

Chapter 5 Rational Expressions and Rational Equations

Section 5.1 Practice Exercises

1. a. rational

b. denominator

c. $\frac{p}{q}$

d. 1; -1

2.

$$f(x) = \frac{5}{x+1}$$

$$f(0) = \frac{5}{0+1} = \frac{5}{1} = 5$$

$$f(2) = \frac{5}{2+1} = \frac{5}{3}$$

$$f(-1) = \frac{5}{-1+1} = \frac{5}{0} \text{ is undefined}$$

$$f(-6) = \frac{5}{-6+1} = \frac{5}{-5} = -1$$

3.

$$k(x) = \frac{-3}{x+4}$$

$$k(0) = \frac{-3}{0+4} = -\frac{3}{4}$$

$$k(-1) = \frac{-3}{-1+4} = -\frac{3}{3} = -1$$

$$k(2) = \frac{-3}{2+4} = -\frac{3}{6} = -\frac{1}{2}$$

$$k(-4) = \frac{-3}{-4+4}$$

$$= -\frac{3}{0} \text{ is undefined}$$

4.

$$m(x) = \frac{x-4}{x+6}$$

$$m(-6) = \frac{-6-4}{-6+6} = \frac{-10}{0} \text{ is undefined}$$

$$m(-4) = \frac{-4-4}{-4+6} = \frac{-8}{2} = -4$$

$$m(0) = \frac{0-4}{0+6} = \frac{-4}{6} = -\frac{2}{3}$$

$$m(4) = \frac{4-4}{4+6} = \frac{0}{10} = 0$$

5.

$$n(a) = \frac{3a+1}{a^2+1}$$

$$n(1) = \frac{3(1)+1}{(1)^2+1} = \frac{3+1}{1+1} = \frac{4}{2} = 2$$

$$n(0) = \frac{3(0)+1}{(0)^2+1} = \frac{0+1}{0+1} = \frac{1}{1} = 1$$

$$n\left(-\frac{1}{3}\right) = \frac{3\left(-\frac{1}{3}\right)+1}{\left(-\frac{1}{3}\right)^2+1} = \frac{-1+1}{\frac{1}{9}+1} = \frac{0}{\frac{10}{9}} = 0$$

6.

$$f(t) = \frac{2t-8}{t^2+9}$$

$$f(4) = \frac{2(4)-8}{(4)^2+9} = \frac{8-8}{16+9} = \frac{0}{25} = 0$$

$$f(-4) = \frac{2(-4)-8}{(-4)^2+9} = \frac{-8-8}{16+9} = \frac{-16}{25} = -\frac{16}{25}$$

$$f(3) = \frac{2(3)-8}{(3)^2+9} = \frac{6-8}{9+9} = \frac{-2}{18} = -\frac{1}{9}$$

$$n(-1) = \frac{3(-1)+1}{(-1)^2+1} = \frac{-3+1}{1+1} = \frac{-2}{2} = -1$$

$$f(-3) = \frac{2(-3)-8}{(-3)^2+9} = \frac{-6-8}{9+9} = \frac{-14}{18} = -\frac{7}{9}$$

7. $f(x) = \frac{9}{x}$
 $x \neq 0$

a. $\{x \mid x \text{ is a real number and } x \neq 0\}$

b. $(-\infty, 0) \cup (0, \infty)$

8. $g(a) = -\frac{10}{a}$
 $a \neq 0$

a. $\{a \mid a \text{ is a real number and } a \neq 0\}$

b. $(-\infty, 0) \cup (0, \infty)$

9. $h(v) = \frac{v+1}{v-7}$
 $v-7 \neq 0$
 $v \neq 7$

a. $\{v \mid v \text{ is a real number and } v \neq 7\}$

b. $(-\infty, 7) \cup (7, \infty)$

10. $p(t) = \frac{t+9}{t+3}$
 $t+3 \neq 0$
 $t \neq -3$

a. $\{t \mid t \text{ is a real number and } t \neq -3\}$

b. $(-\infty, -3) \cup (-3, \infty)$

11. $k(x) = \frac{3x-1}{2x-5}$
 $2x-5 \neq 0$
 $2x \neq 5$
 $x \neq \frac{5}{2}$

a. $\{x \mid x \text{ is a real number and } x \neq \frac{5}{2}\}$

b. $(-\infty, \frac{5}{2}) \cup (\frac{5}{2}, \infty)$

12. $n(t) = \frac{6t+5}{3t+8}$
 $3t+8 \neq 0$
 $3t \neq -8$
 $t \neq -\frac{8}{3}$

a. $\{t \mid t \text{ is a real number and } t \neq -\frac{8}{3}\}$

b. $(-\infty, -\frac{8}{3}) \cup (-\frac{8}{3}, \infty)$

13. $f(q) = \frac{q+1}{q^2+6q-27}$
 $q^2+6q-27 \neq 0$
 $(q+9)(q-3) \neq 0$
 $q+9 \neq 0$ or $q-3 \neq 0$
 $q \neq -9$ or $q \neq 3$

14. $k(a) = \frac{a^2}{2a^2+3a-5}$
 $2a^2+3a-5 \neq 0$
 $(2a+5)(a-1) \neq 0$
 $2a+5 \neq 0$ or $a-1 \neq 0$
 $2a \neq -5$ or $a \neq 1$
 $a \neq -\frac{5}{2}$ or $a \neq 1$

Section 5.1 Rational Expressions and Rational Functions

a. $\left\{ \begin{array}{l} q \mid q \text{ is a real number} \\ \text{and } q \neq -9, q \neq 3 \end{array} \right\}$

b. $(-\infty, -9) \cup (-9, 3) \cup (3, \infty)$

a. $\left\{ \begin{array}{l} a \mid a \text{ is a real number} \\ \text{and } a \neq -\frac{5}{2}, a \neq 1 \end{array} \right\}$

b. $(-\infty, -\frac{5}{2}) \cup (-\frac{5}{2}, 1) \cup (1, \infty)$

15. $h(c) = \frac{c}{c^2 + 25}$

Because c^2 is nonnegative for any real number c , the denominator $c^2 + 25$ cannot equal zero; therefore, no real numbers are excluded from the domain.

a. $\{c \mid c \text{ is a real number}\}$

b. $(-\infty, \infty)$

16. $m(x) = \frac{x}{x^2 + 16}$

Because x^2 is nonnegative for any real number x , the denominator $x^2 + 16$ cannot equal zero; therefore, no real numbers are excluded from the domain.

a. $\{x \mid x \text{ is a real number}\}$

b. $(-\infty, \infty)$

17. $f(x) = \frac{x+5}{x^2-25}$
 $x^2 - 25 \neq 0$
 $(x+5)(x-5) \neq 0$
 $x+5 \neq 0$

or $x-5 \neq 0$

$x \neq -5$

or $x \neq 5$

a. $\left\{ \begin{array}{l} x \mid x \text{ is a real number} \\ \text{and } x \neq -5, x \neq 5 \end{array} \right\}$

b. $(-\infty, -5) \cup (-5, 5) \cup (5, \infty)$

18. $g(t) = \frac{t+4}{t^2-16}$
 $t^2 - 16 \neq 0$
 $(t+4)(t-4) \neq 0$
 $t+4 \neq 0$

or $t-4 \neq 0$

$t \neq -4$

or $t \neq 4$

a. $\left\{ \begin{array}{l} t \mid t \text{ is a real number} \\ \text{and } t \neq -4, t \neq 4 \end{array} \right\}$

b. $(-\infty, -4) \cup (-4, 4) \cup (4, \infty)$

19. $p(x) = \frac{x-5}{3}$

a. $\{x \mid x \text{ is a real number}\}$

b. $(-\infty, \infty)$

20. $r(x) = \frac{x+2}{8}$

a. $\{x \mid x \text{ is a real number}\}$

b. $(-\infty, \infty)$

21. $m(x) = \frac{1}{x+4}$
 $x+4=0$
 $x=-4$

$$D: (-\infty, -4) \cup (-4, \infty)$$

Graph: b

22. $n(x) = \frac{1}{x+1}$
 $x+1=0$
 $x=-1$

$$D: (-\infty, -1) \cup (-1, \infty)$$

Graph: c

23. $q(x) = \frac{1}{x-4}$
 $x-4=0$
 $x=4$

$$D: (-\infty, 4) \cup (4, \infty)$$

Graph: d

24. $p(x) = \frac{1}{x-1}$
 $x-1=0$
 $x=1$

$$D: (-\infty, 1) \cup (1, \infty)$$

Graph: a

25. a. $\frac{8x}{4y} = \frac{2x}{y}$

b. $\frac{8+x}{4+y}$ cannot be simplified

26. a. $\frac{a-21}{14+b}$ cannot be simplified

b. $\frac{-21a}{14b} = \frac{-3a}{2b}$

27. $\frac{x^2+6x+8}{x^2+3x-4}$

a. $\frac{(x+4)(x+2)}{(x+4)(x-1)}$

b. $(x+4)(x-1) \neq 0$
 $x+4 \neq 0$

or $x-1 \neq 0$

$x \neq -4$

or $x \neq 1$

c. $\frac{\cancel{(x+4)}(x+2)}{\cancel{(x+4)}(x-1)} = \frac{x+2}{x-1}$

provided $x \neq -4, x \neq 1$

28. $\frac{x^2-6x}{2x^2-11x-6}$

a. $\frac{x(x-6)}{(2x+1)(x-6)}$

b. $(2x+1)(x-6) \neq 0$
 $2x+1 \neq 0$

or $x-6 \neq 0$

$2x \neq -1$

or $x \neq 6$

$x \neq -\frac{1}{2}$

or $x \neq 6$

c. $\frac{x\cancel{(x-6)}}{(2x+1)\cancel{(x-6)}} = \frac{x}{2x+1}$

provided $x \neq -\frac{1}{2}, x \neq 6$

Section 5.1 Rational Expressions and Rational Functions

$$29. \quad \frac{x^2 - 18x + 81}{x^2 - 81}$$

$$a. \quad \frac{(x-9)(x-9)}{(x+9)(x-9)}$$

$$b. \quad \begin{aligned} (x+9)(x-9) &\neq 0 \\ x+9 &\neq 0 \text{ or } x-9 \neq 0 \\ x &\neq -9 \text{ or } x \neq 9 \end{aligned}$$

$$c. \quad \frac{(x-9)\cancel{(x-9)}}{(x+9)\cancel{(x-9)}} = \frac{x-9}{x+9}$$

provided $x \neq -9, x \neq 9$

$$30. \quad \frac{x^2 + 14x + 49}{x^2 - 49}$$

$$a. \quad \frac{(x+7)(x+7)}{(x+7)(x-7)}$$

$$b. \quad \begin{aligned} (x+7)(x-7) &\neq 0 \\ x+7 &\neq 0 \text{ or } x-7 \neq 0 \\ x &\neq -7 \text{ or } x \neq 7 \end{aligned}$$

$$c. \quad \frac{\cancel{(x+7)}(x+7)}{\cancel{(x+7)}(x-7)} = \frac{x+7}{x-7}$$

provided $x \neq -7, x \neq 7$

$$31. \quad \frac{100x^3y^5}{36xy^8} = \frac{25}{9}x^{3-1}y^{5-8} = \frac{25}{9}x^2y^{-3}$$

$$= \frac{25x^2}{9y^3} \text{ provided } x \neq 0, y \neq 0$$

$$32. \quad \frac{48ab^3c^2}{6a^7bc^0} = 8a^{1-7}b^{3-1}c^{2-0} = 8a^{-6}b^2c^2$$

$$= \frac{8b^2c^2}{a^6} \text{ provided } a \neq 0, b \neq 0, c \neq 0$$

$$33. \quad \frac{7w^{11}z^6}{14w^3z^3} = \frac{1}{2}w^{11-3}z^{6-3} = \frac{1}{2}w^8z^3$$

$$= \frac{w^8z^3}{2} \text{ provided } w \neq 0, z \neq 0$$

$$34. \quad \frac{12r^9s^3}{24r^8s^4} = \frac{1}{2}r^{9-8}s^{3-4} = \frac{1}{2}rs^{-1}$$

$$= \frac{r}{2s} \text{ provided } r \neq 0, s \neq 0$$

$$35. \quad \frac{-3m^4n}{12m^6n^4} = -\frac{1}{4}m^{4-6}n^{1-4} = -\frac{1}{4}m^{-2}n^{-3}$$

$$= -\frac{1}{4m^2n^3} \text{ provided } m \neq 0, n \neq 0$$

$$36. \quad \frac{-5x^3y^2}{20x^4y^2} = -\frac{1}{4}x^{3-4}y^{2-2} = -\frac{1}{4}x^{-1}y^0$$

$$= -\frac{1}{4x} \text{ provided } x \neq 0, y \neq 0$$

$$37. \quad \frac{6a+18}{9a+27} = \frac{6\cancel{(a+3)}}{9\cancel{(a+3)}} = \frac{2}{3} \text{ provided } a \neq -3$$

$$38. \quad \frac{5y-15}{3y-9} = \frac{5\cancel{(y-3)}}{3\cancel{(y-3)}} = \frac{5}{3} \text{ provided } y \neq 3$$

$$39. \quad \frac{x-5}{x^2-25} = \frac{\cancel{x-5}}{(x+5)\cancel{(x-5)}} = \frac{1}{x+5}$$

provided $x \neq -5, x \neq 5$

$$40. \quad \frac{3z-6}{3z^2-12} = \frac{3(z-2)}{3(z^2-4)} = \frac{\cancel{3}\cancel{(z-2)}}{\cancel{3}(z+2)\cancel{(z-2)}}$$

$$= \frac{1}{z+2} \text{ provided } z \neq -2, z \neq 2$$

$$41. \frac{-7c}{21c^2 - 35c} = \frac{-1 \cdot \cancel{7c}}{\cancel{7c}(3c-5)} = -\frac{1}{3c-5}$$

provided $c \neq 0, c \neq \frac{5}{3}$

$$42. \frac{2p+3}{2p^2+7p+6} = \frac{\cancel{2p+3}}{(\cancel{2p+3})(p+2)}$$

$$= \frac{1}{p+2} \text{ provided } p \neq -2, p \neq -\frac{3}{2}$$

$$43. \frac{2t^2+7t-4}{-2t^2-5t+3} = \frac{(2t-1)(t+4)}{-(2t^2+5t-3)}$$

$$= \frac{(\cancel{2t-1})(t+4)}{-(\cancel{2t-1})(t+3)}$$

$$= -\frac{t+4}{t+3} \text{ provided } t \neq \frac{1}{2}, t \neq -3$$

$$44. \frac{y^2+8y-9}{y^2-5y+4} = \frac{(y+9)(\cancel{y-1})}{(y-4)(\cancel{y-1})}$$

$$= \frac{y+9}{y-4} \text{ provided } y \neq 4, y \neq 1$$

$$45. \frac{(p+1)(2p-1)^4}{(p+1)^2(2p-1)^2} = (p+1)^{1-2}(2p-1)^{4-2}$$

$$= (p+1)^{-1}(2p-1)^2$$

$$= \frac{(2p-1)^2}{p+1} \text{ provided } p \neq \frac{1}{2}, p \neq -1$$

$$46. \frac{r(r-3)^5}{r^3(r-3)^2} = r^{1-3}(r-3)^{5-2} = r^{-2}(r-3)^3$$

$$= \frac{(r-3)^3}{r^2} \text{ provided } r \neq 0, r \neq 3$$

$$47. \frac{9-z^2}{2z^2+z-15} = \frac{(\cancel{3+z})(3-z)}{(2z-5)(\cancel{z+3})} = \frac{3-z}{2z-5}$$

provided $z \neq \frac{5}{2}, z \neq -3$

$$48. \frac{2c^2+2c-12}{-8+2c+c^2} = \frac{2(c^2+c-6)}{c^2+2c-8}$$

$$= \frac{2(c+3)(\cancel{c-2})}{(c+4)(\cancel{c-2})} = \frac{2(c+3)}{c+4}$$

provided $c \neq -4, c \neq 2$

$$49. \frac{2z^3+128}{16+8z+z^2} = \frac{2(z^3+64)}{z^2+8z+16}$$

$$= \frac{2(\cancel{z+4})(z^2-4z+16)}{(z+4)(\cancel{z+4})}$$

$$= \frac{2(z^2-4z+16)}{z+4}$$

provided $z \neq -4$

$$50. \frac{p^3-1}{5-10p+5p^2} = \frac{(p-1)(p^2+p+1)}{5(p^2-2p+1)}$$

$$= \frac{(\cancel{p-1})(p^2+p+1)}{5(\cancel{p-1})(p-1)}$$

$$= \frac{p^2+p+1}{5(p-1)}$$

provided $p \neq 1$

Section 5.1 Rational Expressions and Rational Functions

$$\begin{aligned}
 51. \quad & \frac{10x^3 - 25x^2 + 4x - 10}{-4 - 10x^2} \\
 &= \frac{5x^2(2x - 5) + 2(2x - 5)}{-2(5x^2 + 2)} \\
 &= \frac{(2x - 5)\cancel{(5x^2 + 2)}}{-2\cancel{(5x^2 + 2)}} \\
 &= -\frac{2x - 5}{2}
 \end{aligned}$$

$$\begin{aligned}
 52. \quad & \frac{8x^3 - 12x^2 + 6x - 9}{16x^4 - 9} \\
 &= \frac{4x^2(2x - 3) + 3(2x - 3)}{(4x^2 + 3)(4x^2 - 3)} \\
 &= \frac{(2x - 3)\cancel{(4x^2 + 3)}}{\cancel{(4x^2 + 3)}(4x^2 - 3)} \\
 &= \frac{2x - 3}{4x^2 - 3} \\
 &\quad \text{provided } x \neq \frac{\sqrt{3}}{2}, x \neq -\frac{\sqrt{3}}{2}
 \end{aligned}$$

$$\begin{aligned}
 53. \quad & \frac{r+6}{6+r} = \frac{r+6}{r+6} = 1 \\
 &\quad \text{provided } r \neq -6
 \end{aligned}$$

$$\begin{aligned}
 54. \quad & \frac{a+2}{2+a} = \frac{a+2}{a+2} = 1 \\
 &\quad \text{provided } a \neq -2
 \end{aligned}$$

$$\begin{aligned}
 55. \quad & \frac{b+8}{-b-8} = \frac{\cancel{b+8}}{-\cancel{(b+8)}} \\
 &= -1 \\
 &\quad \text{provided } b \neq -8
 \end{aligned}$$

$$\begin{aligned}
 56. \quad & \frac{7+w}{-7-w} = \frac{\cancel{7+w}}{-\cancel{(7+w)}} \\
 &= -1 \\
 &\quad \text{provided } w \neq -7
 \end{aligned}$$

$$\begin{aligned}
 57. \quad & \frac{10-x}{x-10} = \frac{-\cancel{(x-10)}}{\cancel{x-10}} \\
 &= -1 \\
 &\quad \text{provided } x \neq 10
 \end{aligned}$$

$$\begin{aligned}
 58. \quad & \frac{y-14}{14-y} = \frac{\cancel{y-14}}{-\cancel{(y-14)}} \\
 &= -1 \\
 &\quad \text{provided } y \neq 14
 \end{aligned}$$

$$\begin{aligned}
 59. \quad & \frac{2t-2}{1-t} = \frac{2\cancel{(t-1)}}{-\cancel{(t-1)}} \\
 &= -2 \\
 &\quad \text{provided } t \neq 1
 \end{aligned}$$

$$\begin{aligned}
 60. \quad & \frac{5p-10}{2-p} = \frac{5\cancel{(p-2)}}{-\cancel{(p-2)}} \\
 &= -5 \\
 &\quad \text{provided } p \neq 2
 \end{aligned}$$

$$61. \quad \frac{c+4}{c-4} \text{ cannot be simplified}$$

$$62. \quad \frac{b+2}{b-2} \text{ cannot be simplified}$$

$$\begin{aligned}
 63. \quad \frac{y-x}{12x^2-12y^2} &= \frac{-(x-y)}{12(x^2-y^2)} \\
 &= \frac{-(\cancel{x-y})}{12(x+y)(\cancel{x-y})} \\
 &= -\frac{1}{12(x+y)}
 \end{aligned}$$

provided $x \neq y, x \neq -y$

$$\begin{aligned}
 64. \quad \frac{4w^2-49z^2}{14z-4w} &= \frac{(2w+7z)(\cancel{2w-7z})}{-2(\cancel{2w-7z})} \\
 &= -\frac{2w+7z}{2} \\
 &\text{provided } w \neq \frac{7}{2}z
 \end{aligned}$$

$$\begin{aligned}
 65. \quad \frac{t^2-1}{t^2+7t+6} &= \frac{(t-1)(\cancel{t+1})}{(t+6)(\cancel{t+1})} \\
 &= \frac{t-1}{t+6} \text{ provided } t \neq -6, t \neq -1
 \end{aligned}$$

$$\begin{aligned}
 66. \quad \frac{x^2+4x+4}{x^2-4} &= \frac{(\cancel{x+2})(x+2)}{(\cancel{x+2})(x-2)} \\
 &= \frac{x+2}{x-2} \text{ provided } x \neq -2, x \neq 2
 \end{aligned}$$

$$\begin{aligned}
 67. \quad \frac{8p+8}{2p^2-4p-6} &= \frac{8(p+1)}{2(p^2-2p-3)} \\
 &= \frac{\cancel{2} \cdot 4(\cancel{p+1})}{\cancel{2}(p-3)(\cancel{p+1})} \\
 &= \frac{4}{p-3} \\
 &\text{provided } p \neq 3, p \neq -1
 \end{aligned}$$

$$\begin{aligned}
 68. \quad \frac{15y-15}{3y^2+9y-12} &= \frac{15(y-1)}{3(y^2+3y-4)} \\
 &= \frac{\cancel{3} \cdot 5(\cancel{y-1})}{\cancel{3}(y-1)(y+4)} \\
 &= \frac{5}{y+4} \\
 &\text{provided } y \neq -4, y \neq 1
 \end{aligned}$$

$$\begin{aligned}
 69. \quad \frac{-16a^2bc^4}{8ab^2c^4} &= -\frac{16}{8}a^{2-1}b^{1-2}c^{4-4} \\
 &= -2a^1b^{-1}c^0 \\
 &= -\frac{2a}{b} \\
 &\text{provided } a \neq 0, b \neq 0, c \neq 0
 \end{aligned}$$

$$\begin{aligned}
 70. \quad \frac{-9x^3yz^2}{27x^4yz} &= -\frac{9}{27}x^{3-4}y^{1-1}z^{2-1} \\
 &= -\frac{1}{3}x^{-1}y^0z^1 \\
 &= -\frac{z}{3x} \\
 &\text{provided } x \neq 0, y \neq 0, z \neq 0
 \end{aligned}$$

$$\begin{aligned}
 71. \quad \frac{x^2-y^2}{8y-8x} &= \frac{(\cancel{x-y})(x+y)}{-8(\cancel{x-y})} \\
 &= -\frac{x+y}{8} \text{ provided } x \neq y
 \end{aligned}$$

$$\begin{aligned}
 72. \quad \frac{p^2-49}{14-2p} &= \frac{(\cancel{p-7})(p+7)}{-2(\cancel{p-7})} \\
 &= -\frac{p+7}{2} \text{ provided } p \neq 7
 \end{aligned}$$

$$\begin{aligned}
 73. \quad \frac{b+4}{2b^2+5b-12} &= \frac{\cancel{b+4}}{(2b-3)\cancel{(b+4)}} \\
 &= \frac{1}{2b-3} \\
 &\text{provided } b \neq \frac{3}{2}, b \neq -4
 \end{aligned}$$

$$\begin{aligned}
 74. \quad \frac{c-6}{3c^2-17c-6} &= \frac{\cancel{c-6}}{(3c+1)\cancel{(c-6)}} \\
 &= \frac{1}{3c+1} \\
 &\text{provided } c \neq -\frac{1}{3}, c \neq 6
 \end{aligned}$$

$$\begin{aligned}
 75. \quad \frac{-2x+34}{-4x+6} &= \frac{\cancel{2}(x-17)}{\cancel{2}(2x-3)} = \frac{x-17}{2x-3} \\
 &\text{provided } x \neq \frac{3}{2}
 \end{aligned}$$

$$\begin{aligned}
 76. \quad \frac{-9w-3}{3w+12} &= \frac{-\cancel{3}(3w+1)}{\cancel{3}(w+4)} = -\frac{3w+1}{w+4} \\
 &\text{provided } w \neq -4
 \end{aligned}$$

$$\begin{aligned}
 77. \quad \frac{(a-2)^2(a-5)^3}{(a-2)^3(a-5)} &= (a-2)^{2-3}(a-5)^{3-1} \\
 &= (a-2)^{-1}(a-5)^2 \\
 &= \frac{(a-5)^2}{a-2} \\
 &\text{provided } a \neq 2, a \neq 5
 \end{aligned}$$

$$\begin{aligned}
 78. \quad \frac{t^2(t-11)^4}{t^5(t-11)^2} &= t^{2-5}(t-11)^{4-2} \\
 &= t^{-3}(t-11)^2 \\
 &= \frac{(t-11)^2}{t^3} \\
 &\text{provided } t \neq 0, t \neq 11
 \end{aligned}$$

$$\begin{aligned}
 79. \quad \frac{4x-2x^2}{5x-10} &= \frac{-2x\cancel{(x-2)}}{5\cancel{(x-2)}} \\
 &= -\frac{2x}{5} \text{ provided } x \neq 2
 \end{aligned}$$

$$\begin{aligned}
 80. \quad \frac{2y-6}{3y^2-y^3} &= \frac{2\cancel{(y-3)}}{-y^2\cancel{(y-3)}} \\
 &= -\frac{2}{y^2} \text{ provided } y \neq 0, y \neq 3
 \end{aligned}$$

$$\begin{aligned}
 81. \quad \frac{x^3-2x^2-25x+50}{x^3+5x^2-4x-20} &= \frac{x^2(x-2)-25(x-2)}{x^2(x+5)-4(x+5)} \\
 &= \frac{(x-2)(x^2-25)}{(x+5)(x^2-4)} \\
 &= \frac{\cancel{(x-2)}\cancel{(x+5)}(x-5)}{\cancel{(x+5)}(x+2)\cancel{(x-2)}}
 \end{aligned}$$

$$\begin{aligned}
 82. \quad \frac{4y^3+12y^2-y-3}{2y^3+y^2-18y-9} &= \frac{4y^2(y+3)-(y+3)}{y^2(2y+1)-9(2y+1)} \\
 &= \frac{(y+3)(4y^2-1)}{(2y+1)(y^2-9)} \\
 &= \frac{\cancel{(y+3)}\cancel{(2y+1)}(2y-1)}{\cancel{(2y+1)}\cancel{(y+3)}(y-3)}
 \end{aligned}$$

$$= \frac{x-5}{x+2}$$

provided $x \neq -5, x \neq -2, x \neq 2$

$$= \frac{2y-1}{y-3}$$

provided $y \neq -\frac{1}{2}, y \neq -3, y \neq 3$

$$83. \quad \frac{t^3+8}{3t^2+t-10} = \frac{\cancel{(t+2)}(t^2-2t+4)}{(3t-5)\cancel{(t+2)}}$$

$$= \frac{t^2-2t+4}{3t-5}$$

provided $t \neq \frac{5}{3}, t \neq -2$

$$84. \quad \frac{w^3-27}{4w^2-5w-21} = \frac{\cancel{(w-3)}(w^2+3w+9)}{(4w+7)\cancel{(w-3)}}$$

$$= \frac{w^2+3w+9}{4w+7}$$

provided $w \neq -\frac{7}{4}, w \neq 3$

85. For example: $\frac{1}{x-2}$

86. For example: $\frac{1}{x-3}$

87. For example: $f(x) = \frac{1}{x+5}$

88. For example: $f(x) = \frac{1}{x+6}$

Section 5.2 Practice Exercises

1. a. $\frac{pr}{qs}$

b. $\frac{ps}{qr}$

$$2. \quad \frac{3w^2-12}{w^2+5w+6} = \frac{3(w^2-4)}{(w+3)(w+2)}$$

$$= \frac{3(w-2)\cancel{(w+2)}}{(w+3)\cancel{(w+2)}}$$

$$= \frac{3(w-2)}{w+3}$$

$$3. \quad \frac{t^2-5t-6}{t^2-7t+6} = \frac{\cancel{(t-6)}(t+1)}{\cancel{(t-6)}(t-1)}$$

$$= \frac{t+1}{t-1}$$

$$4. \quad \frac{5y+15}{5y^2+16y+3} = \frac{5\cancel{(y+3)}}{(5y+1)\cancel{(y+3)}}$$

$$= \frac{5}{5y+1}$$

$$5. \quad \frac{2-p}{p^2-p-2} = \frac{-1\cancel{(p-2)}}{\cancel{(p-2)}(p+1)}$$

$$= -\frac{1}{p+1}$$

Section 5.2 Multiplication and Division of Rational Expressions

$$\begin{aligned}
 6. \quad \frac{5x^2yz^3}{20xyz} &= \frac{1}{4}x^{2-1}y^{1-1}z^{3-1} \\
 &= \frac{1}{4}xy^0z^2 \\
 &= \frac{xz^2}{4}
 \end{aligned}$$

$$\begin{aligned}
 7. \quad \frac{7x+14}{7x^2-7x-42} &= \frac{7(x+2)}{7(x^2-x-6)} \\
 &= \frac{\cancel{7}(x+2)}{\cancel{7}(x-3)\cancel{(x+2)}} \\
 &= \frac{1}{x-3}
 \end{aligned}$$

$$\begin{aligned}
 8. \quad \frac{25-x^2}{x^2-10x+25} &= \frac{-(x^2-25)}{x^2-10x+25} \\
 &= \frac{-(x+5)\cancel{(x-5)}}{(x-5)\cancel{(x-5)}} \\
 &= -\frac{x+5}{x-5}
 \end{aligned}$$

$$\begin{aligned}
 9. \quad \frac{a^3b^2c^5}{2a^3bc^2} &= \frac{1}{2}a^{3-3}b^{2-1}c^{5-2} \\
 &= \frac{1}{2}a^0bc^3 \\
 &= \frac{bc^3}{2}
 \end{aligned}$$

$$\begin{aligned}
 10. \quad \frac{8w^2}{9} \cdot \frac{3}{2w^4} &= \frac{24w^2}{18w^4} = \frac{4}{3}w^{2-4} \\
 &= \frac{4}{3}w^{-2} \\
 &= \frac{4}{3w^2}
 \end{aligned}$$

$$\begin{aligned}
 11. \quad \frac{16}{z^7} \cdot \frac{z^4}{8} &= \frac{16z^4}{8z^7} = 2z^{4-7} \\
 &= 2z^{-3} \\
 &= \frac{2}{z^3}
 \end{aligned}$$

$$\begin{aligned}
 12. \quad \frac{5p^2q^4}{12pq^3} \cdot \frac{6p^2}{20q^2} &= \frac{\cancel{5}p \cdot \cancel{p} \cdot \cancel{q^4} \cdot \cancel{q}}{\cancel{6} \cdot 2 \cancel{p} \cdot \cancel{q^3}} \cdot \frac{\cancel{6}p^2}{\cancel{5} \cdot 4 \cancel{q} \cdot q} \\
 &= \frac{p^{1+2}}{2 \cdot 4 \cdot q} \\
 &= \frac{p^3}{8q}
 \end{aligned}$$

$$\begin{aligned}
 13. \quad \frac{27r^5}{7s} \cdot \frac{28rs^3}{9r^3s^2} &= \frac{\cancel{9} \cdot 3 \cancel{r^5} \cdot r^2}{\cancel{7} \cancel{s}} \cdot \frac{\cancel{7} \cdot 4 \cancel{r^3} \cdot \cancel{s}}{\cancel{9} \cancel{r^3} \cancel{s^2}} \\
 &= 3 \cdot 4 \cdot r^2 \cdot r \\
 &= 12r^{2+1} \\
 &= 12r^3
 \end{aligned}$$

$$\begin{aligned}
 14. \quad \frac{3z+12}{8z^3} \cdot \frac{16z^3}{9z+36} &= \frac{\cancel{3}(z+4)}{\cancel{8} \cancel{z^3}} \cdot \frac{\cancel{8} \cdot 2 \cancel{z^3}}{3 \cdot \cancel{3}(z+4)} \\
 &= \frac{2}{3}
 \end{aligned}$$

$$\begin{aligned}
 15. \quad \frac{x^2y}{x^2-4x-5} \cdot \frac{2x^2-13x+15}{xy^3} &= \frac{\cancel{x} \cdot x \cdot \cancel{y}}{(\cancel{x-5})(x+1)} \cdot \frac{(2x-3)\cancel{(x-5)}}{\cancel{x} \cdot \cancel{y} \cdot y^2} \\
 &= \frac{x(2x-3)}{y^2(x+1)}
 \end{aligned}$$

$$\begin{aligned}
 16. \quad & \frac{3y^2 + 18y + 15}{6y + 6} \cdot \frac{y - 5}{y^2 - 25} \\
 &= \frac{3(y^2 + 6y + 5)}{6(y + 1)} \cdot \frac{y - 5}{(y + 5)(y - 5)} \\
 &= \frac{\cancel{3}(y + 5)\cancel{(y + 1)}}{2 \cdot \cancel{3}(y + 1)} \cdot \frac{\cancel{y - 5}}{\cancel{(y + 5)}\cancel{(y - 5)}} \\
 &= \frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 17. \quad & \frac{10w - 8}{w + 2} \cdot \frac{3w^2 - w - 14}{25w^2 - 16} \\
 &= \frac{2\cancel{(5w - 4)}}{\cancel{w + 2}} \cdot \frac{(3w - 7)\cancel{(w + 2)}}{(5w + 4)\cancel{(5w - 4)}} \\
 &= \frac{2(3w - 7)}{5w + 4}
 \end{aligned}$$

$$\begin{aligned}
 18. \quad & \frac{x - 5y}{x^2 + xy} \cdot \frac{y^2 - x^2}{10y - 2x} \\
 &= \frac{\cancel{x - 5y} \cancel{-(x^2 - y^2)}}{x(x + y) \cancel{2(x - 5y)}} \\
 &= \frac{\cancel{(x + y)}(x - y)}{2x\cancel{(x + y)}} \\
 &= \frac{x - y}{2x}
 \end{aligned}$$

$$\begin{aligned}
 19. \quad & \frac{3x - 15}{4x^2 - 2x} \cdot \frac{10x - 20x^2}{5 - x} \\
 &= \frac{3\cancel{(x - 5)}}{2x\cancel{(2x - 1)}} \cdot \frac{\cancel{5} \cdot \cancel{2x} \cancel{(2x - 1)}}{\cancel{-(x - 5)}} \\
 &= 15
 \end{aligned}$$

$$\begin{aligned}
 20. \quad & x(x + 5)^2 \cdot \frac{2}{x^2 - 25} = \frac{2x\cancel{(x + 5)}(x + 5)}{\cancel{(x + 5)}(x - 5)} \\
 &= \frac{2x(x + 5)}{x - 5}
 \end{aligned}$$

$$\begin{aligned}
 21. \quad & y(y^2 - 4) \cdot \frac{y}{y + 2} = \frac{y^2\cancel{(y + 2)}(y - 2)}{\cancel{y + 2}} \\
 &= y^2(y - 2)
 \end{aligned}$$

$$\begin{aligned}
 22. \quad & \frac{5x}{7} \div \frac{10x^2}{21} = \frac{5x}{7} \cdot \frac{21}{10x^2} \\
 &= \frac{\cancel{5x} \cdot 3 \cdot \cancel{7}}{\cancel{7} \cdot \cancel{5x} \cdot 2x} \\
 &= \frac{3}{2x}
 \end{aligned}$$

$$\begin{aligned}
 23. \quad & \frac{2a}{7b^3} \div \frac{10a^5}{77} = \frac{2a}{7b^3} \cdot \frac{77}{10a^5} \\
 &= \frac{\cancel{2a} \cdot \cancel{7} \cdot 11}{\cancel{7}b^3 \cdot \cancel{2a} \cdot 5a^4} \\
 &= \frac{11}{5a^4b^3}
 \end{aligned}$$

$$\begin{aligned}
 24. \quad & \frac{6x^2y^2}{x - 2} \div \frac{3xy^2}{(x - 2)^2} \\
 &= \frac{6x^2y^2}{x - 2} \cdot \frac{(x - 2)^2}{3xy^2}
 \end{aligned}$$

$$\begin{aligned}
 25. \quad & \frac{(r + 3)^2}{4r^3s} \div \frac{r + 3}{rs} \\
 &= \frac{(r + 3)^2}{4r^3s} \cdot \frac{rs}{r + 3}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{\cancel{\beta} \cdot 2 \cdot \cancel{\alpha} \cdot x \cdot \cancel{y^2}}{\cancel{x-2}} \cdot \frac{(\cancel{x-2})(x-2)}{\cancel{\beta} \cdot \cancel{\alpha} \cdot \cancel{y^2}} \\
 &= 2x(x-2)
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{(r+3)\cancel{(r+3)} \cdot \cancel{f} \cdot \cancel{\beta}}{4r^2 \cdot \cancel{f} \cdot \cancel{\beta} \cdot \cancel{r+3}} \\
 &= \frac{r+3}{4r^2}
 \end{aligned}$$

$$26. \quad \frac{t^2+5t}{t+1} \div (t+5)$$

$$\begin{aligned}
 &= \frac{t^2+5t}{t+1} \cdot \frac{1}{t+5} \\
 &= \frac{\cancel{t}(t+5)}{t+1} \cdot \frac{1}{\cancel{t+5}} \\
 &= \frac{t}{t+1}
 \end{aligned}$$

$$27. \quad \frac{6p+7}{p+2} \div (36p^2-49)$$

$$\begin{aligned}
 &= \frac{6p+7}{p+2} \cdot \frac{1}{36p^2-49} \\
 &= \frac{\cancel{6p+7}}{p+2} \cdot \frac{1}{(\cancel{6p+7})(6p-7)} \\
 &= \frac{1}{(p+2)(6p-7)}
 \end{aligned}$$

$$28. \quad \frac{a}{a-10} \div \frac{a^3+6a^2-40a}{a^2-100}$$

$$\begin{aligned}
 &= \frac{a}{a-10} \cdot \frac{a^2-100}{a(a^2+6a-40)} \\
 &= \frac{\cancel{a}}{\cancel{a-10}} \cdot \frac{(\cancel{a+10})(a-10)}{\cancel{a}(\cancel{a+10})(a-4)} \\
 &= \frac{1}{a-4}
 \end{aligned}$$

$$29. \quad \frac{b^2-6b+9}{b^2-b-6} \div \frac{b^2-9}{4}$$

$$\begin{aligned}
 &= \frac{b^2-6b+9}{b^2-b-6} \cdot \frac{4}{b^2-9} \\
 &= \frac{(\cancel{b-3})(b-3)}{(\cancel{b-3})(b+2)} \cdot \frac{4}{(b+3)(\cancel{b-3})} \\
 &= \frac{4}{(b+2)(b+3)}
 \end{aligned}$$

$$30. \quad \frac{2x^2+5xy+2y^2}{4x^2-y^2} \div \frac{x^2+xy-2y^2}{2x^2+xy-y^2}$$

$$\begin{aligned}
 &= \frac{2x^2+5xy+2y^2}{4x^2-y^2} \cdot \frac{2x^2+xy-y^2}{x^2+xy-2y^2} \\
 &= \frac{(\cancel{2x+y})(x+2y)}{(\cancel{2x+y})(\cancel{2x-y})} \cdot \frac{(\cancel{2x-y})(x+y)}{(\cancel{x+2y})(x-y)} \\
 &= \frac{x+y}{x-y}
 \end{aligned}$$

$$31. \quad \frac{6s^2+st-2t^2}{6s^2-5st+t^2} \div \frac{3s^2+17st+10t^2}{6s^2+13st-5t^2}$$

$$\begin{aligned}
 &= \frac{6s^2+st-2t^2}{6s^2-5st+t^2} \cdot \frac{6s^2+13st-5t^2}{3s^2+17st+10t^2} \\
 &= \frac{(\cancel{3s+2t})(\cancel{2s-t})}{(\cancel{3s-t})(\cancel{2s-t})} \cdot \frac{(\cancel{3s-t})(2s+5t)}{(\cancel{3s+2t})(s+5t)} \\
 &= \frac{2s+5t}{s+5t}
 \end{aligned}$$

$$\begin{aligned}
 32. \quad \frac{x^4 - x^3 + x^2 - x}{2x^3 + 2x^2 + x + 1} \div \frac{x^3 - 4x^2 + x - 4}{2x^3 - 8x^2 + x - 4} &= \frac{x(x^3 - x^2 + x - 1)}{2x^3 + 2x^2 + x + 1} \cdot \frac{2x^3 - 8x^2 + x - 4}{x^3 - 4x^2 + x - 4} \\
 &= \frac{x[x^2(x-1) + (x-1)]}{2x^2(x+1) + (x+1)} \cdot \frac{2x^2(x-4) + (x-4)}{x^2(x-4) + (x-4)} = \frac{x(x-1)\cancel{(x^2+1)}}{(x+1)\cancel{(2x^2+1)}} \cdot \frac{\cancel{(x-4)}\cancel{(2x^2+1)}}{\cancel{(x-4)}\cancel{(x^2+1)}} \\
 &= \frac{x(x-1)}{x+1}
 \end{aligned}$$

$$\begin{aligned}
 33. \quad \frac{a^3 + a + a^2 + 1}{a^3 + a^2 + ab^2 + b^2} \div \frac{a^3 + a + a^2b + b}{2a^2 + 2ab + ab^2 + b^3} &= \frac{a^3 + a + a^2 + 1}{a^3 + a^2 + ab^2 + b^2} \cdot \frac{2a^2 + 2ab + ab^2 + b^3}{a^3 + a + a^2b + b} \\
 &= \frac{a(a^2 + 1) + (a^2 + 1)}{a^2(a+1) + b^2(a+1)} \cdot \frac{2a(a+b) + b^2(a+b)}{a(a^2 + 1) + b(a^2 + 1)} = \frac{\cancel{(a^2+1)}\cancel{(a+1)}}{\cancel{(a+1)}(a^2 + b^2)} \cdot \frac{\cancel{(a+b)}(2a + b^2)}{\cancel{(a^2+1)}\cancel{(a+b)}} \\
 &= \frac{2a + b^2}{a^2 + b^2}
 \end{aligned}$$

$$\begin{aligned}
 34. \quad \frac{3y - y^2}{y^3 - 27} \div \frac{y}{y^2 + 3y + 9} \\
 &= \frac{3y - y^2}{y^3 - 27} \cdot \frac{y^2 + 3y + 9}{y} \\
 &= \frac{\cancel{y}(y-3)}{(y-3)\cancel{(y^2 + 3y + 9)}} \cdot \frac{\cancel{y^2 + 3y + 9}}{\cancel{y}} \\
 &= -1
 \end{aligned}$$

$$\begin{aligned}
 35. \quad \frac{8x - 4x^2}{xy - 2y + 3x - 6} \div \frac{3x + 6}{y + 3} \\
 &= \frac{8x - 4x^2}{y(x-2) + 3(x-2)} \cdot \frac{y + 3}{3x + 6} \\
 &= \frac{-4x\cancel{(x-2)}}{\cancel{(x-2)}(y+3)} \cdot \frac{\cancel{y+3}}{3(x+2)} \\
 &= \frac{-4x}{3(x+2)}
 \end{aligned}$$

$$\begin{aligned}
 36. \quad \frac{8a^4b^3}{3c} \div \frac{a^7b^2}{9c} &= \frac{8a^4b^3}{3c} \cdot \frac{9c}{a^7b^2} \\
 &= \frac{\cancel{8}a^{\cancel{4}}\cancel{b^2} \cdot b}{\cancel{3}c} \cdot \frac{3 \cdot \cancel{9}c}{\cancel{a^4}a^3\cancel{b^2}} \\
 &= \frac{24b}{a^3}
 \end{aligned}$$

$$\begin{aligned}
 37. \quad \frac{3x^5}{2x^2y^7} \div \frac{4x^3y}{6y^6} &= \frac{3x^5}{2x^2y^7} \cdot \frac{6y^6}{4x^3y} \\
 &= \frac{\cancel{3}x^{\cancel{2}}\cancel{y^6}}{\cancel{2}x^{\cancel{2}}\cancel{y^6}y} \cdot \frac{\cancel{2} \cdot \cancel{3}y^{\cancel{6}}}{4x^{\cancel{3}}y} \\
 &= \frac{9}{4y^2}
 \end{aligned}$$

$$\begin{aligned}
 38. \quad \frac{2}{25x^2} \cdot \frac{5x}{12} \div \frac{2}{15x} &= \frac{2}{25x^2} \cdot \frac{5x}{12} \cdot \frac{15x}{2} \\
 &= \frac{\cancel{2}}{\cancel{5} \cdot \cancel{5}x \cdot \cancel{12}} \cdot \frac{\cancel{5}x}{4 \cdot \cancel{3}} \cdot \frac{\cancel{3} \cdot \cancel{3} \cdot \cancel{15}}{\cancel{2}} = \frac{1}{4}
 \end{aligned}$$

$$\begin{aligned}
 39. \quad \frac{4y}{7} \div \frac{y^2}{14} \cdot \frac{3}{y} &= \frac{4y}{7} \cdot \frac{14}{y^2} \cdot \frac{3}{y} \\
 &= \frac{4\cancel{y}}{\cancel{7}} \cdot \frac{\cancel{14} \cdot 2}{y^2} \cdot \frac{3}{\cancel{y}} = \frac{24}{y^2}
 \end{aligned}$$

$$\begin{aligned}
 40. \quad & \frac{10x^2 - 13xy - 3y^2}{8x^2 - 10xy - 3y^2} \cdot \frac{2y + 8x}{2x^2 + 2y^2} \\
 &= \frac{(5x + y)(\cancel{2x - 3y}) \cancel{2}(4x + y)}{(\cancel{4x + y})(\cancel{2x - 3y}) \cancel{2}(x^2 + y^2)} \\
 &= \frac{5x + y}{x^2 + y^2}
 \end{aligned}$$

$$\begin{aligned}
 41. \quad & \frac{6a^2 + ab - b^2}{10a^2 + 5ab} \cdot \frac{2a^3 + 4a^2b}{3a^2 + 5ab - 2b^2} \\
 &= \frac{(\cancel{3a - b})(\cancel{2a + b}) \cdot 2a^2(\cancel{a + 2b})}{5a(\cancel{2a + b}) \cdot (\cancel{3a - b})(\cancel{a + 2b})} \\
 &= \frac{2a}{5}
 \end{aligned}$$

$$\begin{aligned}
 42. \quad & (3m^2 - 12m) \div \frac{m^2 - 4m}{m^2 - 6m + 8} \\
 &= \frac{3m^2 - 12m}{1} \cdot \frac{m^2 - 6m + 8}{m^2 - 4m} \\
 &= \frac{3\cancel{m}(m - 4) \cdot (\cancel{m - 4})(m - 2)}{1 \cdot \cancel{m}(m - 4)} \\
 &= 3(m - 4)(m - 2)
 \end{aligned}$$

$$\begin{aligned}
 43. \quad & (2x^2 + 8) \div \frac{x^4 - 16}{x^2 + x - 6} \\
 &= \frac{2x^2 + 8}{1} \cdot \frac{x^2 + x - 6}{x^4 - 16} \\
 &= \frac{2(x^2 + 4)}{1} \cdot \frac{(x + 3)(x - 2)}{(x^2 + 4)(x^2 - 4)} \\
 &= \frac{2(\cancel{x^2 + 4}) \cdot (x + 3)(\cancel{x - 2})}{1 \cdot (\cancel{x^2 + 4})(x + 2)(\cancel{x - 2})} \\
 &= \frac{2(x + 3)}{x + 2}
 \end{aligned}$$

$$\begin{aligned}
 44. \quad & \frac{(a + b)^2}{a - b} \cdot \frac{a^3 - b^3}{a^2 - b^2} \div \frac{a^2 + ab + b^2}{(a - b)^2} = \frac{(a + b)^2}{a - b} \cdot \frac{a^3 - b^3}{a^2 - b^2} \cdot \frac{(a - b)^2}{a^2 + ab + b^2} \\
 &= \frac{(a + b)(\cancel{a + b}) \cdot (\cancel{a - b})(\cancel{a^2 + ab + b^2}) \cdot (\cancel{a - b})(a - b)}{\cancel{a - b} \cdot (\cancel{a + b})(\cancel{a - b}) \cdot \cancel{a^2 + ab + b^2}} \\
 &= (a + b)(a - b)
 \end{aligned}$$

$$\begin{aligned}
 45. \quad & \frac{m^2 - n^2}{(m - n)^2} \div \frac{m^2 - 2mn + n^2}{m^2 - mn + n^2} \cdot \frac{(m - n)^4}{m^3 + n^3} \\
 &= \frac{m^2 - n^2}{(m - n)^2} \cdot \frac{m^2 - mn + n^2}{m^2 - 2mn + n^2} \cdot \frac{(m - n)^4}{m^3 + n^3} \\
 &= \frac{(\cancel{m + n})(m - n) \cdot \cancel{m^2 - mn + n^2} \cdot (\cancel{m - n})^2 (\cancel{m - n})^2}{(\cancel{m - n})^2 \cdot (\cancel{m - n})^2 \cdot (\cancel{m + n})(\cancel{m^2 - mn + n^2})} \\
 &= m - n
 \end{aligned}$$

$$\begin{aligned}
 46. \quad \frac{x^2 - 4y^2}{x + 2y} \div (x + 2y) \cdot \frac{2y}{x - 2y} &= \frac{x^2 - 4y^2}{x + 2y} \cdot \frac{1}{x + 2y} \cdot \frac{2y}{x - 2y} \\
 &= \frac{\cancel{(x + 2y)} \cancel{(x - 2y)}}{x + 2y} \cdot \frac{1}{x + 2y} \cdot \frac{2y}{\cancel{x - 2y}} = \frac{2y}{x + 2y}
 \end{aligned}$$

$$\begin{aligned}
 47. \quad \frac{x^2 - 6xy + 9y^2}{x^2 - 4y^2} \cdot \frac{x^2 - 5xy + 6y^2}{3y - x} \div \frac{x^2 - 9y^2}{x + 2y} \\
 &= \frac{x^2 - 6xy + 9y^2}{x^2 - 4y^2} \cdot \frac{x^2 - 5xy + 6y^2}{3y - x} \cdot \frac{x + 2y}{x^2 - 9y^2} \\
 &= \frac{\cancel{(x - 3y)} \cancel{(x - 3y)}}{\cancel{(x + 2y)} \cancel{(x - 2y)}} \cdot \frac{\cancel{(x - 3y)} \cancel{(x - 2y)}}{-\cancel{(x - 3y)}} \cdot \frac{\cancel{x + 2y}}{\cancel{(x + 3y)} \cancel{(x - 3y)}} \\
 &= -\frac{x - 3y}{x + 3y} \text{ or } \frac{3y - x}{x + 3y}
 \end{aligned}$$

$$\begin{aligned}
 48. \quad \frac{8x^3 - 27y^3}{4x^2 - 9y^2} \div \frac{8x^2 + 12xy + 18y^2}{2x + 3y} &= \frac{8x^3 - 27y^3}{4x^2 - 9y^2} \cdot \frac{2x + 3y}{8x^2 + 12xy + 18y^2} \\
 &= \frac{\cancel{(2x - 3y)} \cancel{(4x^2 + 6xy + 9y^2)}}{\cancel{(2x + 3y)} \cancel{(2x - 3y)}} \cdot \frac{\cancel{2x + 3y}}{2 \cancel{(4x^2 + 6xy + 9y^2)}} = \frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 49. \quad \frac{25m^2 - 1}{125m^3 - 1} \div \frac{5m + 1}{25m^2 + 5m + 1} &= \frac{25m^2 - 1}{125m^3 - 1} \cdot \frac{25m^2 + 5m + 1}{5m + 1} \\
 &= \frac{\cancel{(5m + 1)} \cancel{(5m - 1)}}{\cancel{(5m - 1)} \cancel{(25m^2 + 5m + 1)}} \cdot \frac{\cancel{25m^2 + 5m + 1}}{\cancel{5m + 1}} = 1
 \end{aligned}$$

$$\begin{aligned}
 50. \quad \frac{m^3 + 2m^2 - mn^2 - 2n^2}{m^3 - m^2 - 20m} \cdot \frac{m^3 - 25m}{m^3 + m^2n - 4m - 4n} \\
 &= \frac{m^2(m + 2) - n^2(m + 2)}{m(m^2 - m - 20)} \cdot \frac{m(m^2 - 25)}{m^2(m + n) - 4(m + n)} \\
 &= \frac{(m + 2)(m^2 - n^2)}{m(m - 5)(m + 4)} \cdot \frac{m(m + 5)(m - 5)}{(m + n)(m^2 - 4)} \\
 &= \frac{\cancel{(m + 2)} \cancel{(m + n)} (m - n)}{\cancel{m} \cancel{(m - 5)} (m + 4)} \cdot \frac{\cancel{m} \cancel{(m + 5)} \cancel{(m - 5)}}{\cancel{(m + n)} \cancel{(m + 2)} (m - 2)} = \frac{(m - n)(m + 5)}{(m + 4)(m - 2)}
 \end{aligned}$$

Section 5.2 Multiplication and Division of Rational Expressions

$$\begin{aligned}
 51. \quad \frac{2a^2 + ab - 8a - 4b}{2a^2 - 6a + ab - 3b} \cdot \frac{a^2 - 6a + 9}{a^2 - 16} &= \frac{a(2a+b) - 4(2a+b)}{2a(a-3) + b(a-3)} \cdot \frac{(a-3)(a-3)}{(a+4)(a-4)} \\
 &= \frac{\cancel{(2a+b)} \cancel{(a-4)}}{\cancel{(a-3)} \cancel{(2a+b)}} \cdot \frac{(a-3) \cancel{(a-3)}}{(a+4) \cancel{(a-4)}} = \frac{a-3}{a+4}
 \end{aligned}$$

$$52. \quad \frac{7}{3x+15} \cdot (x+5) \div \frac{14}{9x-27} = \frac{\cancel{7}}{\cancel{3}(x+5)} \cdot \frac{\cancel{x+5}}{1} \cdot \frac{\cancel{3} \cdot 3(x-3)}{\cancel{7} \cdot 2} = \frac{3(x-3)}{2}$$

$$53. \quad \frac{45}{2x+1} \cdot (8x+4) \div \frac{27}{4x+2} = \frac{5 \cdot \cancel{9}}{\cancel{2x+1}} \cdot \frac{4 \cancel{(2x+1)}}{1} \cdot \frac{2(2x+1)}{\cancel{9} \cdot 3} = \frac{40(2x+1)}{3}$$

$$\begin{aligned}
 54. \quad \frac{12y+3}{6y^2 - y - 12} \div \frac{4y^2 - 19y - 5}{2y^2 - y - 3} &= \frac{12y+3}{6y^2 - y - 12} \cdot \frac{2y^2 - y - 3}{4y^2 - 19y - 5} \\
 &= \frac{3 \cancel{(4y+1)}}{(3y+4) \cancel{(2y-3)}} \cdot \frac{\cancel{(2y-3)}(y+1)}{\cancel{(4y+1)}(y-5)} \\
 &= \frac{3(y+1)}{(3y+4)(y-5)}
 \end{aligned}$$

$$\begin{aligned}
 55. \quad \frac{2x^2 - 11x - 6}{3x - 2} \div \frac{2x^2 - 5x - 3}{3x^2 - 7x - 6} &= \frac{2x^2 - 11x - 6}{3x - 2} \cdot \frac{3x^2 - 7x - 6}{2x^2 - 5x - 3} \\
 &= \frac{\cancel{(2x+1)}(x-6)}{3x-2} \cdot \frac{(3x+2) \cancel{(x-3)}}{\cancel{(2x+1)} \cancel{(x-3)}} \\
 &= \frac{(x-6)(3x+2)}{3x-2}
 \end{aligned}$$

$$\begin{aligned}
 56. \quad A &= \frac{1}{2} \left(\frac{4a^2}{b} \right) \left(\frac{b^2}{5a} \right) \\
 &= \frac{1}{\cancel{2}} \left(\frac{\cancel{2} \cdot 2a \cdot \cancel{a}}{\cancel{b}} \right) \left(\frac{b \cdot \cancel{b}}{5 \cancel{a}} \right) \\
 &= \frac{2ab}{5} \text{ in}^2
 \end{aligned}$$

$$\begin{aligned}
 57. \quad A &= \frac{1}{2} \left(\frac{k^2}{2h^2} \right) \left(\frac{8}{hk} \right) \\
 &= \frac{1}{\cancel{2}} \left(\frac{k \cdot \cancel{k}}{\cancel{2} h^2} \right) \left(\frac{\cancel{2} \cdot \cancel{2} \cdot 2}{h \cancel{k}} \right) \\
 &= \frac{2k}{h^3} \text{ cm}^2
 \end{aligned}$$

$$58. \quad A = \frac{x+2}{6} \cdot \frac{12x}{x^2-4}$$

$$= \frac{\cancel{x+2}}{\cancel{6}} \cdot \frac{\cancel{6} \cdot 2x}{(\cancel{x+2})(x-2)} = \frac{2x}{x-2} \text{ m}^2$$

$$59. \quad A = \frac{x^2}{x-3} \cdot \frac{5x-15}{4x}$$

$$= \frac{x \cdot \cancel{x}}{\cancel{x-3}} \cdot \frac{5(\cancel{x-3})}{4\cancel{x}}$$

$$= \frac{5x}{4} \text{ ft}^2$$

Section 5.3 Practice Exercises

$$1. \quad \text{a.} \quad \frac{p+r}{q}; \frac{p-r}{q}$$

b. least common denominator

$$2. \quad \frac{x}{x-y} \div \frac{x^2}{y-x} = \frac{x}{x-y} \cdot \frac{y-x}{x^2} = \frac{\cancel{x}}{\cancel{x-y}} \cdot \frac{-(x-y)}{\cancel{x} \cdot x} = -\frac{1}{x}$$

$$3. \quad \frac{9b+9}{4b+8} \cdot \frac{2b+4}{3b-3} = \frac{\cancel{3} \cdot 3(b+1)}{\cancel{2} \cdot 2(b+2)} \cdot \frac{\cancel{2}(b+2)}{\cancel{3}(b-1)} = \frac{3(b+1)}{2(b-1)}$$

$$4. \quad \frac{8a^2}{7b^3} \div \frac{4a}{21b} = \frac{8a^2}{7b^3} \cdot \frac{21b}{4a} = \frac{2 \cdot \cancel{4} \cdot 3 \cdot \cancel{7}}{\cancel{7} \cdot \cancel{4}} a^{2-1} b^{1-3} = 6a^1 b^{-2} = \frac{6a}{b^2}$$

$$5. \quad \frac{(5-a)^2}{10a-2} \cdot \frac{25a^2-1}{a^2-10a+25} = \frac{(-1)^2 (\cancel{a-5})^2}{2(\cancel{5a-1})} \cdot \frac{(5a+1)(\cancel{5a-1})}{(\cancel{a-5})^2} = \frac{5a+1}{2}$$

$$6. \quad \frac{x^2-z^2}{14x^2z^4} \div \frac{x^2+2xz+z^2}{3xz^3} = \frac{x^2-z^2}{14x^2z^4} \cdot \frac{3xz^3}{x^2+2xz+z^2} = \frac{(\cancel{x+z})(x-z)}{14x \cdot \cancel{z} \cdot \cancel{z} \cdot z} \cdot \frac{3\cancel{z}}{(\cancel{x+z})(x+z)}$$

$$= \frac{3(x-z)}{14xz(x+z)}$$

$$7. \quad \frac{3}{5x} + \frac{7}{5x} = \frac{10}{5x} = \frac{\cancel{5} \cdot 2}{\cancel{5}x} = \frac{2}{x}$$

$$8. \quad \frac{1}{2x^2} - \frac{5}{2x^2} = -\frac{4}{2x^2} = -\frac{\cancel{2} \cdot 2}{\cancel{2}x^2} = -\frac{2}{x^2}$$

Section 5.3 Addition and Subtraction of Rational Expressions

$$\begin{aligned}
 9. \quad \frac{x}{x^2-2x-3} - \frac{3}{x^2-2x-3} &= \frac{x-3}{x^2-2x-3} \\
 &= \frac{\cancel{x-3}}{(\cancel{x-3})(x+1)} = \frac{1}{x+1}
 \end{aligned}$$

$$\begin{aligned}
 10. \quad \frac{x}{x^2+4x-12} + \frac{6}{x^2+4x-12} &= \frac{x+6}{x^2+4x-12} \\
 &= \frac{\cancel{x+6}}{(\cancel{x+6})(x-2)} = \frac{1}{x-2}
 \end{aligned}$$

$$\begin{aligned}
 11. \quad \frac{5x-1}{(2x+9)(x-6)} - \frac{3x-6}{(2x+9)(x-6)} \\
 &= \frac{5x-1-(3x-6)}{(2x+9)(x-6)} \\
 &= \frac{5x-1-3x+6}{(2x+9)(x-6)} \\
 &= \frac{2x+5}{(2x+9)(x-6)}
 \end{aligned}$$

$$\begin{aligned}
 12. \quad \frac{4-x}{8x+1} - \frac{5x-6}{8x+1} &= \frac{4-x-(5x-6)}{8x+1} \\
 &= \frac{4-x-5x+6}{8x+1} \\
 &= \frac{10-6x}{8x+1} \\
 &= \frac{-2(3x-5)}{8x+1}
 \end{aligned}$$

$$13. \quad \frac{x+2}{x-5} + \frac{x-12}{x-5} = \frac{2x-10}{x-5} = \frac{2(\cancel{x-5})}{\cancel{x-5}} = 2$$

$$14. \quad \frac{2x-1}{x-2} + \frac{x-5}{x-2} = \frac{3x-6}{x-2} = \frac{3(\cancel{x-2})}{\cancel{x-2}} = 3$$

$$\begin{aligned}
 15. \quad \frac{5}{8} &= \frac{5}{2^3}, & \frac{3}{20x} &= \frac{3}{5 \cdot 2^2 x} \\
 \text{LCD} &= 2^3 \cdot 5 \cdot x = 40x
 \end{aligned}$$

$$\begin{aligned}
 16. \quad \frac{y}{15a} &= \frac{y}{3 \cdot 5 \cdot a}, & \frac{y^2}{35} &= \frac{y^2}{7 \cdot 5} \\
 \text{LCD} &= 3 \cdot 5 \cdot 7 \cdot a = 105a
 \end{aligned}$$

$$\begin{aligned}
 17. \quad \frac{-5}{6m^4} &= \frac{-5}{2 \cdot 3 \cdot m^4}, & \frac{1}{15mn^7} &= \frac{1}{3 \cdot 5 \cdot mn^7} \\
 \text{LCD} &= 2 \cdot 3 \cdot 5 \cdot m^4 n^7 = 30m^4 n^7
 \end{aligned}$$

$$\begin{aligned}
 18. \quad \frac{13}{12cd^5} &= \frac{13}{2^2 \cdot 3 \cdot cd^5}, & \frac{9}{8c^3} &= \frac{9}{2^3 c^3} \\
 \text{LCD} &= 2^3 \cdot 3 \cdot c^3 d^5 = 24c^3 d^5
 \end{aligned}$$

$$\begin{aligned}
 19. \quad \frac{6}{(x-4)(x+2)}, & \quad \frac{-8}{(x-4)(x-6)} \\
 \text{LCD} &= (x-4)(x+2)(x-6)
 \end{aligned}$$

$$\begin{aligned}
 20. \quad \frac{x}{(2x-1)(x-7)}, & \quad \frac{2}{(2x-1)(x+1)} \\
 \text{LCD} &= (2x-1)(x-7)(x+1)
 \end{aligned}$$

$$\begin{aligned}
 21. \quad \frac{3}{x(x-1)(x+7)^2}, & \quad \frac{-1}{x^2(x+7)} \\
 \text{LCD} &= x^2(x-1)(x+7)^2
 \end{aligned}$$

$$\begin{aligned}
 22. \quad \frac{14}{(x-2)^2(x+9)}, & \quad \frac{41}{x(x-2)(x+9)} \\
 \text{LCD} &= x(x-2)^2(x+9)
 \end{aligned}$$

Chapter 5 Rational Expressions and Rational Equations

$$23. \quad \frac{5}{x-6}, \quad \frac{x-5}{x^2-8x+12} = \frac{x-5}{(x-6)(x-2)}$$

LCD = $(x-6)(x-2)$

$$24. \quad \frac{7a}{a+4}, \quad \frac{a+12}{a^2-16} = \frac{a+12}{(a+4)(a-4)}$$

LCD = $(a+4)(a-4)$

$$25. \quad \frac{3a}{a-4}, \quad \frac{5}{4-a} = \frac{5(-1)}{(4-a)(-1)} = \frac{-5}{a-4}$$

LCD = $a-4$ or $4-a$

$$26. \quad \frac{10}{x-6}, \quad \frac{x+1}{6-x} = \frac{(x+1)(-1)}{(6-x)(-1)} = \frac{-x-1}{x-6}$$

LCD = $x-6$ or $6-x$

$$27. \quad \frac{5}{3x} = \frac{5}{9x^2y}$$

$$\frac{5}{3x} \cdot \frac{3xy}{3xy} = \frac{15xy}{9x^2y}$$

$$28. \quad \frac{-5}{xy} = \frac{-5}{4x^2y^3}$$

$$\frac{-5}{xy} \cdot \frac{4xy^2}{4xy^2} = \frac{-20xy^2}{4x^2y^3}$$

$$29. \quad \frac{2x}{x-1} = \frac{2x}{x(x-1)(x+2)}$$

$$\frac{2x}{x-1} \cdot \frac{x(x+2)}{x(x+2)} = \frac{2x^2(x+2)}{x(x-1)(x+2)}$$

$$= \frac{2x^3+4x^2}{x(x-1)(x+2)}$$

$$30. \quad \frac{5x}{2x-5} = \frac{5x}{(2x-5)(x+8)}$$

$$\frac{5x}{2x-5} \cdot \frac{x+8}{x+8} = \frac{5x^2+40x}{(2x-5)(x+8)}$$

$$31. \quad \frac{y}{y+6} = \frac{y}{y^2+5y-6}$$

$$\frac{y}{y+6} = \frac{y}{(y+6)(y-1)}$$

$$\frac{y}{y+6} \cdot \frac{y-1}{y-1} = \frac{y^2-y}{(y+6)(y-1)}$$

$$32. \quad \frac{t^2}{t-8} = \frac{t^2}{t^2-6t-16}$$

$$\frac{t^2}{t-8} = \frac{t^2}{(t-8)(t+2)}$$

$$\frac{t^2}{t-8} \cdot \frac{t+2}{t+2} = \frac{t^3+2t^2}{(t-8)(t+2)}$$

$$33. \quad \frac{4}{3p} - \frac{5}{2p^2} \quad \text{LCD} = 2 \cdot 3 \cdot p^2 = 6p^2$$

$$= \frac{4}{3p} \cdot \frac{2p}{2p} - \frac{5}{2p^2} \cdot \frac{3}{3} = \frac{8p}{6p^2} - \frac{15}{6p^2}$$

$$= \frac{8p-15}{6p^2}$$

$$34. \quad \frac{6}{5a^2b} - \frac{1}{10ab} \quad \text{LCD} = 2 \cdot 5 \cdot a^2b = 10a^2b$$

$$= \frac{6}{5a^2b} \cdot \frac{2}{2} - \frac{1}{10ab} \cdot \frac{a}{a} = \frac{12}{10a^2b} - \frac{a}{10a^2b}$$

$$= \frac{12-a}{10a^2b}$$

$$\begin{aligned}
 35. \quad \frac{s-1}{s} - \frac{t+1}{t} \quad \text{LCD} &= st \\
 &= \frac{s-1}{s} \cdot \frac{t}{t} - \frac{t+1}{t} \cdot \frac{s}{s} = \frac{st-t}{st} - \frac{st+s}{st} \\
 &= \frac{st-t-st-s}{st} = \frac{-t-s}{st}
 \end{aligned}$$

$$\begin{aligned}
 36. \quad \frac{x+2}{x} - \frac{y-2}{y} \quad \text{LCD} &= xy \\
 &= \frac{x+2}{x} \cdot \frac{y}{y} - \frac{y-2}{y} \cdot \frac{x}{x} = \frac{xy+2y}{xy} - \frac{xy-2x}{xy} \\
 &= \frac{xy+2y-xy+2x}{xy} = \frac{2y+2x}{xy}
 \end{aligned}$$

$$\begin{aligned}
 37. \quad \frac{4a-2}{3a+12} - \frac{a-2}{a+4} &= \frac{4a-2}{3(a+4)} - \frac{a-2}{a+4} \\
 \text{LCD} &= 3(a+4) \\
 &= \frac{4a-2}{3(a+4)} - \frac{a-2}{a+4} \cdot \frac{3}{3} \\
 &= \frac{4a-2-3(a-2)}{3(a+4)} \\
 &= \frac{4a-2-3a+6}{3(a+4)} = \frac{\cancel{a}+4}{3(\cancel{a}+4)} = \frac{1}{3}
 \end{aligned}$$

$$\begin{aligned}
 38. \quad \frac{6y+5}{5y-25} - \frac{y+2}{y-5} &= \frac{6y+5}{5(y-5)} - \frac{y+2}{y-5} \\
 \text{LCD} &= 5(y-5) \\
 &= \frac{6y+5}{5(y-5)} - \frac{y+2}{y-5} \cdot \frac{5}{5} \\
 &= \frac{6y+5-5(y+2)}{5(y-5)} \\
 &= \frac{6y+5-5y-10}{5(y-5)} = \frac{\cancel{y}-5}{5(\cancel{y}-5)} = \frac{1}{5}
 \end{aligned}$$

$$\begin{aligned}
 39. \quad \frac{10}{b(b+5)} + \frac{2}{b} \quad \text{LCD} &= b(b+5) \\
 &= \frac{10}{b(b+5)} + \frac{2}{b} \cdot \frac{b+5}{b+5} = \frac{10+2(b+5)}{b(b+5)} \\
 &= \frac{10+2b+10}{b(b+5)} = \frac{2b+20}{b(b+5)}
 \end{aligned}$$

$$\begin{aligned}
 40. \quad \frac{6}{w(w-2)} + \frac{3}{w} \quad \text{LCD} &= w(w-2) \\
 &= \frac{6}{w(w-2)} + \frac{3}{w} \cdot \frac{w-2}{w-2} = \frac{6+3(w-2)}{w(w-2)} \\
 &= \frac{6+3w-6}{w(w-2)} = \frac{\cancel{3}w}{\cancel{w}(w-2)} = \frac{3}{w-2}
 \end{aligned}$$

$$\begin{aligned}
 41. \quad \frac{x-2}{x-6} - \frac{x+2}{6-x} &= \frac{x-2}{x-6} - \frac{x+2}{6-x} \cdot \frac{(-1)}{(-1)} \\
 &= \frac{x-2}{x-6} - \frac{-(x+2)}{x-6} \\
 &= \frac{x-2+x+2}{x-6} \\
 &= \frac{2x}{x-6}
 \end{aligned}$$

$$\begin{aligned}
 42. \quad \frac{x-10}{x-8} - \frac{x+10}{8-x} &= \frac{x-10}{x-8} - \frac{x+10}{8-x} \cdot \frac{(-1)}{(-1)} \\
 &= \frac{x-10}{x-8} - \frac{-(x+10)}{x-8} \\
 &= \frac{x-10+x+10}{x-8} \\
 &= \frac{2x}{x-8}
 \end{aligned}$$

$$\begin{aligned}
 43. \quad \frac{6b}{b-4} - \frac{1}{b+1} \quad \text{LCD} &= (b-4)(b+1) \\
 &= \frac{6b}{b-4} \cdot \frac{b+1}{b+1} - \frac{1}{b+1} \cdot \frac{b-4}{b-4} \\
 &= \frac{6b(b+1) - 1(b-4)}{(b-4)(b+1)} \\
 &= \frac{6b^2 + 6b - b + 4}{(b-4)(b+1)} = \frac{6b^2 + 5b + 4}{(b-4)(b+1)}
 \end{aligned}$$

$$\begin{aligned}
 44. \quad \frac{a}{a-3} - \frac{5}{a+6} \quad \text{LCD} &= (a-3)(a+6) \\
 &= \frac{a}{a-3} \cdot \frac{a+6}{a+6} - \frac{5}{a+6} \cdot \frac{a-3}{a-3} \\
 &= \frac{a(a+6) - 5(a-3)}{(a-3)(a+6)} = \frac{a^2 + 6a - 5a + 15}{(a-3)(a+6)} \\
 &= \frac{a^2 + a + 15}{(a-3)(a+6)}
 \end{aligned}$$

$$\begin{aligned}
 45. \quad \frac{2}{2x+1} + \frac{4}{x-2} \\
 \text{LCD} &= (2x+1)(x-2) \\
 &= \frac{2}{2x+1} \cdot \frac{x-2}{x-2} + \frac{4}{x-2} \cdot \frac{2x+1}{2x+1} \\
 &= \frac{2x-4+8x+4}{(2x+1)(x-2)} = \frac{10x}{(2x+1)(x-2)}
 \end{aligned}$$

$$\begin{aligned}
 46. \quad \frac{3}{y+6} + \frac{1}{3y+1} \\
 \text{LCD} &= (y+6)(3y+1) \\
 &= \frac{3}{y+6} \cdot \frac{3y+1}{3y+1} + \frac{1}{3y+1} \cdot \frac{y+6}{y+6} \\
 &= \frac{9y+3+y+6}{(y+6)(3y+1)} = \frac{10y+9}{(y+6)(3y+1)}
 \end{aligned}$$

$$\begin{aligned}
 47. \quad \frac{y-2}{y-4} + \frac{2y^2-15y+12}{y^2-16} &= \frac{y-2}{y-4} + \frac{2y^2-15y+12}{(y+4)(y-4)} \quad \text{LCD} = (y+4)(y-4) \\
 &= \frac{y-2}{y-4} \cdot \frac{y+4}{y+4} + \frac{2y^2-15y+12}{(y+4)(y-4)} = \frac{y^2+2y-8+2y^2-15y+12}{(y+4)(y-4)} = \frac{3y^2-13y+4}{(y+4)(y-4)} \\
 &= \frac{(3y-1)\cancel{(y-4)}}{(y+4)\cancel{(y-4)}} = \frac{3y-1}{y+4}
 \end{aligned}$$

$$\begin{aligned}
 48. \quad \frac{x^2+13x+18}{x^2-9} + \frac{x+1}{x+3} &= \frac{x^2+13x+18}{(x+3)(x-3)} + \frac{x+1}{x+3} \quad \text{LCD} = (x+3)(x-3) \\
 &= \frac{x^2+13x+18}{(x+3)(x-3)} + \frac{x+1}{x+3} \cdot \frac{x-3}{x-3} = \frac{x^2+13x+18+x^2-2x-3}{(x+3)(x-3)} = \frac{2x^2+11x+15}{(x+3)(x-3)} \\
 &= \frac{(2x+5)\cancel{(x+3)}}{\cancel{(x+3)}(x-3)} \\
 &= \frac{2x+5}{x-3}
 \end{aligned}$$

$$\begin{aligned}
 49. \quad \frac{x+2}{x^2-36} - \frac{x}{x^2+9x+18} &= \frac{x+2}{(x+6)(x-6)} - \frac{x}{(x+6)(x+3)} \quad \text{LCD}=(x+6)(x-6)(x+3) \\
 &= \frac{x+2}{(x+6)(x-6)} \cdot \frac{x+3}{x+3} - \frac{x}{(x+6)(x+3)} \cdot \frac{x-6}{x-6} = \frac{(x+2)(x+3)-x(x-6)}{(x+6)(x-6)(x+3)} \\
 &= \frac{x^2+5x+6-x^2+6x}{(x+6)(x-6)(x+3)} = \frac{11x+6}{(x+6)(x-6)(x+3)}
 \end{aligned}$$

$$\begin{aligned}
 50. \quad \frac{7}{x^2-x-2} + \frac{x}{x^2+4x+3} &= \frac{7}{(x-2)(x+1)} + \frac{x}{(x+1)(x+3)} \quad \text{LCD}=(x-2)(x+1)(x+3) \\
 &= \frac{7}{(x-2)(x+1)} \cdot \frac{x+3}{x+3} + \frac{x}{(x+1)(x+3)} \cdot \frac{x-2}{x-2} \\
 &= \frac{7x+21+x^2-2x}{(x-2)(x+1)(x+3)} = \frac{x^2+5x+21}{(x-2)(x+1)(x+3)}
 \end{aligned}$$

$$51. \quad \frac{5}{w} + \frac{8}{-w} = \frac{5}{w} + \frac{8}{-w} \cdot \frac{(-1)}{(-1)} = \frac{5}{w} + \frac{-8}{w} = -\frac{3}{w} \qquad 52. \quad \frac{4}{y} + \frac{5}{-y} = \frac{4}{y} + \frac{5}{-y} \cdot \frac{(-1)}{(-1)} = \frac{4}{y} + \frac{-5}{y} = -\frac{1}{y}$$

$$\begin{aligned}
 53. \quad \frac{n}{5-n} + \frac{2n-5}{n-5} &= \frac{n}{5-n} \cdot \frac{(-1)}{(-1)} + \frac{2n-5}{n-5} \\
 &= \frac{-n}{n-5} + \frac{2n-5}{n-5} = \frac{n-5}{n-5} = 1 \qquad 54. \quad \frac{c}{7-c} + \frac{2c-7}{c-7} = \frac{c}{7-c} \cdot \frac{(-1)}{(-1)} + \frac{2c-7}{c-7} \\
 &= \frac{-c}{c-7} + \frac{2c-7}{c-7} = \frac{c-7}{c-7} = 1
 \end{aligned}$$

$$\begin{aligned}
 55. \quad \frac{2}{3x-15} + \frac{x}{25-x^2} &= \frac{2}{3x-15} + \frac{x}{25-x^2} \cdot \frac{(-1)}{(-1)} \\
 &= \frac{2}{3(x-5)} + \frac{-x}{(x+5)(x-5)} \quad \text{LCD}=3(x+5)(x-5) \\
 &= \frac{2}{3(x-5)} \cdot \frac{x+5}{x+5} + \frac{-x}{(x+5)(x-5)} \cdot \frac{3}{3} \\
 &= \frac{2x+10-3x}{3(x+5)(x-5)} \\
 &= \frac{10-x}{3(x+5)(x-5)}
 \end{aligned}$$

$$\begin{aligned}
 56. \quad & \frac{5}{9-x^2} - \frac{4}{x^2+4x+3} = \frac{5}{9-x^2} \cdot \frac{(-1)}{(-1)} - \frac{4}{x^2+4x+3} = \frac{-5}{(x+3)(x-3)} - \frac{4}{(x+3)(x+1)} \\
 & \text{LCD} = (x+3)(x-3)(x+1) \\
 & = \frac{-5}{(x+3)(x-3)} \cdot \frac{x+1}{x+1} - \frac{4}{(x+3)(x+1)} \cdot \frac{x-3}{x-3} = \frac{-5(x+1) - 4(x-3)}{(x+3)(x-3)(x+1)} \\
 & = \frac{-5x-5-4x+12}{(x+3)(x-3)(x+1)} = \frac{-9x+7}{(x+3)(x-3)(x+1)}
 \end{aligned}$$

$$\begin{aligned}
 57. \quad & \frac{m}{20+9m+m^2} - \frac{4}{12+7m+m^2} = \frac{m}{(m+5)(m+4)} - \frac{4}{(m+4)(m+3)} \\
 & \text{LCD} = (m+5)(m+4)(m+3) \\
 & = \frac{m}{(m+5)(m+4)} \cdot \frac{m+3}{m+3} - \frac{4}{(m+4)(m+3)} \cdot \frac{m+5}{m+5} \\
 & = \frac{m^2+3m}{(m+5)(m+4)(m+3)} - \frac{4m+20}{(m+5)(m+4)(m+3)} \\
 & = \frac{m^2+3m-4m-20}{(m+5)(m+4)(m+3)} = \frac{m^2-m-20}{(m+5)(m+4)(m+3)} \\
 & = \frac{(m-5)\cancel{(m+4)}}{(m+5)\cancel{(m+4)}(m+3)} = \frac{m-5}{(m+5)(m+3)}
 \end{aligned}$$

$$\begin{aligned}
 58. \quad & \frac{t}{6+5t+t^2} - \frac{2}{2+3t+t^2} = \frac{t}{(t+3)(t+2)} - \frac{2}{(t+2)(t+1)} \\
 & \text{LCD} = (t+3)(t+2)(t+1) \\
 & = \frac{t}{(t+3)(t+2)} \cdot \frac{t+1}{t+1} - \frac{2}{(t+2)(t+1)} \cdot \frac{t+3}{t+3} = \frac{t^2+t}{(t+3)(t+2)(t+1)} - \frac{2t+6}{(t+3)(t+2)(t+1)} \\
 & = \frac{t^2+t-2t-6}{(t+3)(t+2)(t+1)} = \frac{t^2-t-6}{(t+3)(t+2)(t+1)} \\
 & = \frac{(t-3)\cancel{(t+2)}}{(t+3)\cancel{(t+2)}(t+1)} \\
 & = \frac{t-3}{(t+3)(t+1)}
 \end{aligned}$$

Section 5.3 Addition and Subtraction of Rational Expressions

$$\begin{aligned}
 \text{59. } \frac{x+3}{x^2} + \frac{x+5}{2x} \quad \text{LCD} &= 2x^2 \\
 &= \frac{x+3}{x^2} \cdot \frac{2}{2} + \frac{x+5}{2x} \cdot \frac{x}{x} \\
 &= \frac{2x+6}{2x^2} + \frac{x^2+5x}{2x^2} \\
 &= \frac{x^2+7x+6}{2x^2}
 \end{aligned}$$

$$\begin{aligned}
 \text{60. } \frac{x+2}{5x^2} + \frac{x+4}{15x} \quad \text{LCD} &= 3 \cdot 5 \cdot x^2 = 15x^2 \\
 &= \frac{x+2}{5x^2} \cdot \frac{3}{3} + \frac{x+4}{15x} \cdot \frac{x}{x} \\
 &= \frac{3x+6}{15x^2} + \frac{x^2+4x}{15x^2} \\
 &= \frac{x^2+7x+6}{15x^2}
 \end{aligned}$$

$$\begin{aligned}
 \text{61. } w+2 + \frac{1}{w-2} \quad \text{LCD} &= w-2 \\
 &= (w+2) \cdot \frac{w-2}{w-2} + \frac{1}{w-2} \\
 &= \frac{w^2-4+1}{w-2} \\
 &= \frac{w^2-3}{w-2}
 \end{aligned}$$

$$\begin{aligned}
 \text{62. } h-3 + \frac{1}{h+3} \quad \text{LCD} &= h+3 \\
 &= (h-3) \cdot \frac{h+3}{h+3} + \frac{1}{h+3} \\
 &= \frac{h^2-9+1}{h+3} \\
 &= \frac{h^2-8}{h+3}
 \end{aligned}$$

$$\begin{aligned}
 \text{63. } \frac{9}{x^2-2x+1} - \frac{x-3}{x^2-x} &= \frac{9}{(x-1)^2} - \frac{x-3}{x(x-1)} \quad \text{LCD} = x(x-1)^2 \\
 &= \frac{9}{(x-1)^2} \cdot \frac{x}{x} - \frac{x-3}{x(x-1)} \cdot \frac{x-1}{x-1} = \frac{9x - (x^2 - 4x + 3)}{x(x-1)^2} \\
 &= \frac{9x - x^2 + 4x - 3}{x(x-1)^2} = \frac{-x^2 + 13x - 3}{x(x-1)^2}
 \end{aligned}$$

$$\begin{aligned}
 \text{64. } \frac{2}{4z^2-12z+9} - \frac{z+1}{2z^2-3z} &= \frac{2}{(2z-3)^2} - \frac{z+1}{z(2z-3)} \quad \text{LCD} = z(2z-3)^2 \\
 &= \frac{2}{(2z-3)^2} \cdot \frac{z}{z} - \frac{z+1}{z(2z-3)} \cdot \frac{2z-3}{2z-3} \\
 &= \frac{2z - (2z^2 - z - 3)}{z(2z-3)^2} \\
 &= \frac{2z - 2z^2 + z + 3}{z(2z-3)^2} \\
 &= \frac{-2z^2 + 3z + 3}{z(2z-3)^2}
 \end{aligned}$$

$$\begin{aligned}
 65. \quad \frac{t+1}{t+3} - \frac{t-2}{t-3} + \frac{6}{t^2-9} &= \frac{t+1}{t+3} - \frac{t-2}{t-3} + \frac{6}{(t+3)(t-3)} && \text{LCD}=(t+3)(t-3) \\
 &= \frac{t+1}{t+3} \cdot \frac{t-3}{t-3} - \frac{t-2}{t-3} \cdot \frac{t+3}{t+3} + \frac{6}{(t+3)(t-3)} = \frac{t^2-2t-3-(t^2+t-6)+6}{(t+3)(t-3)} \\
 &= \frac{t^2-2t-3-t^2-t+6+6}{(t+3)(t-3)} = \frac{-3t+9}{(t+3)(t-3)} = \frac{-3\cancel{(t-3)}}{(t+3)\cancel{(t-3)}} = -\frac{3}{t+3}
 \end{aligned}$$

$$\begin{aligned}
 66. \quad \frac{y-3}{y-2} - \frac{y+1}{2y-5} + \frac{-4y+7}{2y^2-9y+10} \\
 &= \frac{y-3}{y-2} - \frac{y+1}{2y-5} + \frac{-4y+7}{(2y-5)(y-2)} && \text{LCD}=(2y-5)(y-2) \\
 &= \frac{y-3}{y-2} \cdot \frac{2y-5}{2y-5} - \frac{y+1}{2y-5} \cdot \frac{y-2}{y-2} + \frac{-4y+7}{(2y-5)(y-2)} \\
 &= \frac{2y^2-11y+15-(y^2-y-2)-4y+7}{(2y-5)(y-2)} \\
 &= \frac{2y^2-11y+15-y^2+y+2-4y+7}{(2y-5)(y-2)} \\
 &= \frac{y^2-14y+24}{(2y-5)(y-2)} = \frac{(y-12)\cancel{(y-2)}}{(2y-5)\cancel{(y-2)}} = \frac{y-12}{2y-5}
 \end{aligned}$$

$$\begin{aligned}
 67. \quad (x-1) \cdot \left[\frac{3}{x^2-1} + \frac{x}{2x-2} \right] &= \frac{3\cancel{(x-1)}}{(x+1)\cancel{(x-1)}} + \frac{x\cancel{(x-1)}}{2\cancel{(x-1)}} = \frac{3}{x+1} + \frac{x}{2} && \text{LCD}=2(x+1) \\
 &= \frac{3}{x+1} \cdot \frac{2}{2} + \frac{x}{2} \cdot \frac{x+1}{x+1} = \frac{6+x^2+x}{2(x+1)} = \frac{x^2+x+6}{2(x+1)}
 \end{aligned}$$

$$\begin{aligned}
 68. \quad (3x-2) \cdot \left[\frac{x}{3x^2+x-2} + \frac{2}{x+1} \right] &= \frac{x\cancel{(3x-2)}}{\cancel{(3x-2)}(x+1)} + \frac{2(3x-2)}{x+1} = \frac{x}{x+1} + \frac{6x-4}{x+1} \\
 &= \frac{x}{x+1} + \frac{6x-4}{x+1} = \frac{7x-4}{x+1}
 \end{aligned}$$

$$\begin{aligned}
 69. \quad \frac{3z}{z-3} - \frac{z}{z+4} &&& \text{LCD}=(z-3)(z+4) \\
 &= \frac{3z}{z-3} \cdot \frac{z+4}{z+4} - \frac{z}{z+4} \cdot \frac{z-3}{z-3} = \frac{3z^2+12z-(z^2-3z)}{(z-3)(z+4)} = \frac{3z^2+12z-z^2+3z}{(z-3)(z+4)} = \frac{2z^2+15z}{(z-3)(z+4)}
 \end{aligned}$$

70. $\frac{2p}{p-5} - \frac{p}{p+6}$ LCD = $(p-5)(p+6)$
- $$= \frac{2p}{p-5} \cdot \frac{p+6}{p+6} - \frac{p}{p+6} \cdot \frac{p-5}{p-5} = \frac{2p^2 + 12p - (p^2 - 5p)}{(p-5)(p+6)}$$
- $$= \frac{2p^2 + 12p - p^2 + 5p}{(p-5)(p+6)} = \frac{p^2 + 17p}{(p-5)(p+6)}$$
71. $\frac{2x}{x^2 - y^2} - \frac{1}{x-y} + \frac{1}{y-x}$ LCD = $(x+y)(x-y)$
- $$= \frac{2x}{(x+y)(x-y)} - \frac{1}{x-y} + \frac{1}{y-x}$$
- $$= \frac{2x}{(x+y)(x-y)} - \frac{1}{x-y} \cdot \frac{x+y}{x+y} + \frac{1}{y-x} \cdot \frac{(-1)}{(-1)} \cdot \frac{x+y}{x+y}$$
- $$= \frac{2x - 1(x+y) - 1(x+y)}{(x+y)(x-y)} = \frac{2x - x - y - x - y}{(x+y)(x-y)}$$
- $$= \frac{-2y}{(x+y)(x-y)}$$
72. $\frac{3w-1}{2w^2 + w - 3} - \frac{2-w}{w-1} - \frac{w}{1-w}$ LCD = $(2w+3)(w-1)$
- $$= \frac{3w-1}{(2w+3)(w-1)} - \frac{2-w}{w-1} - \frac{w}{1-w}$$
- $$= \frac{3w-1}{(2w+3)(w-1)} - \frac{2-w}{w-1} \cdot \frac{2w+3}{2w+3} - \frac{w}{1-w} \cdot \frac{(-1)}{(-1)} \cdot \frac{2w+3}{2w+3}$$
- $$= \frac{3w-1 - (-2w^2 + w + 6) - (-2w^2 - 3w)}{(2w+3)(w-1)}$$
- $$= \frac{3w-1 + 2w^2 - w - 6 + 2w^2 + 3w}{(2w+3)(w-1)} = \frac{4w^2 + 5w - 7}{(2w+3)(w-1)}$$
73. $(2p+1) \cdot \left[\frac{2p}{6p+3} - \frac{1}{p+4} \right]$ LCD = $3(p+4)$
- $$= \frac{2p(2p+1)}{3(2p+1)} - \frac{2p+1}{p+4} = \frac{2p}{3} - \frac{2p+1}{p+4}$$
- $$= \frac{2p}{3} \cdot \frac{p+4}{p+4} - \frac{2p+1}{p+4} \cdot \frac{3}{3} = \frac{2p^2 + 8p - (6p+3)}{3(p+4)} = \frac{2p^2 + 8p - 6p - 3}{3(p+4)}$$
- $$= \frac{2p^2 + 2p - 3}{3(p+4)}$$

$$\begin{aligned}
 74. \quad (y+8) \cdot \left[\frac{4}{2y+1} - \frac{y}{2y^2+17y+8} \right] &= \frac{4(y+8)}{2y+1} - \frac{\cancel{y(y+8)}}{(2y+1)\cancel{(y+8)}} \quad \text{LCD} = (2y+1) \\
 &= \frac{4y+32}{2y+1} - \frac{y}{2y+1} = \frac{4y+32-y}{2y+1} = \frac{3y+32}{2y+1}
 \end{aligned}$$

$$\begin{aligned}
 75. \quad \frac{1}{x+5} + \frac{3}{(x+5)^2} - \frac{2}{(x+5)^3} \quad \text{LCD} = (x+5)^3 \\
 &= \frac{1}{x+5} \cdot \frac{(x+5)^2}{(x+5)^2} + \frac{3}{(x+5)^2} \cdot \frac{x+5}{x+5} - \frac{2}{(x+5)^3} = \frac{(x+5)^2}{(x+5)^3} + \frac{3x+15}{(x+5)^3} - \frac{2}{(x+5)^3} \\
 &= \frac{x^2+10x+25+3x+15-2}{(x+5)^3} = \frac{x^2+13x+38}{(x+5)^3}
 \end{aligned}$$

$$\begin{aligned}
 76. \quad \frac{1}{x-2} + \frac{4}{(x-2)^2} - \frac{3}{(x-2)^3} \quad \text{LCD} = (x-2)^3 \\
 &= \frac{1}{x-2} \cdot \frac{(x-2)^2}{(x-2)^2} + \frac{4}{(x-2)^2} \cdot \frac{x-2}{x-2} - \frac{3}{(x-2)^3} \\
 &= \frac{(x-2)^2}{(x-2)^3} + \frac{4x-8}{(x-2)^3} - \frac{3}{(x-2)^3} \\
 &= \frac{x^2-4x+4+4x-8-3}{(x-2)^3} = \frac{x^2-7}{(x-2)^3}
 \end{aligned}$$

$$\begin{aligned}
 77. \quad \frac{-10}{z^2-6z+5} + \frac{15}{z^2-4z-5} &= \frac{-10}{(z-5)(z-1)} + \frac{15}{(z-5)(z+1)} \quad \text{LCD} = (z-5)(z-1)(z+1) \\
 &= \frac{-10}{(z-5)(z-1)} \cdot \frac{z+1}{z+1} + \frac{15}{(z-5)(z+1)} \cdot \frac{z-1}{z-1} = \frac{-10z-10}{(z-5)(z-1)(z+1)} + \frac{15z-15}{(z-5)(z-1)(z+1)} \\
 &= \frac{5z-25}{(z-5)(z-1)(z+1)} \\
 &= \frac{\cancel{5(z-5)}}{\cancel{(z-5)}(z-1)(z+1)} \\
 &= \frac{5}{(z-1)(z+1)}
 \end{aligned}$$

$$\begin{aligned}
78. \quad \frac{-4}{n^2+6n+5} + \frac{3}{n^2+7n+10} &= \frac{-4}{(n+5)(n+1)} + \frac{3}{(n+5)(n+2)} \quad \text{LCD} = (n+5)(n+1)(n+2) \\
&= \frac{-4}{(n+5)(n+1)} \cdot \frac{n+2}{n+2} + \frac{3}{(n+5)(n+2)} \cdot \frac{n+1}{n+1} \\
&= \frac{-4n-8}{(n+5)(n+1)(n+2)} + \frac{3n+3}{(n+5)(n+1)(n+2)} = \frac{-n-5}{(n+5)(n+1)(n+2)} \\
&= \frac{-\cancel{(n+5)}}{\cancel{(n+5)}(n+1)(n+2)} = -\frac{1}{(n+1)(n+2)}
\end{aligned}$$

$$\begin{aligned}
79. \quad \frac{5}{x^2-4} + \frac{2}{x^3-8} \\
&= \frac{5}{(x-2)(x+2)} + \frac{2}{(x-2)(x^2+2x+4)} \\
\text{LCD} &= (x-2)(x+2)(x^2+2x+4) \\
&= \frac{5}{(x-2)(x+2)} \cdot \frac{x^2+2x+4}{x^2+2x+4} + \frac{2}{(x-2)(x^2+2x+4)} \cdot \frac{x+2}{x+2} \\
&= \frac{5x^2+10x+20+2x+4}{(x-2)(x+2)(x^2+2x+4)} \\
&= \frac{5x^2+12x+24}{(x-2)(x+2)(x^2+2x+4)}
\end{aligned}$$

$$\begin{aligned}
80. \quad \frac{-2}{x^2-9} + \frac{3}{x^3-27} \\
&= \frac{-2}{(x-3)(x+3)} + \frac{3}{(x-3)(x^2+3x+9)} \\
\text{LCD} &= (x-3)(x+3)(x^2+3x+9) \\
&= \frac{-2}{(x-3)(x+3)} \cdot \frac{x^2+3x+9}{x^2+3x+9} + \frac{3}{(x-3)(x^2+3x+9)} \cdot \frac{x+3}{x+3} \\
&= \frac{-2x^2-6x-18+3x+9}{(x-3)(x+3)(x^2+3x+9)} \\
&= \frac{-2x^2-3x-9}{(x-3)(x+3)(x^2+3x+9)}
\end{aligned}$$

$$\begin{aligned}
 81. \quad & \frac{2}{3x} + \frac{x+1}{x} + \frac{6}{x^2} \quad \text{LCD} = 3x^2 \\
 & = \frac{2}{3x} \cdot \frac{x}{x} + \frac{x+1}{x} \cdot \frac{3x}{3x} + \frac{6}{x^2} \cdot \frac{3}{3} \\
 & = \frac{2x+3x^2+3x+18}{3x^2} \\
 & = \frac{3x^2+5x+18}{3x^2} \text{ cm}
 \end{aligned}$$

$$\begin{aligned}
 82. \quad & \frac{x+1}{x} + \frac{2}{x^2} + \frac{x-4}{3} \quad \text{LCD} = 3x^2 \\
 & = \frac{x+1}{x} \cdot \frac{3x}{3x} + \frac{2}{x^2} \cdot \frac{3}{3} + \frac{x-4}{3} \cdot \frac{x^2}{x^2} \\
 & = \frac{3x^2+3x+6+x^3-4x^2}{3x^2} \\
 & = \frac{x^3-x^2+3x+6}{3x^2} \text{ yd}
 \end{aligned}$$

$$\begin{aligned}
 83. \quad & 2\left(\frac{5}{x-3}\right) + 2\left(\frac{2x}{x+5}\right) \\
 & \quad \text{LCD} = (x-3)(x+5) \\
 & = \frac{10}{x-3} \cdot \frac{x+5}{x+5} + \frac{4x}{x+5} \cdot \frac{x-3}{x-3} \\
 & = \frac{10x+50+4x^2-12x}{(x-3)(x+5)} \\
 & = \frac{4x^2-2x+50}{(x-3)(x+5)} \text{ m}
 \end{aligned}$$

$$\begin{aligned}
 84. \quad & 2\left(\frac{3}{x+2}\right) + 2\left(\frac{x}{x+1}\right) \\
 & \quad \text{LCD} = (x+2)(x+1) \\
 & = \frac{6}{x+2} \cdot \frac{x+1}{x+1} + \frac{2x}{x+1} \cdot \frac{x+2}{x+2} \\
 & = \frac{6x+6+2x^2+4x}{(x+2)(x+1)} \\
 & = \frac{2x^2+10x+6}{(x+2)(x+1)} \text{ ft}
 \end{aligned}$$

Section 5.4 Practice Exercises

1. complex

$$\begin{aligned}
 2. \quad & \frac{x^3+y^3}{5x+5y} = \frac{\cancel{(x+y)}(x^2-xy+y^2)}{5\cancel{(x+y)}} \\
 & = \frac{x^2-xy+y^2}{5}
 \end{aligned}$$

$$3. \quad \frac{25a^3b^3c}{15a^4bc} = \frac{25}{15}a^{3-4}b^{3-1}c^{1-1} = \frac{5}{3}a^{-1}b^2c^0 = \frac{5b^2}{3a}$$

$$\begin{aligned}
 4. \quad & f(t) = \frac{6t^2-27t+30}{12t-30} \\
 & = \frac{3(2t^2-9t+10)}{6(2t-5)} \\
 & = \frac{\cancel{3}(2t-5)(t-2)}{\cancel{3} \cdot 2(2t-5)} = \frac{t-2}{2}
 \end{aligned}$$

$$\begin{aligned}
 5. \quad & \frac{5}{x^2} + \frac{3}{2x} \quad \text{LCD} = 2x^2 \\
 & = \frac{5}{x^2} \cdot \frac{2}{2} + \frac{3}{2x} \cdot \frac{x}{x} \\
 & = \frac{10+3x}{2x^2}
 \end{aligned}$$

$$\begin{aligned}
 6. \quad \frac{2y-4}{y+1} \cdot \frac{y^2+3y+2}{y^2-4} \\
 = \frac{2\cancel{(y-2)}}{y+1} \cdot \frac{\cancel{(y+2)}(y+1)}{\cancel{(y+2)}(y-2)} = 2
 \end{aligned}$$

$$\begin{aligned}
 7. \quad \frac{3}{a-5} - \frac{1}{a+1} \quad \text{LCD} = (a-5)(a+1) \\
 = \frac{3}{a-5} \cdot \frac{a+1}{a+1} - \frac{1}{a+1} \cdot \frac{a-5}{a-5} \\
 = \frac{3a+3-a+5}{(a-5)(a+1)} = \frac{2a+8}{(a-5)(a+1)}
 \end{aligned}$$

$$\begin{aligned}
 8. \quad \frac{7}{12-6b} \div \frac{14b}{b^2+b-6} = \frac{7}{12-6b} \cdot \frac{b^2+b-6}{14b} \\
 = \frac{\cancel{7}}{-6\cancel{(b-2)}} \cdot \frac{(b+3)\cancel{(b-2)}}{\cancel{7} \cdot 2b} = -\frac{b+3}{12b}
 \end{aligned}$$

$$\begin{aligned}
 9. \quad \frac{5x^2}{9y^2} = \frac{5x^2}{9y^2} \cdot \frac{\cancel{y^2}}{\cancel{y^2}} \cdot \frac{\cancel{3}}{\cancel{3}} = \frac{5x^2}{27y^2}
 \end{aligned}$$

$$\begin{aligned}
 10. \quad \frac{\frac{3w^2}{4rs}}{\frac{15wr}{s^2}} = \frac{3w^2}{4rs} \cdot \frac{s^2}{15wr} = \frac{\cancel{3} \cdot \cancel{w} \cdot w \cdot \cancel{s} \cdot s}{4 \cdot \cancel{3} \cdot 5 \cdot r^2 \cdot \cancel{s} \cdot \cancel{w}} \\
 = \frac{ws}{20r^2}
 \end{aligned}$$

$$\begin{aligned}
 11. \quad \frac{\frac{x-6}{3x}}{\frac{3x-18}{9}} = \frac{x-6}{3x} \cdot \frac{9}{3x-18} \\
 = \frac{\cancel{x-6}}{\cancel{3}x} \cdot \frac{\cancel{3} \cdot \cancel{3}}{\cancel{3}(x-6)} = \frac{1}{x}
 \end{aligned}$$

$$\begin{aligned}
 12. \quad \frac{\frac{a+4}{6}}{\frac{16-a^2}{3}} = \frac{a+4}{6} \cdot \frac{3}{16-a^2} \\
 = \frac{\cancel{a+4}}{\cancel{3} \cdot 2} \cdot \frac{\cancel{3}}{(\cancel{4+a})(4-a)} \\
 = \frac{1}{2(4-a)}
 \end{aligned}$$

$$\begin{aligned}
 13. \quad \frac{\frac{2}{3} + \frac{1}{6}}{\frac{1}{2} - \frac{1}{4}} = \frac{\frac{4}{6} + \frac{1}{6}}{\frac{2}{4} - \frac{1}{4}} = \frac{\frac{5}{6}}{\frac{1}{4}} = \frac{5}{6} \cdot \frac{4}{1} \\
 = \frac{5 \cdot \cancel{2} \cdot 2}{\cancel{2} \cdot 3 \cdot 1} = \frac{10}{3}
 \end{aligned}$$

$$\begin{aligned}
 14. \quad \frac{\frac{7}{8} + \frac{3}{4}}{\frac{1}{3} - \frac{5}{6}} = \frac{\frac{7}{8} + \frac{6}{8}}{\frac{2}{6} - \frac{5}{6}} = \frac{\frac{13}{8}}{\frac{-3}{6}} = \frac{13}{8} \cdot \frac{6}{-3} \\
 = -\frac{13}{4} \cdot \frac{\cancel{2} \cdot \cancel{3}}{\cancel{3}} = -\frac{13}{4}
 \end{aligned}$$

$$\begin{aligned}
 15. \quad \frac{8 - \frac{5}{2x}}{\frac{5}{8x} - 2} = \frac{8 \cdot \frac{2x}{2x} - \frac{5}{2x}}{\frac{5}{8x} - 2 \cdot \frac{8x}{8x}} = \frac{\frac{16x-5}{2x}}{\frac{5-16x}{8x}} \\
 = \frac{\cancel{16x-5}}{\cancel{2}x} \cdot \frac{\cancel{2} \cdot 4x}{-(\cancel{16x-5})} = \frac{4}{-1} = -4
 \end{aligned}$$

$$\begin{aligned}
 16. \quad \frac{10 - \frac{3}{5x}}{\frac{3}{10x} - 5} &= \frac{10 \cdot \frac{5x}{5x} - \frac{3}{5x}}{\frac{3}{10x} - 5 \cdot \frac{10x}{10x}} \\
 &= \frac{\frac{50x - 3}{50x - 3}}{\frac{3 - 50x}{50x - 3}} \\
 &= \frac{10x}{\frac{3 - 50x}{10x}} \\
 &= \frac{50x - 3}{5x} \cdot \frac{2 \cdot 5x}{-(50x - 3)} \\
 &= \frac{2}{-1} = -2
 \end{aligned}$$

$$\begin{aligned}
 17. \quad \frac{\frac{7y}{y+3}}{\frac{1}{4y+12}} &= \frac{\frac{7y}{y+3}}{\frac{1}{4(y+3)}} \quad \text{LCD} = 4(y+3) \\
 &= \frac{4(\cancel{y+3}) \left(\frac{7y}{\cancel{y+3}} \right)}{\cancel{4}(\cancel{y+3}) \left(\frac{1}{\cancel{4}(\cancel{y+3})} \right)} = \frac{4(7y)}{1} = 28y
 \end{aligned}$$

$$\begin{aligned}
 18. \quad \frac{\frac{6x}{x-5}}{\frac{1}{4x-20}} &= \frac{\frac{6x}{x-5}}{\frac{1}{4(x-5)}} \quad \text{LCD} = 4(x-5) \\
 &= \frac{4(\cancel{x-5}) \left(\frac{6x}{\cancel{x-5}} \right)}{\cancel{4}(\cancel{x-5}) \left(\frac{1}{\cancel{4}(\cancel{x-5})} \right)} = \frac{4(6x)}{1} = 24x
 \end{aligned}$$

$$\begin{aligned}
 19. \quad \frac{1 + \frac{1}{3}}{\frac{5}{6} - 1} & \quad \text{LCD} = 6 \\
 &= \frac{6 \left(1 + \frac{1}{3} \right)}{6 \left(\frac{5}{6} - 1 \right)} = \frac{6 \cdot 1 + 6 \left(\frac{1}{3} \right)}{6 \left(\frac{5}{6} \right) - 6 \cdot 1} = \frac{6 + 2}{5 - 6} = \frac{8}{-1} = -8
 \end{aligned}$$

$$\begin{aligned}
 20. \quad \frac{2 + \frac{4}{5}}{-1 + \frac{3}{10}} & \quad \text{LCD} = 10 \\
 &= \frac{10 \left(2 + \frac{4}{5} \right)}{10 \left(-1 + \frac{3}{10} \right)} = \frac{10 \cdot 2 + 10 \left(\frac{4}{5} \right)}{10(-1) + 10 \left(\frac{3}{10} \right)} \\
 &= \frac{20 + 8}{-10 + 3} \\
 &= \frac{28}{-7} = -4
 \end{aligned}$$

$$\begin{aligned}
 21. \quad \frac{\frac{3q}{p} - q}{q - \frac{q}{p}} & \quad \text{LCD} = p \\
 &= \frac{p \left(\frac{3q}{p} - q \right)}{p \left(q - \frac{q}{p} \right)} = \frac{\cancel{p} \left(\frac{3q}{\cancel{p}} \right) - pq}{pq - \cancel{p} \left(\frac{q}{\cancel{p}} \right)} \\
 &= \frac{3q - pq}{pq - q} \\
 &= \frac{\cancel{q}(3-p)}{\cancel{q}(p-1)} = \frac{3-p}{p-1}
 \end{aligned}$$

$$\begin{aligned}
 22. \quad & \frac{\frac{b}{a} + 3b}{b + \frac{2b}{a}} \quad \text{LCD} = a \\
 & \frac{a\left(\frac{b}{a} + 3b\right)}{a\left(b + \frac{2b}{a}\right)} = \frac{\cancel{a}\left(\frac{b}{\cancel{a}}\right) + a(3b)}{a(b) + \cancel{a}\left(\frac{2b}{\cancel{a}}\right)} = \frac{b + 3ab}{ab + 2b} \\
 & = \frac{\cancel{b}(1 + 3a)}{\cancel{b}(a + 2)} = \frac{1 + 3a}{a + 2}
 \end{aligned}$$

$$\begin{aligned}
 23. \quad & \frac{\frac{2}{a} + \frac{3}{a^2}}{\frac{4}{a^2} - \frac{9}{a}} \quad \text{LCD} = a^2 \\
 & \frac{a^2\left(\frac{2}{a} + \frac{3}{a^2}\right)}{a^2\left(\frac{4}{a^2} - \frac{9}{a}\right)} = \frac{a^2\left(\frac{2}{a}\right) + a^2\left(\frac{3}{a^2}\right)}{a^2\left(\frac{4}{a^2}\right) - a^2\left(\frac{9}{a}\right)} \\
 & = \frac{2a + 3}{4 - 9a}
 \end{aligned}$$

$$\begin{aligned}
 24. \quad & \frac{\frac{2}{y^2} + \frac{1}{y}}{\frac{4}{y^2} - \frac{1}{y}} \quad \text{LCD} = y^2 \\
 & \frac{y^2\left(\frac{2}{y^2} + \frac{1}{y}\right)}{y^2\left(\frac{4}{y^2} - \frac{1}{y}\right)} = \frac{y^2\left(\frac{2}{y^2}\right) + y^2\left(\frac{1}{y}\right)}{y^2\left(\frac{4}{y^2}\right) - y^2\left(\frac{1}{y}\right)} = \frac{2 + y}{4 - y}
 \end{aligned}$$

$$\begin{aligned}
 25. \quad & \frac{t^{-1} - 1}{1 - t^{-2}} = \frac{\frac{1}{t} - 1}{1 - \frac{1}{t^2}} \quad \text{LCD} = t^2 \\
 & \frac{t^2\left(\frac{1}{t} - 1\right)}{t^2\left(1 - \frac{1}{t^2}\right)} = \frac{t^2\left(\frac{1}{t}\right) - t^2(1)}{t^2(1) - t^2\left(\frac{1}{t^2}\right)} = \frac{t - t^2}{t^2 - 1} \\
 & = \frac{-t(\cancel{t-1})}{(t+1)(\cancel{t-1})} = -\frac{t}{t+1}
 \end{aligned}$$

$$\begin{aligned}
 26. \quad & \frac{d^{-2} - c^{-2}}{c^{-1} - d^{-1}} = \frac{\frac{1}{d^2} - \frac{1}{c^2}}{\frac{1}{c} - \frac{1}{d}} \quad \text{LCD} = c^2 d^2 \\
 & \frac{c^2 d^2\left(\frac{1}{d^2} - \frac{1}{c^2}\right)}{c^2 d^2\left(\frac{1}{c} - \frac{1}{d}\right)} = \frac{c^2 d^2\left(\frac{1}{d^2}\right) - c^2 d^2\left(\frac{1}{c^2}\right)}{c^2 d^2\left(\frac{1}{c}\right) - c^2 d^2\left(\frac{1}{d}\right)} \\
 & = \frac{c^2 - d^2}{cd^2 - c^2 d} = \frac{(c+d)(\cancel{c-d})}{-cd(\cancel{c-d})} \\
 & = -\frac{c+d}{cd}
 \end{aligned}$$

$$\begin{aligned}
 27. \quad & \frac{-8}{\frac{6w}{w-1} - 4} \quad \text{LCD} = w - 1 \\
 & \frac{(w-1)(-8)}{(w-1)\left(\frac{6w}{w-1} - 4\right)} \\
 & = \frac{(w-1)(-8)}{(\cancel{w-1})\left(\frac{6w}{\cancel{w-1}}\right) - (w-1)4} \\
 & = \frac{-8w + 8}{6w - 4w + 4} = \frac{-8w + 8}{2w + 4} \\
 & = \frac{-4 \cdot \cancel{2}(w-1)}{\cancel{2}(w+2)} = -\frac{4(w-1)}{w+2}
 \end{aligned}$$

$$\begin{aligned}
 28. \quad & \frac{6}{2z - \frac{10}{z-4}} \quad \text{LCD} = z-4 \\
 & \frac{(z-4)(6)}{(z-4)\left(2z - \frac{10}{z-4}\right)} \\
 & = \frac{(z-4)(6)}{(z-4)(2z) - \cancel{(z-4)}\left(\frac{10}{\cancel{z-4}}\right)} \\
 & = \frac{6z-24}{2z^2-8z-10} = \frac{6(z-4)}{2(z^2-4z-5)} \\
 & = \frac{\cancel{2} \cdot 3(z-4)}{\cancel{2}(z-5)(z+1)} \\
 & = \frac{3(z-4)}{(z-5)(z+1)}
 \end{aligned}$$

$$\begin{aligned}
 29. \quad & \frac{y}{y+3} \quad \text{LCD} = y+3 \\
 & \frac{y}{y+3} + y \\
 & \frac{(y+3)\left(\frac{y}{y+3}\right)}{(y+3)\left(\frac{y}{y+3} + y\right)} \\
 & = \frac{\cancel{(y+3)}\left(\frac{y}{\cancel{y+3}}\right)}{\cancel{(y+3)}\left(\frac{y}{\cancel{y+3}} + (y+3)y\right)} \\
 & = \frac{y}{y+y^2+3y} = \frac{y}{y^2+4y} \\
 & = \frac{y}{y(y+4)} = \frac{1}{y+4}
 \end{aligned}$$

$$\begin{aligned}
 30. \quad & \frac{4}{\frac{w-4}{4} - 1} \quad \text{LCD} = w-4 \\
 & \frac{(w-4)\left(\frac{4}{w-4}\right)}{(w-4)\left(\frac{4}{w-4} - 1\right)} \\
 & = \frac{\cancel{(w-4)}\left(\frac{4}{\cancel{w-4}}\right)}{\cancel{(w-4)}\left(\frac{4}{\cancel{w-4}} - (w-4)1\right)} \\
 & = \frac{4}{4-w+4} \\
 & = \frac{4}{8-w}
 \end{aligned}$$

$$\begin{aligned}
 31. \quad & 1 - \frac{1}{x} - \frac{6}{x^2} \quad \text{LCD} = x^2 \\
 & 1 - \frac{4}{x} + \frac{3}{x^2} \\
 & \frac{x^2\left(1 - \frac{1}{x} - \frac{6}{x^2}\right)}{x^2\left(1 - \frac{4}{x} + \frac{3}{x^2}\right)} \\
 & = \frac{x^2(1) - x^2\left(\frac{1}{x}\right) - x^2\left(\frac{6}{x^2}\right)}{x^2(1) - x^2\left(\frac{4}{x}\right) + x^2\left(\frac{3}{x^2}\right)} \\
 & = \frac{x^2 - x - 6}{x^2 - 4x + 3} \\
 & = \frac{\cancel{(x-3)}(x+2)}{\cancel{(x-3)}(x-1)} = \frac{x+2}{x-1}
 \end{aligned}$$

$$\begin{aligned}
 32. \quad & \frac{1 + \frac{1}{x} - \frac{12}{x^2}}{\frac{9}{x^2} + \frac{3}{x} - 2} \quad \text{LCD} = x^2 \\
 & \frac{x^2 \left(1 + \frac{1}{x} - \frac{12}{x^2} \right)}{x^2 \left(\frac{9}{x^2} + \frac{3}{x} - 2 \right)} = \frac{x^2(1) + x^2 \left(\frac{1}{x} \right) - x^2 \left(\frac{12}{x^2} \right)}{x^2 \left(\frac{9}{x^2} \right) + x^2 \left(\frac{3}{x} \right) - x^2(2)} = \frac{x^2 + x - 12}{9 + 3x - 2x^2} = \frac{x^2 + x - 12}{-(2x^2 - 3x - 9)} \\
 & = -\frac{(x+4)(\cancel{x-3})}{(2x+3)(\cancel{x-3})} = -\frac{x+4}{2x+3}
 \end{aligned}$$

$$\begin{aligned}
 33. \quad & \frac{2 - \frac{2}{t+1}}{2 + \frac{2}{t}} \quad \text{LCD} = t(t+1) \\
 & \frac{t(t+1) \left(2 - \frac{2}{t+1} \right)}{t(t+1) \left(2 + \frac{2}{t} \right)} = \frac{t(t+1)(2) - t(\cancel{t+1}) \left(\frac{2}{\cancel{t+1}} \right)}{t(t+1)(2) + \cancel{t}(t+1) \left(\frac{2}{\cancel{t}} \right)} = \frac{2t^2 + 2t - 2t}{2t^2 + 2t + 2t + 2} \\
 & = \frac{\cancel{2}t^2}{\cancel{2}(t^2 + 2t + 1)} = \frac{t^2}{(t+1)^2}
 \end{aligned}$$

$$\begin{aligned}
 34. \quad & \frac{3 + \frac{3}{p-1}}{3 - \frac{3}{p}} \quad \text{LCD} = p(p-1) \\
 & \frac{p(p-1) \left(3 + \frac{3}{p-1} \right)}{p(p-1) \left(3 - \frac{3}{p} \right)} = \frac{p(p-1)(3) + p(\cancel{p-1}) \left(\frac{3}{\cancel{p-1}} \right)}{p(p-1)(3) - \cancel{p}(p-1) \left(\frac{3}{\cancel{p}} \right)} \\
 & = \frac{3p^2 - 3p + 3p}{3p^2 - 3p - 3p + 3} \\
 & = \frac{\cancel{3}p^2}{\cancel{3}(p^2 - 2p + 1)} = \frac{p^2}{(p-1)^2}
 \end{aligned}$$

$$\begin{aligned}
 35. \quad & \frac{\frac{2}{a} - \frac{3}{a+1}}{\frac{2}{a+1} - \frac{3}{a}} \quad \text{LCD} = a(a+1) \\
 & \frac{a(a+1)\left(\frac{2}{a} - \frac{3}{a+1}\right)}{a(a+1)\left(\frac{2}{a+1} - \frac{3}{a}\right)} = \frac{\cancel{a}(a+1)\left(\frac{2}{\cancel{a}}\right) - a(\cancel{a+1})\left(\frac{3}{\cancel{a+1}}\right)}{a(\cancel{a+1})\left(\frac{2}{\cancel{a+1}}\right) - \cancel{a}(a+1)\left(\frac{3}{\cancel{a}}\right)} = \frac{2a+2-3a}{2a-3a-3} = \frac{-a+2}{-a-3}
 \end{aligned}$$

$$\begin{aligned}
 36. \quad & \frac{\frac{5}{b} + \frac{4}{b+1}}{\frac{4}{b} - \frac{5}{b+1}} \quad \text{LCD} = b(b+1) \\
 & \frac{b(b+1)\left(\frac{5}{b} + \frac{4}{b+1}\right)}{b(b+1)\left(\frac{4}{b} - \frac{5}{b+1}\right)} = \frac{\cancel{b}(b+1)\left(\frac{5}{\cancel{b}}\right) + b(\cancel{b+1})\left(\frac{4}{\cancel{b+1}}\right)}{\cancel{b}(b+1)\left(\frac{4}{\cancel{b}}\right) - b(\cancel{b+1})\left(\frac{5}{\cancel{b+1}}\right)} = \frac{5b+5+4b}{4b+4-5b} = \frac{9b+5}{-b+4}
 \end{aligned}$$

$$\begin{aligned}
 37. \quad & \frac{\frac{1}{y+2} + \frac{4}{y-3}}{\frac{2}{y-3} - \frac{7}{y+2}} \quad \text{LCD} = (y+2)(y-3) \\
 & \frac{(y+2)(y-3)\left(\frac{1}{y+2} + \frac{4}{y-3}\right)}{(y+2)(y-3)\left(\frac{2}{y-3} - \frac{7}{y+2}\right)} = \frac{(\cancel{y+2})(y-3)\left(\frac{1}{\cancel{y+2}}\right) + (y+2)(\cancel{y-3})\left(\frac{4}{\cancel{y-3}}\right)}{(y+2)(\cancel{y-3})\left(\frac{2}{\cancel{y-3}}\right) - (\cancel{y+2})(y-3)\left(\frac{7}{\cancel{y+2}}\right)} \\
 & \quad \quad \quad = \frac{y-3+4y+8}{2y+4-7y+21} = \frac{5y+5}{-5y+25} = \frac{\cancel{5}(y+1)}{-\cancel{5}(y-5)} = -\frac{y+1}{y-5}
 \end{aligned}$$

$$\begin{aligned}
 38. \quad & \frac{\frac{1}{t-4} + \frac{1}{t+5}}{\frac{6}{t+5} + \frac{2}{t-4}} \quad \text{LCD} = (t-4)(t+5) \\
 & \frac{(t-4)(t+5)\left(\frac{1}{t-4} + \frac{1}{t+5}\right)}{(t-4)(t+5)\left(\frac{6}{t+5} + \frac{2}{t-4}\right)} = \frac{(\cancel{t-4})(t+5)\left(\frac{1}{\cancel{t-4}}\right) + (t-4)(\cancel{t+5})\left(\frac{1}{\cancel{t+5}}\right)}{(t-4)(\cancel{t+5})\left(\frac{6}{\cancel{t+5}}\right) + (\cancel{t-4})(t+5)\left(\frac{2}{\cancel{t-4}}\right)}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{t+5+t-4}{6t-24+2t+10} \\
 &= \frac{2t+1}{8t-14} = \frac{2t+1}{2(4t-7)}
 \end{aligned}$$

$$39. \quad \frac{\frac{2}{x+h} - \frac{2}{x}}{h} \quad \text{LCD} = x(x+h)$$

$$\begin{aligned}
 \frac{x(x+h)\left(\frac{2}{x+h} - \frac{2}{x}\right)}{x(x+h)(h)} &= \frac{x(\cancel{x+h})\left(\frac{2}{\cancel{x+h}}\right) - \cancel{x}(x+h)\left(\frac{2}{\cancel{x}}\right)}{x(x+h)(h)} = \frac{2x - 2x - 2h}{x(x+h)(h)} \\
 &= \frac{-2\cancel{h}}{x(x+h)(\cancel{h})} = -\frac{2}{x(x+h)}
 \end{aligned}$$

$$40. \quad \frac{\frac{1}{2x+2h} - \frac{1}{2x}}{h} = \frac{\frac{1}{2(x+h)} - \frac{1}{2x}}{h} \quad \text{LCD} = 2x(x+h)$$

$$\begin{aligned}
 \frac{2x(x+h)\left(\frac{1}{2(x+h)} - \frac{1}{2x}\right)}{2x(x+h)(h)} &= \frac{\cancel{2}x(\cancel{x+h})\left(\frac{1}{\cancel{2}(x+h)}\right) - \cancel{2}\cancel{x}(x+h)\left(\frac{1}{\cancel{2}\cancel{x}}\right)}{2x(x+h)(h)} \\
 &= \frac{x-x-h}{2x(x+h)(h)} = \frac{-\cancel{h}}{2x(x+h)(\cancel{h})} = -\frac{1}{2x(x+h)}
 \end{aligned}$$

$$41. \quad \frac{x^{-2}}{x+3x^{-1}} = \frac{\frac{1}{x^2}}{x+\frac{3}{x}} \quad \text{LCD} = x^2$$

$$\begin{aligned}
 \frac{x^2\left(\frac{1}{x^2}\right)}{x^2\left(x+\frac{3}{x}\right)} &= \frac{x^2\left(\frac{1}{x^2}\right)}{x^2(x)+x^2\left(\frac{3}{x}\right)} \\
 &= \frac{1}{x^3+3x} \\
 &= \frac{1}{x(x^2+3)}
 \end{aligned}$$

$$42. \quad \frac{x^{-1}+x^{-2}}{5x^{-2}} = \frac{\frac{1}{x}+\frac{1}{x^2}}{\frac{5}{x^2}} \quad \text{LCD} = x^2$$

$$\begin{aligned}
 \frac{x^2\left(\frac{1}{x}+\frac{1}{x^2}\right)}{x^2\left(\frac{5}{x^2}\right)} &= \frac{x^2\left(\frac{1}{x}\right)+x^2\left(\frac{1}{x^2}\right)}{x^2\left(\frac{5}{x^2}\right)} \\
 &= \frac{x+1}{5}
 \end{aligned}$$

$$\begin{aligned}
 43. \quad \frac{2a^{-1} + 3b^{-2}}{a^{-1} - b^{-1}} &= \frac{\frac{2}{a} + \frac{3}{b^2}}{\frac{1}{a} - \frac{1}{b}} \quad \text{LCD} = ab^2 \\
 \frac{ab^2 \left(\frac{2}{a} + \frac{3}{b^2} \right)}{ab^2 \left(\frac{1}{a} - \frac{1}{b} \right)} &= \frac{ab^2 \left(\frac{2}{a} \right) + ab^2 \left(\frac{3}{b^2} \right)}{ab^2 \left(\frac{1}{a} \right) - ab^2 \left(\frac{1}{b} \right)} \\
 &= \frac{2b^2 + 3a}{b^2 - ab} = \frac{2b^2 + 3a}{b(b-a)}
 \end{aligned}$$

$$\begin{aligned}
 44. \quad \frac{2m^{-1} + n^{-1}}{m^{-2} - 4n^{-1}} &= \frac{\frac{2}{m} + \frac{1}{n}}{\frac{1}{m^2} - \frac{4}{n}} \quad \text{LCD} = m^2n \\
 \frac{m^2n \left(\frac{2}{m} + \frac{1}{n} \right)}{m^2n \left(\frac{1}{m^2} - \frac{4}{n} \right)} &= \frac{m^2n \left(\frac{2}{m} \right) + m^2n \left(\frac{1}{n} \right)}{m^2n \left(\frac{1}{m^2} \right) - m^2n \left(\frac{4}{n} \right)} \\
 &= \frac{2mn + m^2}{n - 4m^2} = \frac{m(2n + m)}{n - 4m^2}
 \end{aligned}$$

$$\begin{aligned}
 45. \quad \frac{\frac{1}{4+h} - \frac{1}{4}}{h} \quad \text{LCD} &= 4(4+h) \\
 \frac{4(4+h) \left(\frac{1}{4+h} - \frac{1}{4} \right)}{4(4+h)h} &= \frac{4 - (4+h)}{4h(4+h)} \\
 &= \frac{-h}{4h(4+h)} \\
 &= \frac{-\cancel{h}}{4\cancel{h}(4+h)} \\
 &= \frac{-1}{4(4+h)}
 \end{aligned}$$

$$\begin{aligned}
 46. \quad \frac{\frac{1}{3+3h} - \frac{1}{3}}{h} &= \frac{\frac{1}{3(1+h)} - \frac{1}{3}}{h} \quad \text{LCD} = 3(1+h) \\
 \frac{3(1+h) \left(\frac{1}{3(1+h)} - \frac{1}{3} \right)}{3(1+h)h} &= \frac{1 - (1+h)}{3h(1+h)} \\
 &= \frac{-\cancel{h}}{3\cancel{h}(1+h)} \\
 &= \frac{-1}{3(1+h)}
 \end{aligned}$$

$$\begin{aligned}
 47. \quad \frac{\frac{6}{x+h} - \frac{6}{x}}{h} \quad \text{LCD} &= x(x+h) \\
 \frac{x(x+h) \left(\frac{6}{x+h} - \frac{6}{x} \right)}{x(x+h)h} &= \frac{6x - 6(x+h)}{xh(x+h)} \\
 &= \frac{6x - 6x - 6h}{xh(x+h)} \\
 &= \frac{-6\cancel{h}}{x\cancel{h}(x+h)} \\
 &= \frac{-6}{x(x+h)}
 \end{aligned}$$

$$\begin{aligned}
 48. \quad \frac{\frac{-3}{x+h} + \frac{3}{x}}{h} \quad \text{LCD} &= x(x+h) \\
 \frac{x(x+h) \left(\frac{-3}{x+h} + \frac{3}{x} \right)}{x(x+h)h} &= \frac{-3x + 3(x+h)}{xh(x+h)} \\
 &= \frac{-3x + 3x + 3h}{xh(x+h)} \\
 &= \frac{3\cancel{h}}{x\cancel{h}(x+h)} \\
 &= \frac{3}{x(x+h)}
 \end{aligned}$$

$$49. \quad m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$50. \quad m = \frac{-2 - \frac{2}{5}}{\frac{1}{4} - 1\frac{1}{2}} = \frac{-2 - \frac{2}{5}}{\frac{1}{4} - \frac{3}{2}} \quad \text{LCD} = 20$$

$$= \frac{20\left(-2 - \frac{2}{5}\right)}{20\left(\frac{1}{4} - \frac{3}{2}\right)} = \frac{-40 - 8}{5 - 30} = \frac{-48}{-25} = \frac{48}{25}$$

$$51. \quad m = \frac{-3 - \frac{3}{5}}{-1 - \left(-\frac{3}{7}\right)} = \frac{-3 - \frac{3}{5}}{-1 + \frac{3}{7}} \quad \text{LCD} = 35$$

$$= \frac{35\left(-3 - \frac{3}{5}\right)}{35\left(-1 + \frac{3}{7}\right)} = \frac{-105 - 21}{-35 + 15}$$

$$= \frac{-126}{-20} = \frac{63}{10}$$

$$52. \quad m = \frac{-\frac{1}{5} - \frac{9}{10}}{-\frac{1}{16} - \frac{5}{8}} \quad \text{LCD} = 80$$

$$= \frac{80\left(-\frac{1}{5} - \frac{9}{10}\right)}{80\left(-\frac{1}{16} - \frac{5}{8}\right)} = \frac{-16 - 72}{-5 - 50}$$

$$= \frac{-88}{-55} = \frac{8}{5}$$

$$53. \quad m = \frac{\frac{1}{6} - \frac{1}{3}}{\frac{1}{8} - \frac{1}{4}} \quad \text{LCD} = 24$$

$$= \frac{24\left(\frac{1}{6} - \frac{1}{3}\right)}{24\left(\frac{1}{8} - \frac{1}{4}\right)} = \frac{4 - 8}{3 - 6} = \frac{-4}{-3} = \frac{4}{3}$$

$$54. \quad (x + x^{-1})^{-1} = \frac{1}{x + x^{-1}} = \frac{1}{x + \frac{1}{x}} \quad \text{LCD} = x$$

$$= \frac{x(1)}{x\left(x + \frac{1}{x}\right)}$$

$$= \frac{x}{x^2 + 1}$$

$$55. \quad (x^{-1} + y^{-1})^{-1} = \frac{1}{x^{-1} + y^{-1}} = \frac{1}{\frac{1}{x} + \frac{1}{y}} \quad \text{LCD} = xy$$

$$= \frac{xy(1)}{xy\left(\frac{1}{x} + \frac{1}{y}\right)} = \frac{xy}{y + x} = \frac{xy}{x + y}$$

$$\begin{aligned}
 56. \quad \frac{x}{1 - \left(1 + \frac{1}{x}\right)^{-1}} &= \frac{x}{1 - \frac{1}{1 + \frac{1}{x}}} = \frac{x}{1 - \frac{1}{\frac{x+1}{x}}} = \frac{x}{1 - \frac{1}{\frac{x+1}{x}}} = \frac{x}{1 - 1 \cdot \frac{x}{x+1}} = \frac{x}{\frac{x+1}{x+1} - \frac{x}{x+1}} = \frac{x}{\frac{1}{x+1}} \\
 &= x \cdot \frac{x+1}{1} = x(x+1)
 \end{aligned}$$

$$\begin{aligned}
 57. \quad \frac{x}{1 - \left(1 - \frac{1}{x}\right)^{-1}} &= \frac{x}{1 - \frac{1}{1 - \frac{1}{x}}} = \frac{x}{1 - \frac{1}{\frac{x-1}{x}}} = \frac{x}{1 - \frac{1}{\frac{x-1}{x}}} = \frac{x}{1 - 1 \cdot \frac{x}{x-1}} = \frac{x}{\frac{x-1}{x-1} - \frac{x}{x-1}} = \frac{x}{\frac{-1}{x-1}} \\
 &= x \cdot \frac{x-1}{-1} = -x(x-1)
 \end{aligned}$$

Problem Recognition Exercises

$$\begin{aligned}
 1. \quad \frac{2}{2y-3} - \frac{3}{2y} + 1 \quad \text{LCD} &= 2y(2y-3) \\
 &= \frac{2}{2y-3} \cdot \frac{2y}{2y} - \frac{3}{2y} \cdot \frac{2y-3}{2y-3} + 1 \cdot \frac{2y(2y-3)}{2y(2y-3)} \\
 &= \frac{4y-3(2y-3)+2y(2y-3)}{2y(2y-3)} \\
 &= \frac{4y-6y+9+4y^2-6y}{2y(2y-3)} \\
 &= \frac{4y^2-8y+9}{2y(2y-3)}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad (x+5) + \left(\frac{7}{x-4}\right) \quad \text{LCD} &= x-4 \\
 &= (x+5) \cdot \frac{x-4}{x-4} + \left(\frac{7}{x-4}\right) \\
 &= \frac{x^2+x-20+7}{x-4} \\
 &= \frac{x^2+x-13}{x-4}
 \end{aligned}$$

$$\begin{aligned}
 3. \quad \frac{5x^2-6x+1}{x^2-1} \div \frac{16x^2-9}{4x^2+7x+3} - \frac{x}{4x-3} \\
 &= \frac{5x^2-6x+1}{x^2-1} \cdot \frac{4x^2+7x+3}{16x^2-9} - \frac{x}{4x-3} \\
 &= \frac{(5x-1)(x-1)}{(x+1)(x-1)} \cdot \frac{(4x+3)(x+1)}{(4x+3)(4x-3)} - \frac{x}{4x-3} \\
 &= \frac{5x-1}{4x-3} - \frac{x}{4x-3} \\
 &= \frac{4x-1}{4x-3}
 \end{aligned}$$

$$\begin{aligned}
 4. \quad \frac{a^2 - 25}{3a^2 + 3ab} \cdot \frac{a^2 + 4a + ab + 4b}{a^2 + 9a + 20} &= \frac{(a+5)(a-5)}{3a(a+b)} \cdot \frac{a(a+4)+b(a+4)}{(a+4)(a+5)} \\
 &= \frac{\cancel{(a+5)}(a-5)}{3a\cancel{(a+b)}} \cdot \frac{\cancel{(a+4)}\cancel{(a+b)}}{\cancel{(a+4)}(a+5)} = \frac{a-5}{3a}
 \end{aligned}$$

$$\begin{aligned}
 5. \quad \frac{4}{y+1} + \frac{y+2}{y^2-1} - \frac{3}{y-1} &= \frac{4}{y+1} + \frac{y+2}{(y+1)(y-1)} - \frac{3}{y-1} \quad \text{LCD} = (y+1)(y-1) \\
 &= \frac{4}{y+1} \cdot \frac{y-1}{y-1} + \frac{y+2}{(y+1)(y-1)} - \frac{3}{y-1} \cdot \frac{y+1}{y+1} \\
 &= \frac{4(y-1) + y+2 - 3(y+1)}{(y+1)(y-1)} \\
 &= \frac{4y-4+y+2-3y-3}{(y+1)(y-1)} \\
 &= \frac{2y-5}{(y+1)(y-1)}
 \end{aligned}$$

$$\begin{aligned}
 6. \quad \frac{8w^2}{w^3-16w} - \frac{4w}{w^2-4w} &= \frac{8w^2}{w(w^2-16)} - \frac{4w}{w(w-4)} \\
 &= \frac{8w^2}{w(w+4)(w-4)} - \frac{4w}{w(w-4)} \quad \text{LCD} = w(w+4)(w-4) \\
 &= \frac{8w^2}{w(w+4)(w-4)} - \frac{4w}{w(w-4)} \cdot \frac{w+4}{w+4} = \frac{8w^2 - 4w(w+4)}{w(w+4)(w-4)} \\
 &= \frac{8w^2 - 4w^2 - 16w}{w(w+4)(w-4)} = \frac{4w^2 - 16w}{w(w+4)(w-4)} \\
 &= \frac{4\cancel{w}(w-4)}{\cancel{w}(w+4)\cancel{(w-4)}} = \frac{4}{w+4}
 \end{aligned}$$

$$\begin{aligned}
 7. \quad \frac{a^2-16}{2x+6} \cdot \frac{x+3}{a-4} &= \frac{(a+4)\cancel{(a-4)}}{2\cancel{(x+3)}} \cdot \frac{\cancel{x+3}}{\cancel{a-4}} \\
 &= \frac{a+4}{2}
 \end{aligned}$$

$$\begin{aligned}
 8. \quad \frac{t^2-9}{t} \div \frac{t+3}{t+2} &= \frac{\cancel{(t+3)}(t-3)}{t} \cdot \frac{t+2}{\cancel{t+3}} \\
 &= \frac{(t-3)(t+2)}{t}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{9.} \quad & \frac{2 + \frac{1}{a}}{4 - \frac{1}{a^2}} \quad \text{LCD} = a^2 \\
 & \frac{a^2 \left(2 + \frac{1}{a} \right)}{a^2 \left(4 - \frac{1}{a^2} \right)} = \frac{a^2(2) + a^2 \left(\frac{1}{a} \right)}{a^2(4) - a^2 \left(\frac{1}{a^2} \right)} \\
 & = \frac{2a^2 + a}{4a^2 - 1} = \frac{\cancel{a(2a+1)}}{\cancel{(2a+1)}(2a-1)} \\
 & = \frac{a}{2a-1}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{10.} \quad & \frac{\frac{6x^2y}{5}}{\frac{3x}{y}} \quad \text{LCD} = 5y \\
 & \frac{5y \left(\frac{6x^2y}{5} \right)}{5y \left(\frac{3x}{y} \right)} = \frac{6x^2y^2}{15x} = \frac{2xy^2}{5}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{11.} \quad & \frac{6xy}{x^2 - y^2} + \frac{x+y}{y-x} = \frac{6xy}{(x+y)(x-y)} + \frac{x+y}{y-x} \\
 & \text{LCD} = (x+y)(x-y) \\
 & = \frac{6xy}{(x+y)(x-y)} + \frac{x+y}{y-x} \cdot \frac{(-1)}{(-1)} \cdot \frac{x+y}{x+y} \\
 & = \frac{6xy - (x^2 + 2xy + y^2)}{(x+y)(x-y)} \\
 & = \frac{6xy - x^2 - 2xy - y^2}{(x+y)(x-y)} \\
 & = \frac{-x^2 + 4xy - y^2}{(x+y)(x-y)}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{12.} \quad & (x^2 - 6x + 8) \left(\frac{3}{x-2} \right) \\
 & = \frac{(x-4)\cancel{(x-2)}}{1} \cdot \frac{3}{\cancel{x-2}} \\
 & = 3(x-4)
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{13.} \quad & \frac{3}{x-2} - \frac{x-2}{6} \quad \text{LCD} = 6(x-2) \\
 & = \frac{3}{x-2} \cdot \frac{6}{6} - \frac{x-2}{6} \cdot \frac{x-2}{x-2} \\
 & = \frac{18 - (x^2 - 4x + 4)}{6(x-2)} \\
 & = \frac{18 - x^2 + 4x - 4}{6(x-2)} \\
 & = \frac{-x^2 + 4x + 14}{6(x-2)}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{14.} \quad & \frac{5}{x+7} + \frac{x+7}{10} \quad \text{LCD} = 10(x+7) \\
 & = \frac{5}{x+7} \cdot \frac{10}{10} + \frac{x+7}{10} \cdot \frac{x+7}{x+7} \\
 & = \frac{50 + x^2 + 14x + 49}{10(x+7)} \\
 & = \frac{x^2 + 14x + 99}{10(x+7)}
 \end{aligned}$$

$$\begin{aligned}
 15. \quad \frac{1}{w-1} - \frac{w+2}{3w-3} &= \frac{1}{w-1} - \frac{w+2}{3(w-1)} \\
 \text{LCD} &= 3(w-1) \\
 &= \frac{3}{3} \cdot \frac{1}{w-1} - \frac{w+2}{3(w-1)} = \frac{3-w-2}{3(w-1)} \\
 &= \frac{1-w}{3(w-1)} = \frac{-1(\cancel{w-1})}{3(\cancel{w-1})} = -\frac{1}{3}
 \end{aligned}$$

$$\begin{aligned}
 16. \quad \frac{3y+6}{y^2-3y-10} \div \frac{27}{y-5} \\
 &= \frac{\cancel{3}(y+2)}{(y+2)(y-5)} \cdot \frac{\cancel{y-5}}{\cancel{3} \cdot 9} = \frac{1}{9}
 \end{aligned}$$

$$\begin{aligned}
 17. \quad \frac{y + \frac{2}{y} - 3}{1 - \frac{2}{y}} \quad \text{LCD} = y \\
 \frac{y\left(y + \frac{2}{y} - 3\right)}{y\left(1 - \frac{2}{y}\right)} = \frac{y(y) + y\left(\frac{2}{y}\right) - y(3)}{y(1) - y\left(\frac{2}{y}\right)} = \frac{y^2 + 2 - 3y}{y - 2} = \frac{\cancel{(y-2)}(y-1)}{\cancel{y-2}} = y-1
 \end{aligned}$$

$$\begin{aligned}
 18. \quad \frac{2}{t-3} - \frac{3}{t+2} + 5 \quad \text{LCD} = (t-3)(t+2) \\
 \left(\frac{t+2}{t+2}\right)\left(\frac{2}{t-3}\right) - \left(\frac{t-3}{t-3}\right)\left(\frac{3}{t+2}\right) + 5\frac{(t-3)(t+2)}{(t-3)(t+2)} = \frac{2t+4-3t+9+5(t^2-t-6)}{(t-3)(t+2)} \\
 = \frac{2t+4-3t+9+5t^2-5t-30}{(t-3)(t+2)} = \frac{5t^2-6t-17}{(t-3)(t+2)}
 \end{aligned}$$

$$\begin{aligned}
 19. \quad \frac{4x^2+22x+24}{4x+4} \cdot \frac{6x+6}{4x^2-9} = \frac{2(2x^2+11x+12)}{4(x+1)} \cdot \frac{6(x+1)}{(2x-3)(2x+3)} \\
 = \frac{\cancel{2}(2x+3)(x+4)}{\cancel{2} \cdot \cancel{2}(x+1)} \cdot \frac{\cancel{2} \cdot 3(x+1)}{(2x-3)(\cancel{2x+3})} = \frac{3(x+4)}{2x-3}
 \end{aligned}$$

$$\begin{aligned}
 20. \quad \frac{12x^3y^5z}{5x^4} \div \frac{16xy^7}{10z^2} = \frac{12x^3y^5z}{5x^4} \cdot \frac{10z^2}{16xy^7} = \frac{120x^3y^5z^3}{80x^5y^7} \\
 = \frac{3}{2}x^{3-5}y^{5-7}z^3 = \frac{3}{2}x^{-2}y^{-2}z^3 = \frac{3z^3}{2x^2y^2}
 \end{aligned}$$

$$\begin{aligned}
 21. \quad \frac{3x-1}{4} + \frac{7}{6x-2} &= \frac{3x-1}{4} + \frac{7}{2(3x-1)} \\
 \text{LCD} &= 4(3x-1) \\
 &= \frac{3x-1}{4} \cdot \frac{3x-1}{3x-1} + \frac{7}{2(3x-1)} \cdot \frac{2}{2} \\
 &= \frac{9x^2 - 6x + 1 + 14}{4(3x-1)} = \frac{9x^2 - 6x + 15}{4(3x-1)}
 \end{aligned}$$

$$\begin{aligned}
 22. \quad \frac{2x^{-1} + 3x^{-2}}{x^{-2} - 5x^{-1}} &= \frac{\frac{2}{x} + \frac{3}{x^2}}{\frac{1}{x^2} - \frac{5}{x}} = \frac{x^2 \left(\frac{2}{x} + \frac{3}{x^2} \right)}{x^2 \left(\frac{1}{x^2} - \frac{5}{x} \right)} \\
 &= \frac{2x+3}{1-5x}
 \end{aligned}$$

$$\begin{aligned}
 23. \quad (y+2) \cdot \frac{2y+1}{y^2-4} - \frac{y-2}{y+3} &= \frac{\cancel{(y+2)}(2y+1)}{\cancel{(y+2)}(y-2)} - \frac{y-2}{y+3} = \frac{2y+1}{y-2} - \frac{y-2}{y+3} \\
 \text{LCD} &= (y-2)(y+3) \\
 \left(\frac{y+3}{y+3} \right) \left(\frac{2y+1}{y-2} \right) - \left(\frac{y-2}{y-2} \right) \left(\frac{y-2}{y+3} \right) &= \frac{2y^2 + 7y + 3 - (y^2 - 4y + 4)}{(y-2)(y+3)} \\
 &= \frac{2y^2 + 7y + 3 - y^2 + 4y - 4}{(y-2)(y+3)} = \frac{y^2 + 11y - 1}{(y-2)(y+3)}
 \end{aligned}$$

$$24. \quad \frac{a^2}{a-10} - \frac{100-20a}{10-a} = \frac{a^2}{a-10} + \frac{100-20a}{a-10} = \frac{a^2 - 20a + 100}{a-10} = \frac{\cancel{(a-10)}(a-10)}{\cancel{a-10}} = a-10$$

Section 5.5 Practice Exercises

1. a. rational.
b. denominator
c. No.

$$\begin{aligned}
 2. \quad \frac{1}{x^2-16} + \frac{1}{x^2+8x+16} &= \frac{1}{(x+4)(x-4)} + \frac{1}{(x+4)^2} \quad \text{LCD} = (x+4)^2(x-4) \\
 &= \frac{1}{(x+4)(x-4)} \cdot \frac{x+4}{x+4} + \frac{1}{(x+4)^2} \cdot \frac{x-4}{x-4} \\
 &= \frac{x+4+x-4}{(x+4)^2(x-4)} = \frac{2x}{(x+4)^2(x-4)}
 \end{aligned}$$

$$\begin{aligned}
 3. \quad \frac{3}{y^2-1} - \frac{2}{y^2-2y+1} &= \frac{3}{(y+1)(y-1)} - \frac{2}{(y-1)^2} & \text{LCD} &= (y-1)^2(y+1) \\
 &= \frac{3}{(y+1)(y-1)} \cdot \frac{y-1}{y-1} - \frac{2}{(y-1)^2} \cdot \frac{y+1}{y+1} = \frac{3y-3-2y-2}{(y-1)^2(y+1)} = \frac{y-5}{(y-1)^2(y+1)}
 \end{aligned}$$

$$\begin{aligned}
 4. \quad \frac{m^2-9}{m^2-3m} \div (m^2-m-12) &= \frac{m^2-9}{m^2-3m} \cdot \frac{1}{m^2-m-12} \\
 &= \frac{(\cancel{m+3})(\cancel{m-3})}{m(\cancel{m-3})} \cdot \frac{1}{(m-4)(\cancel{m+3})} = \frac{1}{m(m-4)}
 \end{aligned}$$

$$5. \quad \frac{2t^2+7t+3}{4t^2-1} \div (t+3) = \frac{2t^2+7t+3}{4t^2-1} \cdot \frac{1}{t+3} = \frac{(\cancel{2t+1})(\cancel{t+3})}{(\cancel{2t+1})(2t-1)} \cdot \frac{1}{\cancel{t+3}} = \frac{1}{2t-1}$$

$$\begin{aligned}
 6. \quad \frac{1+x^{-1}}{1-x^{-2}} &= \frac{1+\frac{1}{x}}{1-\frac{1}{x^2}} & \text{LCD} &= x^2 \\
 \frac{x^2\left(1+\frac{1}{x}\right)}{x^2\left(1-\frac{1}{x^2}\right)} &= \frac{x^2(1)+x^2\left(\frac{1}{x}\right)}{x^2(1)-x^2\left(\frac{1}{x^2}\right)} = \frac{x^2+x}{x^2-1} \\
 &= \frac{x(\cancel{x+1})}{(\cancel{x+1})(x-1)} = \frac{x}{x-1}
 \end{aligned}$$

$$\begin{aligned}
 7. \quad \frac{x+y}{x^{-1}+y^{-1}} &= \frac{x+y}{\frac{1}{x}+\frac{1}{y}} & \text{LCD} &= xy \\
 \frac{xy(x+y)}{xy\left(\frac{1}{x}+\frac{1}{y}\right)} &= \frac{xy(x)+xy(y)}{xy\left(\frac{1}{x}\right)+xy\left(\frac{1}{y}\right)} \\
 &= \frac{x^2y+xy^2}{y+x} \\
 &= \frac{xy(\cancel{x+y})}{\cancel{x+y}} = xy
 \end{aligned}$$

8. A possible solution may make a denominator equal to zero in the original equation, and therefore cannot be a solution.

$$\begin{aligned}
 9. \quad \frac{x+2}{3} - \frac{x-4}{4} &= \frac{1}{2} & \text{LCD} &= 12 \\
 12\left(\frac{x+2}{3} - \frac{x-4}{4}\right) &= 12\left(\frac{1}{2}\right) \\
 12\left(\frac{x+2}{3}\right) - 12\left(\frac{x-4}{4}\right) &= 12\left(\frac{1}{2}\right)
 \end{aligned}$$

$$\begin{aligned}
 10. \quad \frac{x+6}{3} - \frac{x+8}{5} &= 0 & \text{LCD} &= 15 \\
 15\left(\frac{x+6}{3} - \frac{x+8}{5}\right) &= 15(0) \\
 15\left(\frac{x+6}{3}\right) - 15\left(\frac{x+8}{5}\right) &= 0
 \end{aligned}$$

Chapter 5 Rational Expressions and Rational Equations

$$4(x+2) - 3(x-4) = 6$$

$$4x + 8 - 3x + 12 = 6$$

$$x + 20 = 6$$

$$x = -14 \quad \{-14\}$$

$$5(x+6) - 3(x+8) = 0$$

$$5x + 30 - 3x - 24 = 0$$

$$2x + 6 = 0$$

$$2x = -6$$

$$x = -3 \quad \{-3\}$$

11. $\frac{3y}{4} - 2 = \frac{5y}{6}$ LCD = 12

$$12\left(\frac{3y}{4} - 2\right) = 12\left(\frac{5y}{6}\right)$$

$$12\left(\frac{3y}{4}\right) - 12(2) = 12\left(\frac{5y}{6}\right)$$

$$9y - 24 = 10y$$

$$-24 = y \quad \{-24\}$$

12. $\frac{2w}{5} - 8 = \frac{4w}{2}$ LCD = 10

$$10\left(\frac{2w}{5} - 8\right) = 10\left(\frac{4w}{2}\right)$$

$$10\left(\frac{2w}{5}\right) - 10(8) = 10\left(\frac{4w}{2}\right)$$

$$4w - 80 = 20w$$

$$-80 = 16w$$

$$-5 = w \quad \{-5\}$$

13. $\frac{5}{4p} - \frac{7}{6} + 3 = 0$

LCD = $12p$ so $p \neq 0$

$$12p\left(\frac{5}{4p} - \frac{7}{6} + 3\right) = 12p(0)$$

$$12p\left(\frac{5}{4p}\right) - 12p\left(\frac{7}{6}\right) + 12p(3) = 0$$

$$15 - 14p + 36p = 0$$

$$15 + 22p = 0$$

$$22p = -15$$

$$p = -\frac{15}{22} \quad \left\{-\frac{15}{22}\right\}$$

14. $\frac{7}{15w} - \frac{3}{10} - 2 = 0$

LCD = $30w$ so $w \neq 0$

$$30w\left(\frac{7}{15w} - \frac{3}{10} - 2\right) = 30w(0)$$

$$30w\left(\frac{7}{15w}\right) - 30w\left(\frac{3}{10}\right) - 30w(2) = 0$$

$$14 - 9w - 60w = 0$$

$$14 - 69w = 0$$

$$-69w = -14$$

$$w = \frac{14}{69} \quad \left\{\frac{14}{69}\right\}$$

15. $\frac{1}{2} - \frac{3}{2x} = \frac{4}{x} - \frac{5}{12}$

LCD = $12x$ so $x \neq 0$

$$12x\left(\frac{1}{2} - \frac{3}{2x}\right) = 12x\left(\frac{4}{x} - \frac{5}{12}\right)$$

16. $\frac{2}{3x} + \frac{1}{4} = \frac{11}{6x} - \frac{1}{3}$

LCD = $12x$ so $x \neq 0$

$$12x\left(\frac{2}{3x} + \frac{1}{4}\right) = 12x\left(\frac{11}{6x} - \frac{1}{3}\right)$$

Section 5.5 Solving Rational Equations

$$\begin{aligned}
 12x\left(\frac{1}{2}\right) - 12x\left(\frac{3}{2x}\right) &= 12x\left(\frac{4}{x}\right) - 12x\left(\frac{5}{12}\right) \\
 6x - 18 &= 48 - 5x \\
 11x &= 66 \\
 x &= 6 \quad \{6\}
 \end{aligned}$$

$$\begin{aligned}
 12x\left(\frac{2}{3x}\right) + 12x\left(\frac{1}{4}\right) &= 12x\left(\frac{11}{6x}\right) - 12x\left(\frac{1}{3}\right) \\
 8 + 3x &= 22 - 4x \\
 7x &= 14 \\
 x &= 2 \quad \{2\}
 \end{aligned}$$

17.
$$\frac{3}{x-4} + 2 = \frac{5}{x-4}$$

LCD = $x-4$ so $x \neq 4$

$$\begin{aligned}
 (x-4)\left(\frac{3}{x-4} + 2\right) &= (\cancel{x-4})\left(\frac{5}{\cancel{x-4}}\right) \\
 (\cancel{x-4})\left(\frac{3}{\cancel{x-4}}\right) + (x-4)(2) &= 5 \\
 3 + 2x - 8 &= 5 \\
 2x - 5 &= 5 \\
 2x &= 10 \\
 x &= 5 \quad \{5\}
 \end{aligned}$$

18.
$$\frac{5}{x+3} - 2 = \frac{7}{x+3}$$

LCD = $x+3$ so $x \neq -3$

$$\begin{aligned}
 (x+3)\left(\frac{5}{x+3} - 2\right) &= (\cancel{x+3})\left(\frac{7}{\cancel{x+3}}\right) \\
 (\cancel{x+3})\left(\frac{5}{\cancel{x+3}}\right) - (x+3)(2) &= 7 \\
 5 - 2x - 6 &= 7 \\
 -2x - 1 &= 7 \\
 -2x &= 8 \\
 x &= -4 \quad \{-4\}
 \end{aligned}$$

19.
$$\frac{1}{3} + \frac{2}{w-3} = 1$$

LCD = $3(w-3)$ so $w \neq 3$

$$\begin{aligned}
 3(w-3)\left(\frac{1}{3} + \frac{2}{w-3}\right) &= 3(w-3)(1) \\
 \cancel{3}(w-3)\left(\frac{1}{\cancel{3}}\right) + 3(\cancel{w-3})\left(\frac{2}{\cancel{w-3}}\right) &= 3w-9 \\
 w-3+6 &= 3w-9 \\
 w+3 &= 3w-9 \\
 -2w &= -12 \\
 w &= 6 \quad \{6\}
 \end{aligned}$$

20.
$$\frac{3}{5} + \frac{7}{p+2} = 2$$

LCD = $5(p+2)$ so $p \neq -2$

$$\begin{aligned}
 5(p+2)\left(\frac{3}{5} + \frac{7}{p+2}\right) &= 5(p+2)(2) \\
 \cancel{5}(p+2)\left(\frac{3}{\cancel{5}}\right) + 5(\cancel{p+2})\left(\frac{7}{\cancel{p+2}}\right) &= 10p+20 \\
 3p+6+35 &= 10p+20 \\
 3p+41 &= 10p+20 \\
 3p-10p &= 20-41 \\
 -7p &= -21 \\
 p &= 3 \quad \{3\}
 \end{aligned}$$

21. $\frac{12}{x} - \frac{12}{x-5} = \frac{2}{x}$

LCD = $x(x-5)$ so $x \neq 0$ or $x \neq 5$

$$\begin{aligned}
 x(x-5)\left(\frac{12}{x} - \frac{12}{x-5}\right) &= \cancel{x}(x-5)\left(\frac{2}{\cancel{x}}\right) \\
 \cancel{x}(x-5)\left(\frac{12}{\cancel{x}}\right) - x(\cancel{x-5})\left(\frac{12}{\cancel{x-5}}\right) &= 2x-10 \\
 12x-60-12x &= 2x-10 \\
 -60 &= 2x-10 \\
 -50 &= 2x \\
 x &= -25 \quad \{-25\}
 \end{aligned}$$

22. $\frac{25}{y} - \frac{25}{y-2} = \frac{2}{y}$

LCD = $y(y-2)$ so $y \neq 0$ or $y \neq 2$

$$\begin{aligned}
 y(y-2)\left(\frac{25}{y} - \frac{25}{y-2}\right) &= \cancel{y}(y-2)\left(\frac{2}{\cancel{y}}\right) \\
 \cancel{y}(y-2)\left(\frac{25}{\cancel{y}}\right) - y(\cancel{y-2})\left(\frac{25}{\cancel{y-2}}\right) &= 2y-4 \\
 25y-50-25y &= 2y-4 \\
 -50 &= 2y-4 \\
 -46 &= 2y \\
 y &= -23 \quad \{-23\}
 \end{aligned}$$

$$\begin{aligned}
 23. \quad & \frac{3}{a^2} - \frac{4}{a} = -1 \\
 & \text{LCD} = a^2 \quad \text{so } a \neq 0 \\
 & a^2 \left(\frac{3}{a^2} - \frac{4}{a} \right) = a^2(-1) \\
 & \cancel{a^2} \left(\frac{3}{\cancel{a^2}} \right) - a \cdot \cancel{a} \left(\frac{4}{\cancel{a}} \right) = -a^2 \\
 & \quad \quad \quad 3 - 4a = -a^2 \\
 & \quad \quad \quad a^2 - 4a + 3 = 0 \\
 & \quad \quad \quad (a-3)(a-1) = 0 \\
 & \quad \quad \quad a-3=0 \quad \text{or} \quad a-1=0 \\
 & \quad \quad \quad a=3 \quad \text{or} \quad a=1 \\
 & \quad \quad \quad \{3,1\}
 \end{aligned}$$

$$\begin{aligned}
 24. \quad & \frac{3}{w^2} = 2 + \frac{1}{w} \\
 & \text{LCD} = w^2 \quad \text{so } w \neq 0 \\
 & w^2 \left(\frac{3}{w^2} \right) = w^2 \left(2 + \frac{1}{w} \right) \\
 & \cancel{w^2} \left(\frac{3}{\cancel{w^2}} \right) = w^2(2) + \cancel{w} \cdot w \left(\frac{1}{\cancel{w}} \right) \\
 & \quad \quad \quad 3 = 2w^2 + w \\
 & \quad \quad \quad 0 = 2w^2 + w - 3 \\
 & \quad \quad \quad 0 = (2w+3)(w-1) \\
 & \quad \quad \quad 2w+3=0 \quad \text{or} \quad w-1=0 \\
 & \quad \quad \quad 2w=-3 \quad \text{or} \quad w=1 \\
 & \quad \quad \quad w = -\frac{3}{2} \quad \text{or} \quad w=1 \quad \left\{ -\frac{3}{2}, 1 \right\}
 \end{aligned}$$

$$\begin{aligned}
 25. \quad & \frac{1}{4}a - 4a^{-1} = 0 \\
 & \quad \quad \quad \frac{a}{4} - \frac{4}{a} = 0 \\
 & \text{LCD} = 4a \quad \text{so } a \neq 0 \\
 & 4a \left(\frac{a}{4} - \frac{4}{a} \right) = 4a(0) \\
 & \cancel{4}a \left(\frac{a}{\cancel{4}} \right) - 4\cancel{a} \left(\frac{4}{\cancel{a}} \right) = 0 \\
 & \quad \quad \quad a^2 - 16 = 0 \\
 & \quad \quad \quad (a+4)(a-4) = 0 \\
 & \quad \quad \quad a+4=0 \quad \text{or} \quad a-4=0 \\
 & \quad \quad \quad a=-4 \quad \text{or} \quad a=4 \quad \{-4,4\}
 \end{aligned}$$

$$\begin{aligned}
 26. \quad & \frac{1}{3}t - 12t^{-1} = 0 \\
 & \quad \quad \quad \frac{t}{3} - \frac{12}{t} = 0 \\
 & \text{LCD} = 3t \quad \text{so } t \neq 0 \\
 & 3t \left(\frac{t}{3} - \frac{12}{t} \right) = 3t(0) \\
 & \cancel{3}t \left(\frac{t}{\cancel{3}} \right) - 3\cancel{t} \left(\frac{12}{\cancel{t}} \right) = 0 \\
 & \quad \quad \quad t^2 - 36 = 0 \\
 & \quad \quad \quad (t+6)(t-6) = 0 \\
 & \quad \quad \quad t+6=0 \quad \text{or} \quad t-6=0 \\
 & \quad \quad \quad t=-6 \quad \text{or} \quad t=6 \quad \{-6,6\}
 \end{aligned}$$

$$\begin{aligned}
 27. \quad & \frac{y}{y+3} + \frac{2}{y^2+3y} = \frac{6}{y} \\
 & \frac{y}{y+3} + \frac{2}{y(y+3)} = \frac{6}{y} \\
 & \text{LCD} = y(y+3) \quad \text{so } y \neq 0, y \neq -3
 \end{aligned}$$

$$y(y+3)\left(\frac{y}{y+3} + \frac{2}{y(y+3)}\right) = y(y+3)\frac{6}{y}$$

$$y\cancel{(y+3)}\left(\frac{y}{\cancel{y+3}}\right) + \cancel{y}(y+3)\left(\frac{2}{\cancel{y}(y+3)}\right) = \cancel{y}(y+3)\frac{6}{\cancel{y}}$$

$$y^2 + 2 = (y+3)6$$

$$y^2 + 2 = 6y + 18$$

$$y^2 - 6y - 16 = 0$$

$$(y-8)(y+2) = 0$$

$$y-8=0 \text{ or } y+2=0$$

$$y=8 \text{ or } y=-2 \quad \{8, -2\}$$

28.

$$\frac{-8}{t^2-6t} + \frac{t}{t-6} = \frac{1}{t}$$

$$\frac{-8}{t(t-6)} + \frac{t}{t-6} = \frac{1}{t}$$

LCD = $t(t-6)$ so $t \neq 0, t \neq 6$

$$t(t-6)\left(\frac{-8}{t(t-6)} + \frac{t}{t-6}\right) = t(t-6)\left(\frac{1}{t}\right)$$

$$\cancel{t}(t-6)\frac{-8}{\cancel{t}(t-6)} + t\cancel{(t-6)}\frac{t}{\cancel{t-6}} = \cancel{t}(t-6)\frac{1}{\cancel{t}}$$

$$-8 + t^2 = t - 6$$

$$t^2 - t - 2 = 0$$

$$(t-2)(t+1) = 0$$

$$t-2=0 \text{ or } t+1=0$$

$$t=2 \text{ or } t=-1$$

$$\{2, -1\}$$

29.

$$\frac{4}{t-2} - \frac{8}{t^2-2t} = -2$$

$$\frac{4}{t-2} - \frac{8}{t(t-2)} = -2 \quad \text{LCD} = t(t-2) \quad \text{so } t \neq 0 \text{ or } t \neq 2$$

$$t(t-2)\left(\frac{4}{t-2} - \frac{8}{t(t-2)}\right) = t(t-2)(-2)$$

$$\begin{aligned}
 t(\cancel{t-2})\left(\frac{4}{\cancel{t-2}}\right) - \cancel{t}(\cancel{t-2})\left(\frac{8}{\cancel{t}(\cancel{t-2})}\right) &= -2t(t-2) \\
 4t - 8 &= -2t^2 + 4t \\
 2t^2 - 8 &= 0 \\
 2(t^2 - 4) &= 0 \\
 2(t+2)(t-2) &= 0 \\
 2 \neq 0 \text{ or } t+2=0 \text{ or } t-2=0 & \\
 t = -2 \text{ or } t = 2 &
 \end{aligned}$$

$\{-2\}$ is the solution. ($t = 2$ does not check because the denominator is zero.)

30.
$$\frac{x}{x+6} = \frac{72}{x^2-36} + 4$$

$$\frac{x}{x+6} = \frac{72}{(x+6)(x-6)} + 4 \quad \text{LCD} = (x+6)(x-6) \quad \text{so } x \neq -6 \text{ or } x \neq 6$$

$$(x+6)(x-6)\left(\frac{x}{x+6}\right) = (x+6)(x-6)\left(\frac{72}{(x+6)(x-6)} + 4\right)$$

$$(\cancel{x+6})(x-6)\left(\frac{x}{\cancel{x+6}}\right) = (\cancel{x+6})(\cancel{x-6})\left(\frac{72}{(\cancel{x+6})(\cancel{x-6})}\right) + (x+6)(x-6)(4)$$

$$x^2 - 6x = 72 + 4(x^2 - 36)$$

$$x^2 - 6x = 72 + 4x^2 - 144$$

$$0 = 3x^2 + 6x - 72$$

$$3(x^2 + 2x - 24) = 0$$

$$3(x+6)(x-4) = 0$$

$$3 \neq 0 \text{ or } x+6=0 \text{ or } x-4=0$$

$$x = -6 \text{ or } x = 4$$

$\{4\}$ is the solution. ($x = -6$ does not check because the denominator is zero.)

31.
$$\frac{6}{5y+10} - \frac{1}{y-5} = \frac{4}{y^2-3y-10}$$

$$\frac{6}{5(y+2)} - \frac{1}{y-5} = \frac{4}{(y-5)(y+2)} \quad \text{LCD} = 5(y-5)(y+2) \quad \text{so } y \neq 5 \text{ or } y \neq -2$$

$$5(y-5)(y+2)\left(\frac{6}{5(y+2)} - \frac{1}{y-5}\right) = 5(y-5)(y+2)\left(\frac{4}{(y-5)(y+2)}\right)$$

$$\begin{aligned} \cancel{5}(y-5)(\cancel{y+2})\left(\frac{6}{\cancel{5}(\cancel{y+2})}\right) - 5(\cancel{y-5})(y+2)\left(\frac{1}{\cancel{y-5}}\right) \\ = 5(\cancel{y-5})(\cancel{y+2})\left(\frac{4}{(\cancel{y-5})(\cancel{y+2})}\right) \end{aligned}$$

$$6y - 30 - 5y - 10 = 20$$

$$y - 40 = 20$$

$$y = 60$$

{60} is the solution.

32.

$$\frac{-3}{x^2 - 7x + 12} - \frac{2}{x^2 + x - 12} = \frac{10}{x^2 - 16}$$

$$\frac{-3}{(x-3)(x-4)} - \frac{2}{(x+4)(x-3)} = \frac{10}{(x+4)(x-4)}$$

$$\text{LCD} = (x-3)(x-4)(x+4) \quad \text{so } x \neq 3 \text{ or } x \neq 4 \text{ or } x \neq -4$$

$$\begin{aligned} (x-3)(x-4)(x+4)\left(\frac{-3}{(x-3)(x-4)} - \frac{2}{(x+4)(x-3)}\right) \\ = (x-3)(\cancel{x-4})(\cancel{x+4})\left(\frac{10}{(\cancel{x+4})(\cancel{x-4})}\right) \\ (\cancel{x-3})(\cancel{x-4})(x+4)\left(\frac{-3}{(\cancel{x-3})(\cancel{x-4})}\right) - (\cancel{x-3})(x-4)(\cancel{x+4})\left(\frac{2}{(\cancel{x+4})(\cancel{x-3})}\right) \\ = 10(x-3) \\ -3x - 12 - 2x + 8 = 10x - 30 \\ -5x - 4 = 10x - 30 \\ -15x = -26 \\ x = \frac{26}{15} \end{aligned}$$

$\left\{\frac{26}{15}\right\}$ is the solution.

33.

$$\frac{x}{x-5} + \frac{1}{5} = \frac{5}{x-5} \quad \text{LCD} = 5(x-5) \quad \text{so } x \neq 5$$

$$5(x-5)\left(\frac{x}{x-5} + \frac{1}{5}\right) = 5(x-5)\left(\frac{5}{x-5}\right)$$

$$5(\cancel{x-5})\left(\frac{x}{\cancel{x-5}}\right) + \cancel{5}(x-5)\left(\frac{1}{\cancel{5}}\right) = 5(\cancel{x-5})\left(\frac{5}{\cancel{x-5}}\right)$$

$$5x + x - 5 = 25$$

$$6x - 5 = 25$$

$$6x = 30$$

$$x = 5$$

{ } No solution. ($x = 5$ does not check because it makes the denominator zero.)

34. $\frac{x}{x-2} + \frac{2}{3} = \frac{2}{x-2}$ LCD = $3(x-2)$ so $x \neq 2$

$$3(x-2)\left(\frac{x}{x-2} + \frac{2}{3}\right) = 3(x-2)\left(\frac{2}{x-2}\right)$$

$$3(\cancel{x-2})\left(\frac{x}{\cancel{x-2}}\right) + \cancel{3}(x-2)\left(\frac{2}{\cancel{3}}\right) = 3(\cancel{x-2})\left(\frac{2}{\cancel{x-2}}\right)$$

$$3x + 2x - 4 = 6$$

$$5x - 4 = 6$$

$$5x = 10$$

$$x = 2$$

{ } No solution. ($x = 2$ does not check because it makes the denominator zero.)

35. $\frac{6}{x^2-4x+3} - \frac{1}{x-3} = \frac{1}{4x-4}$

$$\frac{6}{(x-3)(x-1)} - \frac{1}{x-3} = \frac{1}{4(x-1)}$$

LCD = $4(x-3)(x-1)$ so $x \neq 3$ or $x \neq 1$

$$4(x-3)(x-1)\left(\frac{6}{(x-3)(x-1)} - \frac{1}{x-3}\right) = \cancel{4}(x-3)(\cancel{x-1})\left(\frac{1}{\cancel{4}(\cancel{x-1})}\right)$$

$$4(\cancel{x-3})(\cancel{x-1})\left(\frac{6}{(\cancel{x-3})(\cancel{x-1})}\right) - 4(\cancel{x-3})(x-1)\left(\frac{1}{\cancel{x-3}}\right) = x-3$$

$$24 - 4x + 4 = x - 3$$

$$-4x + 28 = x - 3$$

$$-5x = -31$$

$$x = \frac{31}{5} \quad \left\{\frac{31}{5}\right\} \text{ is the solution.}$$

$$36. \quad \frac{1}{4x^2-36} - \frac{5}{x+3} + \frac{2}{x-3} = 0$$

$$\frac{1}{4(x+3)(x-3)} - \frac{5}{x+3} + \frac{2}{x-3} = 0 \quad \text{LCD} = 4(x+3)(x-3) \quad \text{so } x \neq -3 \text{ or } x \neq 3$$

$$4(x+3)(x-3) \left(\frac{1}{4(x+3)(x-3)} - \frac{5}{x+3} + \frac{2}{x-3} \right) = 4(x+3)(x-3)(0)$$

$$\cancel{4}(\cancel{x+3})(\cancel{x-3}) \left(\frac{1}{\cancel{4}(\cancel{x+3})(\cancel{x-3})} \right) - 4(\cancel{x+3})(x-3) \left(\frac{5}{\cancel{x+3}} \right)$$

$$+ 4(x+3)(\cancel{x-3}) \left(\frac{2}{\cancel{x-3}} \right) = 0$$

$$1 - 20(x-3) + 8(x+3) = 0$$

$$1 - 20x + 60 + 8x + 24 = 0$$

$$-12x + 85 = 0$$

$$-12x = -85$$

$$x = \frac{85}{12}$$

$\left\{ \frac{85}{12} \right\}$ is the solution.

$$37. \quad \frac{1}{k+2} - \frac{4}{k-2} - \frac{k^2}{4-k^2} = 0$$

$$\frac{1}{k+2} - \frac{4}{k-2} + \frac{k^2}{(k+2)(k-2)} = 0$$

$$\text{LCD} = (k+2)(k-2) \quad \text{so } k \neq -2 \text{ or } k \neq 2$$

$$(k+2)(k-2) \left(\frac{1}{k+2} - \frac{4}{k-2} + \frac{k^2}{(k+2)(k-2)} \right) = (k+2)(k-2)(0)$$

$$(\cancel{k+2})(k-2) \left(\frac{1}{\cancel{k+2}} \right) - (k+2)(\cancel{k-2}) \left(\frac{4}{\cancel{k-2}} \right) + (\cancel{k+2})(\cancel{k-2}) \left(\frac{k^2}{(\cancel{k+2})(\cancel{k-2})} \right) = 0$$

$$k-2-4k-8+k^2 = 0$$

$$k^2-3k-10 = 0$$

$$(k-5)(k+2) = 0$$

$$k-5 = 0 \text{ or } k+2 = 0$$

$$k = 5 \text{ or } k = -2$$

$\{5\}$ is the solution. ($k = -2$ does not check because the denominator is zero.)

38.

$$\begin{aligned} \frac{h}{2} - \frac{h}{h-4} &= \frac{4}{4-h} \\ \frac{h}{2} - \frac{h}{h-4} &= \frac{4}{(-1)(h-4)} \\ \frac{h}{2} - \frac{h}{h-4} &= \frac{-4}{h-4} \\ \text{LCD} &= 2(h-4) \quad \text{so } h \neq 4 \\ 2(h-4)\left(\frac{h}{2} - \frac{h}{h-4}\right) &= 2(h-4)\left(\frac{-4}{h-4}\right) \\ \cancel{2}(h-4)\left(\frac{h}{\cancel{2}}\right) - 2(\cancel{h-4})\left(\frac{h}{\cancel{h-4}}\right) &= 2(\cancel{h-4})\left(\frac{-4}{\cancel{h-4}}\right) \\ h^2 - 4h - 2h &= -8 \\ h^2 - 6h + 8 &= 0 \\ (h-4)(h-2) &= 0 \\ h-4 = 0 \quad \text{or} \quad h-2 &= 0 \\ h = 4 \quad \text{or} \quad h = 2 \end{aligned}$$

{2} is the solution. ($h = 4$ does not check because the denominator is zero.)

39.

$$\begin{aligned} \frac{5}{x^2 - 7x + 12} &= \frac{2}{x-3} + \frac{5}{x-4} \\ \frac{5}{(x-4)(x-3)} &= \frac{2}{x-3} + \frac{5}{x-4} \\ \text{LCD} &= (x-4)(x-3) \quad \text{so } x \neq 4 \quad \text{or } x \neq 3 \\ (x-4)(x-3)\left(\frac{5}{(x-4)(x-3)}\right) &= (x-4)(x-3)\left(\frac{2}{x-3} + \frac{5}{x-4}\right) \\ (\cancel{x-4})(\cancel{x-3})\left(\frac{5}{(\cancel{x-4})(\cancel{x-3})}\right) &= (x-4)(\cancel{x-3})\left(\frac{2}{\cancel{x-3}}\right) + (\cancel{x-4})(x-3)\left(\frac{5}{\cancel{x-4}}\right) \\ 5 &= 2x - 8 + 5x - 15 \\ 5 &= 7x - 23 \\ 28 &= 7x \\ x &= 4 \end{aligned}$$

{ } no solution. ($x = 4$ does not check because the denominator is zero.)

40.

$$\frac{9}{x^2+7x+10} = \frac{5}{x+2} - \frac{3}{x+5}$$

$$\frac{9}{(x+5)(x+2)} = \frac{5}{x+2} - \frac{3}{x+5}$$

LCD = $(x+5)(x+2)$ so $x \neq -5$ or $x \neq -2$

$$(x+5)(x+2) \left(\frac{9}{(x+5)(x+2)} \right) = (x+5)(x+2) \left(\frac{5}{x+2} - \frac{3}{x+5} \right)$$

$$\cancel{(x+5)} \cancel{(x+2)} \left(\frac{9}{\cancel{(x+5)} \cancel{(x+2)}} \right) = (x+5) \cancel{(x+2)} \left(\frac{5}{\cancel{x+2}} \right) - \cancel{(x+5)} (x+2) \left(\frac{3}{\cancel{x+5}} \right)$$

$$9 = 5x + 25 - 3x - 6$$

$$9 = 2x + 19$$

$$-10 = 2x$$

$$x = -5$$

{ } no solution. ($x = -5$ does not check because the denominator is zero.)

41.

$$\frac{4}{x^2+7x+12} - \frac{7}{x^2+8x+15} = \frac{1}{x^2+9x+20}$$

$$\frac{4}{(x+4)(x+3)} - \frac{7}{(x+5)(x+3)} = \frac{1}{(x+5)(x+4)}$$

LCD = $(x+4)(x+3)(x+5)$ so $x \neq -4, x \neq -3, x \neq -5$

$$(x+4)(x+3)(x+5) \left(\frac{4}{(x+4)(x+3)} - \frac{7}{(x+5)(x+3)} \right)$$

$$= (x+4)(x+3)(x+5) \left(\frac{1}{(x+5)(x+4)} \right)$$

$$\cancel{(x+4)} \cancel{(x+3)} (x+5) \frac{4}{\cancel{(x+4)} \cancel{(x+3)}} - (x+4) \cancel{(x+3)} \cancel{(x+5)} \frac{7}{\cancel{(x+5)} \cancel{(x+3)}}$$

$$= \cancel{(x+4)} (x+3) \cancel{(x+5)} \frac{1}{\cancel{(x+5)} \cancel{(x+4)}}$$

$$4x + 20 - 7x - 28 = x + 3$$

$$-3x - 8 = x + 3$$

$$-4x = 11$$

$$x = -\frac{11}{4} \quad \left\{ -\frac{11}{4} \right\} \text{ is the solution.}$$

42.
$$\frac{5}{x^2 - 6x + 8} - \frac{2}{x^2 + 3x - 10} = \frac{8}{x^2 + x - 20}$$

$$\frac{5}{(x-4)(x-2)} - \frac{2}{(x+5)(x-2)} = \frac{8}{(x+5)(x-4)}$$

LCD = $(x-4)(x-2)(x+5)$ so $x \neq 4, x \neq 2, x \neq -5$

$$(x-4)(x-2)(x+5) \left(\frac{5}{(x-4)(x-2)} - \frac{2}{(x+5)(x-2)} \right)$$

$$= (x-4)(x-2)(x+5) \left(\frac{8}{(x+5)(x-4)} \right)$$

$$\cancel{(x-4)} \cancel{(x-2)} (x+5) \frac{5}{\cancel{(x-4)} \cancel{(x-2)}} - (x-4) \cancel{(x-2)} \cancel{(x+5)} \frac{2}{\cancel{(x+5)} \cancel{(x-2)}}$$

$$= \cancel{(x-4)} (x-2) \cancel{(x+5)} \frac{8}{\cancel{(x+5)} \cancel{(x-4)}}$$

$$5x + 25 - 2x + 8 = 8x - 16$$

$$3x + 33 = 8x - 16$$

$$-5x = -49$$

$$x = \frac{49}{5} \quad \left\{ \frac{49}{5} \right\} \text{ is the solution.}$$

43. $K = \frac{ma}{F}$ for m

$$KF = ma$$

$$\frac{KF}{a} = m$$

44. $K = \frac{ma}{F}$ for a

$$KF = ma$$

$$\frac{KF}{m} = a$$

45. $K = \frac{IR}{E}$ for E

$$KE = IR$$

$$E = \frac{IR}{K}$$

46. $K = \frac{IR}{E}$ for R

$$KE = IR$$

$$\frac{KE}{I} = R$$

47. $I = \frac{E}{R+r}$ for R

$$I(R+r) = E$$

48. $I = \frac{E}{R+r}$ for r

$$I(R+r) = E$$

$$IR + Ir = E$$

$$IR = E - Ir$$

$$R = \frac{E - Ir}{I} \text{ or } R = \frac{E}{I} - r$$

$$IR + Ir = E$$

$$Ir = E - IR$$

$$r = \frac{E - IR}{I} \text{ or } r = \frac{E}{I} - R$$

49.

$$h = \frac{2A}{B+b} \text{ for } B$$

$$h(B+b) = 2A$$

$$hB + hb = 2A$$

$$hB = 2A - hb$$

$$B = \frac{2A - hb}{h} \text{ or } B = \frac{2A}{h} - b$$

50.

$$\frac{V}{\pi h} = r^2 \text{ for } h$$

$$V = \pi hr^2$$

$$\frac{V}{\pi r^2} = h$$

51.

$$x = \frac{at+b}{t} \text{ for } t$$

$$xt = at + b$$

$$xt - at = b$$

$$t(x-a) = b$$

$$t = \frac{b}{x-a}$$

52.

$$\frac{T+mf}{m} = g \text{ for } m$$

$$T + mf = mg$$

$$T = mg - mf$$

$$T = m(g-f)$$

$$\frac{T}{g-f} = m$$

53.

$$\frac{x-y}{xy} = z \text{ for } x$$

$$x-y = xyz$$

$$x - xyz = y$$

$$x(1-yz) = y$$

$$x = \frac{y}{1-yz}$$

54.

$$\frac{w-n}{wn} = P \text{ for } w$$

$$w-n = Pwn$$

$$w - Pwn = n$$

$$w(1-Pn) = n$$

$$w = \frac{n}{1-Pn}$$

55.

$$a+b = \frac{2A}{h} \text{ for } h$$

$$h(a+b) = 2A$$

$$h = \frac{2A}{a+b}$$

56.

$$1+rt = \frac{A}{P} \text{ for } P$$

$$P(1+rt) = A$$

$$P = \frac{A}{1+rt}$$

$$57. \quad \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \quad \text{for } R$$

$$RR_1R_2\left(\frac{1}{R}\right) = RR_1R_2\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\cancel{R}R_1\cancel{R_2}\left(\frac{1}{\cancel{R}}\right) = R\cancel{R_1}\cancel{R_2}\left(\frac{1}{\cancel{R_1}}\right) + RR_1\cancel{R_2}\left(\frac{1}{\cancel{R_2}}\right)$$

$$R_1R_2 = RR_2 + RR_1$$

$$R_1R_2 = R(R_2 + R_1)$$

$$\frac{R_1R_2}{R_2 + R_1} = R$$

$$59. \quad v = \frac{s_2 - s_1}{t_2 - t_1} \quad \text{for } t_2$$

$$v(t_2 - t_1) = s_2 - s_1$$

$$vt_2 - vt_1 = s_2 - s_1$$

$$vt_2 = s_2 - s_1 + vt_1$$

$$t_2 = \frac{s_2 - s_1 + vt_1}{v}$$

$$61. \quad \frac{3}{x+2} + \frac{2}{x} = \frac{-4}{x^2 + 2x}$$

$$\frac{3}{x+2} + \frac{2}{x} = \frac{-4}{x(x+2)}$$

$$\text{LCD} = x(x+2) \text{ so } x \neq 0, x \neq -2$$

$$x(x+2)\left(\frac{3}{x+2} + \frac{2}{x}\right) = x(x+2)\frac{-4}{x(x+2)}$$

$$x\cancel{(x+2)}\frac{3}{\cancel{x+2}} + \cancel{(x+2)}\frac{2}{\cancel{x}} = \cancel{(x+2)}\frac{-4}{\cancel{x}\cancel{(x+2)}}$$

$$58. \quad \frac{b+a}{ab} = \frac{1}{f} \quad \text{for } b$$

$$\cancel{ab}f\left(\frac{b+a}{\cancel{ab}}\right) = ab\cancel{f}\left(\frac{1}{\cancel{f}}\right)$$

$$f(b+a) = ab$$

$$fb + fa = ab$$

$$fa = ab - fb$$

$$fa = b(a - f)$$

$$\frac{fa}{a-f} = b$$

$$60. \quad a = \frac{v_2 - v_1}{t_2 - t_1} \quad \text{for } v_1$$

$$a(t_2 - t_1) = v_2 - v_1$$

$$v_1 = v_2 - a(t_2 - t_1)$$

$$62. \quad \frac{1}{y^2 - 3y} + \frac{8}{y} = \frac{2}{y-3}$$

$$\frac{1}{y(y-3)} + \frac{8}{y} = \frac{2}{y-3}$$

$$\text{LCD} = y(y-3) \text{ so } y \neq 0, y \neq 3$$

$$y(y-3)\left(\frac{1}{y(y-3)} + \frac{8}{y}\right) = y(y-3)\left(\frac{2}{y-3}\right)$$

$$\cancel{y}\cancel{(y-3)}\frac{1}{\cancel{y}\cancel{(y-3)}} + \cancel{(y-3)}\frac{8}{\cancel{y}} = y\cancel{(y-3)}\frac{2}{\cancel{y-3}}$$

Chapter 5 Rational Expressions and Rational Equations

$$3x + 2x + 4 = -4$$

$$5x + 4 = -4$$

$$5x = -8$$

$$x = -\frac{8}{5} \quad \left\{ -\frac{8}{5} \right\}$$

$$1 + 8y - 24 = 2y$$

$$8y - 23 = 2y$$

$$-23 = -6y$$

$$y = \frac{23}{6} \quad \left\{ \frac{23}{6} \right\}$$

63. $4c(c+1) = 3(c^2+4)$

$$4c^2 + 4c = 3c^2 + 12$$

$$c^2 + 4c - 12 = 0$$

$$(c+6)(c-2) = 0$$

$$c+6=0 \text{ or } c-2=0$$

$$c = -6 \text{ or } c = 2 \quad \{-6, 2\}$$

64. $3t(2t-2) = 5(t^2-1)$

$$6t^2 - 6t = 5t^2 - 5$$

$$t^2 - 6t + 5 = 0$$

$$(t-5)(t-1) = 0$$

$$t-5=0 \text{ or } t-1=0$$

$$t = 5 \text{ or } t = 1 \quad \{1, 5\}$$

65.

$$\frac{2}{v-1} - \frac{4}{v+5} = \frac{3}{v^2+4v-5}$$

$$\frac{2}{v-1} - \frac{4}{v+5} = \frac{3}{(v+5)(v-1)}$$

LCD = $(v+5)(v-1)$ so $v \neq -5$ or $v \neq 1$

$$(v+5)(v-1)\left(\frac{2}{v-1} - \frac{4}{v+5}\right) = (v+5)(v-1)\left(\frac{3}{(v+5)(v-1)}\right)$$

$$(v+5)\cancel{(v-1)}\left(\frac{2}{\cancel{v-1}}\right) - \cancel{(v+5)}(v-1)\left(\frac{4}{\cancel{v+5}}\right) = \cancel{(v+5)}\cancel{(v-1)}\left(\frac{3}{\cancel{(v+5)}\cancel{(v-1)}}\right)$$

$$2v+10-4v+4=3$$

$$-2v+14=3$$

$$-2v=-11$$

$$v = \frac{11}{2} \quad \left\{ \frac{11}{2} \right\}$$

66.

$$\frac{-2}{a+4} - \frac{3}{a-5} = \frac{6}{a^2-a-20}$$

$$\frac{-2}{a+4} - \frac{3}{a-5} = \frac{6}{(a-5)(a+4)}$$

LCD = $(a-5)(a+4)$ so $a \neq 5$ or $a \neq -4$

$$(a-5)(a+4)\left(\frac{-2}{a+4}-\frac{3}{a-5}\right)=(a-5)(a+4)\left(\frac{6}{(a-5)(a+4)}\right)$$

$$(a-5)\cancel{(a+4)}\left(\frac{-2}{\cancel{a+4}}\right)-\cancel{(a-5)}(a+4)\left(\frac{3}{\cancel{a-5}}\right)=\cancel{(a-5)}\cancel{(a+4)}\left(\frac{6}{\cancel{(a-5)}\cancel{(a+4)}}\right)$$

$$-2a+10-3a-12=6$$

$$-5a-2=6$$

$$-5a=8$$

$$a=-\frac{8}{5} \quad \left\{-\frac{8}{5}\right\}$$

67. $5(x-9)=3(x+4)-2(4x+1)$

$$5x-45=3x+12-8x-2$$

$$5x-45=-5x+10$$

$$10x-45=10$$

$$10x=55$$

$$x=\frac{55}{10}=\frac{11}{2} \quad \left\{\frac{11}{2}\right\}$$

68. $4z-3(5z-3)=z-12$

$$4z-15z+9=z-12$$

$$-11z+9=z-12$$

$$-12z+9=-12$$

$$-12z=-21$$

$$z=\frac{21}{12}=\frac{7}{4} \quad \left\{\frac{7}{4}\right\}$$

69. $\frac{3y}{10}-\frac{5}{2y}=\frac{y}{5}$

LCD = 10y so $y \neq 0$

$$10y\left(\frac{3y}{10}-\frac{5}{2y}\right)=10y\left(\frac{y}{5}\right)$$

$$\cancel{10}y\left(\frac{3y}{\cancel{10}}\right)-5\cdot\cancel{2}y\left(\frac{5}{\cancel{2}y}\right)=\cancel{2}\cdot\cancel{2}y\left(\frac{y}{\cancel{2}}\right)$$

$$3y^2-25=2y^2$$

$$y^2-25=0$$

$$(y-5)(y+5)=0$$

$$y-5=0$$

$$\text{or } y+5=0$$

$$y=5$$

$$\text{or } y=-5 \quad \{-5, 5\}$$

70. $\frac{2h}{3}-\frac{8}{3h}=\frac{h}{2}$

LCD = 6h so $h \neq 0$

$$6h\left(\frac{2h}{3}-\frac{8}{3h}\right)=6h\left(\frac{h}{2}\right)$$

$$2\cdot\cancel{3}h\left(\frac{2h}{\cancel{3}}\right)-2\cdot\cancel{3}h\left(\frac{8}{\cancel{3}h}\right)=\cancel{2}\cdot\cancel{3}h\left(\frac{h}{\cancel{2}}\right)$$

$$4h^2-16=3h^2$$

$$h^2-16=0$$

$$(h-4)(h+4)=0$$

$$h-4=0$$

$$\text{or } h+4=0$$

$$h=4$$

$$\text{or } h=-4 \quad \{-4, 4\}$$

Chapter 5 Rational Expressions and Rational Equations

$$71. \quad \frac{1}{2}(4d-1) + \frac{2}{3}(2d+2) = \frac{5}{6}(4d+1)$$

LCD = 6

$$6 \left[\frac{1}{2}(4d-1) + \frac{2}{3}(2d+2) \right]$$

$$= 6 \left[\frac{5}{6}(4d+1) \right]$$

$$3(4d-1) + 4(2d+2) = 5(4d+1)$$

$$12d - 3 + 8d + 8 = 20d + 5$$

$$20d + 5 = 20d + 5$$

$$5 = 5$$

$$\{d \mid d \text{ is a real number}\}$$

$$72. \quad \frac{2}{5}(2b+5) + \frac{1}{10}(7b-10) = \frac{1}{2}(3b+2)$$

LCD = 10

$$10 \left[\frac{2}{5}(2b+5) + \frac{1}{10}(7b-10) \right]$$

$$= 10 \left[\frac{1}{2}(3b+2) \right]$$

$$4(2b+5) + 7b - 10 = 5(3b+2)$$

$$8b + 20 + 7b - 10 = 15b + 10$$

$$15b + 10 = 15b + 10$$

$$10 = 10$$

$$\{b \mid b \text{ is a real number}\}$$

$$73. \quad 8t^{-1} + 2 = 3t^{-1}$$

$$\frac{8}{t} + 2 = \frac{3}{t} \quad \text{LCD} = t \text{ so } t \neq 0$$

$$t \left(\frac{8}{t} + 2 \right) = t \left(\frac{3}{t} \right)$$

$$\cancel{t} \left(\frac{8}{\cancel{t}} \right) + t(2) = \cancel{t} \left(\frac{3}{\cancel{t}} \right)$$

$$8 + 2t = 3$$

$$2t = -5$$

$$t = -\frac{5}{2} \quad \left\{ -\frac{5}{2} \right\}$$

$$74. \quad 6z^{-2} - 5z^{-1} = 0$$

$$\frac{6}{z^2} - \frac{5}{z} = 0$$

LCD = z^2 so $z \neq 0$

$$z^2 \left(\frac{6}{z^2} - \frac{5}{z} \right) = z^2(0)$$

$$\cancel{z^2} \left(\frac{6}{\cancel{z^2}} \right) - \cancel{z} \cdot z \left(\frac{5}{\cancel{z}} \right) = 0$$

$$6 - 5z = 0$$

$$-5z = -6$$

$$z = \frac{6}{5} \quad \left\{ \frac{6}{5} \right\}$$

$$75. \quad \frac{y-1}{11-3} = \frac{1}{2}$$

$$\frac{y-1}{8} = \frac{1}{2} \quad \text{LCD} = 8$$

$$\cancel{8} \left(\frac{y-1}{\cancel{8}} \right) = 4 \cdot \cancel{2} \left(\frac{1}{\cancel{2}} \right)$$

$$y-1 = 4$$

$$y = 5$$

$$76. \quad \frac{10 - (-5)}{x - (-2)} = 3$$

$$\frac{15}{x+2} = 3 \quad \text{LCD} = x+2$$

$$(\cancel{x+2}) \left(\frac{15}{\cancel{x+2}} \right) = (x+2)(3)$$

$$15 = 3x + 6$$

$$9 = 3x \Rightarrow x = 3$$

Problem Recognition Exercises: Rational Equations vs. Expressions

$$\begin{aligned}
 77. \quad \frac{2 - (-2)}{x - 4} &= 4 \\
 \frac{4}{x - 4} &= 4 \quad \text{LCD} = x - 4 \\
 (\cancel{x - 4}) \left(\frac{4}{\cancel{x - 4}} \right) &= (x - 4)(4) \\
 4 &= 4x - 16 \\
 20 &= 4x \\
 x &= 5
 \end{aligned}$$

$$\begin{aligned}
 78. \quad \frac{y - 2}{-1 - 3} &= -\frac{3}{4} \\
 \frac{y - 2}{-4} &= -\frac{3}{4} \quad \text{LCD} = 4 \\
 \cancel{4} \left(\frac{y - 2}{\cancel{-4}} \right) &= \cancel{4} \left(-\frac{3}{\cancel{4}} \right) \\
 -(y - 2) &= -3 \\
 -y + 2 &= -3 \\
 -y &= -5 \\
 y &= 5
 \end{aligned}$$

Problem Recognition Exercises

$$\begin{aligned}
 1. \quad \text{a.} \quad \frac{3}{w - 5} + \frac{10}{w^2 - 25} - \frac{1}{w + 5} &= \frac{3}{w - 5} + \frac{10}{(w + 5)(w - 5)} - \frac{1}{w + 5} \quad \text{LCD} = (w + 5)(w - 5) \\
 &= \frac{3}{w - 5} \cdot \frac{w + 5}{w + 5} + \frac{10}{(w + 5)(w - 5)} - \frac{1}{w + 5} \cdot \frac{w - 5}{w - 5} = \frac{3w + 15 + 10 - w + 5}{(w + 5)(w - 5)} = \frac{2w + 30}{(w + 5)(w - 5)}
 \end{aligned}$$

$$\begin{aligned}
 \text{b.} \quad \frac{3}{w - 5} + \frac{10}{w^2 - 25} - \frac{1}{w + 5} &= 0 \\
 \frac{2w + 30}{(w + 5)(w - 5)} &= 0 \\
 (\cancel{w + 5})(\cancel{w - 5}) \left(\frac{2w + 30}{(\cancel{w + 5})(\cancel{w - 5})} \right) &= (w + 5)(w - 5)(0) \\
 2w + 30 &= 0 \\
 2w &= -30 \\
 w &= -15 \quad \{-15\}
 \end{aligned}$$

c. The problem in part (a) is an expression, and the problem in part (b) is an equation.

$$\begin{aligned}
 2. \quad \text{a.} \quad \frac{x}{2x + 4} + \frac{2}{3x + 6} - 1 &= \frac{x}{2(x + 2)} + \frac{2}{3(x + 2)} - 1 \quad \text{LCD} = 2 \cdot 3(x + 2) \\
 &= \frac{x}{2(x + 2)} \cdot \frac{3}{3} + \frac{2}{3(x + 2)} \cdot \frac{2}{2} - 1 \cdot \frac{2 \cdot 3(x + 2)}{2 \cdot 3(x + 2)} = \frac{3x + 4 - 6x - 12}{6(x + 2)} = \frac{-3x - 8}{6(x + 2)}
 \end{aligned}$$

b.

$$\begin{aligned} \frac{x}{2x+4} + \frac{2}{3x+6} &= 1 \\ \frac{x}{2(x+2)} + \frac{2}{3(x+2)} &= 1 \\ 6(x+2) \left(\frac{x}{2(x+2)} + \frac{2}{3(x+2)} \right) &= 6(x+2)(1) \\ \cancel{6} \cdot \cancel{3} (\cancel{x+2}) \left(\frac{x}{\cancel{2}(\cancel{x+2})} \right) + 2 \cdot \cancel{3} (\cancel{x+2}) \left(\frac{2}{\cancel{3}(\cancel{x+2})} \right) &= 6(x+2) \\ 3x+4 &= 6x+12 \\ -3x &= 8 \\ x &= -\frac{8}{3} \quad \left\{ -\frac{8}{3} \right\} \end{aligned}$$

c. The problem in part (a) is an expression, and the problem in part (b) is an equation.

$$\begin{aligned} 3. \quad \frac{2}{a^2+4a+3} + \frac{1}{a+3} &= \frac{2}{(a+3)(a+1)} + \frac{1}{a+3} \quad \text{LCD} = (a+3)(a+1) \\ &= \frac{2}{(a+3)(a+1)} + \frac{1}{a+3} \cdot \frac{a+1}{a+1} = \frac{2+a+1}{(a+3)(a+1)} = \frac{\cancel{a+3}}{(\cancel{a+3})(a+1)} = \frac{1}{a+1} \end{aligned}$$

$$\begin{aligned} 4. \quad \frac{1}{c+6} + \frac{4}{c^2+8c+12} &= \frac{1}{c+6} + \frac{4}{(c+6)(c+2)} \quad \text{LCD} = (c+6)(c+2) \\ &= \frac{1}{c+6} \cdot \frac{c+2}{c+2} + \frac{4}{(c+6)(c+2)} = \frac{c+2+4}{(c+6)(c+2)} = \frac{\cancel{c+6}}{(\cancel{c+6})(c+2)} = \frac{1}{c+2} \end{aligned}$$

$$\begin{aligned} 5. \quad \frac{7}{y^2-y-2} + \frac{1}{y+1} - \frac{3}{y-2} &= 0 \\ \frac{7}{(y-2)(y+1)} + \frac{1}{y+1} - \frac{3}{y-2} &= 0 \quad \text{LCD} = (y-2)(y+1) \quad \text{so } y \neq 2 \text{ or } y \neq -1 \\ (y-2)(y+1) \left(\frac{7}{(y-2)(y+1)} + \frac{1}{y+1} - \frac{3}{y-2} \right) &= (y-2)(y+1)(0) \\ (\cancel{y-2})(\cancel{y+1}) \left(\frac{7}{(\cancel{y-2})(\cancel{y+1})} \right) + (y-2)(\cancel{y+1}) \left(\frac{1}{\cancel{y+1}} \right) - (\cancel{y-2})(y+1) \left(\frac{3}{\cancel{y-2}} \right) &= 0 \end{aligned}$$

Problem Recognition Exercises: Rational Equations vs. Expressions

$$\begin{aligned} 7 + y - 2 - 3y - 3 &= 0 \\ -2y + 2 &= 0 \\ -2y &= -2 \\ y &= 1 \end{aligned}$$

{1} is the solution.

6.
$$\frac{3}{b+2} - \frac{1}{b-1} - \frac{5}{b^2+b-2} = 0$$

$$\frac{3}{b+2} - \frac{1}{b-1} - \frac{5}{(b+2)(b-1)} = 0 \quad \text{LCD} = (b+2)(b-1) \quad \text{so } b \neq -2 \text{ or } b \neq 1$$

$$(b+2)(b-1) \left(\frac{3}{b+2} - \frac{1}{b-1} - \frac{5}{(b+2)(b-1)} \right) = (b+2)(b-1)(0)$$

$$\begin{aligned} (\cancel{b+2})(b-1) \left(\frac{3}{\cancel{b+2}} \right) - (b+2)(\cancel{b-1}) \left(\frac{1}{\cancel{b-1}} \right) - (\cancel{b+2})(\cancel{b-1}) \left(\frac{5}{(\cancel{b+2})(\cancel{b-1})} \right) &= 0 \\ 3b - 3 - b - 2 - 5 &= 0 \\ 2b - 10 &= 0 \\ 2b &= 10 \\ b &= 5 \end{aligned}$$

{5} is the solution.

7.
$$\frac{x}{x-1} - \frac{12}{x^2-x} = \frac{x}{x-1} - \frac{12}{x(x-1)}$$

$$\begin{aligned} \text{LCD} &= x(x-1) \\ &= \frac{x}{x-1} \cdot \frac{x}{x} - \frac{12}{x(x-1)} = \frac{x^2-12}{x(x-1)} \end{aligned}$$

8.
$$\frac{3}{5t-20} + \frac{4}{t-4} = \frac{3}{5(t-4)} + \frac{4}{t-4}$$

$$\begin{aligned} \text{LCD} &= 5(t-4) \\ &= \frac{3}{5(t-4)} + \frac{4}{t-4} \cdot \frac{5}{5} = \frac{3+20}{5(t-4)} = \frac{23}{5(t-4)} \end{aligned}$$

9.
$$\frac{3}{w} - 5 = \frac{7}{w} - 1$$

$$\begin{aligned} \text{LCD} &= w \quad \text{so } w \neq 0 \\ w \left(\frac{3}{w} - 5 \right) &= w \left(\frac{7}{w} - 1 \right) \\ \cancel{w} \left(\frac{3}{\cancel{w}} \right) - w(5) &= \cancel{w} \left(\frac{7}{\cancel{w}} \right) - w(1) \\ 3 - 5w &= 7 - w \end{aligned}$$

10.
$$\frac{-3}{y^2} - \frac{1}{y} = -2$$

$$\begin{aligned} \text{LCD} &= y^2 \quad \text{so } y \neq 0 \\ y^2 \left(\frac{-3}{y^2} - \frac{1}{y} \right) &= y^2(-2) \\ \cancel{y^2} \left(\frac{-3}{\cancel{y^2}} \right) - y \cdot \cancel{y} \left(\frac{1}{\cancel{y}} \right) &= -2y^2 \\ -3 - y &= -2y^2 \end{aligned}$$

$$\begin{aligned} -4w &= 4 \\ w &= -1 \quad \{-1\} \end{aligned}$$

$$\begin{aligned} 2y^2 - y - 3 &= 0 \\ (2y-3)(y+1) &= 0 \\ 2y-3 &= 0 \quad \text{or} \quad y+1=0 \\ 2y &= 3 \quad \text{or} \quad y = -1 \\ y &= \frac{3}{2} \quad \text{or} \quad y = -1 \quad \left\{ \frac{3}{2}, -1 \right\} \end{aligned}$$

$$\begin{aligned} 11. \quad \frac{4p+1}{8p-12} + \frac{p-3}{2p-3} &= \frac{4p+1}{4(2p-3)} + \frac{p-3}{2p-3} \\ \text{LCD} &= 4(2p-3) \\ &= \frac{4p+1}{4(2p-3)} + \frac{p-3}{2p-3} \cdot \frac{4}{4} \\ &= \frac{4p+1+4p-12}{4(2p-3)} = \frac{8p-11}{4(2p-3)} \end{aligned}$$

$$\begin{aligned} 12. \quad \frac{x+1}{2x+4} - \frac{x^2}{x+2} &= \frac{x+1}{2(x+2)} - \frac{x^2}{x+2} \\ \text{LCD} &= 2(x+2) \\ &= \frac{x+1}{2(x+2)} - \frac{x^2}{x+2} \cdot \frac{2}{2} = \frac{x+1-2x^2}{2(x+2)} \\ &= \frac{-(2x^2-x-1)}{2(x+2)} = \frac{-(2x+1)(x-1)}{2(x+2)} \end{aligned}$$

$$\begin{aligned} 13. \quad \frac{1}{2x^2} + \frac{1}{6x} \quad \text{LCD} &= 6x^2 \\ &= \frac{1}{2x^2} \cdot \frac{3}{3} + \frac{1}{6x} \cdot \frac{x}{x} = \frac{3+x}{6x^2} \end{aligned}$$

$$\begin{aligned} 14. \quad \frac{5}{4a} + \frac{1}{6a^2} \quad \text{LCD} &= 12a^2 \\ &= \frac{5}{4a} \cdot \frac{3a}{3a} + \frac{1}{6a^2} \cdot \frac{2}{2} = \frac{15a+2}{12a^2} \end{aligned}$$

$$\begin{aligned} 15. \quad \frac{3}{2t} + \frac{2}{3t^2} &= \frac{-1}{t} \\ \text{LCD} &= 6t^2 \quad \text{so } t \neq 0 \\ 6t^2 \left(\frac{3}{2t} + \frac{2}{3t^2} \right) &= 6t^2 \left(\frac{-1}{t} \right) \\ \cancel{2t} \cdot 3t \left(\frac{3}{\cancel{2t}} \right) + 2 \cdot \cancel{3t^2} \left(\frac{2}{\cancel{3t^2}} \right) &= 6t \cdot \cancel{t} \left(\frac{-1}{\cancel{t}} \right) \\ 9t + 4 &= -6t \\ 4 &= -15t \\ -\frac{4}{15} &= t \quad \left\{ -\frac{4}{15} \right\} \end{aligned}$$

$$\begin{aligned} 16. \quad \frac{-3}{b^2} + \frac{1}{5b} &= \frac{1}{2b} \\ \text{LCD} &= 10b^2 \quad \text{so } b \neq 0 \\ 10b^2 \left(\frac{-3}{b^2} + \frac{1}{5b} \right) &= 10b^2 \left(\frac{1}{2b} \right) \\ 10 \cancel{b^2} \left(\frac{-3}{\cancel{b^2}} \right) + 2b \cdot \cancel{5b} \left(\frac{1}{\cancel{5b}} \right) &= 2\cancel{b} \cdot 5b \left(\frac{1}{\cancel{2b}} \right) \\ -30 + 2b &= 5b \\ -30 &= 3b \\ -10 &= b \quad \{-10\} \end{aligned}$$

Problem Recognition Exercises: Rational Equations vs. Expressions

$$\begin{aligned}
 17. \quad & \frac{3}{c^2+4c+3} - \frac{2}{c^2+6c+9} \\
 &= \frac{3}{(c+3)(c+1)} - \frac{2}{(c+3)^2} \\
 \text{LCD} &= (c+3)^2(c+1) \\
 &= \frac{3}{(c+3)(c+1)} \cdot \frac{c+3}{c+3} - \frac{2}{(c+3)^2} \cdot \frac{c+1}{c+1} \\
 &= \frac{3c+9-2c-2}{(c+3)^2(c+1)} \\
 &= \frac{c+7}{(c+3)^2(c+1)}
 \end{aligned}$$

$$\begin{aligned}
 18. \quad & \frac{1}{y^2-10y+25} - \frac{3}{y^2-7y+10} \\
 &= \frac{1}{(y-5)^2} - \frac{3}{(y-5)(y-2)} \\
 \text{LCD} &= (y-5)^2(y-2) \\
 &= \frac{1}{(y-5)^2} \cdot \frac{y-2}{y-2} - \frac{3}{(y-5)(y-2)} \cdot \frac{y-5}{y-5} \\
 &= \frac{y-2-3y+15}{(y-5)^2(y-2)} \\
 &= \frac{-2y+13}{(y-5)^2(y-2)}
 \end{aligned}$$

$$\begin{aligned}
 19. \quad & \frac{4}{w-4} - \frac{36}{2w^2-7w-4} = \frac{3}{2w+1} \\
 & \frac{4}{w-4} - \frac{36}{(2w+1)(w-4)} = \frac{3}{2w+1} \quad \text{LCD} = (2w+1)(w-4) \quad \text{so } w \neq -\frac{1}{2} \text{ or } w \neq 4 \\
 & (2w+1)(w-4) \left(\frac{4}{w-4} - \frac{36}{(2w+1)(w-4)} \right) = (2w+1)(w-4) \left(\frac{3}{2w+1} \right) \\
 & (2w+1) \cancel{(w-4)} \left(\frac{4}{\cancel{w-4}} \right) - \cancel{(2w+1)} \cancel{(w-4)} \left(\frac{36}{\cancel{(2w+1)} \cancel{(w-4)}} \right) = \cancel{(2w+1)} \cancel{(w-4)} \left(\frac{3}{\cancel{2w+1}} \right) \\
 & \quad \quad \quad 8w+4-36=3w-12 \\
 & \quad \quad \quad 8w-32=3w-12 \\
 & \quad \quad \quad 5w-32=-12 \\
 & \quad \quad \quad 5w=20 \\
 & \quad \quad \quad w=4
 \end{aligned}$$

{ } no solution. ($w = 4$ does not check because the denominator is zero.)

$$\begin{aligned}
 20. \quad & \frac{2}{x-3} - \frac{5}{x+2} = \frac{25}{x^2-x-6} \\
 & \frac{2}{x-3} - \frac{5}{x+2} = \frac{25}{(x-3)(x+2)} \\
 \text{LCD} &= (x-3)(x+2) \quad \text{so } x \neq 3 \text{ or } x \neq -2 \\
 & (x-3)(x+2) \left(\frac{2}{x-3} - \frac{5}{x+2} \right) = (x-3)(x+2) \left(\frac{25}{(x-3)(x+2)} \right)
 \end{aligned}$$

$$\cancel{(x-3)}(x+2)\left(\frac{2}{\cancel{x-3}}\right) - (x-3)\cancel{(x+2)}\left(\frac{5}{\cancel{x+2}}\right) = \cancel{(x-3)}\cancel{(x+2)}\left(\frac{25}{\cancel{(x-3)}\cancel{(x+2)}}\right)$$

$$2x + 4 - 5x + 15 = 25$$

$$-3x + 19 = 25$$

$$-3x = 6$$

$$x = -2$$

{ } no solution. ($x = -2$ does not check because the denominator is zero.)

Section 5.6 Practice Exercises

1. a. proportion

b. proportional

2. $3 - \frac{6}{x} = x + 8$

LCD = x so $x \neq 0$

$$x\left(3 - \frac{6}{x}\right) = x(x + 8)$$

$$x(3) - \cancel{x}\left(\frac{6}{\cancel{x}}\right) = x^2 + 8x$$

$$3x - 6 = x^2 + 8x$$

$$0 = x^2 + 5x + 6$$

$$(x + 3)(x + 2) = 0$$

$$x + 3 = 0 \text{ or } x + 2 = 0$$

$$x = -3 \text{ or } x = -2 \quad \{-3, -2\}$$

3. $2 + \frac{6}{x} = x + 7$

LCD = x so $x \neq 0$

$$x\left(2 + \frac{6}{x}\right) = x(x + 7)$$

$$x(2) + \cancel{x}\left(\frac{6}{\cancel{x}}\right) = x^2 + 7x$$

$$2x + 6 = x^2 + 7x$$

$$0 = x^2 + 5x - 6$$

$$(x + 6)(x - 1) = 0$$

$$x + 6 = 0 \text{ or } x - 1 = 0$$

$$x = -6 \text{ or } x = 1 \quad \{-6, 1\}$$

4. $\frac{5}{3x-6} - \frac{3}{4x-8} = \frac{5}{3(x-2)} - \frac{3}{4(x-2)}$

LCD = $12(x-2)$

$$= \frac{5}{3(x-2)} \cdot \frac{4}{4} - \frac{3}{4(x-2)} \cdot \frac{3}{3}$$

$$= \frac{20-9}{12(x-2)}$$

$$= \frac{11}{12(x-2)}$$

5. $\frac{4}{5t-1} + \frac{1}{10t-2} = \frac{4}{5t-1} + \frac{1}{2(5t-1)}$

LCD = $2(5t-1)$

$$= \frac{4}{5t-1} \cdot \frac{2}{2} + \frac{1}{2(5t-1)}$$

$$= \frac{8+1}{2(5t-1)}$$

$$= \frac{9}{2(5t-1)}$$

Section 5.6 Applications of Rational Equations and Proportions

6.

$$\frac{2}{y-1} - \frac{5}{4} = \frac{-1}{y+1}$$

$$\text{LCD} = 4(y+1)(y-1) \quad \text{so } y \neq -1 \text{ or } y \neq 1$$

$$4(y+1)(y-1)\left(\frac{2}{y-1} - \frac{5}{4}\right) = 4(y+1)(y-1)\left(\frac{-1}{y+1}\right)$$

$$4(y+1)\left(\frac{2}{\cancel{y-1}}\right) - \cancel{4}(y+1)(y-1)\left(\frac{5}{\cancel{4}}\right) = -4(y-1)$$

$$8(y+1) - 5(y^2 - 1) = -4y + 4$$

$$8y + 8 - 5y^2 + 5 = -4y + 4$$

$$-5y^2 + 12y + 9 = 0$$

$$-(5y^2 - 12y - 9) = 0$$

$$-(5y+3)(y-3) = 0$$

$$-1 \neq 0 \text{ or } 5y+3=0 \text{ or } y-3=0$$

$$5y = -3 \text{ or } y = 3$$

$$y = -\frac{3}{5} \text{ or } y = 3 \quad \left\{-\frac{3}{5}, 3\right\}$$

7.

$$\frac{5}{w-2} = 7 - \frac{10}{w+2}$$

$$\text{LCD} = (w+2)(w-2) \quad \text{so } w \neq -2 \text{ or } w \neq 2$$

$$(w+2)(w-2)\left(\frac{5}{w-2}\right) = (w+2)(w-2)\left(7 - \frac{10}{w+2}\right)$$

$$(w+2)\left(\frac{5}{\cancel{w-2}}\right) = (w+2)(w-2)(7) - (\cancel{w+2})(w-2)\left(\frac{10}{\cancel{w+2}}\right)$$

$$5w + 10 = 7(w^2 - 4) - 10w + 20$$

$$5w + 10 = 7w^2 - 28 - 10w + 20$$

$$0 = 7w^2 - 15w - 18$$

$$(7w+6)(w-3) = 0$$

$$7w+6=0 \text{ or } w-3=0$$

$$7w = -6 \text{ or } w = 3$$

$$w = -\frac{6}{7} \text{ or } w = 3 \quad \left\{-\frac{6}{7}, 3\right\}$$

8.

$$\frac{ab}{6} \div \frac{a^2}{12} \cdot \frac{a+1}{ab+b} = \frac{ab}{6} \cdot \frac{12}{a^2} \cdot \frac{a+1}{b(a+1)} = \frac{\cancel{a} \cancel{b}}{\cancel{6}} \cdot \frac{\cancel{6} \cdot 2}{\cancel{a} \cdot a} \cdot \frac{\cancel{a+1}}{\cancel{b} (a+1)} = \frac{2}{a}$$

$$\begin{aligned}
 9. \quad \frac{8p^2 - 32}{p^2 - 4p + 4} \cdot \frac{3p^2 - 3p - 6}{2p^2 + 20p + 32} &= \frac{8(p^2 - 4)}{p^2 - 4p + 4} \cdot \frac{3(p^2 - p - 2)}{2(p^2 + 10p + 16)} \\
 &= \frac{4 \cdot \cancel{(p+2)} \cdot \cancel{(p-2)}}{\cancel{(p-2)} \cdot \cancel{(p-2)}} \cdot \frac{3 \cdot \cancel{(p-2)} \cdot (p+1)}{\cancel{(p+8)} \cdot \cancel{(p+2)}} \\
 &= \frac{12(p+1)}{p+8}
 \end{aligned}$$

$$\begin{aligned}
 10. \quad \frac{3t}{t+6} &= t + \frac{2t}{t+6} \quad \text{LCD} = t+6 \quad \text{so } t \neq -6 \\
 (t+6) \left(\frac{3t}{t+6} \right) &= (t+6) \left(t + \frac{2t}{t+6} \right) \\
 3t &= t(t+6) + 2t \\
 3t &= t^2 + 6t + 2t \\
 3t &= t^2 + 8t \\
 0 &= t^2 + 5t \\
 t(t+5) &= 0 \\
 t = 0 \text{ or } t+5 &= 0 \\
 t = 0 \text{ or } t = -5 & \quad \{0, -5\}
 \end{aligned}$$

$$\begin{aligned}
 11. \quad \frac{y}{6} &= \frac{20}{15} \quad \text{LCD} = 30 \\
 30 \left(\frac{y}{6} \right) &= 30 \left(\frac{20}{15} \right) \\
 5y &= 40 \\
 y &= 8 \quad \{8\}
 \end{aligned}$$

$$\begin{aligned}
 12. \quad \frac{12}{18} &= \frac{14}{x} \quad \text{LCD} = 18x \\
 18x \left(\frac{12}{18} \right) &= 18x \left(\frac{14}{x} \right) \\
 12x &= 252 \\
 x &= 21 \quad \{21\}
 \end{aligned}$$

$$\begin{aligned}
 13. \quad \frac{9}{75} &= \frac{m}{50} \quad \text{LCD} = 150 \\
 150 \left(\frac{9}{75} \right) &= 150 \left(\frac{m}{50} \right) \\
 18 &= 3m \\
 m &= 6 \quad \{6\}
 \end{aligned}$$

$$\begin{aligned}
 14. \quad \frac{n}{15} &= \frac{12}{45} \quad \text{LCD} = 45 \\
 45 \left(\frac{n}{15} \right) &= 45 \left(\frac{12}{45} \right) \\
 3n &= 12 \\
 n &= 4 \quad \{4\}
 \end{aligned}$$

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$$\begin{aligned}
 15. \quad \frac{p-1}{4} &= \frac{p+3}{3} & \text{LCD} &= 12 \\
 12\left(\frac{p-1}{4}\right) &= 12\left(\frac{p+3}{3}\right) \\
 3(p-1) &= 4(p+3) \\
 3p-3 &= 4p+12 \\
 -15 &= p & \{-15\}
 \end{aligned}$$

$$\begin{aligned}
 16. \quad \frac{q-5}{2} &= \frac{q+2}{3} & \text{LCD} &= 6 \\
 6\left(\frac{q-5}{2}\right) &= 6\left(\frac{q+2}{3}\right) \\
 3(q-5) &= 2(q+2) \\
 3q-15 &= 2q+4 \\
 q &= 19 & \{19\}
 \end{aligned}$$

$$\begin{aligned}
 17. \quad \frac{x+1}{5} &= \frac{4}{15} & \text{LCD} &= 15 \\
 15\left(\frac{x+1}{5}\right) &= 15\left(\frac{4}{15}\right) \\
 3(x+1) &= 4 \\
 3x+3 &= 4 \\
 3x &= 1 \\
 x &= \frac{1}{3} & \left\{\frac{1}{3}\right\}
 \end{aligned}$$

$$\begin{aligned}
 18. \quad \frac{t-1}{7} &= \frac{2}{21} & \text{LCD} &= 21 \\
 21\left(\frac{t-1}{7}\right) &= 21\left(\frac{2}{21}\right) \\
 3(t-1) &= 2 \\
 3t-3 &= 2 \\
 3t &= 5 \\
 t &= \frac{5}{3} & \left\{\frac{5}{3}\right\}
 \end{aligned}$$

$$\begin{aligned}
 19. \quad \frac{5-2x}{x} &= \frac{1}{4} & \text{LCD} &= 4x \\
 4x\left(\frac{5-2x}{x}\right) &= 4x\left(\frac{1}{4}\right) \\
 4(5-2x) &= x \\
 20-8x &= x \\
 20 &= 9x \\
 x &= \frac{20}{9} & \left\{\frac{20}{9}\right\}
 \end{aligned}$$

$$\begin{aligned}
 20. \quad \frac{2y+3}{y} &= \frac{3}{2} & \text{LCD} &= 2y \\
 2y\left(\frac{2y+3}{y}\right) &= 2y\left(\frac{3}{2}\right) \\
 2(2y+3) &= 3y \\
 4y+6 &= 3y \\
 y &= -6 & \{-6\}
 \end{aligned}$$

$$\begin{aligned}
 21. \quad \frac{2}{y-1} &= \frac{y-3}{4} \\
 \text{LCD} &= 4(y-1) \\
 4\left(\cancel{y-1}\right)\left(\frac{2}{\cancel{y-1}}\right) &= 4(y-1)\left(\frac{y-3}{\cancel{4}}\right) \\
 8 &= y^2 - 4y + 3 \\
 0 &= y^2 - 4y - 5
 \end{aligned}$$

$$\begin{aligned}
 22. \quad \frac{1}{x-5} &= \frac{x-3}{3} \\
 \text{LCD} &= 3(x-5) \\
 3\left(\cancel{x-5}\right)\left(\frac{1}{\cancel{x-5}}\right) &= \cancel{\beta}(x-5)\left(\frac{x-3}{\cancel{\beta}}\right) \\
 3 &= x^2 - 8x + 15 \\
 0 &= x^2 - 8x + 12
 \end{aligned}$$

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$$\begin{aligned}(y-5)(y+1) &= 0 \\ y-5 &= 0 \text{ or } y+1 = 0 \\ y &= 5 \text{ or } y = -1 \quad \{5, -1\}\end{aligned}$$

$$\begin{aligned}(x-6)(x-2) &= 0 \\ x-6 &= 0 \text{ or } x-2 = 0 \\ x &= 6 \text{ or } x = 2 \quad \{6, 2\}\end{aligned}$$

23.

$$\frac{1}{49w} = \frac{w}{9}$$

LCD = $9 \cdot 49w$

$$9 \cdot \cancel{49w} \left(\frac{1}{\cancel{49w}} \right) = \cancel{9} \cdot 49w \left(\frac{w}{\cancel{9}} \right)$$

$$9 = 49w^2$$

$$0 = 49w^2 - 9$$

$$(7w+3)(7w-3) = 0$$

$$7w+3 = 0 \text{ or } 7w-3 = 0$$

$$7w = -3 \text{ or } 7w = 3$$

$$w = -\frac{3}{7} \text{ or } w = \frac{3}{7} \quad \left\{ -\frac{3}{7}, \frac{3}{7} \right\}$$

24.

$$\frac{1}{4z} = \frac{z}{25}$$

LCD = $25 \cdot 4z$

$$25 \cdot \cancel{4z} \left(\frac{1}{\cancel{4z}} \right) = \cancel{25} \cdot 4z \left(\frac{z}{\cancel{25}} \right)$$

$$25 = 4z^2$$

$$0 = 4z^2 - 25$$

$$(2z+5)(2z-5) = 0$$

$$2z+5 = 0 \text{ or } 2z-5 = 0$$

$$2z = -5 \text{ or } 2z = 5$$

$$z = -\frac{5}{2} \text{ or } z = \frac{5}{2} \quad \left\{ -\frac{5}{2}, \frac{5}{2} \right\}$$

25.

$$\frac{x+3}{5x+26} = \frac{2}{x+4}$$

LCD = $(5x+26)(x+4)$

$$\cancel{(5x+26)}(x+4) \left(\frac{x+3}{\cancel{5x+26}} \right)$$

$$= (5x+26) \cancel{(x+4)} \left(\frac{2}{\cancel{x+4}} \right)$$

$$x^2 + 7x + 12 = 10x + 52$$

$$x^2 - 3x - 40 = 0$$

$$(x-8)(x+5) = 0$$

$$x-8 = 0 \text{ or } x+5 = 0$$

$$x = 8 \text{ or } x = -5 \quad \{8, -5\}$$

26.

$$\frac{-2}{x-2} = \frac{x-3}{8x+11}$$

LCD = $(x-2)(8x+11)$

$$\cancel{(x-2)}(8x+11) \left(\frac{-2}{\cancel{x-2}} \right)$$

$$= (x-2) \cancel{(8x+11)} \left(\frac{x-3}{\cancel{8x+11}} \right)$$

$$-16x - 22 = x^2 - 5x + 6$$

$$0 = x^2 + 11x + 28$$

$$0 = (x+7)(x+4)$$

$$x+7 = 0 \text{ or } x+4 = 0$$

$$x = -7 \text{ or } x = -4 \quad \{-7, -4\}$$

27. Let a = the number of adults

$$\frac{3}{1} = \frac{18}{a} \quad \text{LCD} = a$$

$$a \left(\frac{3}{1} \right) = a \left(\frac{18}{a} \right)$$

28. Let c = the number of children

$$\frac{4}{1} = \frac{c}{5} \quad \text{LCD} = 5$$

$$5 \left(\frac{4}{1} \right) = 5 \left(\frac{c}{5} \right)$$

$$20 = c$$

$$3a = 18$$

$$a = 6$$

6 adults must be on the staff.

The maximum number of children that can enroll is 20 children.

- 29.** Let x = the number of grams of fat

$$\frac{3.5}{21.0} = \frac{14}{x} \quad \text{LCD} = 21x$$

$$21x \left(\frac{3.5}{21.0} \right) = 21x \left(\frac{14}{x} \right)$$

$$3.5x = 294$$

$$x = 84$$

The 14-oz box of candy contains 84 g of fat.

- 30.** Let c = the number of calories

$$\frac{6}{350} = \frac{10}{c} \quad \text{LCD} = 350c$$

$$350c \left(\frac{6}{350} \right) = 350c \left(\frac{10}{c} \right)$$

$$6c = 3500$$

$$c = 583.\bar{3}$$

The 10-oz box of candy contains approximately 583 calories.

- 31.** Let x = the number of fish

$$\frac{8}{1840} = \frac{x}{230,000}$$

$$\text{LCD} = 230,000$$

$$230,000 \left(\frac{8}{1840} \right) = 230,000 \left(\frac{x}{230,000} \right)$$

$$1000 = x$$

1000 swordfish were caught.

- 32.** Let c = the cost of 80-oz bottle

$$\frac{64}{12.00} = \frac{80}{c} \quad \text{LCD} = 12.00c$$

$$12.00c \left(\frac{64}{12.00} \right) = 12.00c \left(\frac{80}{c} \right)$$

$$64c = 960$$

$$c = 15.00$$

The 80-oz bottle costs \$15.00.

- 33.** Let x = the number of gallons of gas

$$\frac{243}{4.5} = \frac{621}{x} \quad \text{LCD} = 4.5x$$

$$4.5x \left(\frac{243}{4.5} \right) = 4.5x \left(\frac{621}{x} \right)$$

$$243x = 2794.5$$

$$x = 11.5$$

Pam needs 11.5 gallons of gas.

- 34.** Let x = the actual distance

$$\frac{8}{96} = \frac{7}{x} \quad \text{LCD} = 96x$$

$$96x \left(\frac{8}{96} \right) = 96x \left(\frac{7}{x} \right)$$

$$8x = 672$$

$$x = 84$$

The actual distance is 84 mi.

- 35.** Let x = the total number of bison

$$\frac{x}{200} = \frac{120}{6}$$

$$\text{LCD} = 600$$

$$600 \left(\frac{x}{200} \right) = 600 \left(\frac{120}{6} \right)$$

- 36.** Let x = the total number of manatees

$$\frac{x}{150} = \frac{40}{3}$$

$$\text{LCD} = 150$$

$$150 \left(\frac{x}{150} \right) = 150 \left(\frac{40}{3} \right)$$

$$3x = 12000$$

$$x = 4000$$

There are approximately 4000 bison in the park.

- 37.** Let x = the number of men

$186 - x$ = the number of women

$$\frac{1}{5} = \frac{x}{186 - x}$$

$$\text{LCD} = 186 - x$$

$$5(186 - x)\left(\frac{1}{5}\right) = 5(186 - x)\left(\frac{x}{186 - x}\right)$$

$$186 - x = 5x$$

$$186 = 6x$$

$$x = 31$$

There are 31 men enrolled.

- 39.** Let x = the number of women

$1095 - x$ = the number of men

$$\frac{119}{100} = \frac{1095 - x}{x}$$

$$\text{LCD} = 100x$$

$$100x\left(\frac{119}{100}\right) = 100x\left(\frac{1095 - x}{x}\right)$$

$$119x = 109,500 - 100x$$

$$219x = 109,500$$

$$x = 500$$

$$1095 - x = 1095 - 500$$

$$= 595$$

There are 595 men and 500 women in the group.

$$x = 2000$$

There are approximately 2000 manatees in Florida.

- 38.** Let x = the amount spent on rent

$1640 - x$ = amount spent on car payments

$$\frac{3}{1} = \frac{x}{1640 - x}$$

$$\text{LCD} = 1640 - x$$

$$(1640 - x)\left(\frac{3}{1}\right) = (1640 - x)\left(\frac{x}{1640 - x}\right)$$

$$4920 - 3x = x$$

$$4920 = 4x$$

$$x = 1230$$

$$1640 - x = 1640 - 1230 = 410$$

Rent was \$1230 and car payment was \$410.

- 40.** Let x = the amount alcohol

$450 - x$ = amount of water

$$\frac{7}{8} = \frac{450 - x}{x}$$

$$\text{LCD} = 8x$$

$$8x\left(\frac{7}{8}\right) = 8x\left(\frac{450 - x}{x}\right)$$

$$7x = 3600 - 8x$$

$$15x = 3600$$

$$x = 240$$

$$450 - x = 450 - 240$$

$$= 210$$

There are 240 L of alcohol and 210 L of water in the mixture.

Section 5.6 Applications of Rational Equations and Proportions

41.
$$\frac{11.2}{a} = \frac{14}{10}$$

$$10a\left(\frac{11.2}{a}\right) = 10a\left(\frac{14}{10}\right)$$

$$112 = 14a$$

$$a = 8 \text{ ft}$$

$$\frac{b}{6} = \frac{14}{10}$$

$$30\left(\frac{b}{6}\right) = 30\left(\frac{14}{10}\right)$$

$$5b = 42$$

$$b = 8.4 \text{ ft}$$

42.
$$\frac{18}{x} = \frac{12}{8}$$

$$8x\left(\frac{18}{x}\right) = 8x\left(\frac{12}{8}\right)$$

$$144 = 12x$$

$$x = 12 \text{ cm}$$

$$\frac{y}{13} = \frac{12}{8}$$

$$104\left(\frac{y}{13}\right) = 104\left(\frac{12}{8}\right)$$

$$8y = 156$$

$$y = 19.5 \text{ cm}$$

43.
$$\frac{1.75}{5} = \frac{4.55}{y}$$

$$5y\left(\frac{1.75}{5}\right) = 5y\left(\frac{4.55}{y}\right)$$

$$1.75y = 22.75$$

$$y = 13 \text{ in}$$

$$(1.75)^2 + z^2 = (4.55)^2$$

$$3.0625 + z^2 = 20.7025$$

$$z^2 = 17.64$$

$$z = 4.2 \text{ in}$$

$$\frac{1.75}{5} = \frac{4.2}{x}$$

$$5x\left(\frac{1.75}{5}\right) = 5x\left(\frac{4.2}{x}\right)$$

$$1.75x = 21$$

$$x = 12 \text{ in}$$

44.
$$\frac{3.75}{5} = \frac{a}{12}$$

$$60\left(\frac{3.75}{5}\right) = 60\left(\frac{a}{12}\right)$$

$$45 = 5a$$

$$a = 9 \text{ m}$$

$$(3.75)^2 + (9)^2 = b^2$$

$$14.0625 + 81 = b^2$$

$$b^2 = 95.0625$$

$$b = 9.75 \text{ m}$$

$$\frac{3.75}{5} = \frac{9.75}{c}$$

$$5c\left(\frac{3.75}{5}\right) = 5c\left(\frac{9.75}{c}\right)$$

$$3.75c = 48.75$$

$$c = 13 \text{ m}$$

45. Let x = the number

$$\frac{1}{x} + 5 = \frac{16}{3} \quad \text{LCD} = 3x \text{ so } x \neq 0$$

$$3x\left(\frac{1}{x} + 5\right) = 3x\left(\frac{16}{3}\right)$$

$$3\cancel{x}\left(\frac{1}{\cancel{x}}\right) + 3x(5) = \cancel{3}x\left(\frac{16}{\cancel{3}}\right)$$

$$3 + 15x = 16x$$

$$3 = x$$

46. Let x = the number

$$\frac{1}{x} + \frac{2}{3} = \frac{17}{3} \quad \text{LCD} = 3x \text{ so } x \neq 0$$

$$3x\left(\frac{1}{x} + \frac{2}{3}\right) = 3x\left(\frac{17}{3}\right)$$

$$3\cancel{x}\left(\frac{1}{\cancel{x}}\right) + \cancel{3}x\left(\frac{2}{\cancel{3}}\right) = \cancel{3}x\left(\frac{17}{\cancel{3}}\right)$$

$$3 + 2x = 17x$$

$$3 = 15x$$

$$x = \frac{1}{5}$$

47. Let $x =$ the number

$$7 - \frac{1}{x} = \frac{9}{2}$$

LCD = $2x$ so $x \neq 0$

$$2x\left(7 - \frac{1}{x}\right) = 2x\left(\frac{9}{2}\right)$$

$$2x(7) - 2\cancel{x}\left(\frac{1}{\cancel{x}}\right) = \cancel{2}x\left(\frac{9}{\cancel{2}}\right)$$

$$14x - 2 = 9x$$

$$5x = 2$$

$$x = \frac{2}{5}$$

48. Let $x =$ the number

$$x + \frac{1}{x} = \frac{13}{6}$$

LCD = $6x$ so $x \neq 0$

$$6x\left(x + \frac{1}{x}\right) = 6x\left(\frac{13}{6}\right)$$

$$6x(x) + 6\cancel{x}\left(\frac{1}{\cancel{x}}\right) = \cancel{6}x\left(\frac{13}{\cancel{6}}\right)$$

$$6x^2 + 6 = 13x$$

$$6x^2 - 13x + 6 = 0$$

$$(3x - 2)(2x - 3) = 0$$

$$3x - 2 = 0 \text{ or } 2x - 3 = 0$$

$$3x = 2 \text{ or } 2x = 3$$

$$x = \frac{2}{3} \text{ or } x = \frac{3}{2}$$

49. a. $x + 7$

b. $\frac{48}{x}$

c. $\frac{83}{x+7}$

50. a. $x - 4$

b. $\frac{50}{x}$

c. $\frac{145}{x-4}$

51. Let $x =$ the speed in rainstorm

$x + 20 =$ the speed in sunny weather

	Distance	Rate	Time
<u>Rain</u>	80	x	$80/x$
<u>Sunny</u>	120	$x + 20$	$120/(x + 20)$

(Time rain) = (Time sunny)

$$\frac{80}{x} = \frac{120}{x+20} \quad \text{LCD} = x(x+20)$$

$$x(x+20)\left(\frac{80}{x}\right) = x(x+20)\left(\frac{120}{x+20}\right)$$

$$80x + 1600 = 120x$$

$$1600 = 40x$$

$$x = 40$$

$$x + 20 = 40 + 20 = 60$$

The motorist drives 40 mph in the rainstorm and 60 mph in sunny weather.

52. Let
- $x =$
- Adrianna's speed

 $x - 2 =$ Brooke's speed

	Distance	Rate	Time
Adrianna	18	x	$18/x$
Brooke	12	$x - 2$	$12/(x - 2)$

(Time Adrianna) = (Time Brooke)

$$\frac{12}{x-2} = \frac{18}{x} \quad \text{LCD} = x(x-2)$$

$$x(x-2)\left(\frac{12}{x-2}\right) = x(x-2)\left(\frac{18}{x}\right)$$

$$12x = 18x - 36$$

$$-6x = -36$$

$$x = 6$$

$$x - 2 = 6 - 2$$

$$= 4$$

Adrianna walks at 6 km/hr and Brooke walks at 4 km/hr.

53. Let
- $x =$
- the speed of the Broadmoor truck

 $x + 6.4 =$ the speed of the Wescott truck

	Distance	Rate	Time
Broadmoor	88	x	$88/x$
Wescott	96	$x + 6.4$	$96/(x + 6.4)$

(Time Broadmoor) = (Time Wescott)

$$\frac{88}{x} = \frac{96}{x+6.4}$$

$$\text{LCD} = x(x+6.4)$$

$$x(x+6.4)\left(\frac{88}{x}\right) = x(x+6.4)\left(\frac{96}{x+6.4}\right)$$

$$88x + 563.2 = 96x$$

$$563.2 = 8x$$

$$x = 70.4$$

$$x + 6.4 = 70.4 + 6.4$$

$$= 76.8$$

The Broadmoor truck travels 70.4 mph and the Wescott truck travels 76.8 mph.

54. Let x = Dennis's speed on the bike

$$x - 8 = \text{Kathy's speed}$$

	Distance	Rate	Time
Dennis	7	x	$7/x$
Kathy	3	$x - 8$	$3/(x - 8)$

$$(\text{Time Dennis}) = (\text{Time Kathy})$$

$$\frac{7}{x} = \frac{3}{x-8} \quad \text{LCD} = x(x-8)$$

$$x(x-8)\left(\frac{7}{x}\right) = x(x-8)\left(\frac{3}{x-8}\right)$$

$$7x - 56 = 3x$$

$$-56 = -4x$$

$$x = 14$$

$$x - 8 = 14 - 8 = 6$$

Kathy runs at 6 mi/hr and Dennis bikes at 14 mi/hr.

55. Let x = the speed against the wind

$$x + 5 = \text{the speed with the wind}$$

$$\frac{30}{x} + \frac{30}{x+5} = 5$$

$$\text{LCD} = x(x+5)$$

$$x(x+5)\left(\frac{30}{x} + \frac{30}{x+5}\right) = x(x+5)(5)$$

$$30(x+5) + 30x = 5x(x+5)$$

$$30x + 150 + 30x = 5x^2 + 25x$$

$$5x^2 - 35x - 150 = 0$$

$$5(x^2 - 7x - 30) = 0$$

$$5(x-10)(x+3) = 0$$

$$x - 10 = 0 \quad \text{or} \quad x + 3 = 0$$

$$x = 10 \quad \text{or} \quad x = -3$$

The cyclist rides at a speed of 10 mph against the wind.

56. Let x = the speed going

$$x - 3 = \text{the speed returning}$$

$$\frac{60}{x} + \frac{60}{x-3} = 9$$

$$\text{LCD} = x(x-3)$$

$$x(x-3)\left(\frac{60}{x} + \frac{60}{x-3}\right) = x(x-3)(9)$$

$$60(x-3) + 60x = 9x(x-3)$$

$$60x - 180 + 60x = 9x^2 - 27x$$

$$9x^2 - 147x + 180 = 0$$

$$3(3x^2 - 49x + 60) = 0$$

$$3(3x-4)(x-15) = 0$$

$$3x - 4 = 0 \quad \text{or} \quad x - 15 = 0$$

$$3x = 4 \quad \text{or} \quad x = 15$$

$$x \neq \frac{4}{3} \quad \text{or} \quad x = 15$$

$$x - 3 = 15 - 3 = 12$$

The speed to the island is 15 mph and the speed on the return trip is 12 mph.

57. Let x = Celeste's walking speed

$$x + 2 = \text{speed on moving walkway}$$

	Distance	Rate	Time
Off walkway	100	x	$100/x$
On walkway	140	$x + 2$	$140/(x + 2)$

$$(\text{Time off walkway}) + (\text{Time on walkway}) = 40$$

$$\frac{100}{x} + \frac{140}{x+2} = 40 \quad \text{LCD} = x(x+2)$$

$$x(x+2) \left(\frac{100}{x} + \frac{140}{x+2} \right) = x(x+2)(40)$$

$$100(x+2) + 140x = 40x(x+2)$$

$$100x + 200 + 140x = 40x^2 + 80x$$

$$240x + 200 = 40x^2 + 80x$$

$$0 = 40x^2 - 160x - 200$$

$$0 = 40(x^2 - 4x - 5)$$

$$0 = 40(x-5)(x+1)$$

$$x-5=0 \quad \text{or} \quad x+1=0$$

$$x=5 \quad \text{or} \quad x \neq -1$$

$$x+2=5+2=7$$

Celeste walks 5 ft/sec on the ground and travels 7 ft/sec while on the moving walkway.

58. Let x = riding speed

$$x - 6 = \text{walking speed}$$

	Distance	Rate	Time
Riding	6	x	$6/x$
Walking	1	$x - 6$	$1/(x - 6)$

$$(\text{Time riding}) + (\text{Time walking}) = 1$$

$$\frac{6}{x} + \frac{1}{x-6} = 1 \quad \text{LCD} = x(x-6)$$

$$x(x-6) \left(\frac{6}{x} + \frac{1}{x-6} \right) = x(x-6)(1)$$

$$6(x-6) + 1x = x(x-6)$$

$$6x - 36 + x = x^2 - 6x$$

$$7x - 36 = x^2 - 6x$$

$$0 = x^2 - 13x + 36$$

$$0 = (x-9)(x-4)$$

$$x-9=0 \text{ or } x-4=0$$

$$x=9 \text{ or } x \neq 4$$

$$x-6=9-6=3$$

Julio rides his bike at 9 mph and walks at 3 mph.

59. Let x = Joe's speed

$$x + 2 = \text{Beatrice's speed}$$

	Distance	Rate	Time
Joe	12	x	$12/x$
Beatrice	12	$x + 2$	$12/(x + 2)$

$$(\text{Joe's time}) - (\text{Beatrice's time}) = 0.5$$

$$\frac{12}{x} - \frac{12}{x+2} = \frac{1}{2} \quad \text{LCD} = 2x(x+2)$$

$$2x(x+2)\left(\frac{12}{x} - \frac{12}{x+2}\right) = 2x(x+2)\left(\frac{1}{2}\right)$$

$$24(x+2) - 24x = x(x+2)$$

$$24x + 48 - 24x = x^2 + 2x$$

$$48 = x^2 + 2x$$

$$0 = x^2 + 2x - 48$$

$$0 = (x-6)(x+8)$$

$$x-6=0 \text{ or } x+8=0$$

$$x=6 \text{ or } x \neq -8$$

$$x+2=6+2=8$$

Joe runs at 6 mph and Beatrice runs at 8 mph.

60. Let x = the speed of the bus

$$x + 10 = \text{the speed of the car}$$

	Distance	Rate	Time
Bus	200	x	$200/x$
Car	200	$x + 10$	$200/(x + 10)$

$$(\text{Time bus}) = (\text{Time car}) + 1$$

$$\frac{200}{x} = \frac{200}{x+10} + 1$$

$$\text{LCD} = x(x+10)$$

$$\begin{aligned}
 x(x+10)\left(\frac{200}{x}\right) &= x(x+10)\left(\frac{200}{x+10}+1\right) \\
 200(x+10) &= 200x+x(x+10) \\
 200x+2000 &= 200x+x^2+10x \\
 0 &= x^2+10x-2000 \\
 0 &= (x+50)(x-40) \\
 x+50=0 \text{ or } x-40=0 \\
 x \neq -50 \text{ or } x &= 40 \\
 x+10 &= 40+10 \\
 &= 50
 \end{aligned}$$

The speed of the bus is 40 mph and the speed of the car is 50 mph.

61. Work Rate Time Portion of Job Comp

$$\text{Paint\#1} \quad \frac{1}{6} \quad x \quad \frac{(1/6)x}{\text{-----}}$$

$$\text{Paint\#2} \quad \frac{1}{8} \quad x \quad \frac{(1/8)x}{\text{-----}}$$

$$(\text{Paint\#1 Part}) + (\text{Paint\#2 Part}) = (1 \text{ Job})$$

$$\frac{1}{6}x + \frac{1}{8}x = 1 \quad \text{LCD} = 24$$

$$24\left(\frac{1}{6}x + \frac{1}{8}x\right) = 24(1)$$

$$4x + 3x = 24$$

$$7x = 24$$

$$x = \frac{24}{7} \text{ hr or } 3\frac{3}{7} \text{ hr}$$

Together, the painters can paint the room in $3\frac{3}{7}$ hr.

62. Work Rate Time Portion of Job Comp

$$\text{Karen} \quad \frac{1}{2} \quad x \quad \frac{(1/2)x}{\text{-----}}$$

$$\text{Clarann} \quad \frac{1}{3} \quad x \quad \frac{(1/3)x}{\text{-----}}$$

$$(\text{Karen Part}) + (\text{Clarann Part}) = (1 \text{ Job})$$

$$\frac{1}{2}x + \frac{1}{3}x = 1 \quad \text{LCD} = 6$$

$$6\left(\frac{1}{2}x + \frac{1}{3}x\right) = 6(1)$$

$$3x + 2x = 6$$

$$5x = 6$$

Chapter 5 Rational Expressions and Rational Equations

$$x = \frac{6}{5} \text{ hr or } 1\frac{1}{5} \text{ hr}$$

Together, they can wax the SUV in $1\frac{1}{5}$ hr.

63. Work Rate Time Portion of Job Comp

$$\text{Joel} \quad \frac{1}{12} \quad x \quad \frac{(1/12)x}{\underline{\hspace{2cm}}}$$

$$\text{Michael} \quad \frac{1}{15} \quad x \quad \frac{(1/15)x}{\underline{\hspace{2cm}}}$$

$$(\text{Joel's Part}) + (\text{Michael's Part}) = (1 \text{ Job})$$

$$\frac{1}{12}x + \frac{1}{15}x = 1 \quad \text{LCD} = 60$$

$$60\left(\frac{1}{12}x + \frac{1}{15}x\right) = 60(1)$$

$$5x + 4x = 60$$

$$9x = 60$$

$$x = \frac{60}{9} = \frac{20}{3} \text{ hr or } 6\frac{2}{3} \text{ hr}$$

Together, they can fence the yard in $6\frac{2}{3}$ hr.

64. Work Rate Time Portion of Job Comp

$$\text{Ted} \quad \frac{1}{4} \quad x \quad \frac{(1/4)x}{\underline{\hspace{2cm}}}$$

$$\text{Marie} \quad \frac{1}{5} \quad x \quad \frac{(1/5)x}{\underline{\hspace{2cm}}}$$

$$(\text{Ted's Part}) + (\text{Marie's Part}) = (1 \text{ Job})$$

$$\frac{1}{4}x + \frac{1}{5}x = 1 \quad \text{LCD} = 20$$

$$20\left(\frac{1}{4}x + \frac{1}{5}x\right) = 20(1)$$

$$5x + 4x = 20$$

$$9x = 20$$

$$x = \frac{20}{9} \text{ hr or } 2\frac{2}{9} \text{ hr}$$

Together, they can change the billboard in $2\frac{2}{9}$ hr.

65. a. Work Rate Time Part of Job Comp

$$\text{Old} \quad \frac{1}{30} \quad 12 \quad \frac{(1/30)12}{\underline{\hspace{2cm}}}$$

$$\text{New} \quad \frac{1}{x} \quad 12 \quad \frac{(1/x)12}{\underline{\hspace{2cm}}}$$

$$(\text{Old Part}) + (\text{New Part}) = (1 \text{ Job})$$

$$\frac{1}{30} \cdot 12 + \frac{1}{x} \cdot 12 = 1 \quad \text{LCD} = 30x$$

$$30x \left(\frac{1}{30} \cdot 12 + \frac{1}{x} \cdot 12 \right) = 30x(1)$$

$$12x + 360 = 30x$$

$$360 = 18x$$

$$x = 20 \text{ hr}$$

The new pump will take 20 hr.

- b. The technician should return at noon on Friday.

66. Work Rate Time Part of Job Comp

Carp#1	1/8	4	(1/8) 4
--------	-----	---	---------

Carp#2	1/x	4	(1/x) 4
--------	-----	---	---------

(Carp#1 Part) + (Carp#2 Part) = (1 Job)

$$\frac{1}{8} \cdot 4 + \frac{1}{x} \cdot 4 = 1 \quad \text{LCD} = 8x$$

$$8x \left(\frac{1}{8} \cdot 4 + \frac{1}{x} \cdot 4 \right) = 8x(1)$$

$$4x + 32 = 8x$$

$$32 = 4x$$

$$x = 8 \text{ days}$$

The second carpenter completes the job in 8 days working alone.

67. Work Rate Time Part of Job Comp

Gus	1/x	4	(1/x) 4
-----	-----	---	---------

Sid	1/(2x)	4	(1/(2x)) 4
-----	--------	---	------------

(Gus's Part) + (Sid's Part) = (1 Job)

$$\frac{1}{x} \cdot 4 + \frac{1}{2x} \cdot 4 = 1 \quad \text{LCD} = 2x$$

$$2x \left(\frac{1}{x} \cdot 4 + \frac{1}{2x} \cdot 4 \right) = 2x(1)$$

$$8 + 4 = 2x$$

$$12 = 2x$$

$$x = 6 \text{ hr}$$

$$2x = 2(6) = 12 \text{ hr}$$

Gus would take 6 hr and Sid would take 12 hr to dig the garden.

68. Work Rate Time Part of Job Comp

Adult	$1/x$	1	$(1/x) 1$
-------	-------	---	-----------

Child	$1/(3x)$	1	$(1/(3x)) 1$
-------	----------	---	--------------

$$(\text{Adult's Part}) + (\text{Child's Part}) = (1 \text{ Job})$$

$$\frac{1}{x} \cdot 1 + \frac{1}{3x} \cdot 1 = 1 \quad \text{LCD} = 3x$$

$$3x \left(\frac{1}{x} \cdot 1 + \frac{1}{3x} \cdot 1 \right) = 3x(1)$$

$$3 + 1 = 3x$$

$$4 = 3x$$

$$x = \frac{4}{3}$$

$$= 1\frac{1}{3} \text{ hr}$$

$$3x = 3 \left(\frac{4}{3} \right)$$

$$= 4 \text{ hr}$$

The adult would take $1\frac{1}{3}$ hr and the child would take 4 hr to vacuum the house.

Section 5.7 Practice Exercises

1. a. kx

b. $\frac{k}{x}$

c. kxw

2. $\frac{x}{4} = \frac{13}{10}$

$$20 \left(\frac{x}{4} \right) = 20 \left(\frac{13}{10} \right)$$

$$5x = 26$$

$$x = \frac{26}{5}$$

3. $\frac{8}{y} = \frac{6}{11}$

$$11y \left(\frac{8}{y} \right) = 11y \left(\frac{6}{11} \right)$$

$$88 = 6y$$

$$y = \frac{88}{6} = \frac{44}{3}$$

4. $\frac{3}{8} = \frac{w+2}{6}$

$$24 \left(\frac{3}{8} \right) = 24 \left(\frac{w+2}{6} \right)$$

$$9 = 4w + 8$$

$$1 = 4w$$

$$w = \frac{1}{4}$$

$$\begin{aligned}
 5. \quad \frac{2}{3} &= \frac{x-4}{2} \\
 6\left(\frac{2}{3}\right) &= 6\left(\frac{x-4}{2}\right) \\
 4 &= 3x-12 \\
 16 &= 3x \\
 x &= \frac{16}{3}
 \end{aligned}$$

$$\begin{aligned}
 6. \quad \frac{p-5}{p} &= \frac{2}{7} \\
 7p\left(\frac{p-5}{p}\right) &= 7p\left(\frac{2}{7}\right) \\
 7p-35 &= 2p \\
 -35 &= -5p \\
 p &= 7
 \end{aligned}$$

$$\begin{aligned}
 7. \quad \frac{k}{k+1} &= \frac{1}{9} \\
 9(k+1)\left(\frac{k}{k+1}\right) &= 9(k+1)\left(\frac{1}{9}\right) \\
 9k &= k+1 \\
 8k &= 1 \\
 k &= \frac{1}{8}
 \end{aligned}$$

8. Directly

9. Inversely

10. a. increase
b. decrease

11. $T = kq$

12. $W = kz$

13. $b = \frac{k}{c}$

14. $m = \frac{k}{t}$

15. $Q = \frac{kx}{y}$

16. $d = \frac{kp}{n}$

17. $c = kst$

18. $w = kpf$

19. $L = kw\sqrt{v}$

20. $q = kv\sqrt{w}$

21. $x = \frac{ky^2}{z}$

22. $a = \frac{kn}{d^2}$

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$$\begin{aligned} 23. \quad y &= kx \\ 18 &= k(4) \\ k &= \frac{18}{4} = \frac{9}{2} \end{aligned}$$

$$\begin{aligned} 24. \quad m &= kx \\ 22 &= k(8) \\ k &= \frac{22}{8} = \frac{11}{4} \end{aligned}$$

$$\begin{aligned} 25. \quad p &= \frac{k}{q} \\ 32 &= \frac{k}{16} \\ k &= 32(16) = 512 \end{aligned}$$

$$\begin{aligned} 26. \quad T &= \frac{k}{x} \\ 200 &= \frac{k}{40} \\ k &= 200(40) = 8000 \end{aligned}$$

$$\begin{aligned} 27. \quad y &= kwv \\ 8.75 &= k(50)(0.1) \\ 8.75 &= 5k \\ k &= \frac{8.75}{5} = 1.75 \end{aligned}$$

$$\begin{aligned} 28. \quad N &= ktp \\ 330 &= k(1)(7.5) \\ 330 &= 7.5k \\ k &= \frac{330}{7.5} = 44 \end{aligned}$$

$$\begin{aligned} 29. \quad x &= kp \\ 50 &= k(10) \\ k &= \frac{50}{10} = 5 \\ x &= 5(14) = 70 \end{aligned}$$

$$\begin{aligned} 30. \quad y &= kz \\ 12 &= k(36) \\ k &= \frac{12}{36} = \frac{1}{3} \\ y &= \frac{1}{3}(21) = 7 \end{aligned}$$

$$\begin{aligned} 31. \quad b &= \frac{k}{c} \\ 4 &= \frac{k}{3} \\ k &= 4 \cdot 3 = 12 \\ b &= \frac{12}{2} = 6 \end{aligned}$$

$$\begin{aligned} 32. \quad q &= \frac{k}{w} \\ 8 &= \frac{k}{50} \\ k &= 8 \cdot 50 = 400 \\ q &= \frac{400}{125} = 3.2 \end{aligned}$$

$$\begin{aligned} 33. \quad Z &= kw^2 \\ 14 &= k(4)^2 \\ 14 &= 16k \\ k &= \frac{14}{16} = \frac{7}{8} \\ Z &= \frac{7}{8}(8)^2 = \frac{7}{8}(64) = 56 \end{aligned}$$

$$\begin{aligned} 34. \quad m &= kx^2 \\ 200 &= k(20)^2 \\ 200 &= 400k \\ k &= \frac{200}{400} = \frac{1}{2} \\ m &= \frac{1}{2}(32)^2 = \frac{1}{2}(1024) = 512 \end{aligned}$$

$$\begin{aligned}
 35. \quad Q &= \frac{k}{p^2} \\
 4 &= \frac{k}{3^2} \\
 k &= 4 \cdot 3^2 = 4 \cdot 9 = 36 \\
 Q &= \frac{36}{2^2} = \frac{36}{4} = 9
 \end{aligned}$$

$$\begin{aligned}
 37. \quad L &= ka\sqrt{b} \\
 72 &= k(8)\sqrt{9} \\
 72 &= k \cdot 8 \cdot 3 \\
 72 &= 24k \\
 k &= 3 \\
 L &= 3\left(\frac{1}{2}\right)\sqrt{36} = 3\left(\frac{1}{2}\right)(6) = 9
 \end{aligned}$$

$$\begin{aligned}
 39. \quad B &= \frac{km}{n} \\
 20 &= \frac{k \cdot 10}{3} \\
 k &= \frac{20 \cdot 3}{10} = \frac{60}{10} = 6 \\
 B &= \frac{6 \cdot 15}{12} = \frac{90}{12} = \frac{15}{2}
 \end{aligned}$$

$$\begin{aligned}
 41. \quad m &= kw \\
 3 &= k(150) \\
 k &= \frac{3}{150} = \frac{1}{50} \\
 \text{a.} \quad m &= \frac{1}{50} \cdot 180 = 3.6 \text{ g should} \\
 &\quad \text{be prescribed.} \\
 \text{b.} \quad m &= \frac{1}{50} \cdot 225 = 4.5 \text{ g should} \\
 &\quad \text{be prescribed.} \\
 \text{c.} \quad m &= \frac{1}{50} \cdot 120 = 2.4 \text{ g should} \\
 &\quad \text{be prescribed.}
 \end{aligned}$$

$$\begin{aligned}
 36. \quad z &= \frac{k}{t^2} \\
 15 &= \frac{k}{4^2} \\
 k &= 15 \cdot 4^2 = 15 \cdot 16 = 240 \\
 z &= \frac{240}{10^2} = \frac{240}{100} = 2.4
 \end{aligned}$$

$$\begin{aligned}
 38. \quad Y &= kx^3\sqrt{w} \\
 128 &= k(2^3)\sqrt{16} \\
 k &= \frac{128}{2^3\sqrt{16}} = \frac{128}{8 \cdot 4} = \frac{128}{32} = 4 \\
 Y &= 4\left(\frac{1}{2}\right)^3\sqrt{64} = 4 \cdot \frac{1}{8} \cdot 8 = 4
 \end{aligned}$$

$$\begin{aligned}
 40. \quad R &= \frac{ks}{t} \\
 14 &= \frac{k \cdot 2}{9} \\
 k &= \frac{14 \cdot 9}{2} = \frac{126}{2} = 63 \\
 R &= \frac{63 \cdot 4}{3} = \frac{252}{3} = 84
 \end{aligned}$$

$$\begin{aligned}
 42. \quad p &= kw \\
 10 &= k(15) \\
 k &= \frac{10}{15} = \frac{2}{3} \\
 \text{a.} \quad p &= \frac{2}{3} \cdot 12 = 8 \quad \text{It serves 8 people.} \\
 \text{b.} \quad p &= \frac{2}{3} \cdot 18 = 12 \quad \text{It serves 12 people.} \\
 \text{c.} \quad p &= \frac{2}{3} \cdot 21 = 14 \quad \text{It serves 14 people.}
 \end{aligned}$$

43.
$$c = \frac{k}{n}$$

$$0.48 = \frac{k}{5000}$$

$$k = 0.48(5000) = 2400$$

a. $c = \frac{2400}{6000} = 0.40$ Unit cost is \$0.40

b. $c = \frac{2400}{8000} = 0.30$ Unit cost is \$0.30

c. $c = \frac{2400}{2400} = 1.00$ Unit cost is \$1.00

44.
$$n = \frac{k}{p}$$

$$1500 = \frac{k}{8}$$

$$k = 8(1500) = 12,000$$

a. $n = \frac{12,000}{12} = 1000$ She sells 1000 books.

b. $n = \frac{12,000}{15} = 800$ She sells 800 books.

c. $n = \frac{12,000}{6} = 2000$ She sells 2000 books.

45. Let A = the amount of pollution
 P = the number of people
 $A = kP$

$$56,800 = k(80,000)$$

$$k = \frac{56,800}{80,000} = 0.71$$

$$A = 0.71(500,000)$$

$$= 355,000 \text{ tons}$$

355,000 tons enter the atmosphere.

46. Let A = the area of the picture
 d = the distance from the projector to wall
 $A = kd^2$

$$16 = k(10)^2$$

$$k = \frac{16}{10^2} = \frac{16}{100} = 0.16$$

$$A = 0.16(20)^2 = 0.16(400) = 64 \text{ ft}^2$$

The area is 64 ft^2 .

47. Let I = the intensity of the light source
 d = the distance from the source

$$I = \frac{k}{d^2}$$

$$400 = \frac{k}{5^2}$$

$$k = 400 \cdot 5^2 = 400 \cdot 25$$

$$= 10,000$$

$$I = \frac{10,000}{8^2} = \frac{10,000}{64}$$

$$= 156.25 \text{ lumens/m}^2$$

The intensity is $156.25 \text{ lumens/m}^2$.

48. Let f = the frequency
 L = the length of the string

$$f = \frac{k}{L}$$

$$252 = \frac{k}{24}$$

$$k = 252 \cdot 24$$

$$= 6048$$

$$f = \frac{6048}{18}$$

$$= 336 \text{ cycles/sec}$$

The frequency is 336 cycles/sec .

49. Let I = the current

V = the voltage

R = the resistance

$$I = \frac{kV}{R}$$

$$9 = \frac{k(90)}{10}$$

$$k = \frac{9 \cdot 10}{90} = \frac{90}{90} = 1$$

$$I = \frac{1 \cdot 185}{10} = \frac{185}{10} = 18.5 \text{ A}$$

The current is 18.5 A.

50. Let R = the resistance

L = the length of the wire

d = the diameter of the wire

$$R = \frac{kL}{d^2}$$

$$4 = \frac{k(40)}{(0.1)^2}$$

$$k = \frac{4(0.1)^2}{40} = \frac{4(0.01)}{40} = \frac{0.04}{40} = 0.001$$

$$R = \frac{0.001(50)}{(0.2)^2} = \frac{0.05}{0.04} = 1.25 \Omega$$

The resistance is 1.25 Ω .

51. Let I = the interest

P = the principal

T = the time

$$I = kPt$$

$$500 = k(2500)(4)$$

$$500 = 10,000k$$

$$k = \frac{500}{10,000}$$

$$= 0.05$$

$$I = 0.05(7000)(10)$$

$$= \$3500$$

\$3500 in interest will be earned.

52. Let I = the interest

P = the principal

T = the time

$$I = kPt$$

$$840 = k(6000)(2)$$

$$840 = 12,000k$$

$$k = \frac{840}{12,000}$$

$$= 0.07$$

$$I = 0.07(4500)(8)$$

$$= \$2520$$

\$2520 in interest will be earned.

53. Let d = the stopping distance

s = the speed of the car

$$d = ks^2$$

$$109 = k(40)^2$$

$$k = \frac{109}{40^2}$$

$$= \frac{109}{1600}$$

54. Let w = the weight

r = the radius

$$w = kr^3$$

$$4.32 = k(3)^3$$

$$k = \frac{4.32}{3^3}$$

$$= \frac{4.32}{27}$$

$$= 0.16$$

$$d = \frac{109}{1600}(25)^2$$

$$= \frac{109}{1600}(625) \approx 42.6 \text{ ft}$$

The stopping distance will be 42.6 ft.

$$w = 0.16(5^3)$$

$$= 0.16(125) = 20 \text{ lb}$$

The ball will weigh 20 lb.

- 55.** Let S = the surface area

L = the length of an edge

$$S = kL^2$$

$$24 = k(2)^2$$

$$k = \frac{24}{2^2} = \frac{24}{4} = 6$$

$$S = 6(5)^2 = 6(25) = 150 \text{ ft}^2$$

The surface area is 150 ft².

- 56.** Let P = the period of the pendulum

L = the length of the pendulum

$$P = k\sqrt{L}$$

$$1.8 = k\sqrt{0.81}$$

$$k = \frac{1.8}{\sqrt{0.81}} = \frac{1.8}{0.9} = 2$$

$$P = 2\sqrt{1} = 2 \cdot 1 = 2 \text{ sec}$$

The period is 2 sec.

- 57.** Let P = the power

I = the current

R = the resistance

$$P = kIR^2$$

$$144 = k(4)(6)^2$$

$$k = \frac{144}{4(6)^2} = \frac{144}{144} = 1$$

$$P = 1(3)(10)^2 = 3 \cdot 100 = 300 \text{ W}$$

The power is 300 W.

- 58.** Let S = the strength of the beam

w = the width

t = the thickness

L = the length of the beam

$$S = \frac{kwt^2}{L}$$

$$417 = \frac{k(6)(2^2)}{48}$$

$$k = \frac{417 \cdot 48}{6 \cdot 2^2} = 834$$

$$S = \frac{834(12)(4^2)}{72} = 2224 \text{ lb}$$

The maximum load is 2224 lb.

- 59. a.** $A = kl^2$

b. $A = k(2l)^2 = k(4l^2) = 4kl^2$

The area is 4 times the original.

c. $A = k(3l)^2 = k(9l^2) = 9kl^2$

The area is 9 times the original.

- 60. a.** Direct relationship

b. $d = 1.25F$ or $F = 0.8d$

Chapter 5 Group Activity

1. $R = \$150$

$$I = \frac{0.06}{12}$$

$$= 0.005$$

$$n = 30(12)$$

$$= 360$$

$$S = 150 \left[\frac{(1 + 0.005)^{360} - 1}{0.005} \right]$$

$$= 150 \left[\frac{(1.005)^{360} - 1}{0.005} \right]$$

$$= 150 \left[\frac{6.0223 - 1}{0.005} \right]$$

$$= 150 \left[\frac{5.023}{0.005} \right]$$

$$= 150 [1004.515]$$

$$= \$150,677.25$$

3. $A = 50,000 \div \left[\frac{(1 + 0.09/12)^{180} - 1}{0.09/12} \right]$

$$= \$132.13$$

Approximately \$133 must be saved each month.

2. $S = 100 \left[\frac{(1 + 0.06/12)^{240} - 1}{0.06/12} \right] = \$46,204.09$

$$S = 150 \left[\frac{(1 + 0.06/12)^{240} - 1}{0.06/12} \right] = \$69,306.13$$

$$S = 200 \left[\frac{(1 + 0.06/12)^{240} - 1}{0.06/12} \right] = \$92,408.18$$

$$S = 200 \left[\frac{(1 + 0.024/12)^{240} - 1}{0.024/12} \right] = \$61,529.99$$

$$S = 200 \left[\frac{(1 + 0.036/12)^{240} - 1}{0.036/12} \right] = \$70,148.00$$

$$S = 200 \left[\frac{(1 + 0.048/12)^{240} - 1}{0.048/12} \right] = \$80,335.01$$

$$S = 200 \left[\frac{(1 + 0.06/12)^{300} - 1}{0.06/12} \right] = \$138,598.79$$

$$S = 200 \left[\frac{(1 + 0.06/12)^{360} - 1}{0.06/12} \right]$$

$$= \$200,903.01$$

$$S = 200 \left[\frac{(1 + 0.06/12)^{420} - 1}{0.06/12} \right]$$

$$= \$284,942.06$$

4. $R = \$5000$

$$I = 0.08$$

$$n = 20$$

$$S = 5000 \left[\frac{(1 + 0.08)^{20} - 1}{0.08} \right]$$

$$= \$228,809.82$$

Chapter 5 Review Exercises

Section 5.1

1. $k(y) = \frac{y}{y^2 - 1}$

a. $k(2) = \frac{2}{2^2 - 1} = \frac{2}{4 - 1} = \frac{2}{3}$
 $k(0) = \frac{0}{0^2 - 1} = \frac{0}{0 - 1} = \frac{0}{-1} = 0$
 $k(1) = \frac{1}{1^2 - 1} = \frac{1}{1 - 1} = \frac{1}{0}$ is undefined
 $k(-1) = \frac{-1}{(-1)^2 - 1} = \frac{-1}{1 - 1}$
 $= \frac{-1}{0}$ undefined

$k\left(\frac{1}{2}\right) = \frac{\frac{1}{2}}{\left(\frac{1}{2}\right)^2 - 1} = \frac{\frac{1}{2}}{\frac{1}{4} - \frac{4}{4}} = \frac{\frac{1}{2}}{-\frac{3}{4}}$
 $= \frac{1}{2} \cdot \left(-\frac{4}{3}\right) = -\frac{2}{3}$

b. $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

3. $\frac{28a^3b^3}{14a^2b^3} = 2a^{3-2}b^{3-3}$
 $= 2a^1b^0$
 $= 2a$

5. $\frac{x^2 - 4x + 3}{x - 3} = \frac{(x-3)(x-1)}{x-3}$
 $= x - 1$

2. $h(x) = \frac{x}{x^2 + 1}$

a. $h(1) = \frac{1}{1^2 + 1} = \frac{1}{1 + 1} = \frac{1}{2}$
 $h(0) = \frac{0}{0^2 + 1} = \frac{0}{0 + 1} = \frac{0}{1} = 0$
 $h(-1) = \frac{-1}{(-1)^2 + 1} = \frac{-1}{1 + 1} = -\frac{1}{2}$
 $h(-3) = \frac{-3}{(-3)^2 + 1} = \frac{-3}{9 + 1} = -\frac{3}{10}$
 $h\left(\frac{1}{2}\right) = \frac{\frac{1}{2}}{\left(\frac{1}{2}\right)^2 + 1} = \frac{\frac{1}{2}}{\frac{1}{4} + \frac{4}{4}} = \frac{\frac{1}{2}}{\frac{5}{4}}$
 $= \frac{1}{2} \cdot \frac{4}{5} = \frac{2}{5}$

b. $(-\infty, \infty)$

4. $\frac{25x^2yz^3}{125xyz} = \frac{1}{5}x^{2-1}y^{1-1}z^{3-1}$
 $= \frac{1}{5}xy^0z^2$
 $= \frac{xz^2}{5}$

6. $\frac{k^2 + 3k - 10}{k^2 - 5k + 6} = \frac{(k+5)(k-2)}{(k-3)(k-2)}$
 $= \frac{k+5}{k-3}$

$$\begin{aligned}
 7. \quad \frac{x^3 - 27}{9 - x^2} &= \frac{x^3 - 3^3}{-(x^2 - 9)} \\
 &= \frac{(\cancel{x-3})(x^2 + 3x + 9)}{-(x+3)(\cancel{x-3})} = -\frac{x^2 + 3x + 9}{x+3}
 \end{aligned}$$

$$\begin{aligned}
 8. \quad \frac{a^4 - 81}{3 - a} &= \frac{(a^2 + 9)(a^2 - 9)}{-(a-3)} \\
 &= \frac{(a^2 + 9)(a+3)(\cancel{a-3})}{-(\cancel{a-3})} \\
 &= -(a^2 + 9)(a+3)
 \end{aligned}$$

$$\begin{aligned}
 9. \quad \frac{2t^2 + 3t - 5}{7 - 6t - t^2} &= \frac{(2t+5)(t-1)}{-(t^2 + 6t - 7)} \\
 &= \frac{(2t+5)(\cancel{t-1})}{-(t+7)(\cancel{t-1})} = -\frac{2t+5}{t+7}
 \end{aligned}$$

$$\begin{aligned}
 10. \quad \frac{y^3 - 4y}{y^2 - 5y + 6} &= \frac{y(y^2 - 4)}{(y-3)(y-2)} \\
 &= \frac{y(y+2)(\cancel{y-2})}{(y-3)(\cancel{y-2})} = \frac{y(y+2)}{y-3}
 \end{aligned}$$

$$\begin{aligned}
 11. \quad f(x) &= \frac{1}{x-3} \\
 x-3 &= 0 \\
 x &= 3 \\
 &(-\infty, 3) \cup (3, \infty) \\
 \text{Graph: c}
 \end{aligned}$$

$$\begin{aligned}
 12. \quad m(x) &= \frac{1}{x+2} \\
 x+2 &= 0 \\
 x &= -2 \\
 &(-\infty, -2) \cup (-2, \infty) \\
 \text{Graph: a}
 \end{aligned}$$

$$\begin{aligned}
 13. \quad k(x) &= \frac{6}{x^2 - 3x} = \frac{6}{x(x-3)} \\
 x(x-3) &= 0 \\
 x &= 0 \text{ or } x-3 = 0 \\
 x &= 0 \text{ or } x = 3 \\
 &(-\infty, 0) \cup (0, 3) \cup (3, \infty) \\
 \text{Graph: b}
 \end{aligned}$$

$$\begin{aligned}
 14. \quad p(x) &= \frac{-2}{x^2 + 4} \\
 x^2 + 4 &\text{ is never zero because } x^2 \text{ is} \\
 &\text{nonnegative} \\
 &(-\infty, \infty) \\
 \text{Graph: d}
 \end{aligned}$$

Section 5.2

$$\begin{aligned}
 15. \quad \frac{3a+9}{a^2} \cdot \frac{a^3}{6a+18} &= \frac{\cancel{a}(\cancel{a+3})}{\cancel{a}} \cdot \frac{\cancel{a} \cdot a}{2 \cdot \cancel{a}(\cancel{a+3})} \\
 &= \frac{a}{2}
 \end{aligned}$$

$$\begin{aligned}
 16. \quad \frac{4-y}{5} \div \frac{2y-8}{15} &= \frac{4-y}{5} \cdot \frac{15}{2y-8} \\
 &= \frac{-(\cancel{y-4})}{\cancel{5}} \cdot \frac{\cancel{3} \cdot 3}{2(\cancel{y-4})} = -\frac{3}{2}
 \end{aligned}$$

$$\begin{aligned}
 17. \quad \frac{x-4y}{x^2+xy} \div \frac{20y-5x}{x^2-y^2} &= \frac{x-4y}{x^2+xy} \cdot \frac{x^2-y^2}{20y-5x} \\
 &= \frac{\cancel{x}4y}{x(\cancel{x+y})} \cdot \frac{(\cancel{x+y})(x-y)}{-5(\cancel{x-4y})} \\
 &= -\frac{x-y}{5x}
 \end{aligned}$$

$$\begin{aligned}
 18. \quad (x^2+5x-24) \left(\frac{x+8}{x-3} \right) \\
 &= \frac{(x+8)(\cancel{x-3})}{1} \left(\frac{x+8}{\cancel{x-3}} \right) \\
 &= (x+8)^2
 \end{aligned}$$

$$\begin{aligned}
 19. \quad \frac{7k+28}{2k+4} \cdot \frac{k^2-2k-8}{k^2+2k-8} \\
 &= \frac{7(\cancel{k+4})}{2(\cancel{k+2})} \cdot \frac{(k-4)(\cancel{k+2})}{(\cancel{k+4})(k-2)} \\
 &= \frac{7(k-4)}{2(k-2)}
 \end{aligned}$$

$$\begin{aligned}
 20. \quad \frac{ab+2a+b+2}{ab-3b+2a-6} \cdot \frac{ab-3b+4a-12}{ab-b+4a-4} \\
 &= \frac{a(b+2)+\cancel{(b+2)}}{b(a-3)+2(a-3)} \cdot \frac{b(a-3)+4(a-3)}{b(a-1)+4(a-1)} \\
 &= \frac{(\cancel{b+2})(a+1)}{(\cancel{a-3})(\cancel{b+2})} \cdot \frac{(\cancel{a-3})(\cancel{b+4})}{(a-1)(\cancel{b+4})} \\
 &= \frac{a+1}{a-1}
 \end{aligned}$$

$$\begin{aligned}
 21. \quad \frac{x^2+8x-20}{x^2+6x-16} \div \frac{x^2+6x-40}{x^2+3x-40} &= \frac{x^2+8x-20}{x^2+6x-16} \cdot \frac{x^2+3x-40}{x^2+6x-40} \\
 &= \frac{(\cancel{x+10})(\cancel{x-2})}{(\cancel{x+8})(\cancel{x-2})} \cdot \frac{(\cancel{x+8})(x-5)}{(\cancel{x+10})(x-4)} \\
 &= \frac{x-5}{x-4}
 \end{aligned}$$

$$\begin{aligned}
 22. \quad \frac{2b-b^2}{b^3-8} \cdot \frac{b^2+2b+4}{b^2} &= \frac{-\cancel{b}(\cancel{b-2})}{(\cancel{b-2})(\cancel{b^2+2b+4})} \cdot \frac{\cancel{b^2+2b+4}}{\cancel{b} \cdot b} \\
 &= -\frac{1}{b}
 \end{aligned}$$

$$\begin{aligned}
 23. \quad \frac{2w}{21} \div \frac{3w^2}{7} \cdot \frac{4}{w} &= \frac{2w}{21} \cdot \frac{7}{3w^2} \cdot \frac{4}{w} \\
 &= \frac{\cancel{2} \cancel{w}}{\cancel{7} \cdot 3} \cdot \frac{\cancel{7}}{3w^2} \cdot \frac{4}{\cancel{w}} = \frac{8}{9w^2}
 \end{aligned}$$

24.
$$\frac{5y^2 - 20}{y^3 + 8} \div \frac{7y^2 - 14y}{y^3 + y} = \frac{5(y^2 - 4)}{y^3 + 2^3} \cdot \frac{y^3 + y}{7y^2 - 14y}$$

$$= \frac{5(\cancel{y+2})(\cancel{y-2})}{(\cancel{y+2})(y^2 - 2y + 4)} \cdot \frac{\cancel{y}(y^2 + 1)}{7\cancel{y}(y-2)} = \frac{5(y^2 + 1)}{7(y^2 - 2y + 4)}$$
25.
$$\frac{x^2 + x - 20}{x^2 - 4x + 4} \cdot \frac{x^2 + x - 6}{12 + x - x^2} \div \frac{2x + 10}{10 - 5x} = \frac{x^2 + x - 20}{x^2 - 4x + 4} \cdot \frac{x^2 + x - 6}{-(x^2 - x - 12)} \cdot \frac{10 - 5x}{2x + 10}$$

$$= \frac{(\cancel{x+5})(\cancel{x-4})}{(\cancel{x-2})(\cancel{x-2})} \cdot \frac{(\cancel{x+3})(\cancel{x-2})}{-(\cancel{x-4})(\cancel{x+3})} \cdot \frac{-5(\cancel{x-2})}{2(\cancel{x+5})} = \frac{-5}{-2} = \frac{5}{2}$$
26.
$$(9k^2 - 25) \cdot \left(\frac{k+5}{3k-5}\right) = \frac{(3k+5)(\cancel{3k-5})}{1} \cdot \left(\frac{k+5}{\cancel{3k-5}}\right) = (3k+5)(k+5)$$

Section 5.3

27.
$$\frac{1}{x} + \frac{1}{x^2} - \frac{1}{x^3} \quad \text{LCD} = x^3$$

$$= \frac{1}{x} \cdot \frac{x^2}{x^2} + \frac{1}{x^2} \cdot \frac{x}{x} - \frac{1}{x^3} = \frac{x^2 + x - 1}{x^3}$$
28.
$$\frac{1}{x+2} + \frac{5}{x-2} \quad \text{LCD} = (x+2)(x-2)$$

$$= \frac{1}{x+2} \cdot \frac{x-2}{x-2} + \frac{5}{x-2} \cdot \frac{x+2}{x+2}$$

$$= \frac{x-2+5x+10}{(x+2)(x-2)} = \frac{6x+8}{(x+2)(x-2)}$$

$$= \frac{2(3x+4)}{(x+2)(x-2)}$$
29.
$$\frac{y}{2y-1} + \frac{3}{1-2y}$$

$$= \frac{y}{2y-1} + \frac{3}{1-2y} \cdot \frac{(-1)}{(-1)}$$

$$= \frac{y}{2y-1} + \frac{-3}{2y-1}$$

$$= \frac{y-3}{2y-1}$$
30.
$$\frac{a+2}{2a+6} - \frac{3}{a+3} = \frac{a+2}{2(a+3)} - \frac{3}{a+3}$$

$$\text{LCD} = 2(a+3)$$

$$= \frac{a+2}{2(a+3)} - \frac{3}{a+3} \cdot \frac{2}{2}$$

$$= \frac{a+2-6}{2(a+3)}$$

$$= \frac{a-4}{2(a+3)}$$

$$\begin{aligned}
31. \quad \frac{4k}{k^2+2k+1} + \frac{3}{k^2-1} &= \frac{4k}{(k+1)^2} + \frac{3}{(k+1)(k-1)} \quad \text{LCD} = (k+1)^2(k-1) \\
&= \frac{4k}{(k+1)^2} \cdot \frac{k-1}{k-1} + \frac{3}{(k+1)(k-1)} \cdot \frac{k+1}{k+1} \\
&= \frac{4k^2-4k+3k+3}{(k+1)^2(k-1)} = \frac{4k^2-k+3}{(k+1)^2(k-1)}
\end{aligned}$$

$$32. \quad 4x+3 - \frac{2x+1}{x+4} = (4x+3) \frac{x+4}{x+4} - \frac{2x+1}{x+4} = \frac{4x^2+19x+12-2x-1}{x+4} = \frac{4x^2+17x+11}{x+4}$$

$$\begin{aligned}
33. \quad \frac{2}{a+3} + \frac{2a^2-2a}{a^2-2a-15} &= \frac{2}{a+3} + \frac{2a^2-2a}{(a-5)(a+3)} = \frac{2}{a+3} \cdot \frac{a-5}{a-5} + \frac{2a^2-2a}{(a-5)(a+3)} \\
&= \frac{2a-10+2a^2-2a}{(a-5)(a+3)} = \frac{2a^2-10}{(a-5)(a+3)} = \frac{2(a^2-5)}{(a-5)(a+3)}
\end{aligned}$$

$$\begin{aligned}
34. \quad \frac{6}{x^2+4x+3} + \frac{7}{x^2+5x+6} &= \frac{6}{(x+3)(x+1)} + \frac{7}{(x+3)(x+2)} \quad \text{LCD} = (x+3)(x+1)(x+2) \\
&= \frac{6}{(x+3)(x+1)} \cdot \frac{x+2}{x+2} + \frac{7}{(x+3)(x+2)} \cdot \frac{x+1}{x+1} \\
&= \frac{6x+12+7x+7}{(x+3)(x+1)(x+2)} \\
&= \frac{13x+19}{(x+3)(x+1)(x+2)}
\end{aligned}$$

$$35. \quad \frac{2}{3x-5} - 8 = \frac{2}{3x-5} - 8 \cdot \frac{3x-5}{3x-5} = \frac{2-24x+40}{3x-5} = \frac{-24x+42}{3x-5} = \frac{-6(4x-7)}{3x-5}$$

$$\begin{aligned}
36. \quad \frac{7}{4k^2-k-3} + \frac{1}{4k^2-7k+3} &= \frac{7}{(4k+3)(k-1)} + \frac{1}{(4k-3)(k-1)} \quad \text{LCD} = (4k+3)(4k-3)(k-1) \\
&= \frac{7}{(4k+3)(k-1)} \cdot \frac{4k-3}{4k-3} + \frac{1}{(4k-3)(k-1)} \cdot \frac{4k+3}{4k+3} \\
&= \frac{28k-21+4k+3}{(4k+3)(4k-3)(k-1)} \\
&= \frac{32k-18}{(4k+3)(4k-3)(k-1)} = \frac{2(16k-9)}{(4k+3)(4k-3)(k-1)}
\end{aligned}$$

$$\begin{aligned}
37. \quad & \frac{6a}{3a^2 - 7a + 2} + \frac{2}{1 - 3a} + \frac{3a}{a - 2} \\
&= \frac{6a}{(3a-1)(a-2)} + \frac{-2}{3a-1} + \frac{3a}{a-2} \\
&\text{LCD} = (3a-1)(a-2) \\
&= \frac{6a}{(3a-1)(a-2)} + \frac{-2}{3a-1} \cdot \frac{a-2}{a-2} + \frac{3a}{a-2} \cdot \frac{3a-1}{3a-1} \\
&= \frac{6a}{(3a-1)(a-2)} + \frac{-2a+4}{(3a-1)(a-2)} + \frac{9a^2-3a}{(3a-1)(a-2)} \\
&= \frac{6a-2a+4+9a^2-3a}{(3a-1)(a-2)} \\
&= \frac{9a^2+a+4}{(3a-1)(a-2)}
\end{aligned}$$

$$\begin{aligned}
38. \quad & 4 + \frac{2y-5}{y+2} + \frac{y}{3-y} = 4 + \frac{2y-5}{y+2} + \frac{-y}{y-3} \\
&\text{LCD} = (y-3)(y+2) \\
&= 4 \cdot \frac{(y-3)(y+2)}{(y-3)(y+2)} + \frac{2y-5}{y+2} \cdot \frac{y-3}{y-3} + \frac{-y}{y-3} \cdot \frac{y+2}{y+2} \\
&= \frac{4y^2-4y-24}{(y-3)(y+2)} + \frac{2y^2-11y+15}{(y-3)(y+2)} + \frac{-y^2-2y}{(y-3)(y+2)} \\
&= \frac{5y^2-17y-9}{(y-3)(y+2)}
\end{aligned}$$

Section 5.4

$$\begin{aligned}
39. \quad & \frac{2x}{3x^2-3} = \frac{2x}{3(x^2-1)} \cdot \frac{6x-6}{4x} \\
&= \frac{\cancel{2x}}{\cancel{3}(x+1)(\cancel{x-1})} \cdot \frac{\cancel{3} \cdot \cancel{2}(x-1)}{\cancel{2} \cdot \cancel{2x}} \\
&= \frac{1}{x+1}
\end{aligned}$$

$$\begin{aligned}
40. \quad & \frac{k+2}{\frac{3}{5}} = \frac{k+2}{3} \cdot \frac{k-2}{5} \\
&= \frac{(k+2)(k-2)}{15}
\end{aligned}$$

$$41. \quad \frac{\frac{2}{x} + \frac{1}{xy}}{\frac{4}{x^2}} \quad \text{LCD} = x^2y$$

$$= \frac{x^2y\left(\frac{2}{x} + \frac{1}{xy}\right)}{x^2y\left(\frac{4}{x^2}\right)} = \frac{x^2y\left(\frac{2}{x}\right) + x^2y\left(\frac{1}{xy}\right)}{x^2y\left(\frac{4}{x^2}\right)}$$

$$= \frac{2xy + x}{4y} = \frac{x(2y+1)}{4y}$$

$$42. \quad \frac{\frac{4}{y} - 1}{\frac{1}{y} - \frac{4}{y^2}} \quad \text{LCD} = y^2$$

$$= \frac{y^2\left(\frac{4}{y} - 1\right)}{y^2\left(\frac{1}{y} - \frac{4}{y^2}\right)} = \frac{y^2\left(\frac{4}{y}\right) - y^2(1)}{y^2\left(\frac{1}{y}\right) - y^2\left(\frac{4}{y^2}\right)}$$

$$= \frac{4y - y^2}{y - 4} = \frac{-y(\cancel{y-4})}{\cancel{y-4}} = -y$$

$$43. \quad \frac{\frac{1}{a-1} + 1}{\frac{1}{a+1} - 1} \quad \text{LCD} = (a+1)(a-1)$$

$$= \frac{(a+1)(a-1)\left(\frac{1}{a-1} + 1\right)}{(a+1)(a-1)\left(\frac{1}{a+1} - 1\right)} = \frac{(a+1)(\cancel{a-1})\left(\frac{1}{\cancel{a-1}}\right) + (a+1)(a-1)(1)}{(\cancel{a+1})(a-1)\left(\frac{1}{\cancel{a+1}}\right) - (a+1)(a-1)(1)}$$

$$= \frac{a+1+a^2-1}{a-1-(a^2-1)} = \frac{a^2+a}{a-1-a^2+1} = \frac{a^2+a}{-a^2+a} = \frac{\cancel{a}(a+1)}{-\cancel{a}(a-1)} = -\frac{a+1}{a-1}$$

$$44. \quad \frac{\frac{3}{x-1} - \frac{1}{1-x}}{\frac{2}{x-1} - \frac{2}{x}} = \frac{3}{x-1} + \frac{1}{x-1} \quad \text{LCD} = x(x-1)$$

$$= \frac{x(x-1)\left(\frac{3}{x-1} + \frac{1}{x-1}\right)}{x(x-1)\left(\frac{2}{x-1} - \frac{2}{x}\right)} = \frac{x(\cancel{x-1})\left(\frac{3}{\cancel{x-1}}\right) + x(\cancel{x-1})\left(\frac{1}{\cancel{x-1}}\right)}{x(\cancel{x-1})\left(\frac{2}{\cancel{x-1}}\right) - \cancel{x}(x-1)\left(\frac{2}{\cancel{x}}\right)}$$

$$= \frac{3x+x}{2x-2x+2} = \frac{4x}{2} = 2x$$

$$45. \quad \frac{1+xy^{-1}}{x^2y^{-2}-1} = \frac{1+\frac{x}{y}}{\frac{x^2}{y^2}-1} \quad \text{LCD} = y^2$$

$$= \frac{y^2 \left(1 + \frac{x}{y}\right)}{y^2 \left(\frac{x^2}{y^2} - 1\right)} = \frac{y^2(1) + y \cdot \cancel{\left(\frac{x}{\cancel{y}}\right)}}{\cancel{y^2} \left(\frac{x^2}{\cancel{y^2}}\right) - y^2(1)} = \frac{y^2 + xy}{x^2 - y^2} = \frac{y(\cancel{y+x})}{(\cancel{x+y})(x-y)} = \frac{y}{x-y}$$

$$46. \quad \frac{5a^{-1} + (ab)^{-1}}{3a^{-2}} = \frac{\frac{5}{a} + \frac{1}{ab}}{\frac{3}{a^2}} \quad \text{LCD} = a^2b$$

$$= \frac{a^2b \left(\frac{5}{a} + \frac{1}{ab}\right)}{a^2b \left(\frac{3}{a^2}\right)} = \frac{\cancel{a} \cdot a \cdot b \left(\frac{5}{\cancel{a}}\right) + a \cdot \cancel{ab} \left(\frac{1}{\cancel{ab}}\right)}{\cancel{a^2} b \left(\frac{3}{\cancel{a^2}}\right)} = \frac{5ab + a}{3b} = \frac{a(5b+1)}{3b}$$

$$47. \quad m = \frac{-\frac{5}{3} - \left(-\frac{7}{4}\right)}{\frac{13}{6} - \frac{2}{3}} \quad \text{LCD} = 12$$

$$= \frac{12 \left(-\frac{5}{3} + \frac{7}{4}\right)}{12 \left(\frac{13}{6} - \frac{2}{3}\right)} = \frac{-20 + 21}{26 - 8} = \frac{1}{18}$$

$$48. \quad m = \frac{\frac{9}{5} - \left(-\frac{1}{3}\right)}{\frac{13}{10} - \frac{8}{15}} \quad \text{LCD} = 30$$

$$= \frac{30 \left(\frac{9}{5} + \frac{1}{3}\right)}{30 \left(\frac{13}{10} - \frac{8}{15}\right)} = \frac{54 + 10}{39 - 16} = \frac{64}{23}$$

Section 5.5

$$49. \quad \frac{x+3}{x^2-x} - \frac{8}{x^2-1} = 0$$

$$\frac{x+3}{x(x-1)} - \frac{8}{(x+1)(x-1)} = 0$$

$\text{LCD} = x(x-1)(x+1)$ so $x \neq 0$ or $x \neq 1$ or $x \neq -1$

$$x(x-1)(x+1) \left(\frac{x+3}{x(x-1)} - \frac{8}{(x+1)(x-1)} \right) = x(x-1)(x+1)(0)$$

$$\cancel{x}(\cancel{x-1})(x+1) \left(\frac{x+3}{\cancel{x}(\cancel{x-1})} \right) - x(\cancel{x-1})(\cancel{x+1}) \left(\frac{8}{(\cancel{x+1})(\cancel{x-1})} \right) = 0$$

$$x^2 + 4x + 3 - 8x = 0$$

$$x^2 - 4x + 3 = 0$$

Chapter 5 Rational Expressions and Rational Equations

$$\begin{aligned}(x-3)(x-1) &= 0 \\ x-3 &= 0 \text{ or } x-1 = 0 \\ x &= 3 \text{ or } x = 1\end{aligned}$$

{3} is the solution. ($x = 1$ does not check because the denominator is zero.)

50.

$$\begin{aligned}\frac{y}{y+3} - \frac{3}{3-y} &= \frac{18}{y^2-9} \\ \frac{y}{y+3} + \frac{3}{y-3} &= \frac{18}{y^2-9} \\ \frac{y}{y+3} + \frac{3}{y-3} &= \frac{18}{(y+3)(y-3)} \\ \text{LCD} &= (y+3)(y-3) \quad \text{so } y \neq -3 \text{ or } y \neq 3 \\ (y+3)(y-3) \left(\frac{y}{y+3} + \frac{3}{y-3} \right) &= \cancel{(y+3)} \cancel{(y-3)} \left(\frac{18}{\cancel{(y+3)} \cancel{(y-3)}} \right) \\ \cancel{(y+3)}(y-3) \left(\frac{y}{\cancel{y+3}} \right) + (y+3) \cancel{(y-3)} \left(\frac{3}{\cancel{y-3}} \right) &= 18 \\ y^2 - 3y + 3y + 9 &= 18 \\ y^2 - 9 &= 0 \\ (y-3)(y+3) &= 0 \\ y-3 = 0 \text{ or } y+3 = 0 \\ y = 3 \text{ or } y = -3\end{aligned}$$

{ }, no solution. (The values 3 and -3 do not check because each makes the denominator zero.)

51.

$$\begin{aligned}x-9 &= \frac{72}{x-8} \\ \text{LCD} &= (x-8) \quad \text{so } x \neq 8 \\ (x-8)(x-9) &= \cancel{(x-8)} \left(\frac{72}{\cancel{x-8}} \right) \\ x^2 - 17x + 72 &= 72 \\ x^2 - 17x &= 0 \\ x(x-17) &= 0 \\ x &= 0 \\ \text{or } x-17 &= 0 \\ x &= 17 \\ \text{or } x &= 17 \quad \{0, 17\}\end{aligned}$$

$$52. \quad \frac{3x+1}{x+5} = \frac{x-1}{x+1} + 2 \quad \text{LCD} = (x+5)(x+1) \quad \text{so } x \neq -5 \text{ or } x \neq -1$$

$$(x+5)(x+1)\left(\frac{3x+1}{x+5}\right) = (x+5)(x+1)\left(\frac{x-1}{x+1} + 2\right)$$

$$(\cancel{x+5})(x+1)\left(\frac{3x+1}{\cancel{x+5}}\right) = (x+5)(\cancel{x+1})\left(\frac{x-1}{\cancel{x+1}}\right) + (x+5)(x+1)(2)$$

$$3x^2 + 4x + 1 = x^2 + 4x - 5 + 2(x^2 + 6x + 5)$$

$$3x^2 + 4x + 1 = x^2 + 4x - 5 + 2x^2 + 12x + 10$$

$$3x^2 + 4x + 1 = 3x^2 + 16x + 5$$

$$-12x = 4$$

$$x = -\frac{1}{3} \quad \left\{-\frac{1}{3}\right\}$$

$$53. \quad 5y^{-2} + 1 = 6y^{-1}$$

$$\frac{5}{y^2} + 1 = \frac{6}{y}$$

$$\text{LCD} = y^2 \quad \text{so } y \neq 0$$

$$y^2\left(\frac{5}{y^2} + 1\right) = y^2\left(\frac{6}{y}\right)$$

$$\cancel{y^2}\left(\frac{5}{\cancel{y^2}}\right) + y^2(1) = y \cdot \cancel{y}\left(\frac{6}{\cancel{y}}\right)$$

$$5 + y^2 = 6y$$

$$y^2 - 6y + 5 = 0$$

$$(y-5)(y-1) = 0$$

$$y-5=0 \text{ or } y-1=0$$

$$y=5 \text{ or } y=1 \quad \{5,1\}$$

$$54. \quad 1 + \frac{7}{6}m^{-1} = \frac{13}{6}m^{-1}$$

$$1 + \frac{7}{6m} = \frac{13}{6m}$$

$$\text{LCD} = 6m \quad \text{so } m \neq 0$$

$$6m\left(1 + \frac{7}{6m}\right) = 6m\left(\frac{13}{6m}\right)$$

$$6m(1) + \cancel{6m}\left(\frac{7}{\cancel{6m}}\right) = \cancel{6m}\left(\frac{13}{\cancel{6m}}\right)$$

$$6m + 7 = 13$$

$$6m = 6$$

$$m = 1 \quad \{1\}$$

$$55. \quad c = \frac{ax+b}{x} \quad \text{for } x$$

$$cx = ax + b$$

$$cx - ax = b$$

$$x(c-a) = b$$

$$x = \frac{b}{c-a}$$

$$56. \quad \frac{A}{rt} = P + \frac{P}{rt} \quad \text{for } P$$

$$rt\left(\frac{A}{rt}\right) = rt\left(P + \frac{P}{rt}\right)$$

$$A = rtP + P$$

$$A = P(rt+1)$$

$$\frac{A}{rt+1} = P$$

Section 5.6

$$57. \quad \frac{5}{4} = \frac{x}{6} \quad \text{LCD} = 12$$

$$12\left(\frac{5}{4}\right) = 12\left(\frac{x}{6}\right)$$

$$15 = 2x$$

$$x = \frac{15}{2} \quad \left\{ \frac{15}{2} \right\}$$

$$58. \quad \frac{x}{36} = \frac{6}{7} \quad \text{LCD} = 252$$

$$252\left(\frac{x}{36}\right) = 252\left(\frac{6}{7}\right)$$

$$7x = 216$$

$$x = \frac{216}{7} \quad \left\{ \frac{216}{7} \right\}$$

$$59. \quad \frac{x+2}{3} = \frac{5(x+1)}{4} \quad \text{LCD} = 12$$

$$12\left(\frac{x+2}{3}\right) = 12\left(\frac{5(x+1)}{4}\right)$$

$$4(x+2) = 15(x+1)$$

$$4x+8 = 15x+15$$

$$-11x = 7$$

$$x = -\frac{7}{11} \quad \left\{ -\frac{7}{11} \right\}$$

$$60. \quad \frac{x}{x+2} = \frac{-3}{5} \quad \text{LCD} = 5(x+2)$$

$$5\left(\cancel{x+2}\right)\left(\frac{x}{\cancel{x+2}}\right) = \cancel{5}(x+2)\left(\frac{-3}{\cancel{5}}\right)$$

$$5x = -3x - 6$$

$$8x = -6$$

$$x = -\frac{3}{4} \quad \left\{ -\frac{3}{4} \right\}$$

61. Let y = the number of yards gained

$$\frac{34}{357} = \frac{22}{y} \quad \text{LCD} = 357y$$

$$\cancel{357}y\left(\frac{34}{\cancel{357}}\right) = 357\cancel{y}\left(\frac{22}{\cancel{y}}\right)$$

$$34y = 7854$$

$$y = 231$$

He would gain 231 yd

62. Let c = the number of Canadian dollars

$$\frac{108}{100} = \frac{c}{235} \quad \text{LCD} = 4700$$

$$4700\left(\frac{108}{100}\right) = 4700\left(\frac{c}{235}\right)$$

$$5076 = 20c$$

$$c = 253.80$$

Erik can buy \$253.80 Canadian.

63. Let x = the speed on first day

$x - 5$ = the speed on second day

	Distance	Rate	Time
First Day	100	x	$100/x$
Second Day	75	$x - 5$	$75/(x - 5)$

(Time first) + (Time second) = 10

$$\frac{100}{x} + \frac{75}{x-5} = 10 \quad \text{LCD} = x(x-5)$$

$$x(x-5)\left(\frac{100}{x} + \frac{75}{x-5}\right) = x(x-5)(10)$$

$$100x - 500 + 75x = 10x^2 - 50x$$

$$0 = 10x^2 - 225x + 500$$

$$0 = 2x^2 - 45x + 100$$

$$0 = (2x-5)(x-20)$$

$$2x-5 = 0 \text{ or } x-20 = 0$$

$$2x = 5 \text{ or } x = 20$$

$$x \neq \frac{5}{2} \text{ or } x = 20$$

$$x-5 = 20-5 = 15$$

Tony averaged 20 mph the first day and 15 mph the second day.

64. Let x = the walking speed

$15x$ = the driving speed

	Distance	Rate	Time
Walking	3	x	$3/x$
Driving	45	$15x$	$45/(15x)$

$$(\text{Time walk}) + (\text{Time drive}) = 1.5$$

$$\frac{3}{x} + \frac{45}{15x} = \frac{3}{2} \quad \text{LCD} = 30x$$

$$30x\left(\frac{3}{x} + \frac{45}{15x}\right) = 30x\left(\frac{3}{2}\right)$$

$$90 + 90 = 45x$$

$$180 = 45x$$

$$x = 4$$

$$15x = 15(4) = 60$$

The driving speed was 60 mph.

65.

	Work Rate	Time	Part of Job Comp
Doug	$1/8$	x	$(1/8)x$
Jean	$1/10$	x	$(1/10)x$

$$(\text{Doug's Part}) + (\text{Jean's Part}) = (1 \text{ Job})$$

$$\frac{1}{8} \cdot x + \frac{1}{10} \cdot x = 1 \quad \text{LCD} = 40$$

$$40 \left(\frac{1}{8} \cdot x + \frac{1}{10} \cdot x \right) = 40(1)$$

$$5x + 4x = 40$$

$$9x = 40$$

$$x = \frac{40}{9} \text{ hr} \approx 4.4 \text{ hr}$$

Together it would take about 4.4 hours to complete the job.

66.

Work Rate	Time	Part of Job Comp
-----------	------	------------------

Lg pipe	$1/x$	6	$(1/x) 6$
---------	-------	---	-----------

Sm pipe	$1/(2x)$	6	$(1/(2x)) 6$
---------	----------	---	--------------

(Lg Pipe Part) + (Sm Pipe Part) = (1 Job)

$$\frac{1}{x} \cdot 6 + \frac{1}{2x} \cdot 6 = 1 \quad \text{LCD} = 2x$$

$$2x \left(\frac{1}{x} \cdot 6 + \frac{1}{2x} \cdot 6 \right) = 2x(1)$$

$$12 + 6 = 2x$$

$$18 = 2x$$

$$x = 9 \text{ hr}$$

$$2x = 2(9)$$

$$= 18 \text{ hr}$$

The larger pipe takes 9 hr to fill the tank; the smaller pipe takes 18 hr to fill the tank.

Section 5.7

67. a. $F = kd$

b. $6 = k(2)$

$$k = 3$$

c. $F = 3(4.2) = 12.6 \text{ lb}$

It requires 12.6 lb.

68. $y = \frac{k}{x}$

$$32 = \frac{k}{2}$$

$$k = 32 \cdot 2$$

$$= 64$$

$$y = \frac{64}{4}$$

$$= 16$$

$$\begin{aligned}
 69. \quad y &= kx\sqrt{z} \\
 3 &= k(3)\sqrt{4} \\
 3 &= k(3)(2) \\
 k &= \frac{3}{3 \cdot 2} = \frac{3}{6} = \frac{1}{2} \\
 y &= \frac{1}{2}(8)\sqrt{9} = 4 \cdot 3 = 12
 \end{aligned}$$

$$\begin{aligned}
 70. \quad &\text{Let } d = \text{the distance one can see} \\
 &h = \text{the height above sea level} \\
 d &= k\sqrt{h} \\
 30 &= k\sqrt{25} \\
 30 &= k \cdot 5 \\
 k &= 6 \\
 d &= 6\sqrt{64} = 6 \cdot 8 = 48 \text{ km} \\
 &\text{She can see 48 km.}
 \end{aligned}$$

Chapter 5 Test

$$\begin{aligned}
 1. \quad h(x) &= \frac{2x-14}{x^2-49} \\
 \text{a.} \quad h(0) &= \frac{2(0)-14}{(0)^2-49} = \frac{0-14}{0-49} = \frac{-14}{-49} = \frac{2}{7} \\
 h(5) &= \frac{2(5)-14}{(5)^2-49} = \frac{10-14}{25-49} = \frac{-4}{-24} = \frac{1}{6} \\
 h(7) &= \frac{2(7)-14}{(7)^2-49} = \frac{14-14}{49-49} = \frac{0}{0} \\
 &\text{is undefined} \\
 h(-7) &= \frac{2(-7)-14}{(-7)^2-49} = \frac{-14-14}{49-49} = \frac{-28}{0} \\
 &\text{is undefined}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad k(x) &= \frac{5x-3}{7} \\
 &(-\infty, \infty)
 \end{aligned}$$

$$\text{b.} \quad (-\infty, -7) \cup (-7, 7) \cup (7, \infty)$$

$$\begin{aligned}
 3. \quad f(x) &= \frac{2x+6}{x^2-x-12} \\
 &= \frac{2(x+3)}{(x-4)(x+3)}
 \end{aligned}$$

$$\text{a.} \quad \left\{ x \mid x \text{ is a real number and } x \neq 4, x \neq -3 \right\}$$

$$4. \quad \frac{12m^3n^7}{18mn^8} = \frac{2}{3}m^{3-1}n^{7-8} = \frac{2}{3}m^2n^{-1} = \frac{2m^2}{3n}$$

$$\begin{aligned} \text{b. } f(x) &= \frac{2x+6}{x^2-x-12} = \frac{2(\cancel{x+3})}{(x-4)(\cancel{x+3})} \\ &= \frac{2}{x-4} \end{aligned}$$

$$\begin{aligned} \text{5. } \frac{9x^2-9}{3x^2+2x-5} &= \frac{9(x^2-1)}{(3x+5)(x-1)} \\ &= \frac{9(x+1)(\cancel{x-1})}{(3x+5)(\cancel{x-1})} = \frac{9(x+1)}{3x+5} \end{aligned}$$

$$\begin{aligned} \text{6. } m &= \frac{-\frac{8}{3} - \left(-\frac{3}{4}\right)}{\frac{5}{6} - \frac{1}{12}} \quad \text{LCD} = 12 \\ &= \frac{12\left(-\frac{8}{3} + \frac{3}{4}\right)}{12\left(\frac{5}{6} - \frac{1}{12}\right)} = \frac{-32+9}{10-1} = \frac{-23}{9} = -\frac{23}{9} \end{aligned}$$

$$\begin{aligned} \text{7. } \frac{2x-5}{25-4x^2} \cdot (2x^2-x-15) &= \frac{2x-5}{-(4x^2-25)} \cdot \frac{(2x+5)(x-3)}{1} \\ &= \frac{\cancel{2x-5}}{-(\cancel{2x+5})(\cancel{2x-5})} \cdot \frac{(\cancel{2x+5})(x-3)}{1} \\ &= -(x-3) \end{aligned}$$

$$\begin{aligned} \text{8. } \frac{x^2}{x-4} - \frac{8x-16}{x-4} &= \frac{x^2-8x+16}{x-4} \\ &= \frac{(\cancel{x-4})(x-4)}{\cancel{x-4}} = x-4 \end{aligned}$$

$$\begin{aligned} \text{9. } \frac{4x}{x+1} + x + \frac{2}{x+1} &= \frac{4x}{x+1} + x \cdot \frac{x+1}{x+1} + \frac{2}{x+1} \\ &= \frac{4x+x^2+x+2}{x+1} = \frac{x^2+5x+2}{x+1} \end{aligned}$$

$$\begin{aligned} \text{10. } \frac{3+\frac{3}{k}}{4+\frac{4}{k}} \quad \text{LCD} = k &= \frac{k\left(3+\frac{3}{k}\right)}{k\left(4+\frac{4}{k}\right)} = \frac{3k+3}{4k+4} = \frac{3(\cancel{k+1})}{4(\cancel{k+1})} = \frac{3}{4} \end{aligned}$$

$$\begin{aligned} \text{11. } \frac{2u^{-1}+2v^{-1}}{4u^{-3}+4v^{-3}} &= \frac{\frac{2}{u} + \frac{2}{v}}{\frac{4}{u^3} + \frac{4}{v^3}} \quad \text{LCD} = u^3v^3 \end{aligned}$$

$$\begin{aligned}
&= \frac{u^3 v^3 \left(\frac{2}{u} + \frac{2}{v} \right)}{u^3 v^3 \left(\frac{4}{u^3} + \frac{4}{v^3} \right)} = \frac{2u^2 v^3 + 2u^3 v^2}{4v^3 + 4u^3} = \frac{2u^2 v^2 (v+u)}{4(v^3 + u^3)} \\
&= \frac{\cancel{2} u^2 v^2 (\cancel{v+u})}{\cancel{2} \cdot 2 (\cancel{v+u}) (v^2 - vu + u^2)} = \frac{u^2 v^2}{2(v^2 - vu + u^2)}
\end{aligned}$$

12.
$$\begin{aligned}
&\frac{ax+bx+2a+2b}{ax-3a+bx-3b} \cdot \frac{x-3}{5-x} \div \frac{x+2}{ax-5a} \\
&= \frac{x(a+b)+2(a+b)}{a(x-3)+b(x-3)} \cdot \frac{x-3}{-(x-5)} \cdot \frac{ax-5a}{x+2} \\
&= \frac{(\cancel{a+b})(\cancel{x+2})}{(\cancel{x-3})(\cancel{a+b})} \cdot \frac{\cancel{x-3}}{-(\cancel{x-5})} \cdot \frac{a(\cancel{x-5})}{\cancel{x+2}} \\
&= -a
\end{aligned}$$

13.
$$\begin{aligned}
&\frac{3}{x^2+8x+15} - \frac{1}{x^2+7x+12} - \frac{1}{x^2+9x+20} = \frac{3}{(x+5)(x+3)} - \frac{1}{(x+4)(x+3)} - \frac{1}{(x+5)(x+4)} \\
&\text{LCD} = (x+5)(x+3)(x+4) \\
&= \frac{3}{(x+5)(x+3)} \cdot \frac{x+4}{x+4} - \frac{1}{(x+4)(x+3)} \cdot \frac{x+5}{x+5} - \frac{1}{(x+5)(x+4)} \cdot \frac{x+3}{x+3} \\
&= \frac{3x+12-x-5-x-3}{(x+5)(x+3)(x+4)} \\
&= \frac{\cancel{x+4}}{(x+5)(x+3)(\cancel{x+4})} = \frac{1}{(x+5)(x+3)}
\end{aligned}$$

14.
$$\begin{aligned}
&\frac{7}{z+1} - \frac{z-5}{z^2-1} = \frac{6}{z} \\
&\frac{7}{z+1} - \frac{z-5}{(z+1)(z-1)} = \frac{6}{z}
\end{aligned}$$

LCD = $z(z+1)(z-1)$ so $z \neq 0$ or $z \neq -1$ or $z \neq 1$

$$z(z+1)(z-1) \left(\frac{7}{z+1} - \frac{z-5}{(z+1)(z-1)} \right) = z(z+1)(z-1) \left(\frac{6}{z} \right)$$

$$z(\cancel{z+1})(z-1) \left(\frac{7}{\cancel{z+1}} \right) - z(\cancel{z+1})(\cancel{z-1}) \left(\frac{z-5}{(\cancel{z+1})(\cancel{z-1})} \right) = \cancel{z}(z+1)(z-1) \left(\frac{6}{\cancel{z}} \right)$$

$$\begin{aligned}
 7z(z-1) - z(z-5) &= 6(z^2 - 1) \\
 7z^2 - 7z - z^2 + 5z &= 6z^2 - 6 \\
 6z^2 - 2z &= 6z^2 - 6 \\
 -2z &= -6 \\
 z &= 3 \quad \{3\}
 \end{aligned}$$

15.

$$\begin{aligned}
 \frac{3}{y^2-9} + \frac{4}{y+3} &= 1 \\
 \frac{3}{(y+3)(y-3)} + \frac{4}{y+3} &= 1 \\
 \text{LCD} = (y+3)(y-3) \quad \text{so } y \neq -3 \text{ or } y \neq 3 \\
 (y+3)(y-3) \left(\frac{3}{(y+3)(y-3)} + \frac{4}{y+3} \right) &= (y+3)(y-3)(1) \\
 (\cancel{y+3})(\cancel{y-3}) \left(\frac{3}{(\cancel{y+3})(\cancel{y-3})} \right) + (\cancel{y+3})(y-3) \left(\frac{4}{\cancel{y+3}} \right) &= (y+3)(y-3) \\
 3 + 4y - 12 &= y^2 - 9 \\
 0 &= y^2 - 4y \\
 y(y-4) &= 0 \\
 y &= 0 \\
 &\text{or } y-4 = 0 \\
 y &= 4 \\
 &\text{or } y = 4 \quad \{0, 4\}
 \end{aligned}$$

16.

$$\begin{aligned}
 \frac{4x}{x-4} &= 3 + \frac{16}{x-4} \\
 \text{LCD} = x-4 \quad \text{so } x \neq 4 \\
 (\cancel{x-4}) \left(\frac{4x}{\cancel{x-4}} \right) &= (x-4) \left(3 + \frac{16}{x-4} \right) \\
 4x &= (x-4)(3) + (\cancel{x-4}) \left(\frac{16}{\cancel{x-4}} \right) \\
 4x &= 3x - 12 + 16 \\
 x &= 4
 \end{aligned}$$

{ } no solution. ($x = 4$ does not check because the denominator is zero.)

17.

$$\begin{aligned}
 \frac{1+Tv}{T} &= p \quad \text{for } T \\
 1+Tv &= pT \\
 1 &= pT - Tv \\
 1 &= T(p-v) \\
 \frac{1}{p-v} &= T
 \end{aligned}$$

$$18. \quad F = \frac{Gm_1m_2}{r^2} \quad \text{for } m_1$$

$$Fr^2 = Gm_1m_2$$

$$\frac{Fr^2}{Gm_2} = m_1$$

19. Let $x =$ the number

$$\frac{1}{x} + 3x = \frac{13}{2}$$

LCD = $2x$ so $x \neq 0$

$$2x\left(\frac{1}{x} + 3x\right) = 2x\left(\frac{13}{2}\right)$$

$$2\cancel{x}\left(\frac{1}{\cancel{x}}\right) + 2x(3x) = \cancel{2}x\left(\frac{13}{\cancel{2}}\right)$$

$$2 + 6x^2 = 13x$$

$$6x^2 - 13x + 2 = 0$$

$$(6x - 1)(x - 2) = 0$$

$$6x - 1 = 0 \quad \text{or} \quad x - 2 = 0$$

$$6x = 1 \quad \text{or} \quad x = 2$$

$$x = \frac{1}{6} \quad \text{or} \quad x = 2$$

$$20. \quad \frac{21}{a} = \frac{18}{12}$$

$$12a\left(\frac{21}{a}\right) = 12a\left(\frac{18}{12}\right)$$

$$252 = 18a$$

$$a = 14 \text{ m}$$

$$\frac{b}{10} = \frac{18}{12}$$

$$60\left(\frac{b}{10}\right) = 60\left(\frac{18}{12}\right)$$

$$6b = 90$$

$$b = 15 \text{ m}$$

21. Let $x =$ the actual distance

$$\frac{8.2}{2820} = \frac{5.7}{x} \quad \text{LCD} = 2820x$$

$$2820x\left(\frac{8.2}{2820}\right) = 2820x\left(\frac{5.7}{x}\right)$$

$$8.2x = 16074$$

$$x \approx 1960 \text{ mi}$$

The cities are 1960 mi apart.

22. Let $x =$ the speed against the wind

	Distance	Rate	Time
Against	48	x	$48/x$
With wind	60	$x + 4$	$60/(x + 4)$

(Time against wind) = (Time with wind)

$$\frac{48}{x} = \frac{60}{x + 4} \quad \text{LCD} = x(x + 4)$$

$$\cancel{x}(x + 4)\left(\frac{48}{\cancel{x}}\right) = x(\cancel{x + 4})\left(\frac{60}{\cancel{x + 4}}\right)$$

Chapter 5 Rational Expressions and Rational Equations

$$\begin{aligned}
 48x + 192 &= 60x \\
 192 &= 12x \\
 x &= 16 \\
 x + 4 &= 16 + 4 \\
 &= 20
 \end{aligned}$$

Lance rides 16 mph against the wind and 20 mph with the wind.

23. Work Rate Time Portion of Job Comp

$$\text{Barbara } \frac{1}{4} \quad x \quad \frac{(1/4)x}{x}$$

$$\text{Jack } \frac{1}{10} \quad x \quad \frac{(1/10)x}{x}$$

(Barbara's Part) + (Jack's Part) = (1 Job)

$$\begin{aligned}
 \frac{1}{4}x + \frac{1}{10}x &= 1 \quad \text{LCD} = 20 \\
 20\left(\frac{1}{4}x + \frac{1}{10}x\right) &= 20(1) \\
 5x + 2x &= 20 \\
 7x &= 20 \\
 x &= \frac{20}{7} \text{ hr} \\
 &\text{or } 2\frac{6}{7} \text{ hr}
 \end{aligned}$$

Together, they can type the chapter in $2\frac{6}{7}$ hr.

24. $x = \frac{ky}{t^2}$

25. Let P = the period of the pendulum
 L = the length of the pendulum

$$P = k\sqrt{L}$$

$$2.2 = k\sqrt{4}$$

$$k = \frac{2.2}{\sqrt{4}}$$

$$= \frac{2.2}{2}$$

$$= 1.1$$

$$P = 1.1\sqrt{9}$$

$$= 1.1(3)$$

$$= 3.3 \text{ sec}$$

The period is 3.3 sec.

Chapters 1 – 5 Cumulative Review Exercises

1.		-22	π	6	$-\sqrt{2}$
	Real numbers	-	-	-	-
	Irrational numbers		-		-
	Rational numbers	-		-	
	Integers	-		-	
	Whole numbers			-	
	Natural numbers			-	

2. Multiply the first equation by 2, add to the second equation, and solve for x :

$$\begin{aligned}
 3x + 4y &= -4 \xrightarrow{\times 2} 6x + 8y = -8 \\
 2x - 8y &= -8 \longrightarrow \underline{2x - 8y = -8} \\
 8x &= -16 \\
 x &= -2
 \end{aligned}$$

Substitute and solve for y :

$$\begin{aligned}
 3(-2) + 4y &= -4 \\
 -6 + 4y &= -4 \\
 4y &= 2 \\
 y &= \frac{2}{4} = \frac{1}{2}
 \end{aligned}$$

The solution is: $\left\{(-2, \frac{1}{2})\right\}$.

3. $(2x - 3)(x - 4) - (x - 5)^2$

$$\begin{aligned}
 &= 2x^2 - 8x - 3x + 12 - (x^2 - 10x + 25) \\
 &= 2x^2 - 11x + 12 - x^2 + 10x - 25 \\
 &= x^2 - x - 13
 \end{aligned}$$

4. a. $A = \frac{1}{2}h(b_1 + b_2)$ for b_1

$$\begin{aligned}
 2A &= h(b_1 + b_2) \\
 2A &= hb_1 + hb_2 \\
 2A - hb_2 &= hb_1 \\
 \frac{2A - hb_2}{h} &= b_1
 \end{aligned}$$

b. $b_1 = \frac{2(32) - 4(6)}{4} = \frac{64 - 24}{4}$

$$= \frac{40}{4} = 10 \text{ cm}$$

5. Let w = the width of the pool
 $2w - 10$ = the length of the pool

$$\begin{aligned}
 2w + 2(2w - 10) &= 160 \\
 2w + 4w - 20 &= 160 \\
 6w &= 180 \\
 w &= 30
 \end{aligned}$$

$$\begin{aligned}
 2w - 10 &= 2(30) - 10 \\
 &= 60 - 10 = 50
 \end{aligned}$$

The length is 50 m and the width is 30 m.

Chapter 5 Rational Expressions and Rational Equations

6. $3x - 3y + z = -13$

$$2x - y - 2z = 4$$

$$x + 2y - 3z = 15$$

Multiply the first equation by 2 and add to the second equation to eliminate z :

$$6x - 6y + 2z = -26$$

$$2x - y - 2z = 4$$

$$\hline 8x - 7y = -22$$

Multiply the first equation by 3 and add to the third equation to eliminate z :

$$9x - 9y + 3z = -39$$

$$x + 2y - 3z = 15$$

$$\hline 10x - 7y = -24$$

Multiply the first result by -1 and add to the second result to eliminate y :

$$-8x + 7y = 22$$

$$10x - 7y = -24$$

$$\hline 2x = -2$$

$$x = -1$$

Substitute and solve:

$$8(-1) - 7y = -22$$

$$-8 - 7y = -22$$

$$-7y = -14$$

$$y = 2$$

$$3(-1) - 3(2) + z = -13$$

$$-3 - 6 + z = -13$$

$$-9 + z = -13$$

$$z = -4$$

The solution is $\{(-1, 2, -4)\}$.

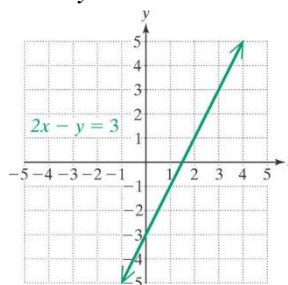
7. The slope of the line is 3 and the slope of the perpendicular line is $-\frac{1}{3}$.

$$y - 5 = -\frac{1}{3}(x - (-3))$$

$$y - 5 = -\frac{1}{3}x - 1$$

$$y = -\frac{1}{3}x + 4$$

8. $2x - y = 3$



9. $s = \frac{k}{t}$

$$60 = \frac{k}{10}$$

$$600 = k$$

$$s = \frac{600}{8} = 75$$

10. $f(x) = -12x^3 + 17x^2 - 6x$

$$= -x(12x^2 - 17x + 6)$$

$$= -x(4x - 3)(3x - 2)$$

$$-x = 0 \text{ or } 4x - 3 = 0 \text{ or } 3x - 2 = 0$$

$$x = 0 \text{ or } 4x = 3 \text{ or } 3x = 2$$

The speed of the car is 75 mph.

$$x = 0 \text{ or } x = \frac{3}{4} \text{ or } x = \frac{2}{3}$$

$$x\text{-intercepts: } (0,0), \left(\frac{3}{4}, 0\right), \left(\frac{2}{3}, 0\right)$$

$$\begin{aligned} 11. \quad 64y^3 - 8z^6 &= 8(8y^3 - z^6) \\ &= 8\left[(2y)^3 - (z^2)^3\right] \\ &= 8(2y - z^2)(4y^2 + 2yz^2 + z^4) \end{aligned}$$

$$12. \quad 10x^2 - x - 2 = (5x + 2)(2x - 1)$$

$$\begin{aligned} 13. \quad \frac{2x^2 + 11x - 21}{4x^2 - 10x + 6} \div \frac{2x^2 - 98}{x^2 - x + xa - a} &= \frac{2x^2 + 11x - 21}{2(2x^2 - 5x + 3)} \cdot \frac{x^2 - x + xa - a}{2x^2 - 98} \\ &= \frac{(2x-3)(x+7)}{2(2x-3)(x-1)} \cdot \frac{x(x-1)+a(x-1)}{2(x^2-49)} \\ &= \frac{\cancel{(2x-3)}\cancel{(x+7)}}{2\cancel{(2x-3)}\cancel{(x-1)}} \cdot \frac{\cancel{(x-1)}(x+a)}{2\cancel{(x+7)}(x-7)} \\ &= \frac{x+a}{4(x-7)} \end{aligned}$$

$$\begin{aligned} 14. \quad \frac{x}{x^2 + 5x - 50} - \frac{1}{x^2 - 7x + 10} + \frac{1}{x^2 + 8x - 20} &= \frac{x}{(x+10)(x-5)} - \frac{1}{(x-5)(x-2)} + \frac{1}{(x+10)(x-2)} \\ \text{LCD} &= (x+10)(x-5)(x-2) \\ &= \frac{x}{(x+10)(x-5)} \cdot \frac{x-2}{x-2} - \frac{1}{(x-5)(x-2)} \cdot \frac{x+10}{x+10} + \frac{1}{(x+10)(x-2)} \cdot \frac{x-5}{x-5} \\ &= \frac{x^2 - 2x - x - 10 + x - 5}{(x+10)(x-5)(x-2)} \\ &= \frac{x^2 - 2x - 15}{(x+10)(x-5)(x-2)} \\ &= \frac{\cancel{(x-5)}(x+3)}{(x+10)\cancel{(x-5)}(x-2)} \\ &= \frac{x+3}{(x+10)(x-2)} \end{aligned}$$

$$\begin{aligned}
 15. \quad & \frac{1 - \frac{49}{c^2}}{\frac{7}{c} + 1} \quad \text{LCD} = c^2 \\
 & = \frac{c^2 \left(1 - \frac{49}{c^2}\right)}{c^2 \left(\frac{7}{c} + 1\right)} = \frac{c^2(1) - \cancel{c^2} \left(\frac{49}{\cancel{c^2}}\right)}{c \cdot \cancel{c} \left(\frac{7}{\cancel{c}}\right) + c^2(1)} = \frac{c^2 - 49}{7c + c^2} \\
 & = \frac{(\cancel{c+7})(c-7)}{c(\cancel{c+7})} = \frac{c-7}{c}
 \end{aligned}$$

$$\begin{aligned}
 16. \quad & y^3 - 5y^2 - y = -5 \\
 & y^3 - 5y^2 - y + 5 = 0 \\
 & y^2(y-5) - 1(y-5) = 0 \\
 & (y-5)(y^2-1) = 0 \\
 & (y-5)(y+1)(y-1) = 0 \\
 & \quad y-5=0 \text{ or } y+1=0 \text{ or } y-1=0 \\
 & \quad y=5 \text{ or } y=-1 \text{ or } y=1 \quad \{5, -1, 1\}
 \end{aligned}$$

$$\begin{aligned}
 17. \quad & \frac{4y}{y+2} - \frac{y}{y-1} = \frac{9}{y^2+y-2} \\
 & \frac{4y}{y+2} - \frac{y}{y-1} = \frac{9}{(y+2)(y-1)} \quad \text{LCD} = (y+2)(y-1) \quad \text{so } y \neq -2 \text{ or } y \neq 1 \\
 & (y+2)(y-1) \left(\frac{4y}{y+2} - \frac{y}{y-1} \right) = (y+2)(y-1) \left(\frac{9}{(y+2)(y-1)} \right) \\
 & (\cancel{y+2})(y-1) \left(\frac{4y}{\cancel{y+2}} \right) - (y+2)(\cancel{y-1}) \left(\frac{y}{\cancel{y-1}} \right) = (\cancel{y+2})(\cancel{y-1}) \left(\frac{9}{(\cancel{y+2})(\cancel{y-1})} \right) \\
 & 4y^2 - 4y - y^2 - 2y = 9 \\
 & 3y^2 - 6y - 9 = 0 \\
 & 3(y^2 - 2y - 3) = 0 \\
 & 3(y-3)(y+1) = 0 \\
 & \quad y-3=0 \text{ or } y+1=0 \\
 & \quad y=3 \text{ or } y=-1 \quad \{3, -1\}
 \end{aligned}$$

18. Let x = the actual distance

$$\frac{6.5}{195} = \frac{9.25}{x} \quad \text{LCD} = 195x$$

$$195x \left(\frac{6.5}{195} \right) = 195x \left(\frac{9.25}{x} \right)$$

$$6.5x = 1803.75$$

$$x \approx 278 \text{ mi}$$

The actual distance is about 278 mi.

20. Let I = the amount of interest

r = the interest rate

t = the time in years

$$I = krt$$

$$1120 = k(0.08)(2)$$

$$1120 = k(0.16)$$

$$k = 7000$$

$$I = 7000(0.10)(5) = \$3500$$

The investment will yield \$3500 in interest.

19. a. $x = -5$ is a vertical line. The slope is undefined.
- b. $2y = 8$ is a horizontal line. The slope of the line is 0.