\sqrt[4]{19}-\sqrt[4]{35-16}=0$

$\sqrt[4]{19}-\sqrt[4]{19}=0$

$0=0$

The solution is $\{7\}$.

74. $\sqrt{t}+\sqrt{t-5}=5$

$\sqrt{t-5}=5-\sqrt{t}$

$\left(\sqrt{t-5}\right)^2=\left(5-\sqrt{t}\right)^2$

$t-5=25-10\sqrt{t}+t$

$-30=-10\sqrt{t}$

$\sqrt{t}=3$

$\left(\sqrt{t}\right)^2=3^2$

$t=9$

Check:

$\sqrt{9}+\sqrt{9-5}=5$

$\sqrt{9}+\sqrt{4}=5$

$3+2=5$

$5=5$

The solution is $\{9\}$.

75. $\sqrt{8x+1}=-\sqrt{x-13}$

$\left(\sqrt{8x+1}\right)^2=\left(-\sqrt{x-13}\right)^2$

$8x+1=x-13$

$7x=-14$

$x=-2$

Check:

$\sqrt{8(-2)+1}=-\sqrt{-2-13}$

$\sqrt{-16+1}=-\sqrt{-15}$

$\sqrt{-15}\neq-\sqrt{-15}$

$\{\ \}$ ($x=-2$ does not check.)

76. $\sqrt{2m^2+4}-\sqrt{9m}=0$

$$\sqrt{2m^2+4}=\sqrt{9m}$$
$$\left(\sqrt{2m^2+4}\right)^2=\left(\sqrt{9m}\right)^2$$
$$2m^2+4=9m$$
$$2m^2-9m+4=0$$
$$(2m-1)(m-4)=0$$
$$2m-1=0 \text{ or } m-4=0$$
$$2m=1 \text{ or } \quad m=4$$
$$m=\frac{1}{2} \text{ or } \quad m=4$$

Check $m=\frac{1}{2}$:

$$\sqrt{2\left(\frac{1}{2}\right)^2+4}-\sqrt{9\left(\frac{1}{2}\right)}=0$$
$$\sqrt{\frac{1}{2}+4}-\sqrt{\frac{9}{2}}=0$$
$$\sqrt{\frac{9}{2}}-\sqrt{\frac{9}{2}}=0$$
$$0=0$$

Check $m=4$:

$$\sqrt{2(4)^2+4}-\sqrt{9(4)}=0$$
$$\sqrt{32+4}-\sqrt{36}=0$$
$$\sqrt{36}-\sqrt{36}=0$$
$$0=0$$

The solution is $\left\{\frac{1}{2},4\right\}$.

77. $\sqrt{x+2}=1-\sqrt{2x+5}$

$$\left(\sqrt{x+2}\right)^2=\left(1-\sqrt{2x+5}\right)^2$$
$$x+2=1-2\sqrt{2x+5}+2x+5$$
$$-x-4=-2\sqrt{2x+5}$$
$$\left(-x-4\right)^2=\left(-2\sqrt{2x+5}\right)^2$$
$$x^2+8x+16=4(2x+5)$$
$$x^2+8x+16=8x+20$$
$$x^2-4=0$$
$$(x+2)(x-2)=0$$
$$x+2=0 \text{ or } x-2=0$$
$$x=-2 \text{ or } \quad x=2$$

Check $x=-2$:

$$\sqrt{-2+2}=1-\sqrt{2(-2)+5}$$
$$\sqrt{0}=1-\sqrt{-4+5}$$
$$0=1-\sqrt{1}$$
$$0=1-1$$
$$0=0$$

Check $x=2$:

$$\sqrt{2+2}=1-\sqrt{2(2)+5}$$
$$\sqrt{4}=1-\sqrt{4+5}$$
$$2=1-\sqrt{9}$$
$$2=1-3$$
$$2\neq -2$$

The solution is $\{-2\}$. ($x=2$ does not check.)

78. Let c = the length of the wire

79. $v(d)=\sqrt{32d}$

(a) $v(20)=\sqrt{32(20)}=\sqrt{640}=\sqrt{64\cdot 10}$

$=8\sqrt{10}$ ft/sec ≈ 25.3 ft/sec

When the water is 20 ft deep, a wave travels about 25.3 ft/sec.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 6^2 + 12^2 &= c^2 \\ 36 + 144 &= c^2 \\ 180 &= c^2 \\ c &= \sqrt{180} \\ &= \sqrt{36 \cdot 5} \\ &= 6\sqrt{5} \text{ m} \\ &\approx 13.4 \text{ m} \end{aligned}$$

(b)
$$\begin{aligned} 16 &= \sqrt{32d} \\ 16^2 &= \left(\sqrt{32d}\right)^2 \\ 256 &= 32d \\ d &= 8 \text{ ft} \end{aligned}$$

Section 6.8

80. $a + bi$, where a and b are real numbers and $i = \sqrt{-1}$.

81. $a + bi$, where $b \neq 0$.

82. To simplify the expression $\dfrac{3}{4+6i}$, first multiply the numerator and denominator by $4-6i$, which is the complex conjugate of the denominator.

83. $\sqrt{-16} = 4i$

84. $-\sqrt{-5} = -i\sqrt{5}$

85.
$$\begin{aligned} \sqrt{-75} \cdot \sqrt{-3} &= \sqrt{-25 \cdot 3} \cdot \sqrt{-3} \\ &= 5i\sqrt{3} \cdot i\sqrt{3} \\ &= 5i^2\sqrt{9} \\ &= 5(-1)(3) \\ &= -15 \end{aligned}$$

86.
$$\begin{aligned} \frac{-\sqrt{-24}}{\sqrt{6}} &= \frac{-\sqrt{-4 \cdot 6}}{\sqrt{6}} \\ &= \frac{-2i\sqrt{6}}{\sqrt{6}} \\ &= -2i \end{aligned}$$

87.
$$\begin{aligned} i^{38} &= i^{36} \cdot i^2 = \left(i^4\right)^9 \cdot i^2 \\ &= 1^9(-1) \\ &= 1(-1) = -1 \end{aligned}$$

88.
$$\begin{aligned} i^{101} &= i^{100} \cdot i = \left(i^4\right)^{25} \cdot i \\ &= 1^{25} \cdot i \\ &= 1 \cdot i = i \end{aligned}$$

89. $i^{19} = i^{16} \cdot i^3 = \left(i^4\right)^4 \cdot i^3$
$= 1^4\left(-i\right)$
$= 1\left(-i\right) = -i$

90. $i^{1000} + i^{1002} = \left(i^4\right)^{250} + \left(i^4\right)^{250} \cdot i^2$
$= 1^{250} + 1^{250}\left(-1\right)$
$= 1 + 1\left(-1\right)$
$= 1 - 1 = 0$

91. $\left(-3+i\right) - \left(2-4i\right) = -3 + i - 2 + 4i$
$= \left(-3-2\right) + \left(1+4\right)i$
$= -5 + 5i$

92. $\left(1+6i\right)\left(3-i\right) = 3 - i + 18i - 6i^2$
$= 3 + 17i - 6\left(-1\right)$
$= 3 + 17i + 6$
$= 9 + 17i$

93. $\left(4-3i\right)\left(4+3i\right) = 4^2 - \left(3i\right)^2 = 16 - 9i^2$
$= 16 - 9\left(-1\right) = 16 + 9$
$= 25 + 0i$

94. $\left(5-i\right)^2 = 5^2 - 2 \cdot 5 \cdot i + i^2$
$= 25 - 10i - 1$
$= 24 - 10i$

95. $\dfrac{17-4i}{-4} = -\dfrac{17}{4} + \dfrac{4}{4}i = -\dfrac{17}{4} + i$

Real part: $-\dfrac{17}{4}$
Imaginary part: 1

96. $\dfrac{-16-8i}{8} = -\dfrac{16}{8} - \dfrac{8}{8}i = -2 - i$

Real part: -2
Imaginary part: -1

97. $\dfrac{2-i}{3+2i} = \dfrac{2-i}{3+2i} \cdot \dfrac{3-2i}{3-2i}$

$= \dfrac{2 \cdot 3 - 2 \cdot 2i - i \cdot 3 + i \cdot 2i}{3^2 - \left(2i\right)^2}$

$= \dfrac{6 - 4i - 3i + 2i^2}{9 - 4i^2}$

$= \dfrac{6 - 7i + 2\left(-1\right)}{9 - 4\left(-1\right)}$

$= \dfrac{6 - 7i - 2}{9 + 4} = \dfrac{4 - 7i}{13}$

$= \dfrac{4}{13} - \dfrac{7}{13}i$

98. $\dfrac{10+5i}{2-i} = \dfrac{10+5i}{2-i} \cdot \dfrac{2+i}{2+i}$

$= \dfrac{10 \cdot 2 + 10 \cdot i + 5i \cdot 2 + 5i \cdot i}{2^2 - i^2}$

$= \dfrac{20 + 10i + 10i + 5i^2}{4 - \left(-1\right)}$

$= \dfrac{20 + 20i + 5\left(-1\right)}{4 + 1}$

$= \dfrac{20 + 20i - 5}{5} = \dfrac{15 + 20i}{5}$

$= \dfrac{5\left(3 + 4i\right)}{5} = 3 + 4i$

99. $\dfrac{5+3i}{-2i} = \dfrac{5+3i}{-2i}\cdot\dfrac{2i}{2i}$

$= \dfrac{5\cdot 2i + 3i\cdot 2i}{-(2i)^2} = \dfrac{10i+6i^2}{-4i^2}$

$= \dfrac{10i+6(-1)}{-4(-1)}$

$= \dfrac{-6+10i}{4}$

$= -\dfrac{6}{4}+\dfrac{10}{4}i$

$= -\dfrac{3}{2}+\dfrac{5}{2}i$

100. $\dfrac{4i}{4-i} = \dfrac{4i}{4-i}\cdot\dfrac{4+i}{4+i}$

$= \dfrac{4i\cdot 4 + 4i\cdot i}{4^2 - i^2}$

$= \dfrac{16i+4i^2}{16-(-1)}$

$= \dfrac{16i+4(-1)}{16+1}$

$= \dfrac{-4+16i}{17}$

$= -\dfrac{4}{17}+\dfrac{16}{17}i$

101. $\dfrac{-8+\sqrt{-40}}{12} = \dfrac{-8+\sqrt{-4\cdot 10}}{12}$

$= \dfrac{-8+2i\sqrt{10}}{12}$

$= \dfrac{2\left(-4+i\sqrt{10}\right)}{12}$

$= \dfrac{-4+i\sqrt{10}}{6}$

$= -\dfrac{4}{6}+\dfrac{\sqrt{10}}{6}i$

$= -\dfrac{2}{3}+\dfrac{\sqrt{10}}{6}i$

102. $\dfrac{6-\sqrt{-144}}{3} = \dfrac{6-12i}{3}$

$= \dfrac{6}{3}-\dfrac{12}{3}i$

$= 2-4i$

Chapter 6 Test

1. **a.** $\sqrt{36}=6$

b. $-\sqrt{36}=-6$

2. **a.** $-\sqrt{100}$ Real

b. $\sqrt{-100}$ Not real

c. $-\sqrt[3]{1000}$ Real

d. $\sqrt[3]{-1000}$ Real

3. **a.** $\sqrt[3]{y^3}=y$

b. $\sqrt[4]{y^4}=\left|y\right|$

4. $\sqrt[4]{81}=3$

5. $\sqrt{\frac{16}{9}}=\frac{4}{3}$

6. $\sqrt[3]{32}=\sqrt[3]{8\cdot 4}$

$=2\sqrt[3]{4}$

7. $\sqrt{a^4b^3c^5}=\sqrt{a^4b^2c^4\cdot bc}$

$=a^2bc^2\sqrt{bc}$

8. $\sqrt{18x^5y^3z^4}=\sqrt{9x^4y^2z^4\cdot 2xy}$

$=3x^2yz^2\sqrt{2xy}$

9. $\sqrt{\frac{32w^6}{2w}}=\sqrt{16w^5}$

$=\sqrt{16w^4\cdot w}$

$=4w^2\sqrt{w}$

10. $\sqrt[3]{\frac{x^6}{125y^3}}=\frac{\sqrt[3]{x^6}}{\sqrt[3]{125y^3}}$

$=\frac{x^2}{5y}$

11. $\frac{2\sqrt{72}}{8}=\frac{2\sqrt{36\cdot 2}}{8}$

$=\frac{2\cdot 6\sqrt{2}}{8}$

$=\frac{12\sqrt{2}}{8}$

$=\frac{3\sqrt{2}}{2}$

12. **a.** $f\left(x\right)=\sqrt{-2x-4}$

$f\left(-8\right)=\sqrt{-2\left(-8\right)-4}=\sqrt{16-4}$

$=\sqrt{12}=\sqrt{4\cdot 3}=2\sqrt{3}$

$f\left(-6\right)=\sqrt{-2\left(-6\right)-4}=\sqrt{12-4}$

$=\sqrt{8}=\sqrt{4\cdot 2}=2\sqrt{2}$

$f\left(-4\right)=\sqrt{-2\left(-4\right)-4}=\sqrt{8-4}$

$=\sqrt{4}=2$

$f\left(-2\right)=\sqrt{-2\left(-2\right)-4}=\sqrt{4-4}$

$=\sqrt{0}=0$

b. $-2x-4\ge 0$

$-2x\ge 4$

$x\le -2 \quad \left(-\infty,-2\right]$

13. $\dfrac{-3-\sqrt{5}}{17}\approx -0.3080$

14. $-27^{1/3}=-\sqrt[3]{27}=-3$

15. $8^{2/3}\cdot\left(\dfrac{25x^4y^6}{z^2}\right)^{1/2}$

$=\left(\sqrt[3]{8}\right)^2\cdot\dfrac{25^{1/2}\left(x^4\right)^{1/2}\left(y^6\right)^{1/2}}{\left(z^2\right)^{1/2}}$

$=2^2\cdot\dfrac{5x^2y^3}{z}$

$=\dfrac{20x^2y^3}{z}$

16. $\sqrt[6]{7}\cdot\sqrt{y}=7^{1/6}\cdot y^{1/2}$

$=7^{1/6}\cdot y^{3/6}$

$=\left(7y^3\right)^{1/6}$

$=\sqrt[6]{7y^3}$

17. $\dfrac{\sqrt[3]{10}}{\sqrt[4]{10}}=\dfrac{10^{1/3}}{10^{1/4}}$

$=10^{(1/3)-(1/4)}$

$=10^{1/12}$

$=\sqrt[12]{10}$

18. $3\sqrt{5}+4\sqrt{5}-2\sqrt{20}=7\sqrt{5}-2\sqrt{4\cdot 5}$

$=7\sqrt{5}-2\cdot 2\sqrt{5}$

$=7\sqrt{5}-4\sqrt{5}$

$=3\sqrt{5}$

19. $3\sqrt{x}\left(\sqrt{2}-\sqrt{5}\right)=3\sqrt{x}\cdot\sqrt{2}-3\sqrt{x}\cdot\sqrt{5}=3\sqrt{2x}-3\sqrt{5x}$

20. $\left(2\sqrt{5}-3\sqrt{x}\right)\left(4\sqrt{5}+\sqrt{x}\right)=2\sqrt{5}\left(4\sqrt{5}\right)+2\sqrt{5}\left(\sqrt{x}\right)-3\sqrt{x}\left(4\sqrt{5}\right)-3\sqrt{x}\left(\sqrt{x}\right)$

$=8\sqrt{25}+2\sqrt{5x}-12\sqrt{5x}-3\sqrt{x^2}$

$=8\cdot 5-10\sqrt{5x}-3x$

$=40-10\sqrt{5x}-3x$

21. $\dfrac{-2}{\sqrt[3]{x}}=\dfrac{-2}{\sqrt[3]{x}}\cdot\dfrac{\sqrt[3]{x^2}}{\sqrt[3]{x^2}}$

$=\dfrac{-2\sqrt[3]{x^2}}{\sqrt[3]{x^3}}$

$=\dfrac{-2\sqrt[3]{x^2}}{x}$

22. $\dfrac{\sqrt{x}+2}{3-\sqrt{x}}=\dfrac{\sqrt{x}+2}{3-\sqrt{x}}\cdot\dfrac{3+\sqrt{x}}{3+\sqrt{x}}$

$=\dfrac{3\sqrt{x}+\sqrt{x}\cdot\sqrt{x}+2\cdot 3+2\sqrt{x}}{3^2-\left(\sqrt{x}\right)^2}$

$=\dfrac{x+5\sqrt{x}+6}{9-x}$

23. **a.** $\sqrt{-8}=\sqrt{-4\cdot 2}=2i\sqrt{2}$

b. $2\sqrt{-16}=2\cdot 4i$

$=8i$

c. $\dfrac{2+\sqrt{-8}}{4}=\dfrac{2+\sqrt{-4\cdot 2}}{4}=\dfrac{2+2i\sqrt{2}}{4}$

$=\dfrac{2\left(1+i\sqrt{2}\right)}{4}=\dfrac{1+i\sqrt{2}}{2}$

$=\dfrac{1}{2}+\dfrac{\sqrt{2}}{2}i$

24. $\left(3-5i\right)-\left(2+6i\right)=3-5i-2-6i$

$=\left(3-2\right)+\left(-5-6\right)i$

$=1-11i$

25. $\left(4+i\right)\left(8+2i\right)=4\cdot 8+4\cdot 2i+i\cdot 8+i\cdot 2i$

$=32+8i+8i+2i^2$

$=32+16i-2$

$=30+16i$

26. $\sqrt{-16}\cdot\sqrt{-49}=4i\cdot 7i$

$=28i^2$

$=28\left(-1\right)$

$=-28$

27. $\left(4-7i\right)^2=4^2-2\cdot 4\cdot 7i+\left(7i\right)^2$

$=16-56i+49i^2$

$=16-56i+49\left(-1\right)$

$=16-56i-49$

$=-33-56i$

28. $\begin{aligned}(2-10i)(2+10i) &= 2^2-(10i)^2\\ &= 4-100i^2\\ &= 4-100(-1)\\ &= 4+100\\ &= 104\end{aligned}$

29. $\dfrac{3-2i}{3-4i}=\dfrac{3-2i}{3-4i}\cdot\dfrac{3+4i}{3+4i}=\dfrac{3\cdot3+3\cdot4i-2i\cdot3-2i\cdot4i}{3^2-(4i)^2}$

$=\dfrac{9+12i-6i-8i^2}{9-16i^2}=\dfrac{9+6i-8(-1)}{9-16(-1)}$

$=\dfrac{9+6i+8}{9+16}=\dfrac{17+6i}{25}=\dfrac{17}{25}+\dfrac{6}{25}i$

30. $\dfrac{6i}{3-5i}=\dfrac{6i}{3-5i}\cdot\dfrac{3+5i}{3+5i}=\dfrac{6i(3+5i)}{3^2-(5i)^2}$

$=\dfrac{6i(3+5i)}{9-25i^2}=\dfrac{6i(3+5i)}{9-25(-1)}=\dfrac{6i(3+5i)}{9+25}$

$=\dfrac{6i(3+5i)}{34}=\dfrac{3i(3+5i)}{17}=\dfrac{9i+15i^2}{17}$

$=\dfrac{9i+15(-1)}{17}=\dfrac{9i-15}{17}=-\dfrac{15}{17}+\dfrac{9}{17}i$

31. $r(V)=\sqrt[3]{\dfrac{3V}{4\pi}}$

$r(10)=\sqrt[3]{\dfrac{3(10)}{4\pi}}$

$=\sqrt[3]{\dfrac{30}{4\pi}}$

≈ 1.34

The radius of a sphere of volume 10 cubic units is approximately 1.34 units.

32. Let c = length of the roof without overhang

$a^2+b^2=c^2$

$2^2+20^2=c^2$

$4+400=c^2$

$404=c^2$

$c=\sqrt{404}$

$=\sqrt{4\cdot101}$

$=2\sqrt{101}$ m

≈ 20.1 ft

With the 8 in overhang included the roof will be approximately 21 ft.

33. $\sqrt[3]{2x+5}=-3$

$\left(\sqrt[3]{2x+5}\right)^3=(-3)^3$

$2x+5=-27$

$2x=-32$

$x=-16$

Check:

$\sqrt[3]{2(-16)+5}=-3$

$\sqrt[3]{-32+5}=-3$

$\sqrt[3]{-27}=-3$

$-3=-3$

The solution is $\{-16\}$.

34.
$$\sqrt{5x+8} = \sqrt{5x-1}+1$$
$$\left(\sqrt{5x+8}\right)^2 = \left(\sqrt{5x-1}+1\right)^2$$
$$5x+8 = 5x-1+2\sqrt{5x-1}+1$$
$$8 = 2\sqrt{5x-1}$$
$$4 = \sqrt{5x-1}$$
$$(4)^2 = \left(\sqrt{5x-1}\right)^2$$
$$16 = 5x-1$$
$$17 = 5x$$
$$x = \frac{17}{5}$$

Check:
$$\sqrt{5\left(\frac{17}{5}\right)+8} = \sqrt{5\left(\frac{17}{5}\right)-1}+1$$
$$\sqrt{17+8} = \sqrt{17-1}+1$$
$$\sqrt{25} = \sqrt{16}+1$$
$$5 = 4+1$$
$$5 = 5$$

The solution is $\left\{\frac{17}{5}\right\}$.

35.
$$\sqrt{t+7}-\sqrt{2t-3} = 2$$
$$\sqrt{t+7} = 2+\sqrt{2t-3}$$
$$\left(\sqrt{t+7}\right)^2 = \left(2+\sqrt{2t-3}\right)^2$$
$$t+7 = 4+4\sqrt{2t-3}+2t-3$$
$$-t+6 = 4\sqrt{2t-3}$$
$$\left(-t+6\right)^2 = \left(4\sqrt{2t-3}\right)^2$$
$$t^2-12t+36 = 16\left(2t-3\right)$$
$$t^2-12t+36 = 32t-48$$
$$t^2-44t+84 = 0$$
$$\left(t-42\right)\left(t-2\right) = 0$$
$$t-42 = 0 \text{ or } t-2 = 0$$
$$t = 42 \text{ or } \quad t = 2$$

Check $t = 42$:
$$\sqrt{42+7}-\sqrt{2\left(42\right)-3} = 2$$
$$\sqrt{49}-\sqrt{84-3} = 2$$
$$7-\sqrt{81} = 2$$
$$7-9 = 2$$
$$-2 \neq 2$$

Check $t = 2$:
$$\sqrt{2+7}-\sqrt{2\left(2\right)-3} = 2$$
$$\sqrt{9}-\sqrt{4-3} = 2$$
$$3-\sqrt{1} = 2$$
$$3-1 = 2$$
$$2 = 2$$

The solution is $\{2\}$. ($t = 42$ does not check.)

Chapters 1 – 6 Cumulative Review Exercises

1. $6^2-2\left[5-8(3-1)+4\div 2\right]$

$=6^2-2\left[5-8(2)+4\div 2\right]$

$=6^2-2\left[5-16+2\right]$

$=6^2-2\left[-9\right]=36-2\left[-9\right]$

$=36+18=54$

2. $3x-3(-2x+5)-4y+2(3x+5)-y$

$=3x+6x-15-4y+6x+10-y$

$=15x-5y-5$

3. $9(2y+8)=20-(y+5)$

$18y+72=20-y-5$

$18y+72=-y+15$

$19y=-57$

$y=-3 \quad \{-3\}$

4. $2a-4<-14$

$2a<-10$

$a<-5 \qquad (-\infty,-5)$

5. $11\ge -x+2>5$

$9\ge -x>3$

$-9\le x<-3 \quad [-9,-3)$

6. $|9-4x|+4<2$

$|9-4x|<-2 \quad \{\ \}$

7. $2x+y=9$

$y=-2x+9$

The slope is –2 so the slope of the parallel line is also –2.

$y-(-1)=-2(x-3)$

$y+1=-2x+6$

$y=-2x+5$

8. $x=-5$

9.
$$\begin{array}{r} 2x-3y=\ \ 0 \\ -4x+3y=-1 \\ \hline -2x\qquad =-1 \end{array}$$

$x=\frac{1}{2}$

$2\left(\frac{1}{2}\right)-3y=0$

$1-3y=0$

$-3y=-1$

$y=\frac{1}{3}$

10. Substitute the ordered triple in each equation to check:

$2(2)+(-2)-4\left(\frac{1}{2}\right)=4-2-2=0$ (checks)

$2-(-2)+2\left(\frac{1}{2}\right)=2+2+1=5$ (checks)

The solution is $\left\{\left(\frac{1}{2},\frac{1}{3}\right)\right\}$.

$3(2)+2(-2)+2\left(\frac{1}{2}\right)=6-4+1=3\neq 4$

$\left(2,-2,\frac{1}{2}\right)$ is not a solution to the system.

11.

	Work Rate	Time	Portion of Job Comp
Bennette	1/3	x	$(1/3)x$
Pepe	1/5	x	$(1/5)x$

(Bennette Part) + (Pepe Part) = (1 Job)

$$\frac{1}{3}x+\frac{1}{5}x=1 \quad \text{LCD}=15$$

$$15\left(\frac{1}{3}x+\frac{1}{5}x\right)=15(1)$$

$$5x+3x=15$$

$$8x=15$$

$$x=\frac{15}{8}\text{ hr}\ \text{ or }1\frac{7}{8}\text{ hr}$$

Together, it will take them $1\frac{7}{8}$ hr.

12. $f(x)=4x-2$

a. $f(-2)=4(-2)-2=-8-2=-10$

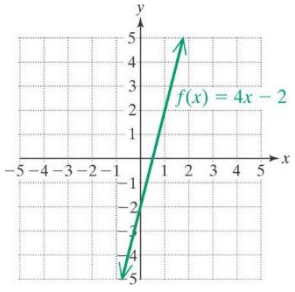
$f(0)=4(0)-2=0-2=-2$

$f(4)=4(4)-2=16-2=14$

$f\left(\frac{1}{2}\right)=4\left(\frac{1}{2}\right)-2=2-2=0$

b. $(-2,-10),(0,-2),(4,14),\left(\frac{1}{2},0\right)$

c.



13. $\left(\dfrac{a^{3/2}b^{-1/4}c^{1/3}}{ab^{-5/4}c^{0}}\right)^{12} = \left(a^{(3/2)-1}b^{(-1/4)-(-5/4)}c^{1/3}\right)^{12}$

$= \left(a^{1/2}b^{1}c^{1/3}\right)^{12}$

$= \left(a^{1/2}\right)^{12}\left(b^{12}\right)\left(c^{1/3}\right)^{12}$

$= a^{6}b^{12}c^{4}$

14. **a.** $\left(3.5\times10^{7}\right)\left(4\times10^{-12}\right)$

$= 3.5\times4\times10^{7+(-12)}$

$= 14\times10^{-5}$

$= 1.4\times10^{1}\times10^{-5}$

$= 1.4\times10^{1+(-5)}$

$= 1.4\times10^{-4}$

b. $\dfrac{6.28\times10^{5}}{2.0\times10^{-4}} = \dfrac{6.28}{2.0}\times10^{5-(-4)}$

$= 3.14\times10^{9}$

15. $(2x+5)(x-3)$

$= 2x\cdot x - 2x\cdot 3 + 5\cdot x - 5\cdot 3$

$= 2x^{2} - 6x + 5x - 15$

$= 2x^{2} - x - 15$

The product is a second degree polynomial.

16. $\sqrt{3}\left(\sqrt{5}+\sqrt{6}+\sqrt{3}\right)$

$= \sqrt{3}\left(\sqrt{5}\right) + \sqrt{3}\left(\sqrt{6}\right) + \sqrt{3}\left(\sqrt{3}\right)$

$= \sqrt{15} + \sqrt{18} + \sqrt{9}$

$= \sqrt{15} + \sqrt{9\cdot 2} + \sqrt{9}$

$= \sqrt{15} + 3\sqrt{2} + 3$

17. $\dfrac{x^{2}-x-12}{x+3} = \dfrac{(x-4)\cancel{(x+3)}}{\cancel{x+3}}$

$= x-4$

18. $\sqrt[4]{\dfrac{1}{16}} - \sqrt[3]{\dfrac{8}{27}} = \dfrac{1}{2} - \dfrac{2}{3}$

$= \dfrac{6}{12} - \dfrac{8}{12}$

$= -\dfrac{2}{12}$

$= -\dfrac{1}{6}$

19. $\sqrt[3]{\dfrac{54c^{4}}{cd^{3}}} = \sqrt[3]{\dfrac{27c^{3}\cdot 2\cancel{c}}{\cancel{c}d^{3}}}$

$= \dfrac{3c\sqrt[3]{2}}{d}$

20. $4\sqrt{45b^{3}} + 5b\sqrt{80b}$

$= 4\sqrt{9b^{2}\cdot 5b} + 5b\sqrt{16\cdot 5b}$

$= 4\cdot 3b\sqrt{5b} + 5b\cdot 4\sqrt{5b}$

$= 12b\sqrt{5b} + 20b\sqrt{5b}$

$= 32b\sqrt{5b}$

21. $\dfrac{13i}{3+2i} = \dfrac{13i}{3+2i} \cdot \dfrac{3-2i}{3-2i}$

$$= \frac{13i(3-2i)}{3^2-(2i)^2}$$

$$= \frac{13i(3-2i)}{9-4i^2}$$

$$= \frac{13(3i-2i^2)}{9-4(-1)}$$

$$= \frac{13(3i-2(-1))}{9+4}$$

$$= \frac{\cancel{13}(3i+2)}{\cancel{13}}$$

$$= 2+3i$$

22.

$$\frac{5}{y-2} - \frac{3}{y-4} = \frac{6}{y^2-6y+8}$$

$$\frac{5}{y-2} - \frac{3}{y-4} = \frac{6}{(y-2)(y-4)} \qquad \text{LCD} = (y-2)(y-4) \qquad y \neq 2 \text{ and } y \neq 4$$

$$(y-2)(y-4)\left(\frac{5}{y-2} - \frac{3}{y-4}\right) = (y-2)(y-4)\left(\frac{6}{(y-2)(y-4)}\right)$$

$$\begin{aligned} 5(y-4)-3(y-2) &= 6 \\ 5y-20-3y+6 &= 6 \\ 2y-14 &= 6 \\ 2y &= 20 \\ y &= 10 \end{aligned} \qquad \{10\}$$

23. $\dfrac{3}{x^2+5x}+\dfrac{-2}{x^2-25}$

$$=\frac{3}{x(x+5)}+\frac{-2}{(x+5)(x-5)}$$

$$\text{LCD}=x(x+5)(x-5)$$

$$=\frac{3}{x(x+5)}\cdot\frac{x-5}{x-5}+\frac{-2}{(x+5)(x-5)}\cdot\frac{x}{x}$$

$$=\frac{3x-15-2x}{x(x+5)(x-5)}=\frac{x-15}{x(x+5)(x-5)}$$

24. $\dfrac{a+10}{2a^2-11a-6}\div\dfrac{a^2+12a+20}{6-a}$

$$=\frac{a+10}{2a^2-11a-6}\cdot\frac{6-a}{a^2+12a+20}$$

$$=\frac{\cancel{a+10}}{(2a+1)\cancel{(a-6)}}\cdot\frac{-1\cancel{(a-6)}}{\cancel{(a+10)}(a+2)}$$

$$=\frac{-1}{(2a+1)(a+2)}$$

25. $(-5x^2-4x+8)-(3x-5)^2$

$$=(-5x^2-4x+8)-(9x^2-30x+25)$$

$$=-5x^2-4x+8-9x^2+30x-25$$

$$=-14x^2+26x-17$$

26. $\dfrac{-4}{\sqrt{3}-\sqrt{5}}=\dfrac{-4}{\sqrt{3}-\sqrt{5}}\cdot\dfrac{\sqrt{3}+\sqrt{5}}{\sqrt{3}+\sqrt{5}}$

$$=\frac{-4(\sqrt{3}+\sqrt{5})}{(\sqrt{3})^2-(\sqrt{5})^2}=\frac{-4(\sqrt{3}+\sqrt{5})}{3-5}$$

$$=\frac{-4(\sqrt{3}+\sqrt{5})}{-2}=2(\sqrt{3}+\sqrt{5})$$
$$=2\sqrt{3}+2\sqrt{5}$$

27. $\dfrac{4}{3-5i}=\dfrac{4}{3-5i}\cdot\dfrac{3+5i}{3+5i}=\dfrac{12+20i}{9-25i^2}$

$$=\frac{12+20i}{9-25(-1)}=\frac{12+20i}{9+25}=\frac{12+20i}{34}$$

$$=\frac{12}{34}+\frac{20}{34}i=\frac{6}{17}+\frac{10}{17}i$$

28. $12x^2+4x-21=0$

$$(2x+3)(6x-7)=0$$

$$2x+3=0 \text{ or } 6x-7=0$$

$$2x=-3 \text{ or } 6x=7$$

$$x=-\frac{3}{2} \text{ or } x=\frac{7}{6}$$

29. $x^2+6x+9-y^2=(x+3)^2-y^2$

$$=(x+3-y)(x+3+y)$$

30. $x^6+8=(x^2)^3+2^3$

$$=(x^2+2)(x^4-2x^2+4)$$