Chapter 4 Polynomials

Section 4.1 Practice Exercises

1. **a.** exponent

b. 1

c. $\left(\frac{1}{b}\right)^n$ or $\frac{1}{b^n}$

d. scientific notation

2. $b^4 \cdot b^3 = (b \cdot b \cdot b \cdot b) \cdot (b \cdot b \cdot b)$
$= b^7$

$(b^4)^3 = b^4 \cdot b^4 \cdot b^4$
$= (b \cdot b \cdot b \cdot b) \cdot (b \cdot b \cdot b \cdot b) \cdot (b \cdot b \cdot b \cdot b)$
$= b^{12}$

3. $ab^3 = a \cdot b \cdot b \cdot b$

$(ab)^3 = (ab) \cdot (ab) \cdot (ab)$
$= a \cdot a \cdot a \cdot b \cdot b \cdot b = a^3 \cdot b^3$

4. For example: $3^2 \cdot 3^4 = 3^6$

$x^8 \cdot x^2 = x^{10}$

5. For example: $(5x)^2 = 5^2 x^2$

$(xy)^3 = x^3 y^3$

6. For example: $(x^2)^4 = x^8$

$(2^3)^5 = 2^{15}$

7. For example: $\frac{x^5}{x^2} = x^3$

$\frac{8^4}{8^2} = 8^2$

8. For example: $\left(\frac{x}{y}\right)^3 = \frac{x^3}{y^3}$

$\left(\frac{2}{7}\right)^2 = \frac{2^2}{7^2}$

9. For example: $6^0 = 1$

$x^0 = 1 \ (x \neq 0)$

10. $\left(\frac{2}{3}\right)^{-1} = \frac{3}{2}$

11. $\left(\frac{1}{3}\right)^{-1} = \left(\frac{3}{1}\right)^1 = 3$

12. $(3)^{-1} = \frac{1}{3}$

13. $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

14. $8^{-2} = \frac{1}{8^2} = \frac{1}{64}$

15. $-5^{-2} = -\frac{1}{5^2} = -\frac{1}{25}$

16. $-8^{-2} = -\frac{1}{8^2} = -\frac{1}{64}$

17. $\left(-5\right)^{-2} = \frac{1}{\left(-5\right)^2} = \frac{1}{25}$

18. $\left(-8\right)^{-2} = \frac{1}{\left(-8\right)^2} = \frac{1}{64}$

19. $\left(-\frac{1}{4}\right)^{-3} = \left(-\frac{4}{1}\right)^{3} = \left(-4\right)^3 = -64$

20. $\left(-\frac{3}{8}\right)^{-1} = \left(-\frac{8}{3}\right)^{1} = -\frac{8}{3}$

21. $\left(-\frac{3}{2}\right)^{-4} = \left(-\frac{2}{3}\right)^{4} = \frac{\left(-2\right)^4}{3^4} = \frac{16}{81}$

22. $\left(-\frac{1}{9}\right)^{-2} = \left(-\frac{9}{1}\right)^{2} = \left(-9\right)^2 = 81$

23. $-\left(\frac{2}{5}\right)^{-3} = -\left(\frac{5}{2}\right)^{3} = -\frac{5^3}{2^3} = -\frac{125}{8}$

24. $-\left(\frac{1}{2}\right)^{-5} = -\left(\frac{2}{1}\right)^{5} = -\left(2\right)^5 = -32$

25. $\left(10ab\right)^0 = 1$

26. $\left(13x\right)^0 = 1$

27. $10ab^0 = 10a \cdot 1 = 10a$

28. $13x^0 = 13 \cdot 1 = 13$

29. $y^3 \cdot y^5 = y^{3+5} = y^8$

30. $x^4 \cdot x^8 = x^{12}$

31. $\frac{13^8}{13^6} = 13^{8-6} = 13^2 = 169$

32. $\frac{5^7}{5^3} = 5^{7-3} = 5^4 = 625$

33. $\left(y^2\right)^4 = y^{2 \cdot 4} = y^8$

34. $\left(z^3\right)^4 = z^{3 \cdot 4} = z^{12}$

35. $\left(3x^2\right)^4 = 3^4\left(x^2\right)^4 = 3^4 x^{2 \cdot 4} = 81x^8$

36. $\left(2y^5\right)^3 = 2^3\left(y^5\right)^3 = 2^3 y^{5 \cdot 3} = 8y^{15}$

37. $p^{-3} = \dfrac{1}{p^3}$

38. $q^{-5} = \dfrac{1}{q^5}$

39. $7^{10} \cdot 7^{-13} = 7^{10+(-13)} = 7^{-3} = \dfrac{1}{7^3} = \dfrac{1}{343}$

40. $11^{-9} \cdot 11^{7} = 11^{-9+7} = 11^{-2} = \dfrac{1}{11^2} = \dfrac{1}{121}$

41. $\dfrac{w^3}{w^5} = w^{3-5} = w^{-2} = \dfrac{1}{w^2}$

42. $\dfrac{t^4}{t^8} = t^{4-8} = t^{-4} = \dfrac{1}{t^4}$

43. $a^{-2}a^{-5} = a^{-2+(-5)} = a^{-7} = \dfrac{1}{a^7}$

44. $b^{-1}b^{-8} = b^{-1+(-8)} = b^{-9} = \dfrac{1}{b^9}$

45. $\dfrac{r}{r^{-1}} = r^{1-(-1)} = r^2$

46. $\dfrac{s^{-1}}{s} = s^{-1-1} = s^{-2} = \dfrac{1}{s^2}$

47. $\dfrac{z^{-6}}{z^{-2}} = z^{-6-(-2)} = z^{-4} = \dfrac{1}{z^4}$

48. $\dfrac{w^{-8}}{w^{-3}} = w^{-8-(-3)} = w^{-5} = \dfrac{1}{w^5}$

49. $\dfrac{a^3}{b^{-2}} = a^3 \cdot \dfrac{1}{b^{-2}} = a^3 b^2$

50. $\dfrac{c^4}{d^{-1}} = c^4 \cdot \dfrac{1}{d^{-1}} = c^4 d^1 = c^4 d$

51. $\left(6xyz^2\right)^0 = 1$

52. $\left(-7ab^3\right)^0 = 1$

53.
$$\begin{aligned} 2^4 + 2^{-2} &= 2^4 + \frac{1}{2^2} \\ &= 16 + \frac{1}{4} \\ &= 16\tfrac{1}{4} \text{ or } \frac{65}{4} \end{aligned}$$

54.
$$\begin{aligned} 3^2 + 3^{-1} &= 3^2 + \frac{1}{3} \\ &= 9 + \frac{1}{3} \\ &= 9\tfrac{1}{3} \text{ or } \frac{28}{3} \end{aligned}$$

55.
$$\begin{aligned} 1^{-2} + 5^{-2} &= \frac{1}{1^2} + \frac{1}{5^2} \\ &= \frac{1}{1} + \frac{1}{25} \\ &= 1\tfrac{1}{25} \text{ or } \frac{26}{25} \end{aligned}$$

56.
$$\begin{aligned} 4^{-2} + 2^{-2} &= \frac{1}{4^2} + \frac{1}{2^2} = \frac{1}{16} + \frac{1}{4} \\ &= \frac{1}{16} + \frac{4}{16} \\ &= \frac{5}{16} \end{aligned}$$

57. $\left(\frac{2}{3}\right)^{-2}-\left(\frac{1}{2}\right)^{2}+\left(\frac{1}{3}\right)^{0}=\left(\frac{3}{2}\right)^{2}-\frac{1}{4}+1$
$=\frac{9}{4}-\frac{1}{4}+\frac{4}{4}$
$=\frac{12}{4}=3$

58. $\left(\frac{1}{6}\right)^{-1}+\left(\frac{2}{3}\right)^{0}-\left(\frac{1}{4}\right)^{-2}=\left(\frac{6}{1}\right)^{1}+1-\left(\frac{4}{1}\right)^{2}$
$=6+1-16$
$=-9$

59. $\left(\frac{4}{5}\right)^{-1}+\left(\frac{3}{2}\right)^{2}-\left(\frac{2}{7}\right)^{0}=\frac{5}{4}+\frac{9}{4}-1$
$=\frac{5}{4}+\frac{9}{4}-\frac{4}{4}$
$=\frac{10}{4}=\frac{5}{2}$

60. $\left(\frac{4}{5}\right)^{0}-\left(\frac{2}{3}\right)^{2}+\left(\frac{9}{5}\right)^{-1}=1-\frac{4}{9}+\frac{5}{9}$
$=\frac{9}{9}-\frac{4}{9}+\frac{5}{9}$
$=\frac{10}{9}$

61. $\frac{p^2q}{p^5q^{-1}}=p^{2-5}q^{1-(-1)}=p^{-3}q^2$
$=\frac{1}{p^3}\cdot q^2=\frac{q^2}{p^3}$

62. $\frac{m^{-1}n^3}{m^4n^{-2}}=m^{-1-4}n^{3-(-2)}=m^{-5}n^5$
$=\frac{1}{m^5}\cdot n^5=\frac{n^5}{m^5}$

63. $\frac{-48ab^{10}}{32a^4b^3}=-\frac{48}{32}a^{1-4}b^{10-3}=-\frac{3}{2}a^{-3}b^7$
$=-\frac{3}{2}\cdot\frac{1}{a^3}\cdot b^7=-\frac{3b^7}{2a^3}$

64. $\frac{25x^2y^{12}}{10x^5y^7}=\frac{25}{10}x^{2-5}y^{12-7}=\frac{5}{2}x^{-3}y^5$
$=\frac{5}{2}\cdot\frac{1}{x^3}\cdot y^5=\frac{5y^5}{2x^3}$

65. $\left(-3x^{-4}y^5z^2\right)^{-4}$
$=(-3)^{-4}\left(x^{-4}\right)^{-4}\left(y^5\right)^{-4}\left(z^2\right)^{-4}$
$=\left(-\frac{1}{3}\right)^4x^{16}y^{-20}z^{-8}$
$=\frac{1}{81}\cdot x^{16}\cdot\frac{1}{y^{20}}\cdot\frac{1}{z^8}$
$=\frac{x^{16}}{81y^{20}z^8}$

66. $\left(-6a^{-2}b^3c\right)^{-2}$
$=(-6)^{-2}\left(a^{-2}\right)^{-2}\left(b^3\right)^{-2}c^{-2}$
$=\left(-\frac{1}{6}\right)^2a^4b^{-6}c^{-2}$
$=\frac{1}{36}\cdot a^4\cdot\frac{1}{b^6}\cdot\frac{1}{c^2}$
$=\frac{a^4}{36b^6c^2}$

67. $\left(4m^{-2}n\right)\left(-m^6n^{-3}\right)$

$= -4m^{-2+6}n^{1+(-3)} = -4m^4n^{-2}$

$= -4m^4 \cdot \frac{1}{n^2} = -\frac{4m^4}{n^2}$

68. $\left(-6pq^{-3}\right)\left(2p^4q\right) = -12p^{1+4}q^{-3+1}$

$= -12p^5q^{-2}$

$= -12p^5 \cdot \frac{1}{q^2} = -\frac{12p^5}{q^2}$

69. $\left(p^{-2}q\right)^3\left(2pq^4\right)^2$

$= \left(p^{-2}\right)^3 q^3 \cdot 2^2 p^2 \left(q^4\right)^2$

$= p^{-6}q^3 \cdot 4p^2q^8 = 4p^{-6+2}q^{3+8}$

$= 4p^{-4}q^{11} = 4 \cdot \frac{1}{p^4} \cdot q^{11}$

$= \frac{4q^{11}}{p^4}$

70. $\left(mn^3\right)^2\left(5m^{-2}n^2\right) = m^2\left(n^3\right)^2\left(5m^{-2}n^2\right)$

$= m^2n^6 \cdot 5m^{-2}n^2$

$= 5m^{2+(-2)}n^{6+2}$

$= 5m^0n^8$

$= 5n^8$

71. $\left(\frac{x^2}{y}\right)^3\left(5x^2y\right) = \frac{x^6}{y^3}\left(5x^2y\right)$

$= 5x^{6+2}y^{1-3} = 5x^8y^{-2}$

$= 5x^8\frac{1}{y^2} = \frac{5x^8}{y^2}$

72. $\left(\frac{a}{b^2}\right)^2\left(3a^2b^3\right) = \frac{a^2}{b^4}\left(3a^2b^3\right)$

$= 3a^{2+2}b^{3-4} = 3a^4b^{-1}$

$= 3a^4 \cdot \frac{1}{b} = \frac{3a^4}{b}$

73. $\frac{\left(-8a^2b^2\right)^4}{\left(16a^3b^7\right)^2} = \frac{\left(-8\right)^4\left(a^2\right)^4\left(b^2\right)^4}{\left(16\right)^2\left(a^3\right)^2\left(b^7\right)^2}$

$= \frac{4096a^8b^8}{256a^6b^{14}}$

$= 16a^{8-6}b^{8-14}$

$= 16a^2b^{-6}$

$= 16a^2 \cdot \frac{1}{b^6}$

$= \frac{16a^2}{b^6}$

74. $\frac{\left(-3x^2y^3\right)^2}{\left(-2xy^4\right)^3} = \frac{\left(-3\right)^2\left(x^2\right)^2\left(y^3\right)^2}{\left(-2\right)^3x^3\left(y^4\right)^3}$

$= \frac{9x^4y^6}{-8x^3y^{12}}$

$= -\frac{9}{8}x^{4-3}y^{6-12}$

$= -\frac{9}{8}xy^{-6}$

$= -\frac{9}{8}x \cdot \frac{1}{y^6}$

$= -\frac{9x}{8y^6}$

75.
$$\left(\frac{-2x^6y^{-5}}{3x^{-2}y^4}\right)^{-3}$$
$$=\left(-\frac{2}{3}x^{6-(-2)}y^{-5-4}\right)^{-3}$$
$$=\left(-\frac{2}{3}x^8y^{-9}\right)^{-3}$$
$$=\left(-\frac{2}{3}\right)^{-3}\left(x^8\right)^{-3}\left(y^{-9}\right)^{-3}$$
$$=\left(-\frac{3}{2}\right)^3x^{-24}y^{27}$$
$$=-\frac{27}{8}\cdot\frac{1}{x^{24}}\cdot y^{27}=-\frac{27y^{27}}{8x^{24}}$$

76.
$$\left(\frac{-6a^2b^{-3}}{5a^{-1}b}\right)^{-2}=\left(-\frac{6}{5}a^{2-(-1)}b^{-3-1}\right)^{-2}$$
$$=\left(-\frac{6}{5}a^3b^{-4}\right)^{-2}$$
$$=\left(-\frac{6}{5}\right)^{-2}\left(a^3\right)^{-2}\left(b^{-4}\right)^{-2}$$
$$=\left(-\frac{5}{6}\right)^2a^{-6}b^8=\frac{25}{36}\cdot\frac{1}{a^6}\cdot b^8$$
$$=\frac{25b^8}{36a^6}$$

77.
$$\left(\frac{2x^{-3}y^0}{4x^6y^{-5}}\right)^{-2}=\left(\frac{1}{2}x^{-3-6}y^{0-(-5)}\right)^{-2}$$
$$=\left(\frac{1}{2}x^{-9}y^5\right)^{-2}$$
$$=\left(\frac{1}{2}\right)^{-2}\left(x^{-9}\right)^{-2}\left(y^5\right)^{-2}$$
$$=(2)^2x^{18}y^{-10}$$
$$=4x^{18}\cdot\frac{1}{y^{10}}$$
$$=\frac{4x^{18}}{y^{10}}$$

78.
$$\left(\frac{a^3b^2c^0}{a^{-1}b^{-2}c^{-3}}\right)^{-2}$$
$$=\left(a^{3-(-1)}b^{2-(-2)}c^{0-(-3)}\right)^{-2}$$
$$=\left(a^4b^4c^3\right)^{-2}$$
$$=\left(a^4\right)^{-2}\left(b^4\right)^{-2}\left(c^3\right)^{-2}$$
$$=a^{-8}b^{-8}c^{-6}$$
$$=\frac{1}{a^8}\cdot\frac{1}{b^8}\cdot\frac{1}{c^6}$$
$$=\frac{1}{a^8b^8c^6}$$

79.
$$3xy^5\left(\frac{2x^4y}{6x^5y^3}\right)^{-2}=3xy^5\left(\frac{1}{3}x^{4-5}y^{1-3}\right)^{-2}=3xy^5\left(\frac{1}{3}x^{-1}y^{-2}\right)^{-2}$$
$$=3xy^5\left(\frac{1}{3}\right)^{-2}\left(x^{-1}\right)^{-2}\left(y^{-2}\right)^{-2}$$
$$=3xy^5(3)^2x^2y^4$$
$$=3\cdot9x^{1+2}y^{5+4}=27x^3y^9$$

80.
$$\begin{aligned}7x^{-3}y^{-4}\left(\frac{3x^{-1}y^5}{9x^3y^{-2}}\right)^{-3} &= 7x^{-3}y^{-4}\left(\frac{1}{3}x^{-1-3}y^{5-(-2)}\right)^{-3}\\ &= 7x^{-3}y^{-4}\left(\frac{1}{3}x^{-4}y^7\right)^{-3} = 7x^{-3}y^{-4}\left(\frac{1}{3}\right)^{-3}\left(x^{-4}\right)^{-3}\left(y^7\right)^{-3}\\ &= 7x^{-3}y^{-4}(3)^3x^{12}y^{-21} = 7\cdot 27x^{-3+12}y^{-4+(-21)}\\ &= 189x^9y^{-25} = 189x^9\cdot\frac{1}{y^{25}}\\ &= \frac{189x^9}{y^{25}}\end{aligned}$$

81. **a.** $\$8{,}000{,}000{,}000 = \$8\times10^9$

b. $3{,}000{,}000 = 3\times10^6$ DVDs

c. $14{,}000{,}000{,}000{,}000 = 1.4\times10^{13}$ eV

d. $0.000\,000\,000\,000\,000\,0001602$
$= 1.602\times10^{-19}$ J

82. **a.** $311{,}000{,}000 = 3.11\times10^8$

b. $0.000\,000\,2 = 2\times10^{-7}$ m

c. $1{,}000{,}000{,}000{,}000 = 1\times10^{12}$

83. **a.** $2\times10^{11} = 200{,}000{,}000{,}000$

b. $4\times10^{-6} = 0.000004$

c. $1.082\times10^{11} = 108{,}200{,}000{,}000$

84. **a.** $3.784\times10^5 = 378{,}400\ \text{m}^3$

b. $3\times10^{-10} = 0.000\,000\,0003$ m

c. $4.1\times10^2 = 410\,\text{m}$

85.
$$\begin{aligned}35\times10^4 &= 3.5\times10^1\times10^4\\ &= 3.5\times10^5\end{aligned}$$

86.
$$\begin{aligned}0.469\times10^{-7} &= 4.69\times10^{-1}\times10^{-7}\\ &= 4.69\times10^{-8}\end{aligned}$$

87. 7.0×10^0 Proper

88. 8.12×10^1 Proper

89. 9×10^1 Proper

90. 6.9×10^0 Proper

91.
$$\begin{aligned}&\left(6.5\times10^3\right)\left(5.2\times10^{-8}\right)\\ &= 33.8\times10^{3+(-8)}\\ &= 3.38\times10^1\times10^{-5}\\ &= 3.38\times10^{-4}\end{aligned}$$

92.
$$\begin{aligned}&\left(3.26\times10^{-6}\right)\left(8.2\times10^9\right)\\ &= 26.732\times10^{-6+9}\\ &= 2.6732\times10^1\times10^3\\ &= 2.6732\times10^4\end{aligned}$$

93. $(0.0000024)(6,700,000,000)$
$= (2.4\times10^{-6})(6.7\times10^{9})$
$= 16.08\times10^{-6+9}$
$= 1.608\times10^{1}\times10^{3} = 1.608\times10^{4}$

94. $(3,400,000,000)(70,000,000,000,000)$
$= (3.4\times10^{9})(7\times10^{13})$
$= 23.8\times10^{9+13}$
$= 2.38\times10^{1}\times10^{22} = 2.38\times10^{23}$

95. $(8.5\times10^{-2})\div(2.5\times10^{-15})$
$= 3.4\times10^{-2-(-15)}$
$= 3.4\times10^{13}$

96. $(3\times10^{9})\div(1.5\times10^{13}) = 2\times10^{9-13}$
$= 2\times10^{-4}$

97. $(900000000)\div(360000)$
$= (9\times10^{8})\div(3.6\times10^{5})$
$= 2.5\times10^{8-5} = 2.5\times10^{3}$

98. $(0.0000000002)\div(8000000)$
$= (2\times10^{-10})\div(8\times10^{6})$
$= 0.25\times10^{-10-6} = 2.5\times10^{-1}\times10^{-16}$
$= 2.5\times10^{-17}$

99. $2\cdot(6.02\times10^{23}) = 12.04\times10^{23}$
$= 1.204\times10^{1}\times10^{23}$
$= 1.204\times10^{24}$ hydrogen atoms
$1\cdot(6.02\times10^{23}) = 6.02\times10^{23}$ oxygen atoms

100. $4.3(6\times10^{9}) = 25.8\times10^{9}$
$= 2.58\times10^{1}\times10^{9}$
$= 2.58\times10^{10}$ mi

101. $2,200,000\div110$
$= (2.2\times10^{6})\div(1.1\times10^{2})$
$= 2\times10^{4}$ or 20,000 people per mi^2

102. $150,000\div400$
$= (1.5\times10^{5})\div(4\times10^{2})$
$= 0.375\times10^{3} = 3.75\times10^{2}$
or 375 people per mi^2

103. $(\$3.5\times10^{9})(15) = \$52.5\times10^{9}$
$= \$5.25\times10^{10}$

104. **a.** $5(6.02\times10^{23}) = 30.1\times10^{23}$
$= 3.01\times10^{24}$ atoms

b. $(4.515\times10^{25})\div(6.02\times10^{23})$
$= 0.75\times10^{2} = 75$ moles

105. **a.** $45 \cdot 12 = 540$ months

b. $\$20(540) = \$10{,}800$

c. $A = \$20\left[\left(1+\frac{0.06}{12}\right)^{540} - 1\right]\left(1+\frac{12}{0.06}\right) = \$55{,}395.45$

106. $x^{a+1}x^{a+5} = x^{a+1+a+5} = x^{2a+6}$

107. $y^{a-5}y^{a+7} = y^{a-5+a+7} = y^{2a+2}$

108. $\frac{y^{2a+1}}{y^{a-1}} = y^{(2a+1)-(a-1)}$

$= y^{2a+1-a+1} = y^{a+2}$

109. $\frac{x^{3a-3}}{x^{a+1}} = x^{(3a-3)-(a+1)}$

$= x^{3a-3-a-1} = x^{2a-4}$

110. $\frac{x^{3b-2}y^{b+1}}{x^{2b+1}y^{2b+2}} = x^{(3b-2)-(2b+1)}y^{(b+1)-(2b+2)} = x^{3b-2-2b-1}y^{b+1-2b-2} = x^{b-3}y^{-b-1}$

111. $\frac{x^{2a-2}y^{a+3}}{x^{a+4}y^{a-3}} = x^{(2a-2)-(a+4)}y^{(a+3)-(a-3)} = x^{2a-2-a-4}y^{a+3-a+3} = x^{a-6}y^{6}$

Section 4.2 Practice Exercises

1. **a.** polynomial

b. coefficient; n

c. 1; 1

d. one

e. binomial

f. trinomial

g. leading; leading coefficient

h. greatest

i. zero

j. exponents

k. polynomial

2. $\left(\frac{6tv^2}{15t^3}\right)^{-1} = \frac{15t^3}{6tv^2} = \frac{5t^{3-1}}{2v^2} = \frac{5t^2}{2v^2}$

3. $\left(2ac^{-2}\right)\left(5a^{-1}c^4\right) = 10a^{1+(-1)}c^{-2+4}$

$= 10a^0c^2 = 10c^2$

4. $\left(\frac{1}{4}\right)^{-2} - \left(\frac{2}{3}\right)^{0} = 4^2 - 1 = 16 - 1 = 15$

5. $\left(3.4 \times 10^5\right)\left(5.0 \times 10^{-2}\right) = 17 \times 10^3$
$= 1.7 \times 10^4$

6. $\frac{6.2 \times 10^3}{3.1 \times 10^5} = 2 \times 10^{3-5} = 2 \times 10^{-2}$

7. $-6a^3 + a^2 - a$

leading coefficient: –6

degree: 3

8. $-b^4 + 5b^2 + 2b$

leading coefficient: –1

degree: 4

9. $3x^4 + 6x^2 - x - 1$

leading coefficient: 3

degree: 4

10. $y^5 - y^2 - 4y + 8$

leading coefficient: 1

degree: 5

11. $-t^2 + 100$

leading coefficient: –1

degree: 2

12. $s^2 - 51$

leading coefficient: 1

degree: 2

13. For example: $3x^5$

14. For example: $2y^4$

15. For example: $x^2 + 2x + 1$

16. For example: $y^3 + y + 6$

17. For example: $6x^4 - x^2$

18. For example: $y^2 - 6$

19. $\left(-4m^2+4m\right)+\left(5m^2+6m\right)$

$=-4m^2+5m^2+4m+6m$

$=m^2+10m$

20. $\left(3n^3+5n\right)+\left(2n^3-2n\right)$

$=3n^3+2n^3+5n+\left(-2n\right)$

$=5n^3+3n$

21. $\left(3x^4-x^3-x^2\right)+\left(3x^3-7x^2+2x\right)$

$=3x^4+\left(-x^3\right)+3x^3+\left(-x^2\right)+\left(-7x^2\right)+2x$

$=3x^4+2x^3-8x^2+2x$

22. $\left(6x^3-2x^2-12\right)+\left(x^2+3x+9\right)$

$=6x^3+\left(-2x^2\right)+x^2+3x+\left(-12\right)+9$

$=6x^3-x^2+3x-3$

23. $\left(\frac{1}{2}w^3+\frac{2}{9}w^2-1.8w\right)+\left(\frac{3}{2}w^3-\frac{1}{9}w^2+2.7w\right)$

$=\frac{1}{2}w^3+\frac{3}{2}w^3+\frac{2}{9}w^2-\frac{1}{9}w^2-1.8w+2.7w$

$=2w^3+\frac{1}{9}w^2+0.9w$

24. $\left(2.9t^4-\frac{7}{8}t+\frac{5}{3}\right)+\left(-8.1t^4-\frac{1}{8}t-\frac{1}{3}\right)$

$=2.9t^4-8.1t^4-\frac{7}{8}t-\frac{1}{8}t+\frac{5}{3}-\frac{1}{3}$

$=-5.2t^4-t+\frac{4}{3}$

25. $\left(9x^2y-5xy+1\right)+\left(8x^2y+xy-15\right)$

$=9x^2y+8x^2y-5xy+xy+1-15$

$=17x^2y-4xy-14$

26. $\left(-x^3y^2+5xy\right)+\left(10x^3y^2+x^2y-10\right)$

$=-x^3y^2+10x^3y^2+x^2y+5xy-10$

$=9x^3y^2+x^2y+5xy-10$

27. $\left(-7a+6a^2+1\right)+\left(-8-4a-2a^2\right)$

$=6a^2-2a^2-7a-4a+1-8$

$=4a^2-11a-7$

28. $\left(1-12p+8p^3\right)+\left(6p^2+p^3-14\right)$

$=8p^3+p^3+6p^2-12p+1-14$

$=9p^3+6p^2-12p-13$

29.
$$\begin{array}{r} 12x^3 \quad +6x-8 \\ +\left(-3x^3-5x^2-4x \quad\right) \\ \hline 9x^3-5x^2+2x-8 \end{array}$$

30.
$$\begin{array}{r} -8y^4-8y^3-6y^2 \quad -9 \\ +\left(4y^4+5y^3 \quad -10y-3\right) \\ \hline -4y^4-3y^3-6y^2-10y-12 \end{array}$$

31. $-\left(-30y^3\right)=30y^3$

32. $-\left(-2x^2\right)=2x^2$

33. $-\left(4p^3+2p-12\right)=-4p^3-2p+12$

34. $-\left(8t^2-4t-3\right)=-8t^2+4t+3$

35. $-\left(-11ab^2+a^2b\right)=11ab^2-a^2b$

36. $-\left(-23rs-4r+9s\right)=23rs+4r-9s$

37.
$$\begin{aligned}&\left(13z^5-z^2\right)-\left(7z^5+5z^2\right)\\&=\left(13z^5-z^2\right)+\left(-7z^5-5z^2\right)\\&=13z^5-7z^5-z^2-5z^2\\&=6z^5-6z^2\end{aligned}$$

38.
$$\begin{aligned}&\left(8w^4+3w^2\right)-\left(12w^4-w^2\right)\\&=\left(8w^4+3w^2\right)+\left(-12w^4+w^2\right)\\&=8w^4-12w^4+3w^2+w^2\\&=-4w^4+4w^2\end{aligned}$$

39.
$$\begin{aligned}&\left(-3x^3+3x^2-x+6\right)-\left(1-x-x^2-x^3\right)\\&=\left(-3x^3+3x^2-x+6\right)+\left(-1+x+x^2+x^3\right)\\&=\left(-3x^3+3x^2-x+6\right)+\left(x^3+x^2+x-1\right)\\&=-3x^3+x^3+3x^2+x^2-x+x+6-1\\&=-2x^3+4x^2+5\end{aligned}$$

40.
$$\begin{aligned}&\left(-8x^3+6x+7\right)-\left(-4-2x-5x^3\right)\\&=\left(-8x^3+6x+7\right)+\left(4+2x+5x^3\right)\\&=\left(-8x^3+6x+7\right)+\left(5x^3+2x+4\right)\\&=-8x^3+5x^3+6x+2x+7+4\\&=-3x^3+8x+11\end{aligned}$$

41.
$$\begin{aligned}&\left(-3xy^3+3x^2y-x+6\right)-\left(-xy^3-xy-x+1\right)\\&=\left(-3xy^3+3x^2y-x+6\right)+\left(xy^3+xy+x-1\right)\\&=-3xy^3+xy^3+3x^2y+xy-x+x+6-1\\&=-2xy^3+3x^2y+xy+5\end{aligned}$$

42.
$$\begin{aligned}&\left(-8x^2y^2+6xy^2+7xy\right)-\left(5xy^2-2xy-4\right)\\&=\left(-8x^2y^2+6xy^2+7xy\right)+\left(-5xy^2+2xy+4\right)\\&=-8x^2y^2+6xy^2-5xy^2+7xy+2xy+4\\&=-8x^2y^2+xy^2+9xy+4\end{aligned}$$

43.
$$\begin{array}{rl}&4t^3-6t^2\quad-18\\-&\left(3t^3+7t^2+9t-5\right)\end{array}\rightarrow\begin{array}{rl}&4t^3-6t^2\quad-18\\+&\left(-3t^3-7t^2-9t+5\right)\\\hline&t^3-13t^2-9t-13\end{array}$$

44.
$$\begin{array}{rl}&5w^3-9w^2+6w+13\\-&\left(7w^3\quad-10w-8\right)\end{array}\rightarrow\begin{array}{rl}&5w^3-9w^2+6w+13\\+&\left(-7w^3\quad+10w+8\right)\\\hline&-2w^3-9w^2+16w+21\end{array}$$

45. $\left(\frac{1}{5}a^2-\frac{1}{2}ab+\frac{1}{10}b^2+3\right)-\left(-\frac{3}{10}a^2+\frac{2}{5}ab-\frac{1}{2}b^2-5\right)$

$$=\left(\frac{1}{5}a^2-\frac{1}{2}ab+\frac{1}{10}b^2+3\right)+\left(\frac{3}{10}a^2-\frac{2}{5}ab+\frac{1}{2}b^2+5\right)$$
$$=\frac{1}{5}a^2+\frac{3}{10}a^2-\frac{1}{2}ab-\frac{2}{5}ab+\frac{1}{10}b^2+\frac{1}{2}b^2+3+5$$
$$=\frac{2}{10}a^2+\frac{3}{10}a^2-\frac{5}{10}ab-\frac{4}{10}ab+\frac{1}{10}b^2+\frac{5}{10}b^2+3+5$$
$$=\frac{1}{2}a^2-\frac{9}{10}ab+\frac{3}{5}b^2+8$$

46. $\left(\frac{4}{7}a^2-\frac{1}{7}ab+\frac{1}{14}b^2-7\right)-\left(\frac{1}{2}a^2-\frac{2}{7}ab-\frac{9}{14}b^2+1\right)$

$$=\left(\frac{4}{7}a^2-\frac{1}{7}ab+\frac{1}{14}b^2-7\right)+\left(-\frac{1}{2}a^2+\frac{2}{7}ab+\frac{9}{14}b^2-1\right)$$
$$=\frac{4}{7}a^2-\frac{1}{2}a^2-\frac{1}{7}ab+\frac{2}{7}ab+\frac{1}{14}b^2+\frac{9}{14}b^2-7-1$$
$$=\frac{8}{14}a^2-\frac{7}{14}a^2-\frac{1}{7}ab+\frac{2}{7}ab+\frac{1}{14}b^2+\frac{9}{14}b^2-7-1$$
$$=\frac{1}{14}a^2+\frac{1}{7}ab+\frac{5}{7}b^2-8$$

47. $\left(8x^2+x-15\right)-\left(9x^2-5x+1\right)$

$$=\left(8x^2+x-15\right)+\left(-9x^2+5x-1\right)$$
$$=8x^2-9x^2+x+5x-15-1$$
$$=-x^2+6x-16$$

48. $\left(10x^3+x^2-10\right)-\left(-x^3+5x\right)$

$$=\left(10x^3+x^2-10\right)+\left(x^3-5x\right)$$
$$=10x^3+x^3+x^2-5x-10$$
$$=11x^3+x^2-5x-10$$

49. $\left(3x^5-2x^3+4\right)-\left(x^4+2x^3-7\right)$

$$=\left(3x^5-2x^3+4\right)+\left(-x^4-2x^3+7\right)$$
$$=3x^5-x^4-2x^3-2x^3+4+7$$
$$=3x^5-x^4-4x^3+11$$

50. $\left(7x^{10}-2x^4-3x\right)-\left(-4x^3-5x^4+x+5\right)$

$$=\left(7x^{10}-2x^4-3x\right)+\left(4x^3+5x^4-x-5\right)$$
$$=7x^{10}-2x^4+5x^4+4x^3-3x-x-5$$
$$=7x^{10}+3x^4+4x^3-4x-5$$

51. $\left(8y^2-4y^3\right)-\left(3y^2-8y^3\right)$

$=\left(8y^2-4y^3\right)+\left(-3y^2+8y^3\right)$

$=-4y^3+8y^3+8y^2-3y^2$

$=4y^3+5y^2$

52. $\left(-9y^2-8\right)-\left(4y^2+3\right)$

$=\left(-9y^2-8\right)+\left(-4y^2-3\right)$

$=-9y^2-4y^2-8-3$

$=-13y^2-11$

53. $\left(-2r-6r^4\right)+\left(-r^4-9r\right)$

$=-6r^4-r^4-2r-9r$

$=-7r^4-11r$

54. $\left(-8s^9+7s^2\right)+\left(7s^9-s^2\right)$

$=-8s^9+7s^9+7s^2-s^2$

$=-s^9+6s^2$

55. $\left(5xy+13x^2+3y\right)-\left(4x^2-8y\right)$

$=\left(5xy+13x^2+3y\right)+\left(-4x^2+8y\right)$

$=13x^2-4x^2+5xy+3y+8y$

$=9x^2+5xy+11y$

56. $\left(6p^2q-2q\right)-\left(-2p^2q+13\right)$

$=\left(6p^2q-2q\right)+\left(2p^2q-13\right)$

$=6p^2q+2p^2q-2q-13$

$=8p^2q-2q-13$

57. $\left(11ab-23b^2\right)+\left(7ab-19b^2\right)$

$=11ab+7ab-23b^2-19b^2$

$=18ab-42b^2$

58. $\left(-4x^2y+9\right)+\left(8x^2y-12\right)$

$=-4x^2y+8x^2y+9-12$

$=4x^2y-3$

59. $\left[2p-\left(3p+5\right)\right]+\left(4p-6\right)+2$

$=\left[2p-3p-5\right]+\left(4p-6\right)+2$

$=-p-5+4p-6+2$

$=-p+4p-5-6+2$

$=3p-9$

60. $-\left(q-2\right)-\left[4-\left(2q-3\right)+5\right]$

$=-q+2-\left[4-2q+3+5\right]$

$=-q+2-4+2q-3-5$

$=-q+2q+2-4-3-5$

$=q-10$

61. $5-\left[2m^2-\left(4m^2+1\right)\right]$

$=5-\left[2m^2-4m^2-1\right]$

$=5-\left[-2m^2-1\right]$

$=5+2m^2+1=2m^2+6$

62. $\left[4n^3-\left(n^3+4\right)\right]+3n^3$

$=\left[4n^3-n^3-4\right]+3n^3$

$=\left[3n^3-4\right]+3n^3$

$=3n^3-4+3n^3=6n^3-4$

63. $\left(6x^3-5\right)-\left(-3x^3+2x\right)-\left(2x^3-6x\right)$

$=6x^3-5+3x^3-2x-2x^3+6x$

$=7x^3+4x-5$

64. $\left(9p^4-2\right)+\left(7p^4+1\right)-\left(8p^4-10\right)$

$=9p^4-2+7p^4+1-8p^4+10$

$=8p^4+9$

65. $\left(-ab+5a^2b\right)-\left[7ab^2-2ab-\left(7a^2b+2ab^2\right)\right]=-ab+5a^2b-\left[7ab^2-2ab-7a^2b-2ab^2\right]$

$=-ab+5a^2b-\left[5ab^2-2ab-7a^2b\right]$

$=-ab+5a^2b-5ab^2+2ab+7a^2b$

$=12a^2b+ab-5ab^2$

66. $\left(m^3n^2+4m^2n\right)-\left[-5m^3n^2-4mn-\left(7m^2n-6mn\right)\right]$

$=m^3n^2+4m^2n-\left[-5m^3n^2-4mn-7m^2n+6mn\right]$

$=m^3n^2+4m^2n-\left[-5m^3n^2-7m^2n+2mn\right]$

$=m^3n^2+4m^2n+5m^3n^2+7m^2n-2mn$

$=6m^3n^2+11m^2n-2mn$

67. $\left(8x^3-x^2+3\right)-\left[5x^2+x-\left(4x^3+x-2\right)\right]$

$=\left(8x^3-x^2+3\right)-\left[5x^2+x-4x^3-x+2\right]$

$=\left(8x^3-x^2+3\right)-\left(-4x^3+5x^2+2\right)$

$=8x^3-x^2+3+4x^3-5x^2-2$

$=8x^3+4x^3-x^2-5x^2+3-2$

$=12x^3-6x^2+1$

68. $\left(y^2+6y-6\right)-\left[\left(2y^3-4y\right)-\left(3y^2+y+1\right)\right]$

$=\left(y^2+6y-6\right)-\left[2y^3-4y-3y^2-y-1\right]$

$=\left(y^2+6y-6\right)-\left(2y^3-3y^2-5y-1\right)$

$=y^2+6y-6-2y^3+3y^2+5y+1$

$=-2y^3+y^2+3y^2+6y+5y-6+1$

$=-2y^3+4y^2+11y-5$

69. $12a^2b-4ab^2-ab \rightarrow \quad 12a^2b-4ab^2-ab$

$\underline{-\left(4a^2b+\ ab^2-5ab\right) \rightarrow +\left(-4a^2b-\ ab^2+5ab\right)}$

$8a^2b-5ab^2+4ab$

70. $2x^2-7xy+3y^2 \rightarrow \quad 2x^2-7xy+3y^2$

$\underline{-\left(9x^2-10xy-\ y^2\right) \rightarrow +\left(-9x^2+10xy+\ y^2\right)}$

$-7x^2+3xy+4y^2$

71. $-5x^4 \quad -11x^2 \quad +6 \rightarrow \quad -5x^4 \quad -11x^2 \quad +6$

$\underline{-\left(-5x^4+3x^3+5x^2-10x+5\right) \rightarrow +\left(5x^4-3x^3-5x^2+10x-5\right)}$

$-3x^3-16x^2+10x+1$

72. $9z^4 \quad +2z^2 \quad +11 \rightarrow \quad 9z^4 \quad +2z^2 \quad +11$

$\underline{-\left(9z^4-4z^3+8z^2-9z-4\right) \rightarrow +\left(-9z^4+4z^3-8z^2+9z+4\right)}$

$4z^3-6z^2+9z+15$

73. $-2.2p^5-9.1p^4 \quad +\ 5.3p^2-7.9p$

$\underline{+\left(\quad -6.4p^4-8.5p^3-10.3p^2 \quad\right)}$

$-2.2p^5-15.5p^4-8.5p^3-\ \ 5p^2-7.9p$

74. $5.5w^4 \quad +4.6w^2-9.3w-8.3$

$\underline{+\left(0.4w^4-7.3w^3 \quad -5.8w+4.6\right)}$

$5.9w^4-7.3w^3+4.6w^2-15.1w-3.7$

75. $P=\left(2x^3+6x\right)+\left(4x^3-5x\right)+\left(6x^3+x\right)$

$=2x^3+6x+4x^3-5x+6x^3+x=12x^3+2x$

76. $P=\left(5x-2\right)+\left(3x-1\right)+\left(2x-6\right)$

$=5x-2+3x-1+2x-6=10x-9$

77. $h\left(x\right)=\frac{2}{3}x^2-5$

It is a polynomial function. The degree is 2.

78. $k\left(x\right)=-7x^4-0.3x+x^3$

It is a polynomial function. The degree is 4.

79. $p(x) = 8x^3 + 2x^2 - \frac{3}{x}$

It is not a polynomial function. The term $-\frac{3}{x} = -3x^{-1}$ and -1 is not a whole number.

80. $q(x) = x^2 - 4x^{-3}$

It is not a polynomial function. In the term $-4x^{-3}$, -3 is not a whole number.

81. $g(x) = -7$

It is a polynomial function. The degree is 0.

82. $g(x) = 4x$

It is a polynomial function. The degree is 1.

83. $M(x) = |x| + 5x$

It is not a polynomial function. The term $|x|$ is not of the form ax^n.

84. $N(x) = x^2 + |x|$

It is not a polynomial function. The term $|x|$ is not of the form ax^n.

85. **a.** $P(x) = -x^4 + 2x - 5$

$P(2) = -(2)^4 + 2(2) - 5$

$= -16 + 4 - 5 = -17$

b. $P(-1) = -(-1)^4 + 2(-1) - 5$

$= -1 - 2 - 5 = -8$

c. $P(0) = -(0)^4 + 2(0) - 5$

$= 0 + 0 - 5 = -5$

d. $P(1) = -(1)^4 + 2(1) - 5$

$= -1 + 2 - 5 = -4$

86. **a.** $N(x) = -x^2 + 5x$

$N(1) = -(1)^2 + 5(1) = -1 + 5 = 4$

b. $N(-1) = -(-1)^2 + 5(-1)$

$= -1 - 5 = -6$

c. $N(2) = -(2)^2 + 5(2)$

$= -4 + 10 = 6$

d. $N(0) = -(0)^2 + 5(0)$

$= 0 + 0 = 0$

87. **a.** $H(x) = \frac{1}{2}x^3 - x + \frac{1}{4}$

$H(0) = \frac{1}{2}(0)^3 - (0) + \frac{1}{4}$

$= 0 - 0 + \frac{1}{4} = \frac{1}{4}$

c. $H(-2) = \frac{1}{2}(-2)^3 - (-2) + \frac{1}{4}$

$= -4 + 2 + \frac{1}{4}$

$= -2 + \frac{1}{4} = -\frac{7}{4}$

b. $H(2)=\frac{1}{2}(2)^3-(2)+\frac{1}{4}$
$=4-2+\frac{1}{4}=2+\frac{1}{4}=\frac{9}{4}$

d. $H(-1)=\frac{1}{2}(-1)^3-(-1)+\frac{1}{4}$
$=-\frac{1}{2}+1+\frac{1}{4}=\frac{3}{4}$

88. a. $K(x)=\frac{2}{3}x^2+\frac{1}{9}$

$K(0)=\frac{2}{3}(0)^2+\frac{1}{9}=0+\frac{1}{9}=\frac{1}{9}$

b. $K(3)=\frac{2}{3}(3)^2+\frac{1}{9}=6+\frac{1}{9}=\frac{55}{9}$

c. $K(-3)=\frac{2}{3}(-3)^2+\frac{1}{9}$
$=6+\frac{1}{9}=\frac{55}{9}$

d. $K(-1)=\frac{2}{3}(-1)^2+\frac{1}{9}=\frac{2}{3}+\frac{1}{9}=\frac{7}{9}$

89. Let x = the width of the garden
$x+3$ = the length of the garden
$P(x)=2x+2(x+3)$
$=2x+2x+6=4x+6$

90. Let x = the width of the room
$2x+4=$ the length of the room
$P(x)=2x+2(2x+4)$
$=2x+4x+8=6x+8$

91. a. $P(x)=R(x)-C(x)$
$=(12x)-(5.40x+99)$
$=12x-5.40x-99$
$=6.6x-99$

b. $P(50)=6.6(50)-99$
$=330-99$
$=231$

The profit will be \$231.

92. a. $P(x)=R(x)-C(x)$
$=(12.99x)-(4.5x+10.1)$
$=12.99x-4.5x-10.1$
$=8.49x-10.1$

b. $P(100)=8.49(100)-10.1$
$=849-10.1$
$=\$838.90$

93. a. $D(x)=5.2x^2+40.4x+1636$

$D(0)=5.2(0)^2+40.4(0)+1636$
$=0+0+1636=1636$

$D(0)=1636$ means that at the beginning of the study, (year 0)the annual dormitory charge was \$1636.

94. a. $P(t)=-0.01t^3+12.96t+10$

$P(0)=-0.01(0)^3+12.96(0)+10$
$=0+0+10=10$

There were 10,000 bacteria initially.

$$D(18) = 5.2(18)^2 + 40.4(18) + 1636$$
$$= 1684.8 + 727.2 + 1636 = 4048$$

In 2008, the annual dormitory charge was \$4048.

b. $$D(25) = 5.2(25)^2 + 40.4(25) + 1636$$
$$= 3250 + 1010 + 1636 = 5896$$

The annual dormitory charge will be \$5896.

$$P(14) = -0.01(14)^3 + 12.96(14) + 10$$
$$= -27.44 + 181.44 + 10 = 164$$

14 hours after the culture was started, 164,000 bacteria were present.

b. $$P(24) = -0.01(24)^3 + 12.96(24) + 10$$
$$= -138.24 + 311.04 + 10 = 182.8$$

24 hours after the culture was started, 182,800 bacteria will be present.

95. a. $W(t) = 143t + 6580$

$$W(0) = 143(0) + 6580 = 6580$$
$$W(5) = 143(5) + 6580$$
$$= 715 + 6580 = 7295$$
$$W(10) = 143(10) + 6580$$
$$= 1430 + 6580 = 8010$$

b. $W(10) = 8010$ means that in Year 10, 8010 thousand (8,010,000) women were due in child support.

96. a. $D(t) = 0.925t + 4.625$

$$D(0) = 0.925(0) + 4.625 = 4.625$$
$$D(4) = 0.925(4) + 4.625$$
$$= 3.7 + 4.625 = 8.325$$
$$D(8) = 0.925(8) + 4.625$$
$$= 7.4 + 4.625 = 12.025$$

b. $D(8) = 12.025$ means that in Year 8, \$12.025 billion (\$12,025,000,000) was due in child support.

97. a. $x(t) = 25t$
$y(t) = -16t^2 + 43.3t$

$$x(0) = 25(0) = 0$$
$$y(0) = -16(0)^2 + 43.3(0) = 0 + 0 = 0$$

(0, 0); at $t = 0$ sec, the position of the rocket is at the origin.

c. $$x(2) = 25(2) = 50$$
$$y(2) = -16(2)^2 + 43.3(2)$$
$$= -64 + 86.6 = 22.6$$

(50, 22.6) At $t = 2$ sec, the position of the rocket is (50, 22.6).

b. $x(1) = 25(1) = 25$

$y(1) = -16(1)^2 + 43.3(1)$

$= -16 + 43.3 = 27.3$

(25, 27.3) At $t = 1$ sec, the position of the rocket is (25, 27.3).

Section 4.3 Practice Exercises

1. **a.** distributive

b. $4x - 7$

c. squares; $a^2 - b^2$

d. perfect; $a^2 + 2ab + b^2$

2. $\left(-4x^2y - 2xy + 3xy^2\right) - \left(2x^2y - 4xy^2\right) + \left(6x^2y + 5xy\right)$

$= -4x^2y - 2xy + 3xy^2 - 2x^2y + 4xy^2 + 6x^2y + 5xy$

$= 3xy + 7xy^2$

3. $\left(-2 - 3x\right) - \left[5 - \left(6x^2 + 4x + 1\right)\right] = -2 - 3x - \left[5 - 6x^2 - 4x - 1\right]$

$= -2 - 3x - \left[-6x^2 - 4x + 4\right]$

$= -2 - 3x + 6x^2 + 4x - 4$

$= 6x^2 + x - 6$

4. **a.** $f(x) = 4x^3 - 5$

$f(3) = 4(3)^3 - 5$

$= 4(27) - 5 = 108 - 5 = 103$

b. $f(0) = 4(0)^3 - 5 = 0 - 5 = -5$

c. $f(-2) = 4(-2)^3 - 5$

$= 4(-8) - 5 = -32 - 5 = -37$

5. **a.** $g(x) = x^4 - x^2 - 3$

$g(-1) = (-1)^4 - (-1)^2 - 3$

$= 1 - 1 - 3$

$= -3$

b. $g(2) = (2)^4 - (2)^2 - 3 = 16 - 4 - 3 = 9$

c. $g(0) = (0)^4 - (0)^2 - 3 = 0 - 0 - 3 = -3$

6. $a(b + c) = ab + ac$

For example: $3(x + 4) = 3x + 12$

7. $\left(7x^4y\right)\left(-6xy^5\right) = 7(-6)\left(x^4 \cdot x\right)\left(y \cdot y^5\right)$

$= -42x^5y^6$

8. $\left(-4a^3b^7\right)\left(-2ab^3\right) = -4(-2)\left(a^3 \cdot a\right)\left(b^7 \cdot b^3\right)$
$= 8a^4b^{10}$

9. $\left(2.2a^6b^4c^7\right)\left(5ab^4c^3\right) = 11a^7b^8c^{10}$

10. $\left(8.5c^4d^5e\right)\left(6cd^2e\right) = 51c^5d^7e^2$

11. $\frac{1}{5}(2a-3) = \frac{1}{5}(2a) + \frac{1}{5}(-3) = \frac{2}{5}a - \frac{3}{5}$

12. $\frac{1}{3}(6b+4) = \frac{1}{3}(6b) + \frac{1}{3}(4) = 2b + \frac{4}{3}$

13. $2m^3n^2\left(m^2n^3 - 3mn^2 + 4n\right)$
$= 2m^3n^2\left(m^2n^3\right) - 2m^3n^2\left(3mn^2\right) + 2m^3n^2(4n)$
$= 2m^5n^5 - 6m^4n^4 + 8m^3n^3$

14. $3p^2q\left(p^3q^3 - pq^2 - 4p\right)$
$= 3p^2q\left(p^3q^3\right) - 3p^2q\left(pq^2\right) - 3p^2q(4p)$
$= 3p^5q^4 - 3p^3q^3 - 12p^3q$

15. $6xy^2\left(\frac{1}{2}x - \frac{2}{3}xy\right) = 6xy^2\left(\frac{1}{2}x\right) - 6xy^2\left(\frac{2}{3}xy\right)$
$= 3x^2y^2 - 4x^2y^3$

16. $12ab\left(\frac{5}{6}a + \frac{1}{4}ab^2\right)$
$= 12ab\left(\frac{5}{6}a\right) + 12ab\left(\frac{1}{4}ab^2\right)$
$= 10a^2b + 3a^2b^3$

17. $(x+y)(x-2y)$
$= x(x) - x(2y) + y(x) - y(2y)$
$= x^2 - 2xy + xy - 2y^2$
$= x^2 - xy - 2y^2$

18. $(3a+5)(a-2)$
$= 3a(a) - 3a(2) + 5(a) - 5(2)$
$= 3a^2 - 6a + 5a - 10$
$= 3a^2 - a - 10$

19. $(6x-1)(5+2x)$
$= 6x(5) + 6x(2x) - 1(5) - 1(2x)$
$= 30x + 12x^2 - 5 - 2x$
$= 12x^2 + 28x - 5$

20. $(7+3x)(x-8)$
$= 7(x) - 7(8) + 3x(x) - 3x(8)$
$= 7x - 56 + 3x^2 - 24x$
$= 3x^2 - 17x - 56$

21. $\left(y^2 - 12\right)\left(2y^2 + 3\right)$
$= y^2\left(2y^2\right) + y^2(3) - 12\left(2y^2\right) - 12(3)$
$= 2y^4 + 3y^2 - 24y^2 - 36$
$= 2y^4 - 21y^2 - 36$

22. $\left(4p^2-1\right)\left(2p^2+5\right)$
$= 4p^2\left(2p^2\right)+4p^2(5)-1\left(2p^2\right)-1(5)$
$= 8p^4+20p^2-2p^2-5$
$= 8p^4+18p^2-5$

23. $(5s+3t)(5s-2t)$
$= 5s(5s)-5s(2t)+3t(5s)-3t(2t)$
$= 25s^2-10st+15st-6t^2$
$= 25s^2+5st-6t^2$

24. $(4a+3b)(4a-b)$
$= 4a(4a)-4a(b)+3b(4a)-3b(b)$
$= 16a^2-4ab+12ab-3b^2$
$= 16a^2+8ab-3b^2$

25. $\left(n^2+10\right)(5n+3)$
$= n^2(5n)+n^2(3)+10(5n)+10(3)$
$= 5n^3+3n^2+50n+30$

26. $\left(m^2+8\right)(3m+7)$
$= m^2(3m)+m^2(7)+8(3m)+8(7)$
$= 3m^3+7m^2+24m+56$

27. $(1.3a-4b)(2.5a+7b)$
$= 1.3a(2.5a)+1.3a(7b)-4b(2.5a)-4b(7b)$
$= 3.25a^2+9.1ab-10ab-28b^2$
$= 3.25a^2-0.9ab-28b^2$

28. $(2.1x-3.5y)(4.7x+2y) = 2.1x(4.7x)+2.1x(2y)-3.5y(4.7x)-3.5y(2y)$
$= 9.87x^2+4.2xy-16.45xy-7y^2$
$= 9.87x^2-12.25xy-7y^2$

29. $(2x+y)\left(3x^2+2xy+y^2\right) = 2x\left(3x^2\right)+2x(2xy)+2x\left(y^2\right)+y\left(3x^2\right)+y(2xy)+y\left(y^2\right)$
$= 6x^3+4x^2y+2xy^2+3x^2y+2xy^2+y^3$
$= 6x^3+7x^2y+4xy^2+y^3$

30. $(h-5k)\left(h^2-2hk+3k^2\right) = h\left(h^2\right)-h(2hk)+h\left(3k^2\right)-5k\left(h^2\right)+5k(2hk)-5k\left(3k^2\right)$
$= h^3-2h^2k+3hk^2-5h^2k+10hk^2-15k^3$
$= h^3-7h^2k+13hk^2-15k^3$

31. $(x-7)\left(x^2+7x+49\right) = x\left(x^2\right)+x(7x)+x(49)-7\left(x^2\right)-7(7x)-7(49)$
$= x^3+7x^2+49x-7x^2-49x-343$
$= x^3-343$

32. $\left(x+3\right)\left(x^2-3x+9\right)=x\left(x^2\right)-x\left(3x\right)+x\left(9\right)+3\left(x^2\right)-3\left(3x\right)+3\left(9\right)$

$=x^3-3x^2+9x+3x^2-9x+27$

$=x^3+27$

33. $\left(4a-b\right)\left(a^3-4a^2b+ab^2-b^3\right)$

$=4a\left(a^3\right)-4a\left(4a^2b\right)+4a\left(ab^2\right)-4a\left(b^3\right)-b\left(a^3\right)+b\left(4a^2b\right)-b\left(ab^2\right)+b\left(b^3\right)$

$=4a^4-16a^3b+4a^2b^2-4ab^3-a^3b+4a^2b^2-ab^3+b^4$

$=4a^4-17a^3b+8a^2b^2-5ab^3+b^4$

34. $\left(3m+2n\right)\left(m^3+2m^2n-mn^2+2n^3\right)$

$=3m\left(m^3\right)+3m\left(2m^2n\right)-3m\left(mn^2\right)+3m\left(2n^3\right)+2n\left(m^3\right)+2n\left(2m^2n\right)-2n\left(mn^2\right)+2n\left(2n^3\right)$

$=3m^4+6m^3n-3m^2n^2+6mn^3+2m^3n+4m^2n^2-2mn^3+4n^4$

$=3m^4+8m^3n+m^2n^2+4mn^3+4n^4$

35. $\left(\frac{1}{2}a-2b+c\right)\left(a+6b-c\right)$

$=\frac{1}{2}a\left(a\right)+\frac{1}{2}a\left(6b\right)-\frac{1}{2}a\left(c\right)-2b\left(a\right)-2b\left(6b\right)+2b\left(c\right)+c\left(a\right)+c\left(6b\right)-c\left(c\right)$

$=\frac{1}{2}a^2+3ab-\frac{1}{2}ac-2ab-12b^2+2bc+ac+6bc-c^2$

$=\frac{1}{2}a^2+ab+\frac{1}{2}ac-12b^2+8bc-c^2$

36. $\left(x+y-2z\right)\left(5x-y+z\right)$

$=x\left(5x\right)-x\left(y\right)+x\left(z\right)+y\left(5x\right)-y\left(y\right)+y\left(z\right)-2z\left(5x\right)+2z\left(y\right)-2z\left(z\right)$

$=5x^2-xy+xz+5xy-y^2+yz-10xz+2yz-2z^2$

$=5x^2+4xy-9xz-y^2+3yz-2z^2$

37. $\left(-x^2+2x+1\right)\left(3x-5\right)=-x^2\left(3x\right)+x^2\left(5\right)+2x\left(3x\right)-2x\left(5\right)+1\left(3x\right)-1\left(5\right)$

$=-3x^3+5x^2+6x^2-10x+3x-5$

$=-3x^3+11x^2-7x-5$

38. $\left(\frac{1}{2}a^2-2ab+b^2\right)(2a+b)=\frac{1}{2}a^2(2a)+\frac{1}{2}a^2(b)-2ab(2a)-2ab(b)+b^2(2a)+b^2(b)$

$=a^3+\frac{1}{2}a^2b-4a^2b-2ab^2+2ab^2+b^3$

$=a^3-\frac{7}{2}a^2b+b^3$

39. $\left(\frac{1}{5}y-10\right)\left(\frac{1}{2}y-15\right)$

$=\frac{1}{5}y\left(\frac{1}{2}y\right)+\frac{1}{5}y(-15)-10\left(\frac{1}{2}y\right)-10(-15)$

$=\frac{1}{10}y^2-3y-5y+150=\frac{1}{10}y^2-8y+150$

40. $\left(\frac{2}{3}x+6\right)\left(\frac{1}{2}x-9\right)$

$=\frac{2}{3}x\left(\frac{1}{2}x\right)+\frac{2}{3}x(-9)+6\left(\frac{1}{2}x\right)+6(-9)$

$=\frac{1}{3}x^2-6x+3x-54\ =\frac{1}{3}x^2-3x-54$

41. $(a-8)(a+8)=a^2-8^2=a^2-64$

42. $(b+2)(b-2)=b^2-2^2=b^2-4$

43. $(3p+1)(3p-1)=(3p)^2-1^2=9p^2-1$

44. $(5q-3)(5q+3)=(5q)^2-3^2=25q^2-9$

45. $\left(x-\frac{1}{3}\right)\left(x+\frac{1}{3}\right)=x^2-\left(\frac{1}{3}\right)^2$

$=x^2-\frac{1}{9}$

46. $\left(\frac{1}{2}x+\frac{1}{3}\right)\left(\frac{1}{2}x-\frac{1}{3}\right)=\left(\frac{1}{2}x\right)^2-\left(\frac{1}{3}\right)^2$

$=\frac{1}{4}x^2-\frac{1}{9}$

47. $(3h-k)(3h+k)=(3h)^2-k^2$

$=9h^2-k^2$

48. $(x-7y)(x+7y)=x^2-(7y)^2$

$=x^2-49y^2$

49. $(3h-k)^2=(3h)^2-2(3h)(k)+k^2$

$=9h^2-6hk+k^2$

50. $(x-7y)^2=x^2-2(x)(7y)+(7y)^2$

$=x^2-14xy+49y^2$

51. $(t-7)^2=t^2-2(t)(7)+7^2$

$=t^2-14t+49$

52. $(w+9)^2=w^2+2(w)(9)+9^2$

$=w^2+18w+81$

53. $(u+3v)^2=u^2+2(u)(3v)+(3v)^2$

$=u^2+6uv+9v^2$

54. $(a-4b)^2=a^2-2(a)(4b)+(4b)^2$

$=a^2-8ab+16b^2$

55. $\left(h+\frac{1}{6}k\right)^2 = h^2+2(h)\left(\frac{1}{6}k\right)+\left(\frac{1}{6}k\right)^2$

$= h^2+\frac{1}{3}hk+\frac{1}{36}k^2$

56. $\left(\frac{2}{5}x+1\right)^2 = \left(\frac{2}{5}x\right)^2+2\left(\frac{2}{5}x\right)(1)+1^2$

$= \frac{4}{25}x^2+\frac{4}{5}x+1$

57. $\left(2z^2-w^3\right)\left(2z^2+w^3\right) = \left(2z^2\right)^2-\left(w^3\right)^2$

$= 4z^4-w^6$

58. $\left(a^4-2b^3\right)\left(a^4+2b^3\right) = \left(a^4\right)^2-\left(2b^3\right)^2$

$= a^8-4b^6$

59. $\left(5x^2-3y\right)^2$

$= \left(5x^2\right)^2-2\left(5x^2\right)(3y)+(3y)^2$

$= 25x^4-30x^2y+9y^2$

60. $t^6+1 = \left(t^2\right)^3+1^3$

$= \left(t^2+1\right)\left[\left(t^2\right)^2-\left(t^2\right)(1)+1^2\right]$

$= \left(t^2+1\right)\left(t^4-t^2+1\right)$

61. a. When two conjugates are multiplied, the resulting binomial is a difference of squares.

$(-5x+4)(5x+4)$

$= -25x^2-20x+20x+16$

$= 16-25x^2$

Since $(-5x+4)(5x+4) = 16-25x^2$ is a difference of squares, the binomials are conjugates.

b. When two conjugates are multiplied, the resulting binomial is a difference of squares.

$(-5x+4)(5x-4)$

$= -25x^2+20x+20x+16$

$= -25x^2+40x+16$

Since $(-5x+4)(5x-4) = -25x^2+40x+16$ is not a difference of squares, the binomials are not conjugates.

62. a. When two conjugates are multiplied, the resulting binomial is a difference of squares.

$(-3-7x)(3+7x)$

$= -9-21x-21x-49x^2$

$= -49x^2-42x-9$

Since $(-3-7x)(3+7x) = -49x^2-42x-9$ is not a difference of squares, the binomials are not conjugates.

b. When two conjugates are multiplied, the resulting binomial is a difference of squares.

$(-3+7x)(3+7x)$

$= -9-21x+21x+49x^2$

$= 49x^2-9$

Since $(-3+7x)(3+7x) = 49x^2-9$ is a difference of squares, the binomials are conjugates.

63. **a.** $(A-B)(A+B)=A^2-B^2$

b. $\big[(x+y)-B\big]\big[(x+y)+B\big]$
$=(x+y)^2-B^2$
$=x^2+2xy+y^2-B^2$

Both are examples of multiplying conjugates to get a difference of squares.

64. **a.** $(A+B)(A-B)=A^2-B^2$

b. $\big[A+(3h+k)\big]\big[A-(3h+k)\big]$
$=A^2-(3h+k)^2$
$=A^2-\left(9h^2+6hk+k^2\right)$
$=A^2-9h^2-6hk-k^2$

Both are examples of multiplying conjugates to get a difference of squares.

65. $\big[(w+v)-2\big]\big[(w+v)+2\big]=(w+v)^2-2^2$
$=w^2+2wv+v^2-4$

66. $\big[(x+y)-6\big]\big[(x+y)+6\big]=(x+y)^2-6^2$
$=x^2+2xy+y^2-36$

67. $\big[2-(x+y)\big]\big[2+(x+y)\big]=2^2-(x+y)^2$
$=4-\left(x^2+2xy+y^2\right)$
$=4-x^2-2xy-y^2$

68. $\big[a-(b+1)\big]\big[a+(b+1)\big]=a^2-(b+1)^2$
$=a^2-\left(b^2+2b+1\right)$
$=a^2-b^2-2b-1$

69. $\big[(3a-4)+b\big]\big[(3a-4)-b\big]$
$=(3a-4)^2-b^2$
$=(3a)^2-2(3a)(4)+4^2-b^2$
$=9a^2-24a+16-b^2$

70. $\big[(5p-7)-q\big]\big[(5p-7)+q\big]$
$=(5p-7)^2-q^2$
$=(5p)^2-2(5p)(7)+7^2-q^2$
$=25p^2-70p+49-q^2$

71. Write $(x+y)^3$ as $(x+y)^2(x+y)$. Square the binomial and then use the distributive property to multiply the resulting trinomial by the remaining factor of $x+y$.

72. Write $(a-b)^3$ as $(a-b)^2(a-b)$. Square the binomial and then use the distributive property to multiply the resulting trinomial by the remaining factor of $a-b$.

73. $\begin{aligned}(2x+y)^3 &= (2x+y)^2(2x+y)\\ &= (4x^2+4xy+y^2)(2x+y)\\ &= 4x^2(2x)+4x^2(y)+4xy(2x)+4xy(y)+y^2(2x)+y^2(y)\\ &= 8x^3+4x^2y+8x^2y+4xy^2+2xy^2+y^3\\ &= 8x^3+12x^2y+6xy^2+y^3\end{aligned}$

74. $\begin{aligned}(x-5y)^3 &= (x-5y)^2(x-5y)\\ &= (x^2-10xy+25y^2)(x-5y)\\ &= x^2(x)-x^2(5y)-10xy(x)+10xy(5y)+25y^2(x)-25y^2(5y)\\ &= x^3-5x^2y-10x^2y+50xy^2+25xy^2-125y^3\\ &= x^3-15x^2y+75xy^2-125y^3\end{aligned}$

75. $\begin{aligned}(4a-b)^3 &= (4a-b)^2(4a-b)\\ &= (16a^2-8ab+b^2)(4a-b)\\ &= 16a^2(4a)-16a^2(b)-8ab(4a)+8ab(b)+b^2(4a)-b^2(b)\\ &= 64a^3-16a^2b-32a^2b+8ab^2+4ab^2-b^3\\ &= 64a^3-48a^2b+12ab^2-b^3\end{aligned}$

76. $\begin{aligned}(3a+4b)^3 &= (3a+4b)^2(3a+4b)\\ &= (9a^2+24ab+16b^2)(3a+4b)\\ &= 9a^2(3a)+9a^2(4b)+24ab(3a)+24ab(4b)+16b^2(3a)+16b^2(4b)\\ &= 27a^3+36a^2b+72a^2b+96ab^2+48ab^2+64b^3\\ &= 27a^3+108a^2b+144ab^2+64b^3\end{aligned}$

77. Multiply the first two binomials and simplify. Then multiply the resulting trinomial and the third binomial, using the distributive property.

78. Multiply the first two binomials whose product simplifies to a^2-b^2. Multiply the last two binomials whose product simplifies to $4a^2-b^2$. Then multiply $(a^2-b^2)(4a^2-b^2)$.

79. $2a^2(a+5)(3a+1)$

$= 2a^2\left[a(3a)+a(1)+5(3a)+5(1)\right]$

$= 2a^2\left[3a^2+a+15a+5\right]$

$= 2a^2\left(3a^2+16a+5\right)$

$= 2a^2\left(3a^2\right)+2a^2(16a)+2a^2(5)$

$= 6a^4+32a^3+10a^2$

80. $-5y(2y-3)(y+3)$

$= -5y\left[2y(y)+2y(3)-3(y)-3(3)\right]$

$= -5y\left[2y^2+6y-3y-9\right]$

$= -5y\left(2y^2+3y-9\right)$

$= -5y\left(2y^2\right)-5y(3y)+5y(9)$

$= -10y^3-15y^2+45y$

81. $(x+3)(x-3)(x+5)$

$= \left(x^2-9\right)(x+5)$

$= x^2(x)+x^2(5)-9(x)-9(5)$

$= x^3+5x^2-9x-45$

82. $(t+2)(t-3)(t+1)$

$= \left[t(t)-t(3)+2(t)-2(3)\right](t+1)$

$= \left[t^2-3t+2t-6\right](t+1)$

$= \left(t^2-t-6\right)(t+1)$

$= t^2(t)+t^2(1)-t(t)-t(1)-6(t)-6(1)$

$= t^3+t^2-t^2-t-6t-6$

$= t^3-7t-6$

83. $128p^6+54q^3 = 2\left(64p^6+27q^3\right)$

$= 2\left[\left(4p^2\right)^3+(3q)^3\right]$

$= 2\left(4p^2+3q\right)\left(16p^4-12p^2q+9q^2\right)$

84. $(p+10)^2-4(p+6)^2$

$= p^2+20p+100-4\left(p^2+12p+36\right)$

$= p^2+20p+100-4p^2-48p-144$

$= -3p^2-28p-44$

85. $(y+1)^2-(2y+3)^2$

$= \left(y^2+2y+1\right)-\left(4y^2+12y+9\right)$

$= y^2+2y+1-4y^2-12y-9$

$= -3y^2-10y-8$

86. $125y^3-8$

$= (5y)^3-2^3$

$= (5y-2)\left(25y^2+10y+4\right)$

87. $(r+t)^2$

88. a^2+b^3

89. x^2-y^3

90. $(3a)^2$

91. The sum of the cube of p and the square of q.

92. The difference of the cube of a and the cube of b.

93. The product of x and the square of y.

94. The cube of the sum of c and d.

95. Let x = the width of the walk

$2x + 20$ = length of garden and walk

$2x + 15$ = width of garden and walk

$$\begin{aligned} A(x) &= (2x+20)(2x+15) \\ &= 2x(2x)+2x(15)+20(2x)+20(15) \\ &= 4x^2+30x+40x+300 \\ &= 4x^2+70x+300 \end{aligned}$$

96. Let x = the width of the frame

$2x + 10$ = length of photograph and frame

$2x + 8$ = width of photograph and frame

80 = area of photograph

(area frame) = (area both) – (area photograph)

$$\begin{aligned} A(x) &= (2x+10)(2x+8)-80 \\ &= 2x(2x)+2x(8)+10(2x)+10(8)-80 \\ &= 4x^2+16x+20x+80-80 \\ &= 4x^2+36x \end{aligned}$$

97. a. Let x = the length of a side of the square

$8 – 2x$ = length and width of base

x = the height of the box

$$\begin{aligned} V(x) &= (8-2x)(8-2x)x \\ &= (64-32x+4x^2)x \\ &= 4x^3-32x^2+64x \end{aligned}$$

b. $$\begin{aligned} V(1) &= 4(1)^3-32(1)^2+64(1) \\ &= 4-32+64 \\ &= 36 \text{ in}^3 \end{aligned}$$

98. a. Let x = the length of a side of the square

$12 – 2x$ = the length of the base

$9 – 2x$ = the width of the base

x = the height of the box

$$\begin{aligned} V(x) &= (12-2x)(9-2x)x \\ &= (108-42x+4x^2)x \\ &= 4x^3-42x^2+108x \end{aligned}$$

b. $$\begin{aligned} V(2) &= 4(2)^3-42(2)^2+108(2) \\ &= 32-168+216 \\ &= 80 \text{ in}^3 \end{aligned}$$

99. $$\begin{aligned} (x-2)^2 &= x^2-2(x)(2)+2^2 \\ &= x^2-4x+4 \end{aligned}$$

100. $(x+3)^2 = x^2+2(x)(3)+3^2 = x^2+6x+9$

101. $(x-2)(x+2) = x^2-2^2 = x^2-4$

102. $(2x+3)(2x-3) = (2x)^2-3^2 = 4x^2-9$

103. $\frac{1}{2}(2x-6)(x+3)=(x-3)(x+3)$
$=x^2-3^2=x^2-9$

104. $\frac{1}{2}(4x)(x-1)=2x(x-1)$
$=2x(x)-2x(1)$
$=2x^2-2x$

105. $x(3x)(3x+10)=3x^2(3x+10)$
$=3x^2(3x)+3x^2(10)$
$=9x^3+30x^2$

106. $2x(x+7)(x+3)=(2x^2+14x)(x+3)$
$=2x^2(x)+2x^2(3)+14x(x)+14x(3)$
$=2x^3+6x^2+14x^2+42x$
$=2x^3+20x^2+42x$

107. $\frac{\left[(x+h)^2-3(x+h)-5\right]-\left(x^2-3x-5\right)}{h}$
$=\frac{x^2+2xh+h^2-3x-3h-5-x^2+3x+5}{h}$
$=\frac{x^2-x^2+2xh+h^2-3x+3x-3h-5+5}{h}$
$=\frac{2xh+h^2-3h}{h}$
$=\frac{h(2x+h-3)}{h}$
$=2x+h-3$

108. $\frac{\left[(x+h)^2-4(x+h)+2\right]-\left(x^2-4x+2\right)}{h}$
$=\frac{x^2+2xh+h^2-4x-4h+2-x^2+4x-2}{h}$
$=\frac{x^2-x^2+2xh+h^2-4x+4x-4h+2-2}{h}$
$=\frac{2xh+h^2-4h}{h}$
$=\frac{h(2x+h-4)}{h}$
$=2x+h-4$

109. Multiply $(x+2)^2(x+2)^2$ by squaring the binomials. Then multiply the resulting trinomials using the distributive property.

110. Multiply $(y-3)^2(y-3)^2$ by squaring the binomials. Then multiply the resulting trinomials using the distributive property.

111. $(5x-6)$
Check:
$(2x-3)(5x-6)$
$=2x(5x)-2x(6)-3(5x)+3(6)$
$=10x^2-12x-15x+18$
$=10x^2-27x+18$

112. $(3x-2)$
Check:
$(4x+1)(3x-2)$
$=4x(3x)-4x(2)+1(3x)-1(2)$
$=12x^2-8x+3x-2$
$=12x^2-5x-2$

113. $(2y-1)$

Check:

$$\begin{aligned}&(4y+3)(2y-1)\\&=4y(2y)-4y(1)+3(2y)-3(1)\\&=8y^2-4y+6y-3\\&=8y^2+2y-3\end{aligned}$$

114. $(y-5)$

Check:

$$\begin{aligned}&(3y-2)(y-5)\\&=3y(y)-3y(5)-2(y)+2(5)\\&=3y^2-15y-2y+10\\&=3y^2-17y+10\end{aligned}$$

Section 4.4 Practice Exercises

1. **a.** division; quotient; remainder

b. Synthetic

2. **a.** $(3x+1)+(2x-5)=3x+2x+1-5=5x-4$

b.
$$\begin{aligned}&(3x+1)(2x-5)\\&=3x(2x)-3x(5)+1(2x)-1(5)\\&=6x^2-15x+2x-5\\&=6x^2-13x-5\end{aligned}$$

3. **a.** $(a-10b)-(5a+b)=a-10b-5a-b=-4a-11b$

b.
$$\begin{aligned}&(a-10b)(5a+b)\\&=a(5a)+a(b)-10b(5a)-10b(b)\\&=5a^2+ab-50ab-10b^2\\&=5a^2-49ab-10b^2\end{aligned}$$

4. **a.**
$$\begin{aligned}&\left(2y^2+1\right)-\left(y^2-5y+1\right)\\&=2y^2+1-y^2+5y-1\\&=y^2+5y\end{aligned}$$

b.
$$\begin{aligned}&\left(2y^2+1\right)\left(y^2-5y+1\right)\\&=2y^2\left(y^2\right)-2y^2(5y)+2y^2(1)\\&\quad+1\left(y^2\right)-1(5y)+1(1)\\&=2y^4-10y^3+2y^2+y^2-5y+1\\&=2y^4-10y^3+3y^2-5y+1\end{aligned}$$

5. **a.**
$$\begin{aligned}&\left(x^2-x\right)+\left(6x^2+x+2\right)\\&=x^2+6x^2-x+x+2\\&=7x^2+2\end{aligned}$$

b.
$$\begin{aligned}&\left(x^2-x\right)\left(6x^2+x+2\right)\\&=x^2\left(6x^2\right)+x^2(x)+x^2(2)\\&\quad-x\left(6x^2\right)-x(x)-x(2)\\&=6x^4+x^3+2x^2-6x^3-x^2-2x\\&=6x^4-5x^3+x^2-2x\end{aligned}$$

6. For example:

$$\begin{aligned}(2y+3)(y-5) &= 2y(y)-2y(5)+3(y)-3(5)\\ &= 2y^2-10y+3y-15\\ &= 2y^2-7y-15\end{aligned}$$

7. For example:

$$\begin{aligned}(5y+1)^2 &= (5y)^2+2(5y)(1)+1^2\\ &= 25y^2+10y+1\end{aligned}$$

8. For example:

$$(6p-1)(6p+1) = (6p)^2-1^2 = 36p^2-1$$

9. $$\begin{aligned}\frac{16t^4-4t^2+20t}{-4t} &= \frac{16t^4}{-4t}-\frac{4t^2}{-4t}+\frac{20t}{-4t}\\ &= -4t^3+t-5\end{aligned}$$

10. $$\begin{aligned}\frac{2x^3+8x^2-2x}{-2x} &= \frac{2x^3}{-2x}+\frac{8x^2}{-2x}-\frac{2x}{-2x}\\ &= -x^2-4x+1\end{aligned}$$

11. $$\begin{aligned}&\left(36y+24y^2+6y^3\right)\div(3y)\\ &= \frac{36y}{3y}+\frac{24y^2}{3y}+\frac{6y^3}{3y}\\ &= 12+8y+2y^2\end{aligned}$$

12. $$\begin{aligned}&\left(6p^2-18p^4+30p^5\right)\div(6p)\\ &= \frac{6p^2}{6p}-\frac{18p^4}{6p}+\frac{30p^5}{6p} = p-3p^3+5p^4\end{aligned}$$

13. $$\begin{aligned}&\left(4x^3y+12x^2y^2-4xy^3\right)\div(4xy)\\ &= \frac{4x^3y}{4xy}+\frac{12x^2y^2}{4xy}-\frac{4xy^3}{4xy} = x^2+3xy-y^2\end{aligned}$$

14. $$\begin{aligned}&\left(25m^5n-10m^4n+m^3n\right)\div\left(5m^3n\right)\\ &= \frac{25m^5n}{5m^3n}-\frac{10m^4n}{5m^3n}+\frac{m^3n}{5m^3n}\\ &= 5m^2-2m+\frac{1}{5}\end{aligned}$$

15. $$\begin{aligned}&\left(-8y^4-12y^3+32y^2\right)\div\left(-4y^2\right)\\ &= \frac{-8y^4}{-4y^2}-\frac{12y^3}{-4y^2}+\frac{32y^2}{-4y^2}\\ &= 2y^2+3y-8\end{aligned}$$

16. $$\begin{aligned}&\left(12y^5-8y^6+16y^4-10y^3\right)\div\left(2y^3\right)\\ &= \frac{12y^5}{2y^3}-\frac{8y^6}{2y^3}+\frac{16y^4}{2y^3}-\frac{10y^3}{2y^3}\\ &= 6y^2-4y^3+8y-5\end{aligned}$$

17. $$\begin{aligned}&\left(3p^4-6p^3+2p^2-p\right)\div(-6p)\\ &= \frac{3p^4}{-6p}-\frac{6p^3}{-6p}+\frac{2p^2}{-6p}-\frac{p}{-6p}\\ &= -\frac{1}{2}p^3+p^2-\frac{1}{3}p+\frac{1}{6}\end{aligned}$$

18. $\left(-4q^3+8q^2-q\right)\div\left(-12q\right)$

$$=\frac{-4q^3}{-12q}+\frac{8q^2}{-12q}-\frac{q}{-12q}$$
$$=\frac{1}{3}q^2-\frac{2}{3}q+\frac{1}{12}$$

19. $\left(a^3+5a^2+a-5\right)\div\left(a\right)$

$$=\frac{a^3}{a}+\frac{5a^2}{a}+\frac{a}{a}-\frac{5}{a}$$
$$=a^2+5a+1-\frac{5}{a}$$

20. $\left(2m^5-3m^4+m^3-m^2+9m\right)\div\left(m^2\right)$

$$=\frac{2m^5}{m^2}-\frac{3m^4}{m^2}+\frac{m^3}{m^2}-\frac{m^2}{m^2}+\frac{9m}{m^2}$$
$$=2m^3-3m^2+m-1+\frac{9}{m}$$

21. $\dfrac{6s^3t^5-8s^2t^4+10st^2}{-2st^4}$

$$=\frac{6s^3t^5}{-2st^4}-\frac{8s^2t^4}{-2st^4}+\frac{10st^2}{-2st^4}$$
$$=-3s^2t+4s-\frac{5}{t^2}$$

22. $\dfrac{-8r^4w^2-4r^3w+2w^3}{-4r^3w}$

$$=\frac{-8r^4w^2}{-4r^3w}-\frac{4r^3w}{-4r^3w}+\frac{2w^3}{-4r^3w}$$
$$=2rw+1-\frac{w^2}{2r^3}$$

23. $\left(8p^4q^7-9p^5q^6-11p^3q-4\right)\div\left(p^2q\right)$

$$=\frac{8p^4q^7}{p^2q}-\frac{9p^5q^6}{p^2q}-\frac{11p^3q}{p^2q}-\frac{4}{p^2q}$$
$$=8p^2q^6-9p^3q^5-11p-\frac{4}{p^2q}$$

24. $\left(20a^5b^5-20a^3b^2+5a^2b+6\right)\div\left(a^2b\right)=\dfrac{20a^5b^5}{a^2b}-\dfrac{20a^3b^2}{a^2b}+\dfrac{5a^2b}{a^2b}+\dfrac{6}{a^2b}$

$$=20a^3b^4-20ab+5+\frac{6}{a^2b}$$

25. a.

$$\begin{array}{r} 2x^2-3x-1 \\ x-2\overline{\left)\begin{array}{r} 2x^3-7x^2+5x-1 \\ \underline{-\left(2x^3-4x^2\right)} \\ -3x^2+5x \\ \underline{-\left(-3x^2+6x\right)} \\ -x-1 \\ \underline{-\left(-x+2\right)} \\ -3 \end{array}\right.} \end{array}$$

b. Multiply the quotient and divisor; then add the remainder. The result should equal the dividend.

Divisor: $(x-2)$ Quotient:

$(2x^2-3x-1)$

Remainder: (–3)

26. **a.**

$$x+3\overline{)\,x^3+4x^2+7x-3}$$

$$\underline{-\left(x^3+3x^2\right)}$$

$$x^2+7x$$

$$\underline{-\left(x^2+3x\right)}$$

$$4x-3$$

$$\underline{-\left(4x+12\right)}$$

$$-15$$

Divisor: $(x+3)$ Quotient:

(x^2+x+4)

Remainder: (–15)

b. Multiply the quotient and divisor; then add the remainder. The result should equal the dividend.

27.

$$x+4\overline{)\,x^2+11x+19}$$

$$\underline{-\left(x^2+4x\right)}$$

$$7x+19$$

$$\underline{-\left(7x+28\right)}$$

$$-9$$

Solution: $x+7-\dfrac{9}{x+4}$

Check:

$$\left(x+4\right)\left(x+7\right)+\left(-9\right)=x^2+11x+28-9$$

$$=x^2+11x+19$$

28.

$$x+2\overline{)\,x^3-7x^2-13x+3}$$

$$\underline{-\left(x^3+2x^2\right)}$$

$$-9x^2-13x$$

$$\underline{-\left(-9x^2-18x\right)}$$

$$5x+3$$

$$\underline{-\left(5x+10\right)}$$

$$-7$$

Solution: $x^2-9x+5+\dfrac{-7}{x+2}$

Check:

29.

$$y-3\overline{)\,3y^3-7y^2-4y+3}$$

$$\underline{-\left(3y^3-9y^2\right)}$$

$$2y^2-4y$$

$$\underline{-\left(2y^2-6y\right)}$$

$$2y+3$$

$$\underline{-\left(2y-6\right)}$$

$$9$$

Solution: $3y^2+2y+2+\dfrac{9}{y-3}$

Check:

$$\begin{aligned}&(x+2)\left(x^2-9x+5\right)+(-7)\\&\quad=x^3-9x^2+5x+2x^2-18x+10-7\\&\quad=x^3-7x^2-13x+3\end{aligned}$$

$$\begin{aligned}&(y-3)\left(3y^2+2y+2\right)+(9)\\&\quad=3y^3+2y^2+2y-9y^2-6y-6+9\\&\quad=3y^3-7y^2-4y+3\end{aligned}$$

30.

$$\begin{array}{r} z^2+2z+10 \\ z-4\overline{)\;z^3-2z^2+2z-5} \\ \underline{-\left(z^3-4z^2\right)} \\ 2z^2+2z \\ \underline{-\left(2z^2-8z\right)} \\ 10z-5 \\ \underline{-(10z-40)} \\ 35 \end{array}$$

Solution: $z^2+2z+10+\dfrac{35}{z-4}$

Check:

$$\begin{aligned}&(z-4)\left(z^2+2z+10\right)+(35)\\&\quad=z^3+2z^2+10z-4z^2-8z-40+35\\&\quad=z^3-2z^2+2z-5\end{aligned}$$

31.

$$\begin{array}{r} -4a+11 \\ 3a-11\overline{)\;-12a^2+77a-121} \\ \underline{-\left(-12a^2+44a\right)} \\ 33a-121 \\ \underline{-(33a-121)} \\ 0 \end{array}$$

Solution: $-4a+11$

Check:

$$\begin{aligned}&(3a-11)(-4a+11)+(0)\\&\quad=-12a^2+33a+44a-121\\&\quad=-12a^2+77a-121\end{aligned}$$

32.

$$\begin{array}{r} 7x-2 \\ 4x-3\overline{)\;28x^2-29x+6} \\ \underline{-\left(28x^2-21x\right)} \\ -8x+6 \\ \underline{-(-8x+6)} \\ 0 \end{array}$$

Solution: $7x-2$

Check:

$$\begin{aligned}&(4x-3)(7x-2)+(0)\\&\quad=28x^2-8x-21x+6\\&\quad=28x^2-29x+6\end{aligned}$$

33.

$$\begin{array}{r} 6y-5 \\ 3y+4\overline{)\ 18y^2+9y-20} \\ \underline{-\left(18y^2+24y\right)} \\ -15y-20 \\ \underline{-\left(-15y-20\right)} \\ 0 \end{array}$$

Solution: $6y-5$

Check:

$$\begin{aligned}&\left(3y+4\right)\left(6y-5\right)+\left(0\right)\\ &=18y^2-15y+24y-20\\ &=18y^2+9y-20\end{aligned}$$

34.

$$\begin{array}{r} 3y+1 \\ y-1\overline{)\ 3y^2-2y-1} \\ \underline{-\left(3y^2-3y\right)} \\ y-1 \\ \underline{-\left(y-1\right)} \\ 0 \end{array}$$

Solution: $3y+1$

Check:

$$\begin{aligned}&\left(y-1\right)\left(3y+1\right)+\left(0\right)\\ &=3y^2+y-3y-1\\ &=3y^2-2y-1\end{aligned}$$

35.

$$\begin{array}{r} 6x^2+4x+5 \\ 3x-2\overline{)\ 18x^3\quad +7x+12} \\ \underline{-\left(18x^3-12x^2\right)} \\ 12x^2+7x \\ \underline{-\left(12x^2-8x\right)} \\ 15x+12 \\ \underline{-\left(15x-10\right)} \\ 22 \end{array}$$

Solution: $6x^2+4x+5+\dfrac{22}{3x-2}$

Check:

$$\begin{aligned}&\left(3x-2\right)\left(6x^2+4x+5\right)+\left(22\right)\\ &=18x^3+12x^2+15x-12x^2-8x-10+22\\ &=18x^3+7x+12\end{aligned}$$

36.

$$\begin{array}{r} 4x^2+2x-2 \\ 2x-1\overline{)\ 8x^3\quad -6x+22} \\ \underline{-\left(8x^3-4x^2\right)} \\ 4x^2-6x \\ \underline{-\left(4x^2-2x\right)} \\ -4x+22 \\ \underline{-\left(-4x+2\right)} \\ 20 \end{array}$$

Solution: $4x^2+2x-2+\dfrac{20}{2x-1}$

Check:

$$\begin{aligned}&\left(2x-1\right)\left(4x^2+2x-2\right)+\left(20\right)\\ &=8x^3+4x^2-4x-4x^2-2x+2+20\\ &=8x^3-6x+22\end{aligned}$$

37.

$$\begin{array}{r} 4a^2-2a+1 \\ 2a+1\overline{\left)\begin{array}{r} 8a^3 \qquad +1 \\ -\left(8a^3+4a^2\right) \\ \hline -4a^2 \\ -\left(-4a^2-2a\right) \\ \hline 2a+1 \\ -\left(2a+1\right) \\ \hline 0 \end{array}\right.} \end{array}$$

Solution: $4a^2-2a+1$

Check:

$$\begin{aligned} &\left(2a+1\right)\left(4a^2-2a+1\right)+\left(0\right) \\ &\quad =8a^3-4a^2+2a+4a^2-2a+1 \\ &\quad =8a^3+1 \end{aligned}$$

38.

$$\begin{array}{r} 27x^3-9x^2+3x-1 \\ 3x+1\overline{\left)\begin{array}{r} 81x^4 \qquad\qquad -1 \\ -\left(81x^4+27x^3\right) \\ \hline -27x^3 \\ -\left(-27x^3-9x^2\right) \\ \hline 9x^2 \\ -\left(9x^2+3x\right) \\ \hline -3x-1 \\ -\left(-3x-1\right) \\ \hline 0 \end{array}\right.} \end{array}$$

Solution: $27x^3-9x^2+3x-1$

Check:

$$\begin{aligned} &\left(3x+1\right)\left(27x^3-9x^2+3x-1\right)+\left(0\right) \\ &\quad =81x^4-27x^3+9x^2-3x+27x^3 \\ &\qquad\qquad -9x^2+3x-1 \\ &\quad =81x^4-1 \end{aligned}$$

39.

$$\begin{array}{r} x^2-2x+2 \\ x^2+x-1\overline{\left)\begin{array}{r} x^4-x^3-x^2+4x-2 \\ -\left(x^4+x^3-x^2\right) \\ \hline -2x^3 \qquad +4x \\ -\left(-2x^3-2x^2+2x\right) \\ \hline 2x^2+2x-2 \\ -\left(2x^2+2x-2\right) \\ \hline 0 \end{array}\right.} \end{array}$$

Solution: x^2-2x+2

Check:

40.

$$\begin{array}{r} a^3-a^2+2a-5 \\ 2a^2-5a+2\overline{\left)\begin{array}{r} 2a^5-7a^4+11a^3-22a^2+29a-10 \\ -\left(2a^5-5a^4+2a^3\right) \\ \hline -2a^4+9a^3-22a^2 \\ -\left(-2a^4+5a^3-2a^2\right) \\ \hline 4a^3-20a^2+29a \\ -\left(4a^3-10a^2+4a\right) \\ \hline -10a^2+25a-10 \\ -\left(-10a^2+25a-10\right) \\ \hline 0 \end{array}\right.} \end{array}$$

Solution: a^3-a^2+2a-5

Check:

$$\begin{aligned}&\left(x^2+x-1\right)\left(x^2-2x+2\right)+(0)\\&\quad=x^4-2x^3+2x^2+x^3-2x^2+2x\\&\qquad\qquad -x^2+2x-2\\&\quad=x^4-x^3-x^2+4x-2\end{aligned}$$

$$\begin{aligned}&\left(2a^2-5a+2\right)\left(a^3-a^2+2a-5\right)+(0)\\&\quad=2a^5-2a^4+4a^3-10a^2-5a^4+5a^3\\&\qquad -10a^2+25a+2a^3-2a^2+4a-10\\&\quad=2a^5-7a^4+11a^3-22a^2+29a-10\end{aligned}$$

41.

$$\begin{array}{r}x^2+2x+5\\ x^2-5\overline{\left)\begin{array}{r}x^4+2x^3\qquad -10x-25\\ -\left(x^4\qquad -5x^2\right)\\ \hline 2x^3+5x^2-10x\\ -\left(2x^3\qquad -10x\right)\\ \hline 5x^2\qquad -25\\ -\left(5x^2\qquad -25\right)\\ \hline 0\end{array}\right.}\end{array}$$

Solution: x^2+2x+5

Check:

$$\begin{aligned}&\left(x^2-5\right)\left(x^2+2x+5\right)+(0)\\&\quad=x^4+2x^3+5x^2-5x^2-10x-25\\&\quad=x^4+2x^3-10x-25\end{aligned}$$

42.

$$\begin{array}{r}x^2-5x-2\\ x^2+2\overline{\left)\begin{array}{r}x^4-5x^3\qquad -10x-4\\ -\left(x^4\qquad +2x^2\right)\\ \hline -5x^3-2x^2-10x\\ -\left(-5x^3\qquad -10x\right)\\ \hline -2x^2\qquad -4\\ -\left(-2x^2\qquad -4\right)\\ \hline 0\end{array}\right.}\end{array}$$

Solution: x^2-5x-2

Check:

$$\begin{aligned}&\left(x^2+2\right)\left(x^2-5x-2\right)+(0)\\&\quad=x^4-5x^3-2x^2+2x^2-10x-4\\&\quad=x^4-5x^3-10x-4\end{aligned}$$

43.

$$\begin{array}{r}x^2-1\\ x^2-2\overline{\left)\begin{array}{r}x^4-3x^2+10\\ -\left(x^4-2x^2\right)\\ \hline -x^2+10\\ -\left(-x^2+2\right)\\ \hline 8\end{array}\right.}\end{array}$$

Solution: $x^2-1+\dfrac{8}{x^2-2}$

44.

$$\begin{array}{r}3y^2-16\\ y^2-3\overline{\left)\begin{array}{r}3y^4-25y^2-18\\ -\left(3y^4-9y^2\right)\\ \hline -16y^2-18\\ -\left(-16y^2+48\right)\\ \hline -66\end{array}\right.}\end{array}$$

Solution: $3y^2-16+\dfrac{-66}{y^2-3}$

Check:

$$\begin{aligned}&\left(x^2-2\right)\left(x^2-1\right)+(8)\\&\quad=x^4-x^2-2x^2+2+8\\&\quad=x^4-3x^2+10\end{aligned}$$

Check:

$$\begin{aligned}&\left(y^2-3\right)\left(3y^2-16\right)+(-66)\\&\quad=3y^4-16y^2-9y^2+48-66\\&\quad=3y^4-25y^2-18\end{aligned}$$

45.

$$\begin{array}{r|l} n-2 & n^4 \qquad\qquad -16 \\ & \underline{-\left(n^4-2n^3\right)} \\ & \quad 2n^3 \\ & \quad \underline{-\left(2n^3-4n^2\right)} \\ & \qquad 4n^2 \\ & \qquad \underline{-\left(4n^2-8n\right)} \\ & \qquad\quad 8n-16 \\ & \qquad\quad \underline{-(8n-16)} \\ & \qquad\qquad 0 \end{array} \quad n^3+2n^2+4n+8$$

Solution: n^3+2n^2+4n+8

Check:

$$\begin{aligned}&(n-2)\left(n^3+2n^2+4n+8\right)+(0)\\&\quad=n^4+2n^3+4n^2+8n-2n^3-4n^2\\&\qquad\qquad -8n-16\\&\quad=n^4-16\end{aligned}$$

46.

$$\begin{array}{r|l} m+3 & m^3 \qquad\quad +27 \\ & \underline{-\left(m^3+3m^2\right)} \\ & \quad -3m^2 \\ & \quad \underline{-\left(-3m^2-9m\right)} \\ & \qquad 9m+27 \\ & \qquad \underline{-(9m+27)} \\ & \qquad\quad 0 \end{array} \quad m^2-3m+9$$

Solution: m^2-3m+9

Check:

$$\begin{aligned}&(m+3)\left(m^2-3m+9\right)+(0)\\&\quad=m^3-3m^2+9m+3m^2-9m+27\\&\quad=m^3+27\end{aligned}$$

47. The divisor must be of the form $x-r$.

48. No, the divisor is not of the form $x-r$.

49. No, the divisor is not of the form $x-r$.

50. Yes, the divisor is of the form $x-r$.

51. **a.** Divisor: $x-5$

b. Quotient: $x^2+3x+11$

c. Remainder: 58

52. **a.** Divisor: $x+2$

b. Quotient: $2x^3-x^2+2x-5$

c. Remainder: 16

53. $8\big|\ 1\ \ -2\ \ -48$

$\qquad\quad 8\ \ \ 48$

$\qquad 1\ \ 6\ \ \big|0$

Quotient: $x+6$

Check:

$$(x-8)(x+6)+(0)=x^2+6x-8x-48$$
$$=x^2-2x-48$$

54. $6\big|\ 1\ \ -4\ \ -12$

$\qquad\quad 6\ \ \ 12$

$\qquad 1\ \ 2\ \ \big|0$

Quotient: $x+2$

Check:

$$(x-6)(x+2)+(0)=x^2+2x-6x-12$$
$$=x^2-4x-12$$

55. $-1\big|\ 1\ \ -3\ \ -4$

$\qquad\quad -1\ \ \ 4$

$\qquad 1\ \ -4\ \ \big|0$

Quotient: $t-4$

Check:

$$(t+1)(t-4)+(0)=t^2-4t+t-4$$
$$=t^2-3t-4$$

56. $-3\big|\ 1\ \ \ 7\ \ \ 12$

$\qquad\quad -3\ \ -12$

$\qquad 1\ \ 4\ \ \big|0$

Quotient: $h+4$

Check:

$$(h+3)(h+4)+(0)=h^2+4h+3h+12$$
$$=h^2+7h+12$$

57. $1\big|\ 5\ \ \ 5\ \ \ 1$

$\qquad\quad 5\ \ \ 10$

$\qquad 5\ \ 10\ \ \big|11$

Quotient: $5y+10+\dfrac{11}{y-1}$

Check:

$$(y-1)(5y+10)+(11)$$
$$=5y^2+10y-5y-10+11$$
$$=5y^2+5y+1$$

58. $-2\big|\ 3\ \ \ 1\ \ -5$

$\qquad\quad -6\ \ \ 10$

$\qquad 3\ \ -5\ \ \big|5$

Quotient: $3w-5+\dfrac{5}{w+2}$

Check:

$$(w+2)(3w-5)+(5)$$
$$=3w^2-5w+6w-10+5$$
$$=3w^2+w-5$$

59. $-3\big|\ 3\ \ \ 7\ \ -4\ \ \ 3$

$\qquad\quad -9\ \ \ 6\ \ -6$

$\qquad 3\ \ -2\ \ \ 2\ \ \big|-3$

Quotient: $3y^2-2y+2+\dfrac{-3}{y+3}$

Check:

60. $-3\big|\ 1\ \ -2\ \ \ 2\ \ -5$

$\qquad\quad -3\ \ \ 15\ \ -51$

$\qquad 1\ \ -5\ \ \ 17\ \ \big|-56$

Quotient: $z^2-5z+17+\dfrac{-56}{z+3}$

Check:

$$\begin{aligned}&(y+3)\left(3y^2-2y+2\right)+(-3)\\&=3y^3-2y^2+2y+9y^2-6y+6-3\\&=3y^3+7y^2-4y+3\end{aligned}$$

$$\begin{aligned}&(z+3)\left(z^2-5z+17\right)+(-56)\\&=z^3-5z^2+17z+3z^2-15z+51-56\\&=z^3-2z^2+2z-5\end{aligned}$$

61.

2	1	−3	0	4
		2	−2	−4
	1	−1	−2	0

Quotient: x^2-x-2

Check:

$$\begin{aligned}&(x-2)\left(x^2-x-2\right)+(0)\\&=x^3-x^2-2x-2x^2+2x+4\\&=x^3-3x^2+4\end{aligned}$$

62.

3	3	0	−25	0	−18
		9	27	6	18
	3	9	2	6	0

Quotient: $3y^3+9y^2+2y+6$

Check:

$$\begin{aligned}&(y-3)\left(3y^3+9y^2+2y+6\right)+(0)\\&=3y^4+9y^3+2y^2+6y-9y^3-27y^2\\&\qquad\qquad -6y-18\\&=3y^4-25y^2-18\end{aligned}$$

63.

2	1	0	0	0	0	−32
		2	4	8	16	32
	1	2	4	8	16	0

Quotient: $a^4+2a^3+4a^2+8a+16$

Check:

$$\begin{aligned}&(a-2)\left(a^4+2a^3+4a^2+8a+16\right)+(0)\\&=a^5+2a^4+4a^3+8a^2+16a\\&\qquad -2a^4-4a^3-8a^2-16a-32\\&=a^5-32\end{aligned}$$

64.

−3	1	0	0	27
		−3	9	−27
	1	−3	9	0

Quotient: b^2-3b+9

Check:

$$\begin{aligned}&(b+3)\left(b^2-3b+9\right)+(0)\\&=b^3-3b^2+9b+3b^2-9b+27\\&=b^3+27\end{aligned}$$

65.

6	1	0	0	−216
		6	36	216
	1	6	36	0

Quotient: $x^2+6x+36$

Check:

$$\begin{aligned}&(x-6)\left(x^2+6x+36\right)+(0)\\&=x^3+6x^2+36x-6x^2-36x-216\\&=x^3-216\end{aligned}$$

66.

−2	1	0	0	0	−16
		−2	4	−8	16
	1	−2	4	−8	0

Quotient: y^3-2y^2+4y-8

Check:

$$(y+2)\left(y^3-2y^2+4y-8\right)+(0)$$
$$= y^4-2y^3+4y^2-8y$$
$$\quad +2y^3-4y^2+8y-16$$
$$= y^4-16$$

67.
$$\begin{array}{r|rrrr} -\frac{2}{3} & 6 & 7 & -1 & 3 \\ & & -4 & -2 & 2 \\ \hline & 6 & 3 & -3 & \underline{5} \end{array}$$

Quotient: $6t^2+3t-3+\dfrac{5}{t+\frac{2}{3}}$

Check:

$$\left(t+\frac{2}{3}\right)\left[(6t^2+3t-3)+\frac{5}{t+\frac{2}{3}}\right]$$
$$=\left(t+\frac{2}{3}\right)(6t^2+3t-3)+\left(t+\frac{2}{3}\right)\left(\frac{5}{t+\frac{2}{3}}\right)$$
$$= 6t^3+3t^2-3t+4t^2+2t-2+5$$
$$= 6t^3+7t^2-t+3$$

68.
$$\begin{array}{r|rrrr} \frac{1}{5} & 5 & -6 & -4 & 4 \\ & & 1 & -1 & -1 \\ \hline & 5 & -5 & -5 & \underline{3} \end{array}$$

Quotient: $5z^2-5z-5+\dfrac{3}{z-\frac{1}{5}}$

Check:

$$\left(z-\frac{1}{5}\right)\left[(5z^2-5z-5)+\frac{3}{z-\frac{1}{5}}\right]$$
$$=\left(z-\frac{1}{5}\right)(5z^2-5z-5)+\left(z-\frac{1}{5}\right)\left(\frac{3}{z-\frac{1}{5}}\right)$$
$$= 5z^3-5z^2-5z-z^2+z+1+3$$
$$= 5z^3-6z^2-4z+4$$

69.
$$\begin{array}{r|rrrrr} \frac{1}{2} & 4 & 0 & -1 & 6 & -3 \\ & & 2 & 1 & 0 & 3 \\ \hline & 4 & 2 & 0 & 6 & \underline{0} \end{array}$$

Quotient: $4w^3+2w^2+6$

Check:

$$\left(w-\frac{1}{2}\right)\left(4w^3+2w^2+6\right)+(0)$$
$$= 4w^4+2w^3+6w-2w^3-w^2-3$$
$$= 4w^4-w^2+6w-3$$

70.
$$\begin{array}{r|rrrrr} -\frac{3}{4} & -12 & -5 & -1 & 1 & 3 \\ & & 9 & -3 & 3 & -3 \\ \hline & -12 & 4 & -4 & 4 & \underline{0} \end{array}$$

Quotient: $-12y^3+4y^2-4y+4$

Check:

$$\left(y+\frac{3}{4}\right)\left(-12y^3+4y^2-4y+4\right)+(0)$$
$$= -12y^4+4y^3-4y^2+4y-9y^3+3y^2$$
$$\quad -3y+3$$
$$= -12y^4-5y^3-y^2+y+3$$

71.
$$\begin{array}{r|rrrr} -4 & -1 & -8 & -3 & -2 \\ & & 4 & 16 & -52 \\ \hline & -1 & -4 & 13 & -54 \end{array}$$

Quotient: $-x^2-4x+13+\dfrac{-54}{x+4}$

72. $\left(8xy^2-9x^2y+6x^2y^2\right)\div\left(x^2y^2\right)$

$$=\frac{8xy^2}{x^2y^2}-\frac{9x^2y}{x^2y^2}+\frac{6x^2y^2}{x^2y^2}=\frac{8}{x}-\frac{9}{y}+6$$

73. $\left(22x^2-11x+33\right)\div\left(11x\right)$

$$=\frac{22x^2}{11x}-\frac{11x}{11x}+\frac{33}{11x}=2x-1+\frac{3}{x}$$

74.
$$\begin{array}{r|rrrr} 3 & 2 & -4 & 5 & -33 \\ & & 6 & 6 & 33 \\ \hline & 2 & 2 & 11 & 0 \end{array}$$

Quotient: $2m^2+2m+11$

75.
$$\begin{array}{r} \qquad\qquad\qquad\qquad 4y-3 \\ 3y^2-2y+5\overline{)\;12y^3-17y^2+30y-10} \\ \underline{-\left(12y^3-8y^2+20y\right)} \\ \qquad\qquad -9y^2+10y-10 \\ \underline{-\left(-9y^2+6y-15\right)} \\ \qquad\qquad\qquad 4y+5 \end{array}$$

Quotient: $4y-3+\dfrac{4y+5}{3y^2-2y+5}$

76. $\left(90h^{12}-63h^9+45h^8-36h^7\right)\div\left(9h^9\right)$

$$=\frac{90h^{12}}{9h^9}-\frac{63h^9}{9h^9}+\frac{45h^8}{9h^9}-\frac{36h^7}{9h^9}$$
$$=10h^3-7+\frac{5}{h}-\frac{4}{h^2}$$

77.
$$\begin{array}{r} \qquad\qquad\qquad\quad 2x^2+3x-1 \\ 2x^2+1\overline{)\;4x^4+6x^3\qquad+3x-1} \\ \underline{-\left(4x^4\qquad+2x^2\right)} \\ \qquad\qquad 6x^3-2x^2+3x \\ \underline{-\left(6x^3\qquad+3x\right)} \\ \qquad\qquad\quad -2x^2\qquad -1 \\ \underline{-\left(-2x^2\qquad -1\right)} \\ \qquad\qquad\qquad\quad 0 \end{array}$$

Quotient: $2x^2+3x-1$

78.
$$\begin{array}{r|rrrrr} -2 & 1 & -3 & -5 & -2 & 5 \\ & & -2 & 10 & -10 & 24 \\ \hline & 1 & -5 & 5 & -12 & 29 \end{array}$$

Quotient: $y^3-5y^2+5y-12+\dfrac{29}{y+2}$

79. $\left(16k^{11}-32k^{10}+8k^8-40k^4\right)\div\left(8k^8\right)$

$$=\frac{16k^{11}}{8k^8}-\frac{32k^{10}}{8k^8}+\frac{8k^8}{8k^8}-\frac{40k^4}{8k^8}$$

$$=2k^3-4k^2+1-\frac{5}{k^4}$$

80.
$$\begin{array}{r} 2m-5 \\ 2m^2-4m+3\overline{)\ 4m^3-18m^2+22m-10} \\ \underline{-\left(4m^3-8m^2+6m\right)} \\ -10m^2+16m-10 \\ \underline{-\left(-10m^2+20m-15\right)} \\ -4m+5 \end{array}$$

Quotient: $2m-5+\dfrac{-4m+5}{2m^2-4m+3}$

81. $\left(5x^3+9x^2+10x\right)\div\left(5x^2\right)$

$$=\frac{5x^3}{5x^2}+\frac{9x^2}{5x^2}+\frac{10x}{5x^2}$$

$$=x+\frac{9}{5}+\frac{2}{x}$$

82.
$$\begin{array}{r} 5k^2+k+3 \\ 3k^2-1\overline{)\ 15k^4+3k^3+4k^2\ \ +4} \\ \underline{-\left(15k^4\ \ \ \ -5k^2\right)} \\ 3k^3+9k^2 \\ \underline{-\left(3k^3\ \ \ \ -k\right)} \\ 9k^2+k+4 \\ \underline{-\left(9k^2\ \ -3\right)} \\ k+7 \end{array}$$

Quotient: $5k^2+k+3+\dfrac{k+7}{3k^2-1}$

83. **a.**
$$P\left(-4\right)=4\left(-4\right)^3+10\left(-4\right)^2-8\left(-4\right)-20$$
$$=4\left(-64\right)+10\left(16\right)+32-20$$
$$=-256+160+32-20$$
$$=-84$$

b.
$$\begin{array}{r|rrrr} -4 & 4 & 10 & -8 & -20 \\ & & -16 & 24 & -64 \\ \hline & 4 & -6 & 16 & -84 \end{array}$$

Quotient: $4x^2-6x+16+\dfrac{-84}{x+4}$

c. The values are the same.

84. **a.**
$$P\left(-6\right)=-3\left(-6\right)^3-12\left(-6\right)^2+5\left(-6\right)-8$$
$$=-3\left(-216\right)-12\left(36\right)-30-8$$
$$=648-432-30-8$$
$$=178$$

b.
$$\begin{array}{r|rrrr} -6 & -3 & -12 & 5 & -8 \\ & & 18 & -36 & 186 \\ \hline & -3 & 6 & -31 & 178 \end{array}$$

Quotient: $-3x^2+6x-31+\dfrac{178}{x+6}$

c. The values are the same.

85. $P(r)$ equals the remainder of $P(x) \div (x - r)$.

86. **a.**
$$\begin{array}{r|rrr} 1 & 7 & -16 & 9 \\ & & 7 & -9 \\ \hline & 7 & -9 & 0 \end{array}$$
Quotient: $7x - 9$

b. Yes

87. **a.**
$$\begin{array}{r|rrr} -1 & 8 & 13 & 5 \\ & & -8 & -5 \\ \hline & 8 & 5 & 0 \end{array}$$
Quotient: $8x + 5$

b. Yes

Problem Recognition Exercises

1. **a.**
$$\begin{aligned}(3x+1)^2 &= (3x)^2 + 2(3x)(1) + 1^2 \\ &= 9x^2 + 6x + 1\end{aligned}$$

b
$$\begin{aligned}(3x+1)(3x-1) &= (3x)^2 - 1^2 \\ &= 9x^2 - 1\end{aligned}$$

c.
$$\begin{aligned}(3x+1) - (3x-1) &= 3x + 1 - 3x + 1 \\ &= 2\end{aligned}$$

2. **a.**
$$\begin{aligned}(9m-5) - (9m+5) &= 9m - 5 - 9m - 5 \\ &= -10\end{aligned}$$

b.
$$\begin{aligned}(9m-5)(9m+5) &= (9m)^2 - 5^2 \\ &= 81m^2 - 25\end{aligned}$$

c.
$$\begin{aligned}(9m-5)^2 &= (9m)^2 - 2(9m)(5) + 5^2 \\ &= 81m^2 - 90m + 25\end{aligned}$$

3. **a.**
$$\begin{aligned}\frac{4x^2 + 8x - 10}{2x} &= \frac{4x^2}{2x} + \frac{8x}{2x} - \frac{10}{2x} \\ &= 2x + 4 - \frac{5}{x}\end{aligned}$$

b.
$$\begin{array}{r} 2x + 5 \\ 2x - 1 \overline{)\ 4x^2 + 8x - 10} \\ \underline{-(4x^2 - 2x)} \\ 10x - 10 \\ \underline{-(10x - 5)} \\ -5 \end{array}$$

Solution: $2x + 5 + \dfrac{-5}{2x - 1}$

4. **a.**
$$\begin{aligned}\frac{3y^2 - 15y + 4}{3y} &= \frac{3y^2}{3y} - \frac{15y}{3y} + \frac{4}{3y} \\ &= y - 5 + \frac{4}{3y}\end{aligned}$$

b.
$$\begin{array}{r} y - 7 \\ 3y + 6 \overline{)\ 3y^2 - 15y + 4} \\ \underline{-(3y^2 + 6y)} \\ -21y + 4 \\ \underline{-(-21y - 42)} \\ 46 \end{array}$$

Solution: $y - 7 + \dfrac{46}{3y + 6}$

c.
$$\begin{array}{r|rrr} 1 & 4 & 8 & -10 \\ & & 4 & 12 \\ \hline & 4 & 12 & 2 \end{array}$$

Quotient: $4x+12+\dfrac{2}{x-1}$

c.
$$\begin{array}{r|rrr} -6 & 3 & -15 & 4 \\ & & -18 & 198 \\ \hline & 3 & -33 & 202 \end{array}$$

Quotient: $3y-33+\dfrac{202}{y+6}$

5. a. $(p-5)(p+5)-(p^2+5)$
$= p^2-25-p^2-5=-30$

b. $(p-5)(p+5)-(p+5)^2$
$= p^2-25-p^2-10p-25$
$= -10p-50$

c. $(p-5)(p+5)-(p^2-25)$
$= p^2-25-p^2+25=0$

6. a. $(x+4)(x-4)-(x+4)^2$
$= x^2-16-x^2-8x-16$
$= -8x-32$

b. $(x+4)(x-4)-(x^2+4)$
$= x^2-16-x^2-4=-20$

c. $(x+4)(x-4)-(x^2-16)$
$= x^2-16-x^2+16=0$

7. $(5t^2-6t+2)-(3t^2-7t+3)$
$= 5t^2-6t+2-3t^2+7t-3$
$= 2t^2+t-1$

8. $-5x^2(3x^2+x-2)$
$= -5x^2(3x^2)-5x^2(x)+5x^2(2)$
$= -15x^4-5x^3+10x^2$

9. $(6z+5)(6z-5)=(6z)^2-5^2=36z^2-25$

10. $(6y^3+2y^2+y-2)+(3y^3-4y+3)$
$= 9y^3+2y^2-3y+1$

11. $(3b-4)(2b-1)$
$= 3b(2b)-3b(1)-4(2b)+4(1)$
$= 6b^2-3b-8b+4$
$= 6b^2-11b+4$

12. $(5a+2)(2a^2+3a+1)$
$= 10a^3+15a^2+5a+4a^2+6a+2$
$= 10a^3+19a^2+11a+2$

13. $(t^3-4t^2+t-9)+(t+12)-(2t^2-6t)$
$= t^3-4t^2+t-9+t+12-2t^2+6t$
$= t^3-6t^2+8t+3$

14.
$$\begin{array}{r|rrrr} 2 & 2 & 0 & -3 & -10 \\ & & 4 & 8 & 10 \\ \hline & 2 & 4 & 5 & 0 \end{array}$$

Quotient: $2b^2+4b+5$

15. $\left(k+4\right)^2+\left(-4k+9\right)$

$=k^2+2\left(k\right)\left(4\right)+4^2-4k+9$

$=k^2+8k+16-4k+9$

$=k^2+4k+25$

16.

$\underline{4}\vert$	3	−11	−4	−5	20
		12	4	0	−20
	3	1	0	−5	$\vert\underline{0}$

Quotient: $3x^3+x^2-5$

17. $-2t\left(t^2+6t-3\right)+t\left(3t+2\right)\left(3t-2\right)$

$=-2t^3-12t^2+6t+t\left(9t^2-4\right)$

$=-2t^3-12t^2+6t+9t^3-4t$

$=7t^3-12t^2+2t$

18. $\dfrac{7x^2y^3-14xy^2-x^2}{-7xy}=\dfrac{7x^2y^3}{-7xy}-\dfrac{14xy^2}{-7xy}-\dfrac{x^2}{-7xy}$

$=-xy^2+2y+\dfrac{x}{7y}$

19. $\left(\dfrac{1}{4}p^3-\dfrac{1}{6}p^2+5\right)-\left(-\dfrac{2}{3}p^3+\dfrac{1}{3}p^2-\dfrac{1}{5}p\right)$

$=\dfrac{3}{12}p^3-\dfrac{1}{6}p^2+5+\dfrac{8}{12}p^3-\dfrac{2}{6}p^2+\dfrac{1}{5}p$

$=\dfrac{11}{12}p^3-\dfrac{1}{2}p^2+\dfrac{1}{5}p+5$

20. $-6w^3\left(1.2w-2.6w^2+5.1w^3\right)$

$=-7.2w^4+15.6w^5-30.6w^6$

$=-30.6w^6+15.6w^5-7.2w^4$

21. $\left(6a^2-4b\right)^2=\left(6a^2\right)^2-2\left(6a^2\right)\left(4b\right)+\left(4b\right)^2$

$=36a^4-48a^2b+16b^2$

22. $\left(\dfrac{1}{2}z^2-\dfrac{1}{3}\right)\left(\dfrac{1}{2}z^2+\dfrac{1}{3}\right)=\left(\dfrac{1}{2}z^2\right)^2-\left(\dfrac{1}{3}\right)^2$

$=\dfrac{1}{4}z^4-\dfrac{1}{9}$

23. $\left(m-3\right)^2-2\left(m+8\right)$

$=m^2-6m+9-2m-16$

$=m^2-8m-7$

24. $\left(2x-5\right)\left(x+1\right)-\left(x-3\right)^2$

$=2x^2+2x-5x-5-\left(x^2-6x+9\right)$

$=2x^2-3x-5-x^2+6x-9$

$=x^2+3x-14$

25. $\left(m^2-6m+7\right)\left(2m^2+4m-3\right)=m^2\left(2m^2+4m-3\right)-6m\left(2m^2+4m-3\right)+7\left(2m^2+4m-3\right)$

$=2m^4+4m^3-3m^2-12m^3-24m^2+18m+14m^2+28m-21$

$=2m^4-8m^3-13m^2+46m-21$

26.
$$\begin{array}{r|cccc} 4 & 1 & 0 & 0 & -64 \\ & & 4 & 16 & 64 \\ \hline & 1 & 4 & 16 & 0 \end{array}$$
Quotient: $x^2+4x+16$

27. $\left[5-(a+b)\right]^2 = 5^2-2(5)(a+b)+(a+b)^2$
$= 25-10a-10b+a^2+2ab+b^2$

28. $\left[a-(x-y)\right]\left[a+(x-y)\right]$
$= (a)^2-(x-y)^2$
$= a^2-\left(x^2-2xy+y^2\right)$
$= a^2-x^2+2xy-y^2$

29. $(x+y)^2-(x-y)^2$
$= x^2+2xy+y^2-\left(x^2-2xy+y^2\right)$
$= x^2+2xy+y^2-x^2+2xy-y^2$
$= 4xy$

30. $(a-4)^3 = (a-4)^2(a-4)$
$= \left(a^2-8a+16\right)(a-4)$
$= a^3-4a^2-8a^2+32a+16a-64$
$= a^3-12a^2+48a-64$

31. $\left(-\frac{1}{2}x+\frac{1}{3}\right)\left(\frac{1}{4}x-\frac{1}{2}\right)$
$= -\frac{1}{8}x^2+\frac{1}{4}x+\frac{1}{12}x-\frac{1}{6}$
$= -\frac{1}{8}x^2+\frac{1}{3}x-\frac{1}{6}$

32. $-3x^2y^3z^4\left(\frac{1}{6}x^4yzw^3\right) = -\frac{1}{2}x^6y^4z^5w^3$

Section 4.5 Practice Exercises

1. **a.** product
b. greatest common factor
c. greatest common factor
d. grouping

2. $\left(-4a^3b^5c\right)\left(-2a^7c^2\right) = 8a^{10}b^5c^3$

3. $\left(7t^4+5t^3-9t\right)-\left(-2t^4+6t^2-3t\right)$
$= 7t^4+5t^3-9t+2t^4-6t^2+3t$
$= 9t^4+5t^3-6t^2-6t$

4. $\left(5x^3-9x+5\right)+\left(4x^3+3x^2-2x+1\right)-\left(6x^3-3x^2+x+1\right)$
$= 5x^3-9x+5+4x^3+3x^2-2x+1-6x^3+3x^2-x-1$
$= 3x^3+6x^2-12x+5$

5. $\left(5y^2-3\right)\left(y^2+y+2\right)$
$=5y^4+5y^3+10y^2-3y^2-3y-6$
$=5y^4+5y^3+7y^2-3y-6$

6. $\left(a+6b\right)^2=a^2+2\left(a\right)\left(6b\right)+\left(6b\right)^2$
$=a^2+12b+36b^2$

7. $\dfrac{6v^3-12v^2+2v}{-2v}=\dfrac{6v^3}{-2v}-\dfrac{12v^2}{-2v}+\dfrac{2v}{-2v}$
$=-3v^2+6v-1$

8.
$$\begin{array}{r|cccc} -2 & 3 & 2 & 0 & -4 \\ & & -6 & 8 & -16 \\ \hline & 3 & -4 & 8 & -20 \end{array}$$
Quotient: $3x^2-4x+8+\dfrac{-20}{x+2}$

9. $3x+12=3\left(x\right)+3\left(4\right)=3\left(x+4\right)$

10. $15x-10=5\left(3x\right)-5\left(2\right)=5\left(3x-2\right)$

11. $6z^2+4z=2z\left(3z\right)+2z\left(2\right)=2z\left(3z+2\right)$

12. $49y^3-35y^2=7y^2\left(7y\right)-7y^2\left(5\right)$
$=7y^2\left(7y-5\right)$

13. $4p^6-4p=4p\left(p^5\right)-4p\left(1\right)=4p\left(p^5-1\right)$

14. $5q^2-5q=5q\left(q\right)-5q\left(1\right)=5q\left(q-1\right)$

15. $12x^4-36x^2=12x^2\left(x^2\right)-12x^2\left(3\right)$
$=12x^2\left(x^2-3\right)$

16. $51w^4-34w^3=17w^3\left(3w\right)-17w^3\left(2\right)$
$=17w^3\left(3w-2\right)$

17. $9st^2+27t=9t\left(st\right)+9t\left(3\right)=9t\left(st+3\right)$

18. $8a^2b^3+12a^2b=4a^2b\left(2b^2\right)+4a^2b\left(3\right)$
$=4a^2b\left(2b^2+3\right)$

19. $9a^4b^3+27a^3b^4-18a^2b^5$
$=9a^2b^3\left(a^2\right)+9a^2b^3\left(3ab\right)-9a^2b^3\left(2b^2\right)$
$=9a^2b^3\left(a^2+3ab-2b^2\right)$

20. $3x^5y^4-15x^4y^5+9x^2y^7$
$=3x^2y^4\left(x^3\right)-3x^2y^4\left(5x^2y\right)+3x^2y^4\left(3y^3\right)$
$=3x^2y^4\left(x^3-5x^2y+3y^3\right)$

21. $10x^2y+15xy^2-5xy$
$=5xy\left(2x\right)+5xy\left(3y\right)-5xy\left(1\right)$
$=5xy\left(2x+3y-1\right)$

22. $12c^3d-15c^2d+3cd$
$=3cd\left(4c^2\right)-3cd\left(5c\right)+3cd\left(1\right)$
$=3cd\left(4c^2-5c+1\right)$

23. $13b^2 - 11a^2b - 12ab$
$= b(13b) - b(11a^2) - b(12a)$
$= b(13b - 11a^2 - 12a)$

24. $6a^3 - 2a^2b + 5a^2 = a^2(6a) - a^2(2b) + a^2(5)$
$= a^2(6a - 2b + 5)$

25. $-x^2 - 10x + 7 = -1(x^2 + 10x - 7)$

26. $-5y^2 + 10y + 3 = -1(5y^2 - 10y - 3)$

27. $-12x^3y - 6x^2y - 3xy$
$= -3xy(4x^2) - 3xy(2x) - 3xy(1)$
$= -3xy(4x^2 + 2x + 1)$

28. $-32a^4b^2 + 24a^3b + 16a^2b$
$= -8a^2b(4a^2b) - 8a^2b(-3a) - 8a^2b(-2)$
$= -8a^2b(4a^2b - 3a - 2)$

29. $-2t^3 + 11t^2 - 3t$
$= -t(2t^2) - t(-11t) - t(3)$
$= -t(2t^2 - 11t + 3)$

30. $-7y^2z - 5yz - z$
$= -z(7y^2) - z(5y) - z(1)$
$= -z(7y^2 + 5y + 1)$

31. $2a(3z - 2b) - 5(3z - 2b)$
$= (3z - 2b)(2a - 5)$

32. $5x(3x + 4) + 2(3x + 4)$
$= (3x + 4)(5x + 2)$

33. $2x^2(2x - 3) + (2x - 3) = (2x - 3)(2x^2 + 1)$

34. $z(w - 9) + (w - 9) = (w - 9)(z + 1)$

35. $y(2x + 1)^2 - 3(2x + 1)^2 = (2x + 1)^2(y - 3)$

36. $a(b - 7)^2 + 5(b - 7)^2 = (b - 7)^2(a + 5)$

37. $3y(x - 2)^2 + 6(x - 2)^2$
$= 3\left[y(x - 2)^2 + 2(x - 2)^2\right]$
$= 3(x - 2)^2(y + 2)$

38. $10z(z + 3)^2 - 2(z + 3)^2$
$= 2\left[5z(z + 3)^2 - (z + 3)^2\right]$
$= 2(z + 3)^2(5z - 1)$

39. For example: $3x^3 + 6x^2 + 12x^4$

40. For example: $10x^2y^3 + 5x^3y^3 - 5x^2y^4$;

$5x^2y^3z - 25x^4y^4 + 15x^2y^3z^2$

41. For example: $6(c+d)+y(c+d)$

42. Factor by grouping

43. **a.** $2ax-ay+6bx-3by$
$=a(2x-y)+3b(2x-y)$
$=(2x-y)(a+3b)$

b. $10w^2-5w-6bw+3b$
$=5w(2w-1)-3b(2w-1)$
$=(2w-1)(5w-3b)$

c. In part (b), $-3b$ was factored out so that the signs in the last two terms were changed. The resulting binomial factor matches the binomial factor in the first two terms.

44. **a.** $3xy+2bx+6by+4b^2$
$=x(3y+2b)+2b(3y+2b)$
$=(3y+2b)(x+2b)$

b. $15ac+10ab-6bc-4b^2$
$=5a(3c+2b)-2b(3c+2b)$
$=(3c+2b)(5a-2b)$

c. In part (b), $-2b$ was factored out so that the signs in the last two terms were changed. The resulting binomial factor matches the binomial factor in the first two terms.

45. $y^3+4y^2+3y+12=y^2(y+4)+3(y+4)$
$=(y+4)(y^2+3)$

46. $ab+b+2a+2=b(a+1)+2(a+1)$
$=(a+1)(b+2)$

47. $6p-42+pq-7q=6(p-7)+q(p-7)$
$=(p-7)(6+q)$

48. $2t-8+st-4s=2(t-4)+s(t-4)$
$=(t-4)(2+s)$

49. $2mx+2nx+3my+3ny$
$=2x(m+n)+3y(m+n)$
$=(m+n)(2x+3y)$

50. $4x^2+6xy-2xy-3y^2$
$=2x(2x+3y)-y(2x+3y)$
$=(2x+3y)(2x-y)$

51. $10ax-15ay-8bx+12by$
$=5a(2x-3y)-4b(2x-3y)$
$=(2x-3y)(5a-4b)$

52. $35a^2-15a+14a-6$
$=5a(7a-3)+2(7a-3)$
$=(7a-3)(5a+2)$

53. $x^3-x^2-3x+3=x^2(x-1)-3(x-1)$
$=(x-1)(x^2-3)$

54. $2rs+4s-r-2=2s(r+2)-1(r+2)$
$=(r+2)(2s-1)$

55. $6p^2q+18pq-30p^2-90p$

$$\begin{aligned}&=6p\left[pq+3q-5p-15\right]\\&=6p\left[q\left(p+3\right)-5\left(p+3\right)\right]\\&=6p\left(p+3\right)\left(q-5\right)\end{aligned}$$

56. $5s^2t+20st-15s^2-60s$

$$\begin{aligned}&=5s\left[st+4t-3s-12\right]\\&=5s\left[t\left(s+4\right)-3\left(s+4\right)\right]\\&=5s\left(s+4\right)\left(t-3\right)\end{aligned}$$

57. $100x^3-300x^2+200x-600$

$$\begin{aligned}&=100\left[x^3-3x^2+2x-6\right]\\&=100\left[x^2\left(x-3\right)+2\left(x-3\right)\right]\\&=100\left(x-3\right)\left(x^2+2\right)\end{aligned}$$

58. $2x^5-10x^4+6x^3-30x^2$

$$\begin{aligned}&=2x^2\left[x^3-5x^2+3x-15\right]\\&=2x^2\left[x^2\left(x-5\right)+3\left(x-5\right)\right]\\&=2x^2\left(x-5\right)\left(x^2+3\right)\end{aligned}$$

59. $6ax-by+2bx-3ay$

$$\begin{aligned}&=6ax+2bx-3ay-by\\&=2x\left(3a+b\right)-y\left(3a+b\right)\\&=\left(3a+b\right)\left(2x-y\right)\end{aligned}$$

60. $5pq-12-4q+15p$

$$\begin{aligned}&=5pq+15p-4q-12\\&=5p\left(q+3\right)-4\left(q+3\right)\\&=\left(q+3\right)\left(5p-4\right)\end{aligned}$$

61. $4a-3b-ab+12$

$$\begin{aligned}&=4a-ab+12-3b\\&=a\left(4-b\right)+3\left(4-b\right)\\&=\left(4-b\right)\left(a+3\right)\end{aligned}$$

62. $x^2y+6x-3x^3-2y$

$$\begin{aligned}&=x^2y-3x^3-2y+6x\\&=x^2\left(y-3x\right)-2\left(y-3x\right)\\&=\left(y-3x\right)\left(x^2-2\right)\end{aligned}$$

63. $7y^3-21y^2+5y-10$ cannot be factored.

64. $5ax+10bx-2ac+4bc$ cannot be factored.

65. It is not possible to get a common binomial factor regardless of the order of the terms.

66. It is not possible to get a common binomial factor regardless of the order of the terms.

67.

$$\begin{aligned}U&=Av+Acw\\U&=A\left(v+cw\right)\\\frac{U}{v+cw}&=A\end{aligned}$$

68.

$$\begin{aligned}S&=rt+wt\\S&=t\left(r+w\right)\\\frac{S}{r+w}&=t\end{aligned}$$

69. $ay + bx = cy$

$bx = cy - ay$

$bx = y(c - a)$

$y = \dfrac{bx}{c-a}$ or $y = \dfrac{-bx}{a-c}$

70. $cd + 2x = ac$

$2x = ac - cd$

$2x = c(a - d)$

$c = \dfrac{2x}{a-d}$ or $c = \dfrac{-2x}{d-a}$

71. $A = 2w^2 + w$

$A = w(2w + 1)$

The length of the rectangle is $2w + 1$.

72. **a.** $A = P + Prt$

$A = P(1 + rt)$

$P = \dfrac{A}{1+rt}$

b. $P = \dfrac{12,705}{1 + 0.07(3)} = \dfrac{12,705}{1.21} = \$10,500$

73. $(a+3)^4 + 6(a+3)^5$

$= (a+3)^4[1 + 6(a+3)]$

$= (a+3)^4[1 + 6a + 18]$

$= (a+3)^4(6a + 19)$

74. $(4-b)^4 - 2(4-b)^3$

$= (4-b)^3[(4-b) - 2]$

$= (4-b)^3(2-b)$

75. $24(3x+5)^3 - 30(3x+5)^2$

$= 6(3x+5)^2[4(3x+5) - 5]$

$= 6(3x+5)^2[12x + 20 - 5]$

$= 6(3x+5)^2(12x + 15)$

$= 6(3x+5)^2 3(4x+5)$

$= 18(3x+5)^2(4x+5)$

76. $10(2y+3)^2 + 15(2y+3)^3$

$= 5(2y+3)^2[2 + 3(2y+3)]$

$= 5(2y+3)^2[2 + 6y + 9]$

$= 5(2y+3)^2(6y + 11)$

77. $(t+4)^2 - (t+4) = (t+4)[(t+4) - 1]$

$= (t+4)(t+3)$

78. $(p+6)^2 - (p+6) = (p+6)[(p+6) - 1]$

$= (p+6)(p+5)$

79. $15w^2(2w-1)^3+5w^3(2w-1)^2$

$=5w^2(2w-1)^2\left[3(2w-1)+w\right]$

$=5w^2(2w-1)^2\left[6w-3+w\right]$

$=5w^2(2w-1)^2(7w-3)$

80. $8z^4(3z-2)^2+12z^3(3z-2)^3$

$=4z^3(3z-2)^2\left[2z+3(3z-2)\right]$

$=4z^3(3z-2)^2\left[2z+9z-6\right]$

$=4z^3(3z-2)^2(11z-6)$

Section 4.6 Practice Exercises

1. **a.** positive

b. opposite

c. $(2x+3)(x-4)=2x^2-8x+3x-12$

$=2x^2-5x-12$

$(x-4)(2x+3)=2x^2+3x-8x-12$

$=2x^2-5x-12$

Both are correct.

d. $6x^2-4x-10=2\left(3x^2-2x-5\right)$

$=2\left(3x^2+3x-5x-5\right)$

$=2\left[3x(x+1)-5(x+1)\right]$

$=2(3x-5)(x+1)$

e. $(a+b)^2$; $(a-b)^2$

2. Multiply the factors and determine if the product equals the original polynomial.

3. $36c^2d^7e^{11}+12c^3d^5e^{15}-6c^2d^4e^7$

$=6c^2d^4e^7\left(6d^3e^4+2cde^8-1\right)$

4. $5x^3y^3+15x^4y^2-35x^2y^4$

$=5x^2y^2\left(xy+3x^2-7y^2\right)$

5. $2x(3a-b)-(3a-b)=(3a-b)(2x-1)$

6. $6(v-8)-3u(v-8)=3(v-8)(2-u)$

7. $wz^2+2wz-33az-66a$

$=wz(z+2)-33a(z+2)$

$=(z+2)(wz-33a)$

8. $3a^2x+9ab-abx-3b^2$

$=3a(ax+3b)-b(ax+3b)$

$=(ax+3b)(3a-b)$

9. $b^2-12b+32=b^2-4b-8b+32$

$=b(b-4)-8(b-4)$

$=(b-4)(b-8)$

10. $a^2-12a+27=a^2-9a-3a+27$

$=a(a-9)-3(a-9)$

$=(a-9)(a-3)$

11. $y^2+10y-24 = y^2+12y-2y-24$
$= y(y+12)-2(y+12)$
$= (y+12)(y-2)$

12. $w^2+3w-54 = w^2+9w-6w-54$
$= w(w+9)-6(w+9)$
$= (w+9)(w-6)$

13. $x^2+13x+30 = x^2+10x+3x+30$
$= x(x+10)+3(x+10)$
$= (x+10)(x+3)$

14. $t^2+9t+8 = t^2+8t+t+8$
$= t(t+8)+1(t+8)$
$= (t+8)(t+1)$

15. $c^2-6c-16 = c^2-8c+2c-16$
$= c(c-8)+2(c-8)$
$= (c-8)(c+2)$

16. $z^2-3z-28 = z^2-7z+4z-28$
$= z(z-7)+4(z-7)$
$= (z-7)(z+4)$

17. $2x^2-7x-15 = 2x^2-10x+3x-15$
$= 2x(x-5)+3(x-5)$
$= (x-5)(2x+3)$

18. $2y^2-13y+15 = 2y^2-10y-3y+15$
$= 2y(y-5)-3(y-5)$
$= (y-5)(2y-3)$

19. $a+6a^2-5 = 6a^2+a-5$
$= 6a^2+6a-5a-5$
$= 6a(a+1)-5(a+1)$
$= (a+1)(6a-5)$

20. $10b^2-3-29b = 10b^2-29b-3$
$= 10b^2-30b+b-3$
$= 10b(b-3)+1(b-3)$
$= (b-3)(10b+1)$

21. $s^2+st-6t^2 = s^2+3st-2st-6t^2$
$= s(s+3t)-2t(s+3t)$
$= (s+3t)(s-2t)$

22. $p^2-pq-20q^2 = p^2-5pq+4pq-20q^2$
$= p(p-5q)+4q(p-5q)$
$= (p-5q)(p+4q)$

23. $3x^2-60x+108 = 3(x^2-20x+36)$
$= 3(x^2-18x-2x+36)$
$= 3[x(x-18)-2(x-18)]$
$= 3(x-18)(x-2)$

24. $4c^2+12c-72 = 4(c^2+3c-18)$
$= 4(c^2+6c-3c-18)$
$= 4[c(c+6)-3(c+6)]$
$= 4(c+6)(c-3)$

25. $2c^2-2c-24 = 2(c^2-c-12)$
$= 2(c^2-4c+3c-12)$

26. $3x^2+12x-15 = 3(x^2+4x-5)$
$= 3(x^2+5x-x-5)$

$$\begin{aligned}&= 2\left[c(c-4)+3(c-4)\right]\\ &= 2(c-4)(c+3)\end{aligned}$$

$$\begin{aligned}&= 3\left[x(x+5)-(x+5)\right]\\ &= 3(x+5)(x-1)\end{aligned}$$

27. $2x^2+8xy-10y^2 = 2\left(x^2+4xy-5y^2\right)$

$$\begin{aligned}&= 2\left(x^2+5xy-xy-5y^2\right)\\ &= 2\left[x(x+5y)-y(x+5y)\right]\\ &= 2(x+5y)(x-y)\end{aligned}$$

28. $20z^2+26zw-28w^2 = 2\left(10z^2+13zw-14w^2\right)$

$$\begin{aligned}&= 2\left(10z^2+20zw-7zw-14w^2\right)\\ &= 2\left[10z(z+2w)-7w(z+2w)\right]\\ &= 2(z+2w)(10z-7w)\end{aligned}$$

29. $33t^2-18t+2$

Since there are not two factors of 66 whose sum is –18, the polynomial is prime.

30. $5p^2-10p+7$

Since there are not two factors of 35 whose sum is –10, the polynomial is prime.

31. $3x^2+14xy+15y^2 = 3x^2+9xy+5xy+15y^2$

$$\begin{aligned}&= 3x(x+3y)+5y(x+3y)\\ &= (x+3y)(3x+5y)\end{aligned}$$

32. $2a^2+15ab-27b^2 = 2a^2+18ab-3ab-27b^2$

$$\begin{aligned}&= 2a(a+9b)-3b(a+9b)\\ &= (a+9b)(2a-3b)\end{aligned}$$

33. $5u^3v-30u^2v^2+45uv^3 = 5uv\left(u^2-6uv+9v^2\right)$

$$\begin{aligned}&= 5uv\left(u^2-3uv-3uv+9v^2\right)\\ &= 5uv\left[u(u-3v)-3v(u-3v)\right]\\ &= 5uv(u-3v)(u-3v)\\ &= 5uv(u-3v)^2\end{aligned}$$

34. $3a^3+30a^2b+75ab^2 = 3a\left(a^2+10ab+25b^2\right)$

$$\begin{aligned}&= 3a\left(a^2+5ab+5ab+25b^2\right)\\ &= 3a\left[a(a+5b)+5b(a+5b)\right]\\ &= 3a(a+5b)(a+5b)\\ &= 3a(a+5b)^2\end{aligned}$$

35. $x^3-5x^2-14x = x\left(x^2-5x-14\right)$

$$\begin{aligned}&= x\left(x^2-7x+2x-14\right)\\ &= x\left[x(x-7)+2(x-7)\right]\\ &= x(x-7)(x+2)\end{aligned}$$

36. $p^3+2p^2-24p = p\left(p^2+2p-24\right)$

$$\begin{aligned}&= p\left(p^2+6p-4p-24\right)\\ &= p\left[p(p+6)-4(p+6)\right]\\ &= p(p+6)(p-4)\end{aligned}$$

37. $-23z-5+10z^2 = 10z^2-23z-5$

$$= 10z^2-25z+2z-5$$

38. $3+16y^2+14y = 16y^2+14y+3$

$$= 16y^2+8y+6y+3$$

$$= 5z(2z-5)+(2z-5)$$
$$= (2z-5)(5z+1)$$

$$= 8y(2y+1)+3(2y+1)$$
$$= (2y+1)(8y+3)$$

39. $b^2+2b+15$

Since there are not two factors of 15 whose sum is 2, the polynomial is prime.

40. x^2-x-1

Since there are not two factors of –1 whose sum is –1, the polynomial is prime.

41. $-2t^2+12t+80 = -2(t^2-6t-40)$
$$= -2(t^2-10t+4t-40)$$
$$= -2[t(t-10)+4(t-10)]$$
$$= -2(t-10)(t+4)$$

42. $-3c^2+33c-72 = -3(c^2-11c+24)$
$$= -3(c^2-8c-3c+24)$$
$$= -3[c(c-8)-3(c-8)]$$
$$= -3(c-8)(c-3)$$

43. $14a^2+13a-12 = 14a^2+21a-8a-12$
$$= 7a(2a+3)-4(2a+3)$$
$$= (2a+3)(7a-4)$$

44. $12x^2-16x+5 = 12x^2-6x-10x+5$
$$= 6x(2x-1)-5(2x-1)$$
$$= (2x-1)(6x-5)$$

45. $6a^2b+22ab+12b = 2b(3a^2+11a+6)$
$$= 2b(3a^2+9a+2a+6)$$
$$= 2b[3a(a+3)+2(a+3)]$$
$$= 2b(a+3)(3a+2)$$

46. $6cd^2+9cd-42c = 3c(2d^2+3d-14)$
$$= 3c(2d^2+7d-4d-14)$$
$$= 3c[d(2d+7)-2(2d+7)]$$
$$= 3c(2d+7)(d-2)$$

47. a. $(x+5)(x+5) = x^2+5x+5x+25$
$$= x^2+10x+25$$

b. $x^2+10x+25 = (x+5)^2$

48. a. $(2w-5)(2w-5)$
$$= 4w^2-10w-10w+25$$
$$= 4w^2-20w+25$$

b. $4w^2-20w+25 = (2w-5)^2$

49. a. $(3x-2y)(3x-2y)$
$$= 9x^2-6xy-6xy+4y^2$$
$$= 9x^2-12xy+4y^2$$

b. $9x^2-12xy+4y^2 = (3x-2y)^2$

50. a. $(x+7y)(x+7y)$
$$= x^2+7xy+7xy+49y^2$$
$$= x^2+14xy+49y^2$$

b. $x^2+14xy+49y^2 = (x+7y)^2$

51. $9x^2+(\_)+25=(3x)^2+2(3x)(5)+5^2$
$=9x^2+(\underline{30x})+25$

52. $16x^4-(\_)+1=(4x^2)^2-2(4x^2)(1)+1^2$
$=16x^4-(\underline{8x^2})+1$

53. $64z^4+(\_)+t^2=(8z^2)^2+2(8z^2)(t)+t^2$
$=64z^4+(\underline{16z^2t})+t^2$

54. $9m^4-(\_)+49n^2$
$=(3m^2)^2-2(3m^2)(7n)+(7n)^2$
$=9m^4-(\underline{42m^2n})+49n^2$

55. $y^2-8y+16=y^2-2(y)(4)+4^2=(y-4)^2$

56. $x^2+10x+25=x^2+2(x)(5)+5^2=(x+5)^2$

57. $64m^2+80m+25=(8m)^2+2(8m)(5)+5^2$
$=(8m+5)^2$

58. $100c^2-140c+49=(10c)^2-2(10c)(7)+7^2$
$=(10c-7)^2$

59. $w^2-5w+9=w^2-2(w)(3)+3^2$
Not a perfect square trinomial.

60. $2a^2+14a+98=2(a^2+7a+49)$
$=2(a^2-2(a)(7)+7^2)$
Not a perfect square trinomial.

61. $9a^2-30ab+25b^2$
$=(3a)^2-2(3a)(5b)+(5b)^2$
$=(3a-5b)^2$

62. $16x^4-48x^2y+9y^2$
$=(4x^2)^2-2(4x^2)(3y)+(3y)^2$
Not a perfect square trinomial.

63. $16t^2-80tv+20v^2=4(4t^2-20tv+5v^2)$
Not a perfect square trinomial.

64. $12x^2-12xy+3y^2=3(4x^2-4xy+y^2)$
$=3((2x)^2-2(2x)(y)+y^2)$
$=3(2x-y)^2$

65. $5b^4 - 20b^2 + 20 = 5\left(b^4 - 4b^2 + 4\right)$
$= 5\left(\left(b^2\right)^2 - 2\left(b^2\right)(2) + 2^2\right)$
$= 5\left(b^2 - 2\right)^2$

66. $a^4 + 12a^2 + 36 = \left(a^2\right)^2 + 2\left(a^2\right)(6) + 6^2$
$= \left(a^2 + 6\right)^2$

67. **a.** $u^2 - 10u + 25 = u^2 - 2(u)(5) + 5^2$
$= (u-5)^2$

b. $x^4 - 10x^2 + 25 = \left(x^2\right)^2 - 10x^2 + 25$
Let $u = x^2$
$u^2 - 10u + 25 = (u-5)^2$
$= \left(x^2 - 5\right)^2$

c. $(a+1)^2 - 10(a+1) + 25$
Let $u = a+1$
$u^2 - 10u + 25 = (u-5)^2$
$= ((a+1)-5)^2$
$= (a-4)^2$

68. **a.** $u^2 + 12u + 36 = u^2 + 2(u)(6) + 6^2$
$= (u+6)^2$

b. $y^4 + 12y^2 + 36 = \left(y^2\right)^2 + 12y^2 + 36$
Let $u = y^2$
$u^2 + 12u + 36 = (u+6)^2$
$= \left(y^2 + 6\right)^2$

c. $(b-2)^2 + 12(b-2) + 36$
Let $u = b-2$
$u^2 + 12u + 36 = (u+6)^2$
$= ((b-2)+6)^2$
$= (b+4)^2$

69. **a.** $u^2 + 11u - 26 = u^2 + 13u - 2u - 26$
$= u(u+13) - 2(u+13)$
$= (u+13)(u-2)$

b. $w^6 + 11w^3 - 26 = \left(w^3\right)^2 + 11w^3 - 26$
Let $u = w^3$
$u^2 + 11u - 26 = (u+13)(u-2)$
$= \left(w^3 + 13\right)\left(w^3 - 2\right)$

c. $(y-4)^2 + 11(y-4) - 26$
Let $u = y-4$

70. **a.** $u^2 + 17u + 30 = u^2 + 15u + 2u + 30$
$= u(u+15) + 2(u+15)$
$= (u+15)(u+2)$

b. $z^6 + 17z^3 + 30 = \left(z^3\right)^2 + 17z^3 + 30$
Let $u = z^3$
$u^2 + 17u + 30 = (u+15)(u+2)$
$= \left(z^3 + 15\right)\left(z^3 + 2\right)$

c. $(x+3)^2 + 17(x+3) + 30$
Let $u = x+3$

$$u^2+11u-26=(u+13)(u-2)$$
$$=((y-4)+13)((y-4)-2)$$
$$=(y+9)(y-6)$$

$$u^2+17u+30=(u+15)(u+2)$$
$$=((x+3)+15)((x+3)+2)$$
$$=(x+18)(x+5)$$

71. $(3x-1)^2-(3x-1)-6$

Let $u=3x-1$

$$u^2-u-6=u^2-3u+2u-6$$
$$=u(u-3)+2(u-3)$$
$$=(u-3)(u+2)$$
$$=((3x-1)-3)((3x-1)+2)$$
$$=(3x-4)(3x+1)$$

72. $(2x+5)^2-(2x+5)-12$

Let $u=2x+5$

$$u^2-u-12=u^2-4u+3u-12$$
$$=u(u-4)+3(u-4)$$
$$=(u-4)(u+3)$$
$$=((2x+5)-4)((2x+5)+3)$$
$$=(2x+1)(2x+8)$$
$$=2(2x+1)(x+4)$$

73. $2(x-5)^2+9(x-5)+4$

Let $u=x-5$

$$2u^2+9u+4=2u^2+8u+u+4$$
$$=2u(u+4)+(u+4)$$
$$=(u+4)(2u+1)$$
$$=((x-5)+4)(2(x-5)+1)$$
$$=(x-1)(2x-10+1)$$
$$=(x-1)(2x-9)$$

74. $4(x-3)^2+7(x-3)+3$

Let $u=x-3$

$$4u^2+7u+3=4u^2+4u+3u+3$$
$$=4u(u+1)+3(u+1)$$
$$=(u+1)(4u+3)$$
$$=((x-3)+1)(4(x-3)+3)$$
$$=(x-2)(4x-12+3)$$
$$=(x-2)(4x-9)$$

75. $3(y+4)^2+5(y+4)-2$

Let $u=y+4$

$$3u^2+5u-2=3u^2+6u-u-2$$
$$=3u(u+2)-(u+2)$$
$$=(u+2)(3u-1)$$
$$=((y+4)+2)(3(y+4)-1)$$
$$=(y+6)(3y+12-1)$$
$$=(y+6)(3y+11)$$

76. $(3t-2)^2-(3t-2)-20$

Let $u=3t-2$

$$u^2-u-20=u^2+4u-5u-20$$
$$=u(u+4)-5(u+4)$$
$$=(u+4)(u-5)$$
$$=((3t-2)+4)((3t-2)-5)$$
$$=(3t+2)(3t-7)$$

77. $3y^6+11y^3+6$

Let $u=y^3$

$$\begin{aligned}3u^2+11u+6&=3u^2+9u+2u+6\\&=3u(u+3)+2(u+3)\\&=(u+3)(3u+2)\\&=(y^3+3)(3y^3+2)\end{aligned}$$

78. $3x^4-5x^2-12$

Let $u=x^2$

$$\begin{aligned}3u^2-5u-12&=3u^2-9u+4u-12\\&=3u(u-3)+4(u-3)\\&=(u-3)(3u+4)\\&=(x^2-3)(3x^2+4)\end{aligned}$$

79. $4p^4+5p^2+1$

Let $u=p^2$

$$\begin{aligned}4u^2+5u+1&=4u^2+4u+u+1\\&=4u(u+1)+(u+1)\\&=(u+1)(4u+1)\\&=(p^2+1)(4p^2+1)\end{aligned}$$

80. t^4+3t^2+2

Let $u=t^2$

$$\begin{aligned}u^2+3u+2&=u^2+2u+u+2\\&=u(u+2)+(u+2)\\&=(u+2)(u+1)\\&=(t^2+2)(t^2+1)\end{aligned}$$

81. x^4+15x^2+36

Let $u=x^2$

$$\begin{aligned}u^2+15u+36&=u^2+12u+3u+36\\&=u(u+12)+3(u+12)\\&=(u+12)(u+3)\\&=(x^2+12)(x^2+3)\end{aligned}$$

82. t^6-16t^3+63

Let $u=t^3$

$$\begin{aligned}u^2-16u+63&=u^2-9u-7u+63\\&=u(u-9)-7(u-9)\\&=(u-9)(u-7)\\&=(t^3-9)(t^3-7)\end{aligned}$$

83. The factorization $(2y-1)(2y-4)$ is not factored completely because the factor $2y-4$ has a greatest common factor of 2.

84. The factorization $(3w+6)^2$ is not factored completely because the factor $3w+6$ has a greatest common factor of 3.

85. $$\begin{aligned}w^4+12w^2+36&=(w^2)^2+2(w^2)(6)+6^2\\&=(w^2+6)^2\end{aligned}$$

86. $$\begin{aligned}9-6t^2+t^4&=3^2-2(3)(t^2)+(t^2)^2\\&=(3-t^2)^2\end{aligned}$$

87. $81w^2+90w+25=(9w)^2+2(9w)(5)+5^2$
$=(9w+5)^2$

88. $49a^2-28ab+4b^2$
$=(7a)^2-2(7a)(2b)+(2b)^2$
$=(7a-2b)^2$

89. $3x(a+b)-6(a+b)=(a+b)(3x-6)$
$=3(a+b)(x-2)$

90. $4p(t-8)+2(t-8)=2\left[2p(t-8)+(t-8)\right]$
$=2(t-8)(2p+1)$

91. $12a^2bc^2+4ab^2c^2-6abc^3$
$=2abc^2(6a+2b-3c)$

92. $18x^2z-6xyz+30xz^2=6xz(3x-y+5z)$

93. $-20x^3+74x^2-60x$
$=-2x(10x^2-37x+30)$
$=-2x(10x^2-25x-12x+30)$
$=-2x\left[5x(2x-5)-6(2x-5)\right]$
$=-2x(2x-5)(5x-6)$

94. $-24y^3+90y^2-75y$
$=-3y(8y^2-30y+25)$
$=-3y(8y^2-20y-10y+25)$
$=-3y\left[4y(2y-5)-5(2y-5)\right]$
$=-3y(2y-5)(4y-5)$

95. $2y^2-9y-4$
Since there are not two factors of –8 whose sum is –9, the polynomial is prime.

96. $3w^2-12w+4$
Since there are not two factors of 12 whose sum is –12, the polynomial is prime.

97. $2(w^2-5)^2+(w^2-5)-15$
Let $u=w^2-5$
$2u^2+u-15=2u^2+6u-5u-15$
$=2u(u+3)-5(u+3)$
$=(u+3)(2u-5)$
$=\left[(w^2-5)+3\right]\left[2(w^2-5)-5\right]$
$=\left[w^2-5+3\right]\left[2w^2-10-5\right]$
$=(w^2-2)(2w^2-15)$

98. $5(t^2+3)^2+21(t^2+3)+4$
Let $u=t^2+3$
$5u^2+21u+4=5u^2+20u+u+4$
$=5u(u+4)+(u+4)$
$=(u+4)(5u+1)$
$=\left[(t^2+3)+4\right]\left[5(t^2+3)+1\right]$
$=\left[t^2+3+4\right]\left[5t^2+15+1\right]$
$=(t^2+7)(5t^2+16)$

99. $1-4d+3d^2 = 1-3d-d+3d^2$
$= (1-3d)-d(1-3d)$
$= (1-3d)(1-d)$ or $(3d-1)(d-1)$

100. $2-5a+2a^2 = 2-4a-a+2a^2$
$= 2(1-2a)-a(1-2a)$
$= (1-2a)(2-a)$ or $(2a-1)(a-2)$

101. $ax-5a^2+2bx-10ab$
$= a(x-5a)+2b(x-5a)$
$= (x-5a)(a+2b)$

102. $my+y^2-3xm-3xy = y(m+y)-3x(m+y)$
$= (m+y)(y-3x)$

103. $8z^2+24zw-224w^2 = 8(z^2+3zw-28w^2)$
$= 8(z^2+7zw-4zw-28w^2)$
$= 8[z(z+7w)-4w(z+7w)]$
$= 8(z+7w)(z-4w)$

104. $9x^2-18xy-135y^2 = 9(x^2-2xy-15y^2)$
$= 9(x^2-5xy+3xy-15y^2)$
$= 9[x(x-5y)+3y(x-5y)]$
$= 9(x-5y)(x+3y)$

105. $ay+ax-5cy-5cx = a(y+x)-5c(y+x)$
$= (y+x)(a-5c)$

106. $f(x) = 2x^2+13x-7 = 2x^2+14x-x-7$
$= 2x(x+7)-(x+7)$
$= (x+7)(2x-1)$

107. $g(x) = 3x^2+14x+8$
$= 3x^2+12x+2x+8$
$= 3x(x+4)+2(x+4)$
$= (x+4)(3x+2)$

108. $m(t) = t^2-22t+121$
$= t^2-2(t)(11)+11^2$
$= (t-11)^2$

109. $n(t) = t^2+20t+100$
$= t^2+2(t)(10)+10^2$
$= (t+10)^2$

110. $P(x) = x^3+4x^2+3x = x(x^2+4x+3)$
$= x(x^2+3x+x+3)$
$= x[x(x+3)+(x+3)]$
$= x(x+3)(x+1)$

111. $Q(x) = x^4 + 6x^3 + 8x^2$

$= x^2\left(x^2 + 6x + 8\right)$

$= x^2\left(x^2 + 4x + 2x + 8\right)$

$= x^2\left[x(x+4) + 2(x+4)\right]$

$= x^2(x+4)(x+2)$

112. $h(a) = a^3 + 5a^2 - 6a - 30$

$= a^2(a+5) - 6(a+5)$

$= (a+5)\left(a^2 - 6\right)$

113. $k(a) = a^3 - 4a^2 + 2a - 8$

$= a^2(a-4) + 2(a-4)$

$= (a-4)\left(a^2 + 2\right)$

114. $f(x) = 3x^3 - 9x^2 + 5x - 15$

$= 3x^2(x-3) + 5(x-3)$

$= (x-3)\left(3x^2 + 5\right)$

Section 4.7 Practice Exercises

1. **a.** difference; $(a+b)(a-b)$

b. sum

c. is not

d. square

e. sum; cubes

f. difference; cubes

g. $a-b$; $a^2 + ab + b^2$

h. $a+b$; $a^2 - ab + b^2$

2. **a.** Any expression raised to an even power (multiple of 2) is a perfect square. The expressions that can be written as an expression raised to an even power are:

$4 = 2^2$, $16 = 4^2$, $25 = 5^2$, $64 = 8^2$, x^2, and $x^4 = \left(x^2\right)^2$.

b. Any expression raised to a power that is a multiple of 3 is a perfect cube. The expressions that can be written as an expression raised to a power of 3 are:

$8 = 2^3$, $64 = 4^3$, x^3, and $x^9 = \left(x^3\right)^3$.

3. $4x^2 - 20x + 25 = (2x)^2 - 2(2x)(5) + 5^2$

$= (2x-5)^2$

4. $9t^2 - 42t + 49 = (3t)^2 - 2(3t)(7) + 7^2$

$= (3t-7)^2$

5. $10x + 6xy + 5 + 3y = 2x(5+3y) + (5+3y)$

$= (5+3y)(2x+1)$

6. $21a + 7ab - 3b - b^2 = 7a(3+b) - b(3+b)$

$= (3+b)(7a-b)$

7. $32p^2-28p-4=4\left(8p^2-7p-1\right)$
$=4\left(8p^2-8p+p-1\right)$
$=4\left[8p\left(p-1\right)+\left(p-1\right)\right]$
$=4\left(p-1\right)\left(8p+1\right)$

8. $6q^2+37q-35=6q^2+42q-5q-35$
$=6q\left(q+7\right)-5\left(q+7\right)$
$=\left(q+7\right)\left(6q-5\right)$

9. Look for a binomial of the form a^2-b^2 ; $a^2-b^2=\left(a+b\right)\left(a-b\right)$

10. No; a sum of squares is prime.

11. $x^2-9=x^2-3^2=\left(x+3\right)\left(x-3\right)$

12. $y^2-25=y^2-5^2=\left(y+5\right)\left(y-5\right)$

13. $16-49w^2=4^2-\left(7w\right)^2=\left(4+7w\right)\left(4-7w\right)$

14. $81-64b^2=9^2-\left(8b\right)^2=\left(9+8b\right)\left(9-8b\right)$

15. $8a^2-162b^2=2\left(4a^2-81b^2\right)$
$=2\left[\left(2a\right)^2-\left(9b\right)^2\right]$
$=2\left(2a+9b\right)\left(2a-9b\right)$

16. $50c^2-72d^2=2\left(25c^2-36d^2\right)$
$=2\left[\left(5c\right)^2-\left(6d\right)^2\right]$
$=2\left(5c+6d\right)\left(5c-6d\right)$

17. $25u^2+1$ Prime

18. w^2+4 Prime

19. $2a^4-32=2\left(a^4-16\right)$
$=2\left(a^2+4\right)\left(a^2-4\right)$
$=2\left(a^2+4\right)\left(a+2\right)\left(a-2\right)$

20. $5y^4-5=5\left(y^4-1\right)$
$=5\left(y^2+1\right)\left(y^2-1\right)$
$=5\left(y^2+1\right)\left(y+1\right)\left(y-1\right)$

21. $49-k^6=7^2-\left(k^3\right)^2=\left(7+k^3\right)\left(7-k^3\right)$

22. $4-h^6=2^2-\left(h^3\right)^2=\left(2+h^3\right)\left(2-h^3\right)$

23. $x^3-x^2-16x+16=x^2\left(x-1\right)-16\left(x-1\right)$
$=\left(x-1\right)\left(x^2-16\right)$
$=\left(x-1\right)\left(x^2-4^2\right)$
$=\left(x-1\right)\left(x+4\right)\left(x-4\right)$

24. $x^3+5x^2-x-5=x^2\left(x+5\right)-1\left(x+5\right)$
$=\left(x+5\right)\left(x^2-1\right)$
$=\left(x+5\right)\left(x+1\right)\left(x-1\right)$

25. $4x^3+12x^2-x-3=4x^2\left(x+3\right)-\left(x+3\right)$
$=\left(x+3\right)\left(4x^2-1\right)$
$=\left(x+3\right)\left(\left(2x\right)^2-1^2\right)$
$=\left(x+3\right)\left(2x+1\right)\left(2x-1\right)$

26. $5x^3-x^2-45x+9=x^2\left(5x-1\right)-9\left(5x-1\right)$
$=\left(5x-1\right)\left(x^2-9\right)$
$=\left(5x-1\right)\left(x+3\right)\left(x-3\right)$

27. $9y^3+7y^2-36y-28$
$=y^2\left(9y+7\right)-4\left(9y+7\right)$
$=\left(9y+7\right)\left(y^2-4\right)$
$=\left(9y+7\right)\left(y^2-2^2\right)$
$=\left(9y+7\right)\left(y+2\right)\left(y-2\right)$

28. $9z^3-5z^2-36z+20=z^2\left(9z-5\right)-4\left(9z-5\right)$
$=\left(9z-5\right)\left(z^2-4\right)$
$=\left(9z-5\right)\left(z+2\right)\left(z-2\right)$

29. $49x^2+28x+4-y^2=\left(49x^2+28x+4\right)-y^2$
$=\left(7x+2\right)^2-y^2$
$=\left(7x+2+y\right)\left(7x+2-y\right)$

30. $100y^2+140y+49-z^2$
$=\left(100y^2+140y+49\right)-z^2$
$=\left(10y+7\right)^2-z^2$
$=\left(10y+7+z\right)\left(10y+7-z\right)$

31. $w^2-9n^2+6n-1=w^2-\left(9n^2-6n+1\right)$
$=w^2-\left(3n-1\right)^2$
$=\left[w+\left(3n-1\right)\right]\left[w-\left(3n-1\right)\right]$
$=\left(w+3n-1\right)\left(w-3n+1\right)$

32. $m^2-25c^2+20c-4=m^2-\left(25c^2-20c+4\right)$
$=m^2-\left(5c-2\right)^2$
$=\left[m+\left(5c-2\right)\right]\left[m-\left(5c-2\right)\right]$
$=\left(m+5c-2\right)\left(m-5c+2\right)$

33. $p^4-10p^2+25-t^4=\left(p^4-10p^2+25\right)-t^4$
$=\left(p^2-5\right)^2-\left(t^2\right)^2$
$=\left(p^2-5+t^2\right)\left(p^2-5-t^2\right)$

34. $m^4-14m^2+49-z^4=\left(m^4-14m^2+49\right)-z^4$
$=\left(m^2-7\right)^2-\left(z^2\right)^2$
$=\left(m^2-7+z^2\right)\left(m^2-7-z^2\right)$

35. $9u^4 - 4v^4 + 20v^2 - 25$

$= 9u^4 - \left(4v^4 - 20v^2 + 25\right)$

$= \left(3u^2\right)^2 - \left(2v^2 - 5\right)^2$

$= \left[3u^2 + \left(2v^2 - 5\right)\right]\left[3u^2 - \left(2v^2 - 5\right)\right]$

$= \left(3u^2 + 2v^2 - 5\right)\left(3u^2 - 2v^2 + 5\right)$

36. $x^4 - 9y^4 - 42y^2 - 49 = x^4 - \left(9y^4 + 42y^2 + 49\right)$

$= \left(x^2\right)^2 - \left(3y^2 + 7\right)^2$

$= \left[x^2 + \left(3y^2 + 7\right)\right]\left[x^2 - \left(3y^2 + 7\right)\right]$

$= \left(x^2 + 3y^2 + 7\right)\left(x^2 - 3y^2 - 7\right)$

37. Look for a binomial of the form $a^3 + b^3$;

$a^3 + b^3 = \left(a + b\right)\left(a^2 - ab + b^2\right)$

38. Look for a binomial of the form $a^3 - b^3$;

$a^3 - b^3 = \left(a - b\right)\left(a^2 + ab + b^2\right)$

39. $8x^3 - 1 = \left(2x\right)^3 - 1^3$

$= \left(2x - 1\right)\left[\left(2x\right)^2 + \left(2x\right)\left(1\right) + 1^2\right]$

$= \left(2x - 1\right)\left(4x^2 + 2x + 1\right)$

Check:

$\left(2x - 1\right)\left(4x^2 + 2x + 1\right)$

$= 8x^3 + 4x^2 + 2x - 4x^2 - 2x - 1$

$= 8x^3 - 1$

40. $y^3 + 64 = y^3 + 4^3$

$= \left(y + 4\right)\left[y^2 - \left(y\right)\left(4\right) + 4^2\right]$

$= \left(y + 4\right)\left(y^2 - 4y + 16\right)$

Check:

$\left(y + 4\right)\left(y^2 - 4y + 16\right)$

$= y^3 - 4y^2 + 16y + 4y^2 - 16y + 64$

$= y^3 + 64$

41. $125c^3 + 27 = \left(5c\right)^3 + 3^3$

$= \left(5c + 3\right)\left[\left(5c\right)^2 - \left(5c\right)\left(3\right) + 3^2\right]$

$= \left(5c + 3\right)\left(25c^2 - 15c + 9\right)$

42. $216u^3 - v^3 = \left(6u\right)^3 - v^3$

$= \left(6u - v\right)\left[\left(6u\right)^2 + \left(6u\right)\left(v\right) + v^2\right]$

$= \left(6u - v\right)\left(36u^2 + 6uv + v^2\right)$

43. $x^3 - 1000 = x^3 - 10^3$

$= \left(x - 10\right)\left[x^2 + \left(x\right)\left(10\right) + 10^2\right]$

$= \left(x - 10\right)\left(x^2 + 10x + 100\right)$

44. $y^3 - 27 = y^3 - 3^3$

$= \left(y - 3\right)\left[y^2 + y\left(3\right) + 3^2\right]$

$= \left(y - 3\right)\left(y^2 + 3y + 9\right)$

45. $64t^6+1=\left(4t^2\right)^3+1^3$
$=\left(4t^2+1\right)\left[\left(4t^2\right)^2-\left(4t^2\right)(1)+1^2\right]$
$=\left(4t^2+1\right)\left(16t^4-4t^2+1\right)$

46. $125r^6+1=\left(5r^2\right)^3+1^3$
$=\left(5r^2+1\right)\left[\left(5r^2\right)^2-\left(5r^2\right)(1)+1^2\right]$
$=\left(5r^2+1\right)\left(25r^4-5r^2+1\right)$

47. $2000y^6+2x^3=2\left(1000y^6+x^3\right)$
$=2\left[\left(10y^2\right)^3+x^3\right]$
$=2\left(10y^2+x\right)\left[\left(10y^2\right)^2-\left(10y^2\right)(x)+x^2\right]$
$=2\left(10y^2+x\right)\left(100y^4-10y^2x+x^2\right)$

48. $3a^6+24b^3=3\left(a^6+8b^3\right)$
$=3\left[\left(a^2\right)^3+(2b)^3\right]$
$=3\left(a^2+2b\right)\left[\left(a^2\right)^2-a^2(2b)+(2b)^2\right]$
$=3\left(a^2+2b\right)\left(a^4-2a^2b+4b^2\right)$

49. $16z^4-54z=2z\left(8z^3-27\right)$
$=2z\left[(2z)^3-3^3\right]$
$=2z(2z-3)\left[(2z)^2+(2z)(3)+3^2\right]$
$=2z(2z-3)\left(4z^2+6z+9\right)$

50. $x^5-64x^2=x^2\left(x^3-64\right)$
$=x^2\left(x^3-4^3\right)$
$=x^2(x-4)\left(x^2+x(4)+4^2\right)$
$=x^2(x-4)\left(x^2+4x+16\right)$

51. $p^{12}-125=\left(p^4\right)^3-5^3$
$=\left(p^4-5\right)\left[\left(p^4\right)^2+p^4(5)+5^2\right]$
$=\left(p^4-5\right)\left(p^8+5p^4+25\right)$

52. $t^9-8=\left(t^3\right)^3-2^3$
$=\left(t^3-2\right)\left[\left(t^3\right)^2+t^3(2)+2^2\right]$
$=\left(t^3-2\right)\left(t^6+2t^3+4\right)$

53. $36y^2-\frac{1}{25}=(6y)^2-\left(\frac{1}{5}\right)^2$
$=\left(6y+\frac{1}{5}\right)\left(6y-\frac{1}{5}\right)$

54. $16p^2-\frac{1}{9}=(4p)^2-\left(\frac{1}{3}\right)^2$
$=\left(4p+\frac{1}{3}\right)\left(4p-\frac{1}{3}\right)$

55. $18d^{12}-32=2\left(9d^{12}-16\right)=2\left[\left(3d^6\right)^2-4^2\right]$
$=2\left(3d^6+4\right)\left(3d^6-4\right)$

56. $3z^8-12=3\left(z^8-4\right)=3\left[\left(z^4\right)^2-2^2\right]$
$=3\left(z^4+2\right)\left(z^4-2\right)$

57. $242v^2+32=2\left(121v^2+16\right)$

58. $8p^2+200=8\left(p^2+25\right)$

59. $4x^2-16=4\left(x^2-4\right)=4\left(x^2-2^2\right)$
$=4\left(x+2\right)\left(x-2\right)$

60. $9m^2-81n^2=9\left(m^2-9n^2\right)=9\left[m^2-\left(3n\right)^2\right]$
$=9\left(m+3n\right)\left(m-3n\right)$

61. $25-49q^2=5^2-\left(7q\right)^2$
$=\left(5+7q\right)\left(5-7q\right)$

62. $1-25p^2=1^2-\left(5p\right)^2$
$=\left(1+5p\right)\left(1-5p\right)$

63. $\left(t+2s\right)^2-36=\left(t+2s\right)^2-6^2$
$=\left(t+2s+6\right)\left(t+2s-6\right)$

64. $\left(5x+4\right)^2-y^2=\left(5x+4+y\right)\left(5x+4-y\right)$

65. $27-t^3=3^3-t^3$
$=\left(3-t\right)\left[3^2+\left(3\right)\left(t\right)+t^2\right]$
$=\left(3-t\right)\left(9+3t+t^2\right)$

66. $8+y^3=2^3+y^3$
$=\left(2+y\right)\left[2^2-\left(2\right)\left(y\right)+y^2\right]$
$=\left(2+y\right)\left(4-2y+y^2\right)$

67. $27a^3+\frac{1}{8}=\left(3a\right)^3+\left(\frac{1}{2}\right)^3$
$=\left(3a+\frac{1}{2}\right)\left[\left(3a\right)^2-\left(3a\right)\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2\right]$
$=\left(3a+\frac{1}{2}\right)\left(9a^2-\frac{3}{2}a+\frac{1}{4}\right)$

68. $b^3+\frac{27}{125}=b^3+\left(\frac{3}{5}\right)^3$
$=\left(b+\frac{3}{5}\right)\left[b^2-\left(b\right)\left(\frac{3}{5}\right)+\left(\frac{3}{5}\right)^2\right]$
$=\left(b+\frac{3}{5}\right)\left(b^2-\frac{3}{5}b+\frac{9}{25}\right)$

69. $2m^3+16=2\left(m^3+8\right)=2\left(m^3+2^3\right)$
$=2\left(m+2\right)\left[m^2-\left(m\right)\left(2\right)+2^2\right]$
$=2\left(m+2\right)\left(m^2-2m+4\right)$

70. $3x^3-375=3\left(x^3-125\right)=3\left(x^3-5^3\right)$
$=3\left(x-5\right)\left[x^2+\left(x\right)\left(5\right)+5^2\right]$
$=3\left(x-5\right)\left(x^2+5x+25\right)$

71. $x^4 - y^4 = \left(x^2\right)^2 - \left(y^2\right)^2$
$= \left(x^2 + y^2\right)\left(x^2 - y^2\right)$
$= \left(x^2 + y^2\right)(x + y)(x - y)$

72. $81u^4 - 16v^4 = \left(9u^2\right)^2 - \left(4v^2\right)^2$
$= \left(9u^2 + 4v^2\right)\left(9u^2 - 4v^2\right)$
$= \left(9u^2 + 4v^2\right)\left((3u)^2 - (2v)^2\right)$
$= \left(9u^2 + 4v^2\right)(3u + 2v)(3u - 2v)$

73. $a^9 + b^9 = \left(a^3\right)^3 + \left(b^3\right)^3$
$= \left(a^3 + b^3\right)\left[\left(a^3\right)^2 - \left(a^3\right)\left(b^3\right) + \left(b^3\right)^2\right]$
$= \left(a^3 + b^3\right)\left(a^6 - a^3b^3 + b^6\right)$
$= (a + b)\left[a^2 - (a)(b) + b^2\right]\left(a^6 - a^3b^3 + b^6\right)$
$= (a + b)\left(a^2 - ab + b^2\right)\left(a^6 - a^3b^3 + b^6\right)$

74. $27m^9 - 8n^9 = \left(3m^3\right)^3 - \left(2n^3\right)^3$
$= \left(3m^3 - 2n^3\right)\left[\left(3m^3\right)^2 + \left(3m^3\right)\left(2n^3\right) + \left(2n^3\right)^2\right]$
$= \left(3m^3 - 2n^3\right)\left(9m^6 + 6m^3n^3 + 4n^6\right)$

75. $\frac{1}{8}p^3 - \frac{1}{125} = \left(\frac{1}{2}p\right)^3 - \left(\frac{1}{5}\right)^3$
$= \left(\frac{1}{2}p - \frac{1}{5}\right)\left[\left(\frac{1}{2}p\right)^2 + \left(\frac{1}{2}p\right)\left(\frac{1}{5}\right) + \left(\frac{1}{5}\right)^2\right]$
$= \left(\frac{1}{2}p - \frac{1}{5}\right)\left(\frac{1}{4}p^2 + \frac{1}{10}p + \frac{1}{25}\right)$

76. $1 - \frac{1}{27}d^3 = 1^3 - \left(\frac{1}{3}d\right)^3$
$= \left(1 - \frac{1}{3}d\right)\left[1^2 + 1\left(\frac{1}{3}d\right) + \left(\frac{1}{3}d\right)^2\right]$
$= \left(1 - \frac{1}{3}d\right)\left(1 + \frac{1}{3}d + \frac{1}{9}d^2\right)$

77. $4w^2 + 25$ Prime

78. $64 + a^2$ Prime

79. $\frac{1}{25}x^2 - \frac{1}{4}y^2 = \left(\frac{1}{5}x\right)^2 - \left(\frac{1}{2}y\right)^2$
$= \left(\frac{1}{5}x + \frac{1}{2}y\right)\left(\frac{1}{5}x - \frac{1}{2}y\right)$

80. $\frac{1}{100}a^2 - \frac{4}{49}b^2 = \left(\frac{1}{10}a\right)^2 - \left(\frac{2}{7}b\right)^2$
$= \left(\frac{1}{10}a + \frac{2}{7}b\right)\left(\frac{1}{10}a - \frac{2}{7}b\right)$

81. $a^6 - b^6$

$= \left(a^3\right)^2 - \left(b^3\right)^2$

$= \left(a^3 + b^3\right)\left(a^3 - b^3\right)$

$= (a+b)\left(a^2 - ab + b^2\right)(a-b)\left(a^2 + ab + b^2\right)$

82. $64x^6 - y^6$

$= \left(8x^3\right)^2 - \left(y^3\right)^2$

$= \left(8x^3 + y^3\right)\left(8x^3 - y^3\right)$

$= \left[(2x)^3 + y^3\right]\left[(2x)^3 - y^3\right]$

$= (2x+y)\left(4x^2 - 2xy + y^2\right)$

$(2x-y)\left(4x^2 + 2xy + y^2\right)$

83. $64 - y^6 = 8^2 - \left(y^3\right)^2$

$= \left(8 + y^3\right)\left(8 - y^3\right)$

$= \left[2^3 + y^3\right]\left[2^3 - y^3\right]$

$= (2+y)\left(4 - 2y + y^2\right)(2-y)\left(4 + 2y + y^2\right)$

84. $1 - p^6 = 1^2 - \left(p^3\right)^2$

$= \left(1 + p^3\right)\left(1 - p^3\right)$

$= \left[1^3 + p^3\right]\left[1^3 - p^3\right]$

$= (1+p)\left(1 - p + p^2\right)(1-p)\left(1 + p + p^2\right)$

85. $h^6 + k^6 = \left(h^2\right)^3 + \left(k^2\right)^3$

$= \left(h^2 + k^2\right)\left(h^4 - h^2k^2 + k^4\right)$

86. $27q^6 + 125p^6 = \left(3q^2\right)^3 + \left(5p^2\right)^3$

$= \left(3q^2 + 5p^2\right)\left[\left(3q^2\right)^2 - \left(3q^2\right)\left(5p^2\right) + \left(5p^2\right)^2\right]$

$= \left(3q^2 + 5p^2\right)\left(9q^4 - 15q^2p^2 + 25p^4\right)$

87. $8x^6 + 125 = \left(2x^2\right)^3 + 5^3$

$= \left(2x^2 + 5\right)\left[\left(2x^2\right)^2 - \left(2x^2\right)(5) + 5^2\right]$

$= \left(2x^2 + 5\right)\left(4x^4 - 10x^2 + 25\right)$

88. $t^6 + 1 = \left(t^2\right)^3 + 1^3$

$= \left(t^2 + 1\right)\left[\left(t^2\right)^2 - \left(t^2\right)(1) + 1^2\right]$

$= \left(t^2 + 1\right)\left(t^4 - t^2 + 1\right)$

89. $(2x+3)(2x-3) = (2x)^2 - 3^2 = 4x^2 - 9$

90. $(4-p)(4+p) = 4^2 - p^2 = 16 - p^2$

91. $\left(4a^2 + 6a + 9\right)(2a-3) = (2a)^3 - 3^3$

$= 8a^3 - 27$

92. $\left(25c^2 - 10cd + 4d^2\right)(5c + 2d) = (5c)^3 + (2d)^3$

$= 125c^3 + 8d^3$

93. $\left(4x^2+y\right)\left(16x^4-4x^2y+y^2\right)=\left(4x^2\right)^3+y^3$
$=64x^6+y^3$

94. $\left(3t-r^2\right)\left(9t^2+3tr^2+r^4\right)=\left(3t\right)^3-\left(r^2\right)^3$
$=27t^3-r^6$

95. **a.** $A=x^2-y^2$

b. $x^2-y^2=\left(x+y\right)\left(x-y\right)$

c. $A=x^2-y^2$
$=6^2-4^2=36-16=20\,\text{in}^2$

96. **a.** $A=\pi R^2-\pi r^2$

b. $\pi R^2-\pi r^2=\pi\left(R^2-r^2\right)$
$=\pi\left(R+r\right)\left(R-r\right)$

c. $A=\pi R^2-\pi r^2$
$=\pi\left(\frac{1}{2}\right)^2-\pi\left(\frac{1}{4}\right)^2=\frac{1}{4}\pi-\frac{1}{16}\pi$
$=\frac{3}{16}\pi\,\text{in}^2\approx 0.59\,\text{in}^2$

97. $x^2-y^2+x+y=\left(x+y\right)\left(x-y\right)+\left(x+y\right)$
$=\left(x+y\right)\left(x-y+1\right)$

98. $64m^2-25n^2+8m+5n$
$=\left(8m+5n\right)\left(8m-5n\right)+\left(8m+5n\right)$
$=\left(8m+5n\right)\left(8m-5n+1\right)$

99. x^3+y^3+x+y
$=\left(x+y\right)\left(x^2-xy+y^2\right)+x+y$
$=\left(x+y\right)\left(x^2-xy+y^2+1\right)$

100. $4pu^3-4pv^3-7yu^3+7yv^3$
$=4p\left(u^3-v^3\right)-7y\left(u^3-v^3\right)$
$=\left(u^3-v^3\right)\left(4p-7y\right)$
$=\left(u-v\right)\left(u^2+uv+v^2\right)\left(4p-7y\right)$

101. $576a^5-9a^2-64a^3c^2+c^2$
$=9a^2(64a^3-1)-c^2(64a^3-1)$
$=(9a^2-c^2)(64a^3-1)$
$=(3a-c)(3a+c)(4a-1)(16a^2+4a+1)$

102. $32t^5-108t^2-72t^3v^2+243v^2$
$=4t^2(8t^3-27)-9v^2(8t^3-27)$
$=(4t^2-9v^2)(8t^3-27)$
$=(2t-3v)(2t+3v)(2t-3)(4t^2+6t+9)$

Problem Recognition Exercises:

1. A prime factor is an expression whose only factors are 1 and itself.

2. The first step in factoring is to factor out the greatest common factor.

3. When factoring binomials, look for:
Difference of squares: $a^2 - b^2$;
Difference of cubes: $a^3 - b^3$; or
Sums of cubes: $a^3 + b^3$.

4. Look for a perfect square trinomial $a^2 + 2ab + b^2$ or $a^2 - 2ab + b^2$.

5. Try factoring by grouping (2 terms and two terms) or grouping 3 terms and one term.

6. Let $u = 4x^2 + 1$. The polynomial becomes $3u^2 + 20u + 12$. Factor this simpler expression and then back substitute.

7. **a.** Trinomial

b.
$$\begin{aligned} 6x^2 - 21x - 45 &= 3\left(2x^2 - 7x - 15\right) \\ &= 3\left(2x^2 - 10x + 3x - 15\right) \\ &= 3\left[2x\left(x-5\right) + 3\left(x-5\right)\right] \\ &= 3\left(x-5\right)\left(2x+3\right) \end{aligned}$$

8. **a.** Trinomial

b.
$$\begin{aligned} 8m^3 - 10m^2 - 3m &= m\left(8m^2 - 10m - 3\right) \\ &= m\left(8m^2 - 12m + 2m - 3\right) \\ &= m\left[4m\left(2m-3\right) + \left(2m-3\right)\right] \\ &= m\left(2m-3\right)\left(4m+1\right) \end{aligned}$$

9. **a.** Difference of squares

b.
$$\begin{aligned} 8a^2 - 50 &= 2\left(4a^2 - 25\right) = 2\left[\left(2a\right)^2 - 5^2\right] \\ &= 2\left(2a+5\right)\left(2a-5\right) \end{aligned}$$

10. **a.** Grouping

b.
$$\begin{aligned} ab + ay - b^2 - by &= a\left(b+y\right) - b\left(b+y\right) \\ &= \left(b+y\right)\left(a-b\right) \end{aligned}$$

11. **a.** Trinomial

b.
$$\begin{aligned} &14u^2 - 11uv + 2v^2 \\ &= 14u^2 - 7uv - 4uv + 2v^2 \\ &= 7u\left(2u-v\right) - 2v\left(2u-v\right) \\ &= \left(2u-v\right)\left(7u-2v\right) \end{aligned}$$

12. **a.** Perfect square trinomial

b.
$$\begin{aligned} &9p^2 - 12pq + 4q^2 \\ &= \left(3p\right)^2 - 2\left(3p\right)\left(2q\right) + \left(2q\right)^2 \\ &= \left(3p - 2q\right)^2 \end{aligned}$$

13. **a.** Difference of cubes

b.
$$\begin{aligned} 16x^3 - 2 &= 2\left(8x^3 - 1\right) = 2\left[\left(2x\right)^3 - 1^3\right] \\ &= 2\left(2x-1\right)\left(4x^2 + 2x + 1\right) \end{aligned}$$

14. **a.** Sum of squares

b. $9m^2 + 16n^2$ is prime.

15. **a.** Sum of cubes

b. $27y^3+125=(3y)^3+5^3$
$=(3y+5)(9y^2-15y+25)$

16. **a.** None of these

b. $3x^2-16$ is prime.

17. **a.** Sum of cubes

b. $128p^6+54q^3=2(64p^6+27q^3)$
$=2\left[(4p^2)^3+(3q)^3\right]$
$=2(4p^2+3q)(16p^4-12p^2q+9q^2)$

18. **a.** Perfect square trinomial

b. $5b^2-30b+45$
$=5(b^2-6b+9)$
$=5\left[b^2-2(b)(3)+3^2\right]$
$=5(b-3)^2$

19. **a.** Difference of squares

b. $16a^4-1=(4a^2)-1^2$
$=(4a^2+1)(4a^2-1)$
$=(4a^2+1)(2a+1)(2a-1)$

20. **a.** Perfect square trinomial

b. $81u^2-90uv+25v^2$
$=(9u)^2-2(9u)(5v)+(5v)^2$
$=(9u-5v)^2$

21. **a.** Grouping

b. $p^2-12p+36-c^2=(p-6)^2-c^2$
$=(p-6+c)(p-6-c)$

22. **a.** Sum of squares

b. $4x^2+16=4(x^2+4)$

23. **a.** Grouping

b. $12ax-6ay+4bx-2by$
$=2(6ax-3ay+2bx-by)$
$=2\left[3a(2x-y)+b(2x-y)\right]$
$=2(2x-y)(3a+b)$

24. **a.** Difference of cubes

b. $125y^3-8=(5y)^3-2^3$
$=(5y-2)(25y^2+10y+4)$

25. **a.** Trinomial

26. **a.** Difference of squares

b. $5y^2+14y-3=5y^2+15y-y-3$
$=5y(y+3)-(y+3)$
$=(y+3)(5y-1)$

b. $2m^4-128=2(m^4-64)$
$=2\left[(m^2)^2-8^2\right]$
$=2(m^2+8)(m^2-8)$

27. **a.** Difference of squares

b. $t^2-100=t^2-10^2=(t-10)(t+10)$

28. **a.** Difference of squares

b. $4m^2-49n^2=(2m)^2-(7n)^2$
$=(2m-7n)(2m+7n)$

29. **a.** Sum of cubes

b. $y^3+27=y^3+3^3$
$=(y+3)(y^2-3y+9)$

30. **a.** Sum of cubes

b. $x^3+1=x^3+1^3$
$=(x+1)(x^2-x+1)$

31. **a.** Trinomial

b. $d^2+3d-28=(d+7)(d-4)$

32. **a.** Trinomial

b. $c^2+5c-24=(c+8)(c-3)$

33. **a.** Perfect square trinomial

b. $x^2-12x+36=x^2-2(x)(6)+(6)^2$
$=(x-6)^2$

34. **a.** Perfect square trinomial

b. $p^2+16p+64=p^2+2(p)(8)+(8)^2$
$=(p+8)^2$

35. **a.** Grouping

b. $2ax^2-5ax+2bx-5b$
$=ax(2x-5)+b(2x-5)$
$=(ax+b)(2x-5)$

36. **a.** Grouping

b. $8x^2-4bx+2ax-ab$
$=4x(2x-b)+a(2x-b)$
$=(4x+a)(2x-b)$

37. **a.** Trinomial

b. $10y^2+3y-4=(2y-1)(5y+4)$

38. **a.** Trinomial

b. $12z^2+11z+2=(4z+1)(3z+2)$

39. **a.** Difference of squares

b. $10p^2-640=10(p^2-64)$
$=10(p-8)(p+8)$

40. **a.** Difference of squares

b. $50a^2-72=2(25a^2-36)$
$=2(5a-6)(5a+6)$

41. **a.** Difference of cubes

b. $z^4 - 64z = z(z^3 - 64)$
$= z(z-4)(z^2 + 4z + 16)$

42. **a.** Difference of cubes

b. $t^4 - 8t = t(t^3 - 8)$
$= t(t-2)(t^2 + 2t + 4)$

43. **a.** Trinomial

b. $b^3 - 4b^2 - 45b = b(b^2 - 4b - 45)$
$= b(b-9)(b+5)$

44. **a.** Trinomial

b. $y^3 - 14y^2 + 40y = y(y^2 - 14y + 40)$
$= y(y-10)(y-4)$

45. **a.** Perfect square trinomial

b. $9w^2 + 24wx + 16x^2$
$= (3w)^2 + 2(3w)(4x) + (4x)^2$
$= (3w + 4x)^2$

46. **a.** Perfect square trinomial

b. $4k^2 - 20kp + 25p^2$
$= (2k)^2 - 2(2k)(5p) + (5p)^2$
$= (2k - 5p)^2$

47. **a.** Grouping

b. $60x^2 - 20x + 30ax - 10a$
$= 10(6x^2 - 2x + 3ax - a)$
$= 10[2x(3x-1) + a(3x-1)]$
$= 10(2x + a)(3x - 1)$

48. **a.** Grouping

b. $50x^2 - 200x + 10cx - 40c$
$= 10(5x^2 - 20x + cx - 4c)$
$= 10[5x(x-4) + c(x-4)]$
$= 10(5x + c)(x - 4)$

49. **a.** Difference of squares

b. $w^4 - 16 = (w^2 - 4)(w^2 + 4)$
$= (w-2)(w+2)(w^2 + 4)$

50. **a.** Difference of squares

b. $k^4 - 81 = (k^2 - 9)(k^2 + 9)$
$= (k-3)(k+3)(k^2 + 9)$

51. **a.** Difference of cubes

b. $t^6 - 8 = (t^2)^3 - 2^3$
$= (t^2 - 2)(t^4 + 2t^2 + 4)$

52. **a.** Sum of cubes

b. $p^6 + 27 = (p^2)^3 + 3^3$
$= (p^2 + 3)(p^4 - 3p^2 + 9)$

53. **a.** Trinomial

b. $8p^2 - 22p + 5 = (4p - 1)(2p - 5)$

54. **a.** Trinomial

b. $9m^2 - 3m - 20 = (3m + 4)(3m - 5)$

55. **a.** Perfect square trinomial

56. **a.** Perfect square trinomial

b. $36y^2 - 12y + 1$
$= (6y)^2 - 2(6y)(1) + (1)^2$
$= (6y - 1)^2$

b. $9a^2 + 42a + 49$
$= (3a)^2 + 2(3a)(7) + 7^2$
$= (3a + 7)^2$

57. **a.** Sum of squares

b. $2x^2 + 50 = 2(x^2 + 25)$

58. **a.** Sum of squares

b. $4y^2 + 64 = 4(y^2 + 16)$

59. **a.** Trinomial

b. $12r^2s^2 + 7rs^2 - 10s^2$
$= s^2(12r^2 + 7r - 10)$
$= s^2(4r + 5)(3r - 2)$

60. **a.** Trinomial

b. $7z^2w^2 - 10zw^2 - 8w^2$
$= w^2(7z^2 - 10z - 8)$
$= w^2(7z + 4)(z - 2)$

61. **a.** Trinomial

b. $x^2 + 8xy - 33y^2 = (x - 3y)(x + 11y)$

62. **a.** Trinomial

b. $s^2 - 9st - 36t^2 = (s - 12t)(s + 3t)$

63. **a.** Sum of cubes

b. $m^6 + n^3 = (m^2)^3 + n^3$
$= (m^2 + n)(m^4 - m^2n + n^2)$

64. **a.** Difference of cubes

b. $a^3 - b^6 = a^3 - (b^2)^3$
$= (a - b^2)(a^2 + ab^2 + b^4)$

65. **a.** None of these

b. $x^2 - 4x = x(x - 4)$

66. **a.** None of these

b. $y^2 - 9y = y(y - 9)$

67. $x^2(x + y) - y^2(x + y)$
$= (x + y)(x^2 - y^2)$
$= (x + y)(x + y)(x - y)$
$= (x + y)^2(x - y)$

68. $u^2(u - v) - v^2(u - v)$
$= (u - v)(u^2 - v^2)$
$= (u - v)(u - v)(u + v)$
$= (u - v)^2(u + v)$

69. $(a + 3)^4 + 6(a + 3)^5 = (a + 3)^4(1 + 6(a + 3))$
$= (a + 3)^4(1 + 6a + 18)$
$= (a + 3)^4(6a + 19)$

70. $(4 - b)^4 - 2(4 - b)^3 = (4 - b)^3(4 - b - 2)$
$= (4 - b)^3(2 - b)$

71. $24(3x+5)^3 - 30(3x+5)^2$

$= 6(3x+5)^2[4(3x+5)-5]$

$= 6(3x+5)^2[12x+15]$

$= 6(3x+5)^2 3(4x+5)$

$= 18(3x+5)^2(4x+5)$

72. $10(2y+3)^2 + 15(2y+3)^3$

$= 5(2y+3)^2[2+3(2y+3)]$

$= 5(2y+3)^2[2+6y+9]$

$= 5(2y+3)^2(6y+11)$

73. $\frac{1}{100}x^2 + \frac{1}{35}x + \frac{1}{49}$

$= \left(\frac{1}{10}x\right)^2 + 2\left(\frac{1}{10}x\right)\left(\frac{1}{7}\right) + \left(\frac{1}{7}\right)^2$

$= \left(\frac{1}{10}x + \frac{1}{7}\right)^2$

74. $\frac{1}{25}a^2 + \frac{1}{15}a + \frac{1}{36}$

$= \left(\frac{1}{5}a\right)^2 + 2\left(\frac{1}{5}a\right)\left(\frac{1}{6}\right) + \left(\frac{1}{6}\right)^2$

$= \left(\frac{1}{5}a + \frac{1}{6}\right)^2$

75. $\left(5x^2-1\right)^2 - 4\left(5x^2-1\right) - 5$

Let $u = 5x^2 - 1$

$u^2 - 4u - 5 = (u-5)(u+1)$

$= \left(5x^2-1-5\right)\left(5x^2-1+1\right)$

$= \left(5x^2-6\right)\left(5x^2\right)$

76. $\left(x^3+4\right)^2 - 10\left(x^3+4\right) + 24$

Let $u = x^3 + 4$

$u^2 - 10u + 24 = (u-6)(u-4)$

$= \left(x^3+4-6\right)\left(x^3+4-4\right)$

$= \left(x^3-2\right)\left(x^3\right)$

77. $16p^4 - q^4 = (4p^2)^2 - (q^2)^2$

$= (4p^2+q^2)(4p^2-q^2)$

$= (4p^2+q^2)(2p+q)(2p-q)$

78. $s^4t^4 - 81 = (s^2t^2+9)(s^2t^2-9)$

$= (s^2t^2+9)(st+3)(st-3)$

79. $y^3 + \frac{1}{64} = y^3 + \left(\frac{1}{4}\right)^3$

$= \left(y + \frac{1}{4}\right)\left(y^2 - \frac{1}{4}y + \frac{1}{16}\right)$

80. $z^3 + \frac{1}{125} = z^3 + \left(\frac{1}{5}\right)^3$

$= \left(z + \frac{1}{5}\right)\left(z^2 - \frac{1}{5}z + \frac{1}{25}\right)$

81. $6a^3 + a^2b - 6ab^2 - b^3$

$= a^2(6a+b) - b^2(6a+b)$

$= (6a+b)(a^2-b^2)$

$= (6a+b)(a+b)(a-b)$

82. $4p^3 + 12p^2q - pq^2 - 3q^3$

$= 4p^2(p+3q) - q^2(p+3q)$

$= (p+3q)(4p^2-q^2)$

$= (p+3q)(2p+q)(2p-q)$

83. $\frac{1}{9}t^2+\frac{1}{6}t+\frac{1}{16}$

$$=\left(\frac{1}{3}t\right)^2+2\left(\frac{1}{3}t\right)\left(\frac{1}{4}\right)+\left(\frac{1}{4}\right)^2$$

$$=\left(\frac{1}{3}t+\frac{1}{4}\right)^2$$

84. $\frac{1}{25}y^2+\frac{1}{5}y+\frac{1}{4}$

$$=\left(\frac{1}{5}y\right)^2+2\left(\frac{1}{5}y\right)\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2$$

$$=\left(\frac{1}{5}y+\frac{1}{2}\right)^2$$

85. $x^2+12x+36-a^2=(x+6)^2-a^2$

$$=(x+6+a)(x+6-a)$$

86. $a^2+10a+25-b^2=(a+5)^2-b^2$

$$=(a+5+b)(a+5-b)$$

87. $p^2+2pq+q^2-81$

$$=(p+q)^2-9^2$$

$$=(p+q+9)(p+q-9)$$

88. $m^2-2mn+n^2-9$

$$=(m-n)^2-3^2$$

$$=(m-n+3)(m-n-3)$$

89. $b^2-(x^2+4x+4)$

$$=b^2-(x+2)^2$$

$$=(b+(x+2))(b-(x+2))$$

$$=(b+x+2)(b-x-2)$$

90. $p^2-(y^2-6y+9)$

$$=p^2-(y-3)^2$$

$$=(p+(y-3))(p-(y-3))$$

$$=(p+y-3)(p-y+3)$$

91. $4-u^2+2uv-v^2$

$$=4-\left(u^2-2uv+v^2\right)$$

$$=4-(u-v)^2$$

$$=(2+(u-v))(2-(u-v))$$

$$=(2+u-v)(2-u+v)$$

92. $25-a^2-2ab-b^2$

$$=25-\left(a^2+2ab+b^2\right)$$

$$=25-(a+b)^2$$

$$=(5+(a+b))(5-(a+b))$$

$$=(5+a+b)(5-a-b)$$

93. $6ax-by+2bx-3ay$

$$=6ax+2bx-by-3ay$$

$$=2x(3a+b)-y(3a+b)$$

$$=(3a+b)(2x-y)$$

94. $5pq-12-4q+15p$

$$=5pq-4q+15p-12$$

$$=q(5p-4)+3(5p-4)$$

$$=(5p-4)(q+3)$$

95. $u^6 - 64$
$= (u^3)^2 - (8)^2$
$= (u^3 + 8)(u^3 - 8)$
$= (u+2)(u^2 - 2u + 4)(u-2)(u^2 + 2u + 4)$
$= (u+2)(u-2)(u^2 - 2u + 4)(u^2 + 2u + 4)$

96. $1 - v^6 = 1 - (v^3)^2$
$= (1 - v^3)(1 + v^3)$
$= (1-v)(1+v+v^2)(1+v)(1-v+v^2)$
$= (1-v)(1+v)(1+v+v^2)(1-v+v^2)$

97. $x^8 - 1 = (x^4)^2 - 1^2$
$= (x^4 + 1)(x^4 - 1)$
$= (x^4 + 1)(x^2 + 1)(x^2 - 1)$
$= (x^4 + 1)(x^2 + 1)(x+1)(x-1)$

98. $y^8 - 256 = (y^4)^2 - 16^2$
$= (y^4 + 16)(y^4 - 16)$
$= (y^4 + 16)(y^2 + 4)(y^2 - 4)$
$= (y^4 + 16)(y^2 + 4)(y+2)(y-2)$

99. $a^2 - b^2 + a + b = (a+b)(a-b) + (a+b)$
$= (a+b)(a-b+1)$

100. $25c^2 - 9d^2 + 5c - 3d$
$= (5c + 3d)(5c - 3d) + (5c - 3d)$
$= (5c - 3d)(5c + 3d + 1)$

101. $5wx^3 + 5wy^3 - 2zx^3 - 2zy^3$
$= 5w(x^3 + y^3) - 2z(x^3 + y^3)$
$= (x^3 + y^3)(5w - 2z)$
$= (x+y)(x^2 - xy + y^2)(5w - 2z)$

Section 4.8 Practice Exercises

1.
a. quadratic
b. 0; 0
c. Pythagorean; c^2
d. quadratic
e. $f(x) = 0$; y
f. $x+1$; $x+2$; $x+2$
g. lw
h. $\frac{1}{2}bh$

2.
a. $x^2 - y^2 = (x-y)(x+y)$
b. $x^2 + y^2$ is prime.
c. $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$
d. $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$

3. $10x^2 + 3x = x(10x + 3)$

4. $7x^2 - 28 = 7(x^2 - 4) = 7(x+2)(x-2)$

5. $2p^2-9p-5=2p^2-10p+p-5$
$=2p(p-5)+(p-5)$
$=(p-5)(2p+1)$

6. $3q^2-4q-4=3q^2-6q+2q-4$
$=3q(q-2)+2(q-2)$
$=(q-2)(3q+2)$

7. $t^3-1=t^3-1^3=(t-1)\left(t^2+t+1\right)$

8. $z^2-11z+30=z^2-6z-5z+30$
$=z(z-6)-5(z-6)$
$=(z-6)(z-5)$

9. The equation must be set equal to 0, and the polynomial must be factored.

10. If $a\cdot b=0$, then either $a=0$ or $b=0$.

11. $2x(x-3)=0$ Correct form.

12. $(u+1)(u-3)=10$ Incorrect form. The equation is not set equal to 0.

13. $3p^2-7p+4=0$ Incorrect form. The polynomial is not factored.

14. $t^2-t-12=0$ Incorrect form. The polynomial is not factored.

15. $a(a+3)^2=5$ Incorrect form. The equation is not set equal to 0.

16. $\left(\frac{2}{3}x-5\right)\left(x+\frac{1}{2}\right)=0$ Correct form.

17. **a.** $w^2-81=(w+9)(w-9)$

b. $w^2-81=0$
$(w+9)(w-9)=0$
$w+9=0$ or $w-9=0$
$w=-9$ or $w=9$ $\{-9,9\}$

18. **a.** $p^2-25=(p+5)(p-5)$

b. $p^2-25=0$
$(p+5)(p-5)=0$
$p+5=0$ or $p-5=0$
$p=-5$ or $p=5$ $\{-5,5\}$

19. **a.** $3x^2+14x-5=(3x-1)(x+5)$

b. $3x^2+14x-5=0$
$(3x-1)(x+5)=0$
$3x-1=0$ or $x+5=0$
$x=\frac{1}{3}$ or $x=-5$ $\left\{\frac{1}{3},-5\right\}$

20. **a.** $2y^2-y-3=(2y-3)(y+1)$

b. $2y^2-y-3=0$
$(2y-3)(y+1)=0$
$2y-3=0$ or $y+1=0$
$y=\frac{3}{2}$ or $y=-1$ $\left\{\frac{3}{2},-1\right\}$

21. $(x+3)(x+5)=0$

$x+3=0$ or $x+5=0$

$x=-3$ or $x=-5$ $\{-3,-5\}$

22. $(x+7)(x-4)=0$

$x+7=0$ or $x-4=0$

$x=-7$ or $x=4$ $\{-7,4\}$

23. $(2w+9)(5w-1)=0$

$2w+9=0$ or $5w-1=0$

$2w=-9$ or $5w=1$

$w=-\frac{9}{2}$ or $w=\frac{1}{5}$ $\left\{-\frac{9}{2},\frac{1}{5}\right\}$

24. $(3a+1)(4a-5)=0$

$3a+1=0$ or $4a-5=0$

$3a=-1$ or $4a=5$

$a=-\frac{1}{3}$ or $a=\frac{5}{4}$ $\left\{-\frac{1}{3},\frac{5}{4}\right\}$

25. $x(x+4)(10x-3)=0$

$x=0$ or $x+4=0$ or $10x-3=0$

$x=0$ or $x=-4$ or $10x=3$

$x=0$ or $x=-4$ or $x=\frac{3}{10}$

$\left\{0,-4,\frac{3}{10}\right\}$

26. $t(t-6)(3t-11)=0$

$t=0$ or $t-6=0$ or $3t-11=0$

$t=0$ or $t=6$ or $3t=11$

$t=0$ or $t=6$ or $t=\frac{11}{3}$

$\left\{0,6,\frac{11}{3}\right\}$

27. $0=5(y-0.4)(y+2.1)$

$5=0$ or $y-0.4=0$ or $y+2.1=0$

no solution $y=0.4$ or $y=-2.1$

$\{0.4,-2.1\}$

28. $0=-4(z-7.5)(z-9.3)$

$-4=0$ or $z-7.5=0$ or $z-9.3=0$

no solution $z=7.5$ or $z=9.3$

$\{7.5,9.3\}$

29. $x^2+6x-27=0$

$(x+9)(x-3)=0$

$x+9=0$ or $x-3=0$

$x=-9$ or $x=3$

$\{-9,3\}$

30. $2x^2+x-15=0$

$2x^2+6x-5x-15=0$

$2x(x+3)-5(x+3)=0$

$(x+3)(2x-5)=0$

$x+3=0$ or $2x-5=0$

$x=-3$ or $2x=5$

$x=-3$ or $x=\frac{5}{2}$

$\left\{-3,\frac{5}{2}\right\}$

31.
$$2x^2+5x=3$$
$$2x^2+5x-3=0$$
$$2x^2+6x-x-3=0$$
$$2x(x+3)-(x+3)=0$$
$$(x+3)(2x-1)=0$$
$$x+3=0 \text{ or } 2x-1=0$$
$$x=-3 \text{ or } x=\frac{1}{2} \quad \left\{-3,\frac{1}{2}\right\}$$

32.
$$-11x=3x^2-4$$
$$3x^2+11x-4=0$$
$$3x^2+12x-x-4=0$$
$$3x(x+4)-(x+4)=0$$
$$(x+4)(3x-1)=0$$
$$x+4=0 \text{ or } 3x-1=0$$
$$x=-4 \text{ or } 3x=1$$
$$x=-4 \text{ or } x=\frac{1}{3} \quad \left\{-4,\frac{1}{3}\right\}$$

33.
$$10x^2=15x$$
$$10x^2-15x=0$$
$$5x(2x-3)=0$$
$$5x=0 \text{ or } 2x-3=0$$
$$x=0 \text{ or } 2x=3$$
$$x=0 \text{ or } x=\frac{3}{2} \quad \left\{0,\frac{3}{2}\right\}$$

34.
$$5x^2=7x$$
$$5x^2-7x=0$$
$$x(5x-7)=0$$
$$x=0 \text{ or } 5x-7=0$$
$$x=0 \text{ or } 5x=7$$
$$x=0 \text{ or } x=\frac{7}{5} \quad \left\{0,\frac{7}{5}\right\}$$

35.
$$6(y-2)-3(y+1)=8$$
$$6y-12-3y-3=8$$
$$3y-15=8$$
$$3y=23$$
$$y=\frac{23}{3} \quad \left\{\frac{23}{3}\right\}$$

36.
$$4x+3(x-9)=6x+1$$
$$4x+3x-27=6x+1$$
$$7x-27=6x+1$$
$$7x-6x-27=6x-6x+1$$
$$x-27=1$$
$$x=28 \quad \{28\}$$

37.
$$-9=y(y+6)$$
$$-9=y^2+6y$$
$$y^2+6y+9=0$$
$$(y+3)^2=0$$
$$y+3=0$$
$$y=-3 \quad \{-3\}$$

38.
$$-62=t(t-16)+2$$
$$-62=t^2-16t+2$$
$$t^2-16t+64=0$$
$$(t-8)^2=0$$
$$t-8=0$$
$$t=8 \quad \{8\}$$

39.
$$9p^2-15p-6=0$$
$$3\left(3p^2-5p-2\right)=0$$
$$3\left(3p^2-6p+p-2\right)=0$$
$$3\left[3p\left(p-2\right)+\left(p-2\right)\right]=0$$
$$3\left(p-2\right)\left(3p+1\right)=0$$
$3=0$ or $p-2=0$ or $3p+1=0$

$p=2$ or $3p=-1$

no solution $p=2$ or $p=-\frac{1}{3}$

$$\left\{2,-\frac{1}{3}\right\}$$

40.
$$6y^2+2y=48$$
$$6y^2+2y-48=0$$
$$2\left(3y^2+y-24\right)=0$$
$$2\left(3y^2+9y-8y-24\right)=0$$
$$2\left[3y\left(y+3\right)-8\left(y+3\right)\right]=0$$
$$2\left(y+3\right)\left(3y-8\right)=0$$
$2=0$ or $y+3=0$ or $3y-8=0$

$y=-3$ or $3y=8$

no solution $y=-3$ or $y=\frac{8}{3}$

$$\left\{-3,\frac{8}{3}\right\}$$

41.
$$\left(x+1\right)\left(2x-1\right)\left(x-3\right)=0$$
$x+1=0$ or $2x-1=0$ or $x-3=0$

$x=-1$ or $2x=1$ or $x=3$

$x=-1$ or $x=\frac{1}{2}$ or $x=3$

$$\left\{-1,\frac{1}{2},3\right\}$$

42.
$$2x\left(x-4\right)^2\left(4x+3\right)=0$$
$2x=0$ or $x-4=0$ or $4x+3=0$

$x=0$ or $x=4$ or $4x=-3$

$x=0$ or $x=4$ or $x=-\frac{3}{4}$

$$\left\{0,4,-\frac{3}{4}\right\}$$

43.
$$\left(y-3\right)\left(y+4\right)=8$$
$$y^2+y-12=8$$
$$y^2+y-20=0$$
$$\left(y+5\right)\left(y-4\right)=0$$
$y+5=0$ or $y-4=0$

$y=-5$ or $y=4$

$$\left\{-5,4\right\}$$

44.
$$\left(t+10\right)\left(t+5\right)=6$$
$$t^2+15t+50=6$$
$$t^2+15t+44=0$$
$$\left(t+11\right)\left(t+4\right)=0$$
$t+11=0$ or $t+4=0$

$t=-11$ or $t=-4$

$$\left\{-11,-4\right\}$$

45. $(2a-1)(a-1)=6$

$2a^2-3a+1=6$

$2a^2-3a-5=0$

$(2a-5)(a+1)=0$

$2a-5=0$ or $a+1=0$

$2a=5$ or $a=-1$

$a=\frac{5}{2}$ or $a=-1$

$\left\{\frac{5}{2},-1\right\}$

46. $w(6w+1)=2$

$6w^2+w=2$

$6w^2+w-2=0$

$(3w+2)(2w-1)=0$

$3w+2=0$ or $2w-1=0$

$3w=-2$ or $2w=1$

$w=-\frac{2}{3}$ or $w=\frac{1}{2}$

$\left\{-\frac{2}{3},\frac{1}{2}\right\}$

47. $p^2+(p+7)^2=169$

$p^2+p^2+14p+49=169$

$2p^2+14p-120=0$

$2(p^2+7p-60)=0$

$2(p+12)(p-5)=0$

$2\neq 0$ or $p+12=0$ or $p-5=0$

$p=-12$ or $p=5$

$\{-12,5\}$

48. $x^2+(x+2)^2=100$

$x^2+x^2+4x+4=100$

$2x^2+4x-96=0$

$2(x^2+2x-48)=0$

$2(x+8)(x-6)=0$

$2\neq 0$ or $x+8=0$ or $x-6=0$

$x=-8$ or $x=6$

$\{-8,6\}$

49. $3t(t+5)-t^2=2t^2+4t-1$

$3t^2+15t-t^2=2t^2+4t-1$

$11t=-1$

$t=-\frac{1}{11}$ $\left\{-\frac{1}{11}\right\}$

50. $a^2-4a-2=(a+3)(a-5)$

$a^2-4a-2=a^2-2a-15$

$-2a=-13$

$a=\frac{13}{2}$ $\left\{\frac{13}{2}\right\}$

51. $2x^3-8x^2-24x=0$

$2x(x^2-4x-12)=0$

$2x(x-6)(x+2)=0$

$2x=0$ or $x-6=0$ or $x+2=0$

$x=0$ or $x=6$ or $x=-2$

$\{0,6,-2\}$

52. $2p^3+20p^2+42p=0$

$2p(p^2+10p+21)=0$

$2p(p+7)(p+3)=0$

$2p=0$ or $p+7=0$ or $p+3=0$

$p=0$ or $p=-7$ or $p=-3$

$\{0,-7,-3\}$

53.
$$w^3 = 16w$$
$$w^3 - 16w = 0$$
$$w\left(w^2 - 16\right) = 0$$
$$w(w+4)(w-4) = 0$$
$$w = 0 \text{ or } w+4 = 0 \text{ or } w-4 = 0$$
$$w = 0 \text{ or } x = -4 \text{ or } x = 4$$
$$\{0, -4, 4\}$$

54.
$$12x^3 = 27x$$
$$12x^3 - 27x = 0$$
$$3x\left(4x^2 - 9\right) = 0$$
$$3x(2x+3)(2x-3) = 0$$
$$3x = 0 \text{ or } 2x+3 = 0 \text{ or } 2x-3 = 0$$
$$x = 0 \text{ or } 2x = -3 \text{ or } 2x = 3$$
$$x = 0 \text{ or } x = -\frac{3}{2} \text{ or } x = \frac{3}{2} \quad \left\{0, -\frac{3}{2}, \frac{3}{2}\right\}$$

55.
$$0 = 2x^3 + 5x^2 - 18x - 45$$
$$0 = x^2(2x+5) - 9(2x+5)$$
$$0 = (2x+5)\left(x^2 - 9\right)$$
$$0 = (2x+5)(x+3)(x-3)$$
$$2x+5 = 0 \text{ or } x+3 = 0 \text{ or } x-3 = 0$$
$$2x = -5 \text{ or } x = -3 \text{ or } x = 3$$
$$x = -\frac{5}{2} \text{ or } x = -3 \text{ or } x = 3$$
$$\left\{-\frac{5}{2}, -3, 3\right\}$$

56.
$$0 = 3y^3 + y^2 - 48y - 16$$
$$0 = y^2(3y+1) - 16(3y+1)$$
$$0 = (3y+1)\left(y^2 - 16\right)$$
$$0 = (3y+1)(y+4)(y-4)$$
$$3y+1 = 0 \text{ or } y+4 = 0 \text{ or } y-4 = 0$$
$$3y = -1 \text{ or } y = -4 \text{ or } y = 4$$
$$y = -\frac{1}{3} \text{ or } y = -4 \text{ or } y = 4$$
$$\left\{-\frac{1}{3}, -4, 4\right\}$$

57. Let x = the number
$$x^2 + 5 = 30$$
$$x^2 - 25 = 0$$
$$(x+5)(x-5) = 0$$
$$x+5 = 0 \text{ or } x-5 = 0$$
$$x = -5 \text{ or } x = 5$$

58. Let x = the number
$$x^2 - 4 = 77$$
$$x^2 - 81 = 0$$
$$(x+9)(x-9) = 0$$
$$x+9 = 0 \text{ or } x-9 = 0$$
$$x = -9 \text{ or } x = 9$$

59. Let x = the number
$$x^2 = x + 12$$
$$x^2 - x - 12 = 0$$
$$(x+3)(x-4) = 0$$
$$x+3 = 0 \text{ or } x-4 = 0$$
$$x = -3 \text{ or } x = 4$$

60. Let x = the number
$$x^2 = x + 20$$
$$x^2 - x - 20 = 0$$
$$(x+4)(x-5) = 0$$
$$x+4 = 0 \text{ or } x-5 = 0$$
$$x = -4 \text{ or } x = 5$$

61. Let x = the first consecutive integer

$x + 1$ = the second consecutive integer

$$x(x+1) = 42$$
$$x^2 + x = 42$$
$$x^2 + x - 42 = 0$$
$$(x+7)(x-6) = 0$$
$$x + 7 = 0 \text{ or } x - 6 = 0$$
$$x = -7 \text{ or } x = 6$$
$$x + 1 = -7 + 1 = -6 \text{ or } x + 1 = 6 + 1 = 7$$

The consecutive integers are –7 and –6 or 6 and 7.

62. Let x = the first consecutive integer

$x + 1$ = the second consecutive integer

$$x(x+1) = 110$$
$$x^2 + x = 110$$
$$x^2 + x - 110 = 0$$
$$(x+11)(x-10) = 0$$
$$x + 11 = 0 \text{ or } x - 10 = 0$$
$$x = -11 \text{ or } x = 10$$
$$x + 1 = -11 + 1 = -10 \text{ or } x + 1 = 10 + 1 = 11$$

The consecutive integers are –11 and –10 or 10 and 11.

63. Let x = the first consecutive odd integer

$x + 2$ = second consecutive odd integer

$$x(x+2) = 63$$
$$x^2 + 2x = 63$$
$$x^2 + 2x - 63 = 0$$
$$(x+9)(x-7) = 0$$
$$x + 9 = 0 \text{ or } x - 7 = 0$$
$$x = -9 \text{ or } x = 7$$
$$x + 2 = -9 + 2 = -7 \text{ or } x + 2 = 7 + 2 = 9$$

The consecutive odd integers are –9 and –7 or 7 and 9.

64. Let x = the first consecutive even integer

$x + 2$ = second consecutive even integer

$$x(x+2) = 120$$
$$x^2 + 2x = 120$$
$$x^2 + 2x - 120 = 0$$
$$(x+12)(x-10) = 0$$
$$x + 12 = 0 \text{ or } x - 10 = 0$$
$$x = -12 \text{ or } x = 10$$
$$x + 2 = -12 + 2 = -10 \text{ or } x + 2 = 10 + 2 = 12$$

The consecutive even integers are –12 and –10 or 10 and 12.

65. Let x = the length

$x - 2$ = the width

66. Let x = the width

$x + 7$ = the length

$$x(x-2)=35$$
$$x^2-2x=35$$
$$x^2-2x-35=0$$
$$(x+5)(x-7)=0$$
$$x+5=0 \quad \text{or} \quad x-7=0$$
$$x \neq -5 \quad \text{or} \quad x=7$$
$$\text{or} \quad x-2=7-2=5$$

The length is 7 ft and the width is 5 ft.

$$x(x+7)=78$$
$$x^2+7x=78$$
$$x^2+7x-78=0$$
$$(x+13)(x-6)=0$$
$$x+13=0 \quad \text{or} \quad x-6=0$$
$$x \neq -13 \quad \text{or} \quad x=6$$
$$\text{or} \quad x+7=6+7=13$$

The width is 6 in and the length is 13 in.

67. Let x = the width

$x+5$ = the length

$$x(x+5)=300$$
$$x^2+5x=300$$
$$x^2+5x-300=0$$
$$(x+20)(x-15)=0$$
$$x+20=0 \quad \text{or} \quad x-15=0$$
$$x \neq -20 \quad \text{or} \quad x=15$$
$$\text{or} \quad x+5=15+5=20$$

The width is 15 yd and the length is 20 yd.

68. Let x = the width

$2x$ = the length

$$x(2x)=18$$
$$2x^2=18$$
$$2x^2-18=0$$
$$2(x^2-9)=0$$
$$2(x+3)(x-3)=0$$
$$x+3=0 \quad \text{or} \quad x-3=0$$
$$x \neq -3 \quad \text{or} \quad x=3$$
$$\text{or} \quad 2x=2(3)=6$$

The table is 3 ft wide and 6 ft long.

69. a. Let b = the base of the triangle

$b+1$ = the height of the triangle

$$\frac{1}{2}b(b+1+2)=20$$
$$b(b+3)=40$$
$$b^2+3b=40$$
$$b^2+3b-40=0$$

70. a. Let h = the height of the triangle

$h+2$ = the base of the triangle

$$\frac{1}{2}(h+2+4)(h)=56$$
$$(h+6)h=112$$
$$h^2+6h=112$$
$$h^2+6h-112=0$$

b. $(b+8)(b-5)=0$

$b+8=0 \quad \text{or} \quad b-5=0$

$b \neq -8 \quad \text{or} \quad b=5$

$b+1=5+1=6$

The base is 5 in and the height is 6 in.

$A=\frac{1}{2}(5)(6)=15 \text{ in}^2$

The area is 15 in^2.

b. $(h+14)(h-8)=0$

$h+14=0 \quad \text{or} \quad h-8=0$

$h \neq -14 \quad \text{or} \quad h=8$

$h+2=8+2=10$

The base is 10 cm and the height is 8 cm.

$A=\frac{1}{2}(10)(8)=40 \text{ cm}^2$

The area is 40 cm^2.

71. Let h = the height of the triangle

$2h$ = the base of the triangle

$\frac{1}{2}(2h)(h)=25$

$h^2=25$

$h^2-25=0$

$(h+5)(h-5)=0$

$h+5=0 \quad \text{or} \quad h-5=0$

$h \neq -5 \quad \text{or} \quad h=5$

$2h=2(5)=10$

The height is 5 ft and the base is 10 ft.

72. Let b = the base of the triangle

$2b+1$ = the height of the triangle

$\frac{1}{2}(b)(2b+1)=18$

$2b^2+b=36$

$2b^2+b-36=0$

$(2b+9)(b-4)=0$

$2b+9=0 \quad \text{or} \quad b-4=0$

$2b=-9 \quad \text{or} \quad b=4$

$b \neq -\frac{9}{2} \quad 2b+1=2(4)+1=9$

The base is 4 in and the height is 9 in.

73. Let x = the first positive consecutive integer

$x+1$ = second pos consecutive integer

$x^2+(x+1)^2=41$

$x^2+x^2+2x+1=41$

$2x^2+2x-40=0$

$2(x^2+x-20)=0$

$2(x+5)(x-4)=0$

$x+5=0 \quad \text{or} \quad x-4=0$

$x \neq -5 \quad \text{or} \quad x=4$

$x+1=4+1=5$

74. Let x = first positive consecutive even integer

$x+2$ = second pos consecutive even integer

$x^2+(x+2)^2=164$

$x^2+x^2+2x+4=164$

$2x^2+2x-160=0$

$2(x^2+2-80)=0$

$2(x+10)(x-8)=0$

$x+10=0 \quad \text{or} \quad x-8=0$

$x \neq -10 \quad \text{or} \quad x=8$

$x+2=8+2=10$

The consecutive positive integers are 4 and 5.

The consecutive positive even integers are 8 and 10.

75. **a.** Let x = the northern leg

$x-2$ = the eastern leg

$$x^2+(x-2)^2=10^2$$
$$x^2+x^2-4x+4=100$$
$$2x^2-4x-96=0$$
$$2(x^2-2x-48)=0$$
$$2(x+6)(x-8)=0$$
$$x+6=0 \text{ or } x-8=0$$
$$x\neq -6 \text{ or } x=8$$
$$x-2=8-2=6$$

b. The alternative route is 8 mi + 6 mi =14 mi.

$$t=\frac{d}{r}=\frac{10}{40}=\frac{1}{4}=0.25 \text{ hr}$$
$$t=\frac{d}{r}=\frac{14}{60}=\frac{7}{30}\approx 0.23 \text{ hr}$$

The alternative route using superhighways takes less time.

76. Let h = the distance up the wall

$h-7$ = the distance on the ground

$$h^2+(h-7)^2=17^2$$
$$h^2+h^2-14h+49=289$$
$$2h^2-14h-240=0$$
$$2(h^2-7h-120)=0$$
$$2(h+8)(h-15)=0$$
$$h+8=0 \text{ or } h-15=0$$
$$h\neq -8 \text{ or } h=15$$
$$h-7=15-7=8$$

The distance from the base of the ladder to the base of the wall is 8 ft.

77. Let x = the first consecutive even integer

$x+2$ = second consecutive even integer

$x+4$ = third consecutive even integer

78. Let x = the length of the shorter leg

$2x+2$ = the length of the longer leg

$2x+3$ = the length of the hypotenuse

$$x^2+(2x+2)^2=(2x+3)^2$$
$$x^2+4x^2+8x+4=4x^2+12x+9$$
$$x^2-4x-5=0$$
$$(x+1)(x-5)=0$$
$$x+1=0 \text{ or } x-5=0$$
$$x=-1 \text{ or } x=5$$
$$2x+2=2(5)+2=12$$
$$2x+3=2(5)+3=13$$

$$x^2+(x+2)^2=(x+4)^2$$
$$x^2+x^2+4x+4=x^2+8x+16$$
$$x^2-4x-12=0$$
$$(x+2)(x-6)=0$$
$$x+2=0 \text{ or } x-6=0$$
$$x\neq -2 \text{ or } x=6$$
$$x+2=6+2=8$$
$$x+4=6+4=10$$

The lengths of the sides are 6 m, 8 m, and 10 m.

The lengths of the sides are 5 m, 12 m, and 13 m.

79. Let r = the radius of the circle

$$\pi r^2=2\pi r$$
$$r^2=2r$$
$$r^2-2r=0$$
$$r(r-2)=0$$
$$r=0 \text{ or } r-2=0$$
$$r=0 \text{ or } r=2$$

The radius is 2 units.

80. Let r = the radius of the circle

$$\pi r^2=2(2\pi r)$$
$$\pi r^2=4\pi r$$
$$r^2=4r$$
$$r^2-4r=0$$
$$r(r-4)=0$$
$$r=0 \text{ or } r-4=0$$
$$r=0 \text{ or } r=4$$

The radius is 4 units.

81. **a.** $f(x)=x^2-3x=0$

$$x(x-3)=0$$
$$x=0 \text{ or } x-3=0$$
$$x=0 \text{ or } x=3$$

b. $f(0)=0^2-3(0)=0-0=0$

82. **a.** $f(x)=4x^2+2x=0$

$$2x(2x+1)=0$$
$$2x=0 \text{ or } 2x+1=0$$
$$x=0 \text{ or } 2x=-1$$
$$x=0 \text{ or } x=-\frac{1}{2}$$

b. $f(0)=4(0^2)+2(0)=0+0=0$

83. **a.** $f(x)=x^2-6x-7=0$

$(x-7)(x+1)=0$

$x-7=0$ or $x+1=0$

$x=7$ or $x=-1$

b. $f(0)=0^2-6(0)-7=0-0-7=-7$

84. **a.** $f(x)=2x^2+11x+5=0$

$(2x+1)(x+5)=0$

$2x+1=0$ or $x+5=0$

$2x=-1$ or $x=-5$

$x=-\frac{1}{2}$ or $x=-5$

b. $f(0)=2(0)^2+11(0)+5=0+0+5=5$

85. $f(x)=\frac{1}{2}(x-2)(x+1)(2x)=0$

$\frac{1}{2}\neq 0$ or $x-2=0$ or $x+1=0$ or $2x=0$

$x=2$ or $x=-1$ or $x=0$

$f(0)=\frac{1}{2}(0-2)(0+1)(2\cdot 0)$

$=\frac{1}{2}(-2)(1)(0)=0$

x-intercepts: (2, 0), (–1, 0), (0, 0)

y-intercept: (0, 0)

86. $f(x)=(x+1)(x-2)(x+3)^2=0$

$x+1=0$ or $x-2=0$ or $x+3=0$

$x=-1$ or $x=2$ or $x=-3$

$f(0)=(0+1)(0-2)(0+3)^2$

$=(1)(-2)(3)^2=-2(9)=-18$

x-intercepts: (–1, 0), (2, 0), (–3, 0)

y-intercept: (0, –18)

87. $f(x)=x^2-2x+1=0$

$(x-1)^2=0$

$x-1=0$

$x=1$

$f(0)=0^2-2(0)+1=0-0+1=1$

x-intercepts: (1, 0)

y-intercept: (0, 1)

88. $f(x)=x^2+4x+4=0$

$(x+2)^2=0$

$x+2=0$

$x=-2$

$f(0)=0^2+4(0)+4=0+0+4=4$

x-intercepts: (–2, 0)

y-intercept: (0, 4)

89. $g(x)=(x+3)(x-3)=0$

$(x+3)=0$ or $x-3=0$

$x=-3$ or $x=3$

x-intercepts: (–3, 0), (3,0)

Graph d.

90. $h(x)=x(x-2)(x+4)=0$

$x=0$ or $x-2=0$ or $x+4=0$

$x=0$ or $x=2$ or $x=-4$

x-intercepts: (0, 0), (2,0), (–4, 0)

Graph c.

91. $f(x)=4(x+1)=0$

$4\neq 0$ or $x+1=0$

$x=-1$

92. $k(x)=(x+1)(x+3)(x-2)(x-1)=0$

$x+1=0$ or $x+3=0$ or $x-2=0$ or $x-1=0$

$x=-1$ or $x=-3$ or $x=2$ or $x=1$

x-intercepts: (–1, 0)
Graph a.

x-intercepts: (–1, 0), (–3,0), (2, 0), (1, 0)
Graph b.

93. **a.** The function is in the form $s(t) = at^2 + bt + c$

b.
$$s(t) = -4.9t^2 + 490t = 0$$
$$-4.9t(t - 100) = 0$$
$$-4.9t = 0 \text{ or } t - 100 = 0$$
$$t = 0 \text{ or } t = 100$$

c. t-intercepts (0, 0), (100, 0)

d. At 0 sec and 100 sec, the rocket is at ground level (height = 0).

$$s(t) = -4.9t^2 + 490t = 485.1$$
$$-4.9t^2 + 490t - 485.1 = 0$$
$$-4.9(t^2 - 100t + 99) = 0$$
$$-4.9(t - 1)(t - 99) = 0$$
$$-4.9 \neq 0 \text{ or } t - 1 = 0 \text{ or } t - 99 = 0$$
$$t = 1 \text{ or } t = 99$$

The height is 485.1 m at 1 sec and 99 sec.

94. **a.** $P(x) = -2x^2 + 1000x$ is a quadratic function.

b.
$$P(x) = -2x^2 + 1000x = 0$$
$$-2x(x - 500) = 0$$
$$-2x = 0 \text{ or } x - 500 = 0$$
$$x = 0 \text{ or } x = 500$$

c. 0 or 500 units produce zero profit.

d. The points on the graph represented are the x-intercepts (0, 0) and (500, 0).

$$P(x) = -2x^2 + 1000x = 80{,}000$$
$$-2x^2 + 1000x - 80{,}000 = 0$$
$$-2(x^2 - 500x + 40{,}000) = 0$$
$$-2(x - 100)(x - 400) = 0$$
$$-2 \neq 0 \text{ or } x - 100 = 0 \text{ or } x - 400 = 0$$
$$x = 100 \text{ or } x = 400$$

There is a profit of \$80,000 when 100 or 400 systems are produced.

95.
$$f(x) = x^2 - 7x + 10 = 0$$
$$f(x) = (x - 5)(x - 2) = 0$$
$$x - 5 = 0 \text{ or } x - 2 = 0$$
$$x = 5 \text{ or } x = 2$$

$x = 5$ and $x = 2$ represent the x-intercepts.

96.
$$f(x) = x^2 - 2x - 3 = 0$$
$$f(x) = (x - 3)(x + 1) = 0$$
$$x - 3 = 0 \text{ or } x + 1 = 0$$
$$x = 3 \text{ or } x = -1$$

$x = 3$ and $x = -1$ represent the x-intercepts.

97.
$$f(x) = x^2 + 2x + 1 = 0$$
$$f(x) = (x + 1)^2 = 0$$
$$x + 1 = 0$$
$$x = -1$$

$x = -1$represents the x-intercept.

98.
$$f(x) = x^2 - 8x + 16 = 0$$
$$f(x) = (x - 4)^2 = 0$$
$$x - 4 = 0$$
$$x = 4$$

$x = 4$ represents the x-intercept.

99. $f(x) = -x^2 - 6x - 5 = 0$

$f(x) = -(x^2 + 6x + 5) = 0$

$f(x) = -(x+1)(x+5) = 0$

$x + 1 = 0 \quad \text{or} \quad x + 5 = 0$

$x = -1 \quad \text{or} \quad x = -5$

$x = -1$ and $x = -5$ represent the x-intercepts.

100. $f(x) = -x^2 + 6x - 8 = 0$

$f(x) = -(x^2 - 6x + 8) = 0$

$f(x) = -(x-4)(x-2) = 0$

$x - 4 = 0 \quad \text{or} \quad x - 2 = 0$

$x = 4 \quad \text{or} \quad x = 2$

$x = 4$ and $x = 2$ represent the x-intercepts.

101. $SA = 2\pi r^2 + 2\pi rh$

$2\pi r^2 + 2\pi r(7) = 156\pi$

$r^2 + 7r = 78$

$r^2 + 7r - 78 = 0$

$(r+13)(r-6) = 0$

$r + 13 = 0 \quad \text{or} \quad r - 6 = 0$

$r = -13 \quad \text{or} \quad r = 6$

The radius is 6 ft.

102. Let l = the length

w = the width

$2l + 2w = 20$

$2w = 20 - 2l$

$w = 10 - l$

$A = l(10 - l) = 16$

$10l - l^2 = 16$

$0 = l^2 - 10l + 16$

$0 = (l-8)(l-2)$

$l - 8 = 0 \quad \text{or} \quad l - 2 = 0$

$l = 8 \quad \text{or} \quad l = 2$

$w = 10 - 8 = 2$

The length is 8 yd and the width is 2 yd.

103. Let l = the length

w = the width

$2l + 2w = 28$

$2w = 28 - 2l$

$w = 14 - l$

$$A = l(14 - l) = 48$$
$$14l - l^2 = 48$$
$$0 = l^2 - 14l + 48$$
$$0 = (l - 8)(l - 6)$$
$$l - 8 = 0 \text{ or } l - 6 = 0$$
$$l = 8 \text{ or } l = 6$$
$$w = 14 - 8 = 6$$

The length is 8 ft and the width is 6 ft.

104. $x = -3$ and $x = 1$

$$(x + 3)(x - 1) = 0$$
$$x^2 + 2x - 3 = 0$$

105. $x = 2$ and $x = -2$

$$(x - 2)(x + 2) = 0$$
$$x^2 - 4 = 0$$

106. $x = 0$ and $x = -5$

$$(x - 0)(x + 5) = 0$$
$$x^2 + 5x = 0$$

107. $x = 0$ and $x = -3$

$$(x - 0)(x + 3) = 0$$
$$x^2 + 3x = 0$$

Chapter 4 Group Activity

1. $(a + b)^0 = 1$

2. **a.** $(a + b)^1 = a + b$

b. $(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2$

c. $(a + b)^3 = (a + b)(a + b)(a + b)$
$= a^3 + 3a^2b + 3ab^2 + b^3$

3. **a.** 3 **d.** 6
b. 4 **e.** $n+1$
c. 5

4. The exponent on a begins at n and decreases for each successive term. The exponent on b begins at zero and increases.

5.
$(a+b)^4$ 1 4 6 4 1
$(a+b)^5$ 1 5 10 10 5 1
$(a+b)^6$ 1 6 15 20 15 6 1
$(a+b)^7$ 1 7 21 35 35 21 7 1

6. **a.** $(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

b. $(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$

c. $(a+b)^6 = a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$

Chapter 4 Review Exercises

Section 4.1

1. $(3x)^3(3x)^2 = (3x)^{2+3} = (3x)^5 = 3^5x^5 = 243x^5$

2. $(-6x^{-4})(3x^{-8}) = -18x^{-4+(-8)} = -18x^{-12}$
$= -18 \cdot \frac{1}{x^{12}} = -\frac{18}{x^{12}}$

3. $\frac{24x^5y^3}{-8x^4y} = -3x^{5-4}y^{3-1} = -3xy^2$

4. $\frac{-18x^{-2}y^3}{-12x^{-5}y^5} = \frac{3}{2}x^{-2-(-5)}y^{3-5} = \frac{3}{2}x^3y^{-2}$
$= \frac{3}{2}x^3 \cdot \frac{1}{y^2} = \frac{3x^3}{2y^2}$

5. $(-2a^2b^{-5})^{-3} = (-2)^{-3}(a^2)^{-3}(b^{-5})^{-3}$
$= \left(-\frac{1}{2}\right)^3 a^{-6}b^{15} = -\frac{1}{8} \cdot \frac{1}{a^6} \cdot b^{15} = -\frac{b^{15}}{8a^6}$

6. $(-4a^{-2}b^3)^{-2} = (-4)^{-2}(a^{-2})^{-2}(b^3)^{-2}$
$= \left(-\frac{1}{4}\right)^2 a^4b^{-6} = \frac{1}{16}a^4 \cdot \frac{1}{b^6} = \frac{a^4}{16b^6}$

7. $\left(\dfrac{-4x^4y^{-2}}{5x^{-1}y^4}\right)^{-4} = \left(-\dfrac{4}{5}x^{4-(-1)}y^{-2-4}\right)^{-4}$

$= \left(-\dfrac{4}{5}x^5y^{-6}\right)^{-4} = \left(-\dfrac{4}{5}\right)^{-4}\left(x^5\right)^{-4}\left(y^{-6}\right)^{-4}$

$= \left(-\dfrac{5}{4}\right)^4 x^{-20}y^{24} = \dfrac{5^4}{4^4}\cdot\dfrac{1}{x^{20}}\cdot y^{24} = \dfrac{5^4y^{24}}{4^4x^{20}}$

8. $\left(\dfrac{25x^2y^{-3}}{5x^4y^{-2}}\right)^{-5} = \left(5x^{2-4}y^{-3-(-2)}\right)^{-5} = \left(5x^{-2}y^{-1}\right)^{-5}$

$= (5)^{-5}\left(x^{-2}\right)^{-5}\left(y^{-1}\right)^{-5} = \left(\dfrac{1}{5}\right)^5 x^{10}y^5$

$= \dfrac{1}{5^5}x^{10}y^5 = \dfrac{x^{10}y^5}{5^5}$

9. **a.** $3{,}686{,}600{,}000 = 3.6866\times10^9$

b. $0.000001 = 1.0\times10^{-6}$

10. **a.** $0.001 = 1.0\times10^{-3}$

b. $5{,}155{,}700{,}000 = 5.1557\times10^9$

11. **a.** $1\times10^{-3} = 0.001$

b. $1\times10^{-9} = 0.000000001$

12. **a.** $5.23\times10^9 = 5{,}230{,}000{,}000\,\text{ft}^2$

b. $1.091\times10^{12} = \$1{,}091{,}000{,}000{,}000$

13. $\dfrac{2{,}500{,}000}{0.0004} = \dfrac{2.5\times10^6}{4\times10^{-4}} = 0.625\times10^{6-(-4)}$

$= 6.25\times10^{-1}\times10^{10} = 6.25\times10^9$

14. $\dfrac{0.0005}{25{,}000} = \dfrac{5\times10^{-4}}{2.5\times10^4} = 2\times10^{-4-4} = 2\times10^{-8}$

5. $\left(3.6\times10^8\right)\left(9.0\times10^{-2}\right) = 32.4\times10^{8+(-2)}$

$= 3.24\times10^1\times10^6 = 3.24\times10^7$

16. $\left(7.0\times10^{-12}\right)\left(5.2\times10^3\right) = 36.4\times10^{-12+3}$

$= 3.64\times10^1\times10^{-9} = 3.64\times10^{-8}$

Section 4.2

17. $6x^4 + 10x - 1$ Trinomial; degree 4

18. 18 Monomial; degree 0

19. **a.** $g(x) = 4x - 7$

$g(0) = 4(0) - 7 = 0 - 7 = -7$

b. $g(-4) = 4(-4) - 7 = -16 - 7 = -23$

c. $g(3) = 4(3) - 7 = 12 - 7 = 5$

20. **a.** $p(x) = -x^4 - x + 12$

$p(0) = -(0)^4 - 0 + 12 = 0 - 0 + 12 = 12$

b. $p(1) = -(1)^4 - 1 + 12 = -1 - 1 + 12 = 10$

c. $p(-2) = -(-2)^4 - (-2) + 12$

$= -16 + 2 + 12 = -2$

21. **a.** $A(x) = 0.047x^2 + 1.46x + 16.8$

$A(5) = 0.047(5^2) + 1.46(5) + 16.8$

$= 0.047(25) + 1.46(5) + 16.8$

$= 1.175 + 7.3 + 16.8 = 25.275$

b. $A(15) = 0.047(15^2) + 1.46(15) + 16.8$

$= 0.047(225) + 1.46(15) + 16.8$

$= 10.575 + 21.9 + 16.8 = 49.275$

$A(5) \approx 25$ means that in year 5, each American on average consumed approximately 25 gal of bottled water.

$A(15) \approx 49$ means that in year 15, each American will consume approximately 49 gal of bottled water if this trend continues.

22. $\begin{aligned}(x^2 - 2x - 3xy - 7) + (-3x^2 - x + 2xy + 6) &= -2x^2 - 3x - xy - 1\end{aligned}$

23. $\begin{array}{r} 7xy - 3xz + 5yz \\ \underline{+13xy - 15xz - 8yz} \\ 20xy - 18xz - 3yz \end{array}$

24. $\begin{array}{r} -4a^3 + 8a^2 - 3a \\ \underline{-(-7a^3 + 3a^2 - 9a)} \end{array} \rightarrow \begin{array}{r} -4a^3 + 8a^2 - 3a \\ \underline{+(7a^3 - 3a^2 + 9a)} \\ 3a^3 + 5a^2 + 6a \end{array}$

25. $\begin{aligned}(3a^2 - 2a - a^3) - (5a^2 - a^3 - 8a) &= 3a^2 - 2a - a^3 - 5a^2 + a^3 + 8a \\ &= -2a^2 + 6a\end{aligned}$

26. $\begin{aligned}\left(\frac{5}{8}x^4 - \frac{1}{4}x^2 - \frac{1}{2}\right) - \left(-\frac{3}{8}x^4 + \frac{3}{4}x^2 + \frac{1}{2}\right) &= \frac{5}{8}x^4 - \frac{1}{4}x^2 - \frac{1}{2} + \frac{3}{8}x^4 - \frac{3}{4}x^2 - \frac{1}{2} \\ &= x^4 - x^2 - 1\end{aligned}$

27. $\begin{aligned}\left(\frac{5}{6}x^4 + \frac{1}{2}x^2 - \frac{1}{3}\right) - \left(-\frac{1}{6}x^4 - \frac{1}{4}x^2 - \frac{1}{3}\right) &= \frac{5}{6}x^4 + \frac{1}{2}x^2 - \frac{1}{3} + \frac{1}{6}x^4 + \frac{1}{4}x^2 + \frac{1}{3} \\ &= x^4 + \frac{3}{4}x^2\end{aligned}$

28. $\begin{aligned}(7x - y) - [-(2x + y) - (-3x - 6y)] &= 7x - y - [-2x - y + 3x + 6y] \\ &= 7x - y + 2x + y - 3x - 6y \\ &= 6x - 6y\end{aligned}$

29. $\begin{aligned}-(4x - 4y) - [(4x + 2y) - (3x + 7y)] &= -4x + 4y - [4x + 2y - 3x - 7y] \\ &= -4x + 4y - 4x - 2y + 3x + 7y \\ &= -5x + 9y\end{aligned}$

30. $(-4x + 6) + (-7x - 5) = -11x + 1$

31. $(2x^2 - 4x) + (2x^2 - 7x) = 4x^2 - 11x$

32. $\begin{aligned}(-7x - 5) - (-4x + 6) &= -7x - 5 + 4x - 6 \\ &= -3x - 11\end{aligned}$

33. $\begin{aligned}(2x^2 - 7x) - (2x^2 - 4x) &= 2x^2 - 7x - 2x^2 + 4x \\ &= -3x\end{aligned}$

Section 4.3

34. $2x(x^2 - 7x - 4) = 2x^3 - 14x^2 - 8x$

35. $-3x(6x^2 - 5x + 4) = -18x^3 + 15x^2 - 12x$

36. $(x+6)(x-7) = x^2 - 7x + 6x - 42$
$= x^2 - x - 42$

37. $(x-2)(x-9) = x^2 - 9x - 2x + 18$
$= x^2 - 11x + 18$

38. $\left(\frac{1}{2}x+1\right)\left(\frac{1}{2}x-5\right) = \frac{1}{4}x^2 - \frac{5}{2}x + \frac{1}{2}x - 5$
$= \frac{1}{4}x^2 - 2x - 5$

39. $\left(-\frac{1}{5}+2y\right)\left(\frac{1}{5}+y\right) = -\frac{1}{25} - \frac{1}{5}y + \frac{2}{5}y + 2y^2$
$= 2y^2 + \frac{1}{5}y - \frac{1}{25}$

40. $(3x+5)(9x^2-15x+25)$
$= 27x^3 - 45x^2 + 75x + 45x^2 - 75x + 125$
$= 27x^3 + 125$

41. $(x-y)(x^2+xy+y^2)$
$= x^3 + x^2y + xy^2 - x^2y - xy^2 - y^3$
$= x^3 - y^3$

42. $(2x-5)^2 = (2x)^2 - 2(2x)(5) + (5)^2$
$= 4x^2 - 20x + 25$

43. $\left(\frac{1}{2}x+4\right)^2 = \left(\frac{1}{2}x\right)^2 + 2\left(\frac{1}{2}x\right)(4) + (4)^2$
$= \frac{1}{4}x^2 + 4x + 16$

44. $(3y-11)(3y+11) = (3y)^2 - (11)^2$
$= 9y^2 - 121$

45. $(6w-1)(6w+1) = (6w)^2 - (1)^2$
$= 36w^2 - 1$

46. $\left(\frac{2}{3}t+4\right)\left(\frac{2}{3}t-4\right) = \left(\frac{2}{3}t\right)^2 - (4)^2$
$= \frac{4}{9}t^2 - 16$

47. $\left(z+\frac{1}{4}\right)\left(z-\frac{1}{4}\right) = (z)^2 - \left(\frac{1}{4}\right)^2$
$= z^2 - \frac{1}{16}$

48. $[(x+2)-b][(x+2)+b]$
$=(x+2)^2 - b^2$
$=x^2 + 4x + 4 - b^2$

49. $[c-(w+3)][c+(w+3)]$
$= c^2 - (w+3)^2$
$= c^2 - (w^2 + 6w + 9)$
$= c^2 - w^2 - 6w - 9$

50. $(2x+1)^3 = (2x+1)(2x+1)^2$
$= (2x+1)(4x^2+4x+1)$
$= 8x^3 + 8x^2 + 2x + 4x^2 + 4x + 1$
$= 8x^3 + 12x^2 + 6x + 1$

51. $(y^2-3)^3 = (y^2-3)(y^2-3)^2$
$= (y^2-3)(y^4 - 6y^2 + 9)$
$= y^6 - 6y^4 + 9y^2 - 3y^4 + 18y^2 - 27$
$= y^6 - 9y^4 + 27y^2 - 27$

52. **a.** $A = (2x+3)^2$
$= (2x)^2 + 2(2x)(3) + (3)^2$
$= 4x^2 + 12x + 9$

b. $A = (2x+3+2x)^2 = (4x+3)^2$
$= (4x)^2 + 2(4x)(3) + (3)^2$
$= 16x^2 + 24x + 9$

c. $(16x^2 + 24x + 9) - (4x^2 + 12x + 9)$
$= 16x^2 + 24x + 9 - 4x^2 - 12x - 9$
$= 12x^2 + 12x$

53. **a.** Let x = the width of the rectangle
$3x + 2$ = the length of the rectangle
$P(x) = 2(3x+2) + 2x$
$= 6x + 4 + 2x = 8x + 4$

b. $A(x) = (3x+2)(x) = 3x^2 + 2x$

54. **a.** $2x^2 + 5x = 7x^3$
False; cannot add *unlike* terms.
$(2x^2)(5x) = 10x^3$
True

b. $4x - 7x = -3x$
True
$4x - 7x = -3$
False; when subtracting like terms, keep the variable.

Section 4.4

55. $(6x^3y + 12x^2y^2 - 9xy^3) \div (3xy)$
$\frac{6x^3y}{3xy} + \frac{12x^2y^2}{3xy} - \frac{9xy^3}{3xy}$
$= 2x^2 + 4xy - 3y^2$

56. $(10x^4 + 15x^3 - 20x^2) \div (-5x^2)$
$\frac{10x^4}{-5x^2} + \frac{15x^3}{-5x^2} - \frac{20x^2}{-5x^2}$
$= -2x^2 - 3x + 4$

57. **a.**

$$\begin{array}{r} 3y^3-2y^2+6y-4 \\ 3y+2\overline{)\ 9y^4\quad +14y^2\quad -8} \\ \underline{-\left(9y^4+6y^3\right)} \\ -6y^3+14y^2 \\ \underline{-\left(-6y^3-4y^2\right)} \\ 18y^2 \\ \underline{-\left(18y^2+12y\right)} \\ -12y-8 \\ \underline{-\left(-12y-8\right)} \\ 0 \end{array}$$

Quotient: $3y^3-2y^2+6y-4$

b. Remainder: 0

c. Multiply the quotient and the divisor.

58.

$$\begin{array}{r} x+2 \\ x+5\overline{)\ x^2+7x+10} \\ \underline{-\left(x^2+5x\right)} \\ 2x+10 \\ \underline{-\left(2x+10\right)} \\ 0 \end{array}$$

The quotient is $x+2$.

59.

$$\begin{array}{r} x+4 \\ x+4\overline{)\ x^2+8x-16} \\ \underline{-\left(x^2+4x\right)} \\ 4x-16 \\ \underline{-\left(4x+16\right)} \\ -32 \end{array}$$

The quotient is $x+4+\dfrac{-32}{x+4}$.

60.

$$\begin{array}{r} 2x^3+2x^2+8x+24 \\ x^2-3x\overline{)\ 2x^5-4x^4+2x^3\qquad -4} \\ \underline{-\left(2x^5-6x^4\right)} \\ 2x^4+2x^3 \\ \underline{-\left(2x^4-6x^3\right)} \\ 8x^3 \\ \underline{-\left(8x^3-24x^2\right)} \\ 24x^2 \\ \underline{-\left(24x^2-72x\right)} \\ 72x-4 \end{array}$$

The quotient is

$$2x^3+2x^2+8x+24+\frac{72x-4}{x^2-3x}$$

61.

$$\begin{array}{r} 2x^3-2x^2+5x-4 \\ x^2+x\overline{)\ 2x^5\quad +3x^3+x^2\quad -4} \\ \underline{-\left(2x^5+2x^4\right)} \\ -2x^4+3x^3 \\ \underline{-\left(-2x^4-2x^3\right)} \\ 5x^3+x^2 \\ \underline{-\left(5x^3+5x^2\right)} \\ -4x^2 \\ \underline{-\left(-4x^2-4x\right)} \\ 4x-4 \end{array}$$

The quotient is

$$2x^3-2x^2+5x-4+\frac{4x-4}{x^2+x}.$$

62. Synthetic division can be used with a divisor in the form $x-r$.

63. **a.** Divisor: $x-3$

b. Quotient: $2x^3+11x^2+31x+99$

c. Remainder: 298

64.

$$\begin{array}{r|rrrr} 2 & 1 & -3 & 8 & -12 \\ & & 2 & -2 & 12 \\ \hline & 1 & -1 & 6 & 0 \end{array}$$

Quotient: t^2-t+6

65.

$$\begin{array}{r|rrr} -5 & 1 & 7 & 14 \\ & & -5 & -10 \\ \hline & 1 & 2 & 4 \end{array}$$

Quotient: $x+2+\dfrac{4}{x+5}$

66.

$$\begin{array}{r|rrr} -4 & 1 & 8 & 20 \\ & & -4 & -16 \\ \hline & 1 & 4 & 4 \end{array}$$

Quotient: $x+4+\dfrac{4}{x+4}$

67.
$$\begin{array}{r|rrrr} 3 & 1 & -6 & 0 & 8 \\ & & 3 & -9 & -27 \\ \hline & 1 & -3 & -9 & \underline{-19} \end{array}$$

Quotient: $w^2 - 3w - 9 + \dfrac{-19}{w-3}$

68.
$$\begin{array}{r|rrrrr} 2 & 1 & 0 & 0 & 0 & -16 \\ & & 2 & 4 & 8 & 16 \\ \hline & 1 & 2 & 4 & 8 & \underline{0} \end{array}$$

Quotient: $p^3 + 2p^2 + 4p + 8$

Section 4.5

69. $-x^3 - 4x^2 + 11x = -x\left(x^2 + 4x - 11\right)$
or $x\left(-x^2 - 4x + 11\right)$

70. $21w^3 - 7w + 14 = 7\left(3w^3 - w + 2\right)$

71. $5x\left(x-7\right) - 2\left(x-7\right) = \left(x-7\right)\left(5x-2\right)$

72. $3t\left(t+4\right) + 5\left(t+4\right) = \left(t+4\right)\left(3t+5\right)$

73.
$$\begin{aligned} m^3 - 8m^2 + m - 8 &= m^2\left(m-8\right) + \left(m-8\right) \\ &= \left(m-8\right)\left(m^2+1\right) \end{aligned}$$

74.
$$\begin{aligned} 24x^3 - 36x^2 + 72x - 108 &= 12\left(2x^3 - 3x^2 + 6x - 9\right) \\ &= 12\left[x^2\left(2x-3\right) + 3\left(2x-3\right)\right] \\ &= 12\left(2x-3\right)\left(x^2+3\right) \end{aligned}$$

75.
$$\begin{aligned} 4ax^2 + 2bx^2 - 6ax - 3xb &= x\left(4ax + 2bx - 6a - 3b\right) \\ &= x\left[2x\left(2a+b\right) - 3\left(2a+b\right)\right] \\ &= x\left(2a+b\right)\left(2x-3\right) \end{aligned}$$

76.
$$\begin{aligned} y^3 - 6y^2 + y - 6 &= y^2\left(y-6\right) + \left(y-6\right) \\ &= \left(y-6\right)\left(y^2+1\right) \end{aligned}$$

Section 4.6

77. The trinomial must be of the form $a^2 + 2ab + b^2$ or $a^2 - 2ab + b^2$.

78.
$$\begin{aligned} 18x^2 + 27xy + 10y^2 &= 18x^2 + 15xy + 12xy + 10y^2 \\ &= 3x\left(6x+5y\right) + 2y\left(6x+5y\right) \\ &= \left(6x+5y\right)\left(3x+2y\right) \end{aligned}$$

79. $3m^2 + mt - 10t^2 = 3m^2 + 6mt - 5mt - 10t^2$
$= 3m(m + 2t) - 5t(m + 2t)$
$= (m + 2t)(3m - 5t)$

80. $60a^2 + 65a^3 - 20a^4 = -5a^2(4a^2 - 13a - 12)$
$= -5a^2(4a^2 - 16a + 3a - 12)$
$= -5a^2[4a(a - 4) + 3(a - 4)]$
$= -5a^2(a - 4)(4a + 3)$

81. $2k^2 + 7k^3 + 6k^4 = k^2(6k^2 + 7k + 2)$
$= k^2(6k^2 + 4k + 3k + 2)$
$= k^2[2k(3k + 2) + (3k + 2)]$
$= k^2(3k + 2)(2k + 1)$

82. $49x^2 + 36 - 84x = 49x^2 - 84x + 36$
$= (7x)^2 - 2(7x)(6) + 6^2$
$= (7x - 6)^2$

83. $80z + 32 + 50z^2 = 50z^2 + 80z + 32$
$= 2(25z^2 + 40z + 16)$
$= 2[(5z)^2 + 2(5z)(4) + 4^2]$
$= 2(5z + 4)^2$

84. $(9w + 2)^2 + 4(9w + 2) - 5$
Let $u = 9w + 2$
$u^2 + 4u - 5 = u^2 + 5u - u - 5$
$= u(u + 5) - (u + 5)$
$= (u + 5)(u - 1)$
$= (9w + 2 + 5)(9w + 2 - 1)$
$= (9w + 7)(9w + 1)$

85. $(4x + 3)^2 - 12(4x + 3) + 36$
Let $u = 4x + 3$
$u^2 - 12u + 36 = u^2 - 2(u)(6) + 6^2$
$= (u - 6)^2$
$= (4x + 3 - 6)^2$
$= (4x - 3)^2$

86. $18a^4 + 39a^2 - 15 = 3(6a^4 + 13a^2 - 5)$
$= 3(6a^4 + 15a^2 - 2a^2 - 5)$
$= 3[3a^2(2a^2 + 5) - (2a^2 + 5)]$
$= 3(2a^2 + 5)(3a^2 - 1)$

87. $3w^4 - 2w^2 - 5 = 3w^4 - 5w^2 + 3w^2 - 5$
$= w^2\left(3w^2 - 5\right) + \left(3w^2 - 5\right)$
$= \left(3w^2 - 5\right)\left(w^2 + 1\right)$

Section 4.7

88. $25 - y^2 = 5^2 - y^2 = (5 + y)(5 - y)$

89. $x^3 - \frac{1}{27} = x^3 - \left(\frac{1}{3}\right)^3 = \left(x - \frac{1}{3}\right)\left(x^2 + \frac{1}{3}x + \frac{1}{9}\right)$

90. $b^2 + 64$ is prime.

91. $h^3 + 9h = h\left(h^2 + 9\right)$

92. $a^3 + 64 = a^3 + 4^3 = (a + 4)\left(a^2 - 4a + 16\right)$

93. $k^4 - 16 = \left(k^2\right)^2 - 4^2 = \left(k^2 + 4\right)\left(k^2 - 4\right)$
$= \left(k^2 + 4\right)(k + 2)(k - 2)$

94. $9y^3 - 4y = y\left(9y^2 - 4\right) = y\left[(3y)^2 - 2^2\right]$
$= y(3y + 2)(3y - 2)$

95. $x^2 - 8xy + 16y^2 - 9 = (x - 4y)^2 - 3^2$
$= (x - 4y + 3)(x - 4y - 3)$

96. $a^2 + 12a + 36 - b^2 = (a + 6)^2 - b^2$
$= (a + 6 + b)(a + 6 - b)$

97. $t^2 + 16t + 64 - 25c^2 = (t + 8)^2 - (5c)^2$
$= (t + 8 + 5c)(t + 8 - 5c)$

98. $y^2 - 6y + 9 - 16x^2 = (y - 3)^2 - (4x)^2$
$= (y - 3 + 4x)(y - 3 - 4x)$

Section 4.8

99. A quadratic equation can be written in the form $ax^2 + bx + c = 0 \ (a \neq 0)$.

100. The graph of a quadratic function is a parabola.

101. $x^2 + 6x = 7$

$x^2 + 6x - 7 = 0$ Quadratic

102. $(x-3)(x+4) = 9$

$x^2 + 4x - 3x - 12 = 9$

$x^2 + x - 21 = 0$ Quadratic

103. $2x - 5 = 3$

$2x - 8 = 0$ Linear

104. $x + 3 = 5x^2$

$0 = 5x^2 - x - 3$ Quadratic

105. a. $5x^2 + 6x - 8 = (5x - 4)(x + 2)$

b. $5x^2 + 6x - 8 = 0$

$(5x - 4)(x + 2) = 0$

$5x - 4 = 0$ or $x + 2 = 0$

$x = \frac{4}{5}$ or $x = -2$ $\left\{\frac{4}{5}, -2\right\}$

106. a. $3x^2 - 19x + 28 = (3x - 7)(x - 4)$

b. $3x^2 - 19x + 28 = 0$

$(3x - 7)(x - 4) = 0$

$3x - 7 = 0$ or $x - 4 = 0$

$x = \frac{7}{3}$ or $x = 4$ $\left\{\frac{7}{3}, 4\right\}$

107. $x^2 - 2x - 15 = 0$

$(x-5)(x+3) = 0$

$x - 5 = 0$ or $x + 3 = 0$

$x = 5$ or $x = -3$

$\{5, -3\}$

108. $8x^2 = 59x - 21$

$8x^2 - 59x + 21 = 0$

$8x^2 - 56x - 3x + 21 = 0$

$8x(x-7) - 3(x-7) = 0$

$(x-7)(8x-3) = 0$

$x - 7 = 0$ or $8x - 3 = 0$

$x = 7$ or $8x = 3$

$x = 7$ or $x = \frac{3}{8}$ $\left\{7, \frac{3}{8}\right\}$

109. $2t(t+5)+1=3t-3-t^2$

$2t^2+10t+1=3t-3-t^2$

$3t^2+7t+4=0$

$(3t+4)(t+1)=0$

$3t+4=0 \text{ or } t+1=0$

$3t=-4 \text{ or } t=-1$

$t=-\frac{4}{3} \text{ or } t=-1 \quad \left\{-\frac{4}{3},-1\right\}$

110. $3(x-1)(x+5)(2x-9)=0$

$3\neq 0 \text{ or } x-1=0 \text{ or } x+5=0 \text{ or } 2x-9=0$

$x=1 \text{ or } x=-5 \text{ or } 2x=9$

$x=1 \text{ or } x=-5 \text{ or } x=\frac{9}{2}$

$\left\{1,-5,\frac{9}{2}\right\}$

111. $f(x)=-4x^2+4=0$

$-4(x^2-1)=0$

$-4(x+1)(x-1)=0$

$-4\neq 0 \text{ or } (x+1)=0 \text{ or } x-1=0$

$x=-1 \text{ or } x=1$

$f(0)=-4(0)^2+4$

$=0+4$

$=4$

x-intercepts: (–1, 0), (1,0)

y-intercept: (0, 4)

Graph b.

112. $g(x)=2x^2-2=0$

$2(x^2-1)=0$

$2(x+1)(x-1)=0$

$2\neq 0 \text{ or } (x+1)=0 \text{ or } x-1=0$

$x=-1 \text{ or } x=1$

$g(0)=2(0)^2-2=0-2=-2$

x-intercepts: (–1, 0), (1,0)

y-intercept: (0, –2)

Graph d.

113. $h(x)=5x^3-10x^2-20x+40=0$

$5(x^3-2x^2-4x+8)=0$

$5[x^2(x-2)-4(x-2)]=0$

$5(x-2)(x^2-4)=0$

$5(x-2)(x-2)(x+2)=0$

$5\neq 0 \text{ or } (x-2)=0 \text{ or } x-2=0 \text{ or } x+2=0$

$x=2 \text{ or } x=2 \text{ or } x=-2$

$h(0)=5(0)^3-10(0)^2-20(0)+40$

$=40$

x-intercepts: (–2, 0), (2,0)

114. $k(x)=-\frac{1}{8}x^2+\frac{1}{2}=0$

$-\frac{1}{8}(x^2-4)=0$

$-\frac{1}{8}(x+2)(x-2)=0$

$-\frac{1}{8}\neq 0 \text{ or } x+2=0 \text{ or } x-2=0$

$x=-2 \text{ or } x=2$

$k(0)=-\frac{1}{8}(0)^2+\frac{1}{2}$

$=0+\frac{1}{2}$

$=\frac{1}{2}$

y-intercept: (0, 40)

Graph c.

x-intercepts: (–2, 0), (2,0)

y-intercept: $\left(0,\frac{1}{2}\right)$

Graph a.

115. Let x = the width of the truck

$2x - 1$ = the length of the truck

$$V = 10(x)(2x-1) = 1200$$
$$x(2x-1) = 120$$
$$2x^2 - x - 120 = 0$$
$$2x^2 - 16x + 15x - 120 = 0$$
$$2x(x-8) + 15(x-8) = 0$$
$$(x-8)(2x+15) = 0$$
$$x - 8 = 0 \text{ or } 2x + 15 = 0$$
$$x = 8 \text{ or } 2x = -15$$
$$x = 8 \text{ or } x \neq -\frac{15}{2}$$
$$2x - 1 = 2(8) - 1$$
$$= 15$$

The width of the truck is 8 ft, the length of the truck is 15 ft, and the height is 10 ft.

116. **a.**

Time	Height
0	–1280
1	–624
3	592
10	3840
20	5760
30	4480
42	–1280

b. The position of the missile is below sea level.

c. $h(t) = -16t^2 + 672t - 1280 = 0$

$$-16(t^2 - 42t + 80) = 0$$
$$-16(t-40)(t-2) = 0$$
$$t - 40 = 0 \text{ or } t - 2 = 0$$
$$t = 40 \text{ or } t = 2$$

The missile will be at sea level after 2 sec and again after 40 sec.

Chapter 4 Test

1. $\dfrac{20a^7}{4a^{-6}} = 5a^{7-(-6)}$

$= 5a^{13}$

2. $\dfrac{x^6x^3}{x^{-2}} = \dfrac{x^{6+3}}{x^{-2}}$

$= \dfrac{x^9}{x^{-2}}$

$= x^{9-(-2)}$

$= x^{11}$

3. $\left(\dfrac{-3x^6}{5y^7}\right)^2 = \dfrac{(-3)^2\left(x^6\right)^2}{(5)^2\left(y^7\right)^2}$

$= \dfrac{9x^{12}}{25y^{14}}$

4. $\dfrac{\left(2^{-1}xy^{-2}\right)^{-3}\left(x^{-4}y\right)}{\left(x^0y^5\right)^{-1}} = \dfrac{\left(2^{-1}\right)^{-3}x^{-3}\left(y^{-2}\right)^{-3}\left(x^{-4}y\right)}{\left(x^0\right)^{-1}\left(y^5\right)^{-1}}$

$= \dfrac{2^3x^{-3}y^6x^{-4}y}{x^0y^{-5}} = \dfrac{8x^{-3+(-4)}y^{6+1}}{1\cdot y^{-5}}$

$= \dfrac{8x^{-7}y^7}{y^{-5}} = 8x^{-7}y^{7-(-5)}$

$= 8x^{-7}y^{12} = 8\cdot\dfrac{1}{x^7}\cdot y^{12}$

$= \dfrac{8y^{12}}{x^7}$

5. $\left(8.0\times10^{-6}\right)\left(7.1\times10^5\right) = 56.8\times10^{-6+5}$

$= 5.68\times10^1\times10^{-1}$

$= 5.68\times10^0$

$= 5.68$

6. $\left(9{,}200{,}000\right)\div\left(0.004\right) = \dfrac{9.2\times10^6}{4\times10^{-3}}$

$= 2.3\times10^{6-(-3)}$

$= 2.3\times10^9$

7. $F(x)=5x^3-2x^2+8$

$F(-1)=5(-1)^3-2(-1)^2+8=-5-2+8=$

$F(2)=5(2)^3-2(2)^2+8=40-8+8=4($

$F(0)=5(0)^3-2(0)^2+8=0-0+8=8$

8. $\left(5x^2-7x+3\right)-\left(x^2+5x-25\right)+\left(4x^2+4x-20\right)$

$=5x^2-7x+3-x^2-5x+25+4x^2+4x-2($

$=8x^2-8x+8$

9. $(2a-5)\left(a^2-4a-9\right)$

$=2a^3-8a^2-18a-5a^2+20a+45$

$=2a^3-13a^2+2a+45$

10. $\left(\frac{1}{3}x-\frac{3}{2}\right)(6x+4)=2x^2+\frac{4}{3}x-9x-6$

$=2x^2-\frac{23}{3}x-6$

11. $\left(5x-4y^2\right)\left(5x+4y^2\right)=(5x)^2-\left(4y^2\right)^2$

$=25x^2-16y^4$

12. The expression $25x^2+49$ does not account for the middle term $70x$.

13. $(7x-4)^2=(7x)^2-2(7x)(4)+4^2$

$=49x^2-56x+16$

14. $\left(2x^3y^4+5x^2y^2-6xy^3-xy\right)\div(2xy)$

$\frac{2x^3y^4}{2xy}+\frac{5x^2y^2}{2xy}-\frac{6xy^3}{2xy}-\frac{xy}{2xy}$

$=x^2y^3+\frac{5}{2}xy-3y^2-\frac{1}{2}$

15.

$$
\begin{array}{r}
5p^2-p+1 \\
2p+3\overline{)\;10p^3+13p^2-\;p+3} \\
\underline{-\left(10p^3+15p^2\right)} \\
-2p^2-\;p \\
\underline{-\left(-2p^2-3p\right)} \\
2p+3 \\
\underline{-(2p+3)} \\
0
\end{array}
$$

Quotient: $5p^2-p+1$

16.

$$
\begin{array}{r|rrrrr}
2 & 1 & 0 & 0 & -2 & 5 \\
 & & 2 & 4 & 8 & 12 \\
\hline
 & 1 & 2 & 4 & 6 & 17
\end{array}
$$

Quotient: $y^3+2y^2+4y+6+\frac{17}{y-2}$

17. $17y + 3y^2 - 28 = 3y^2 + 17y - 28$
$= 3y^2 + 21y - 4y - 28$
$= 3y(y + 7) - 4(y + 7)$
$= (3y - 4)(y + 7)$

18. $x^2 - 5x - 4$ is a prime.

19. $3a^2 + 27ab + 54b^2 = 3(a^2 + 9ab + 18b^2)$
$= 3(a + 6b)(a + 3b)$

20. $c^4 - 1 = (c^2 + 1)(c^2 - 1)$
$= (c^2 + 1)(c + 1)(c - 1)$

21. $xy - 7x + 3y - 21 = x(y - 7) + 3(y - 7)$
$= (y - 7)(x + 3)$

22. $49 + p^2$ is prime.

23. $-10u^2 + 30u - 20 = -10(u^2 - 3u + 2)$
$= -10(u - 2)(u - 1)$

24. $12t^2 - 75 = 3(4t^2 - 25)$
$= 3(2t + 5)(2t - 5)$

25. $5y^2 - 50y + 125 = 5(y^2 - 10y + 25)$
$= 5(y - 5)(y - 5)$
$= 5(y - 5)^2$

26. $21q^2 + 14q = 7q(3q + 2)$

27. $2x^3 + x^2 - 8x - 4 = x^2(2x + 1) - 4(2x + 1)$
$= (2x + 1)(x^2 - 4)$
$= (2x + 1)(x + 2)(x - 2)$

28. $y^3 - 125 = (y - 5)(y^2 + 5y + 25)$

29. $x^2 + 8x + 16 - y^2 = (x + 4)^2 - y^2$
$= (x + 4 + y)(x + 4 - y)$

30. $r^6 - 256r^2 = r^2(r^4 - 256)$
$= r^2(r^2 + 16)(r^2 - 16)$
$= r^2(r^2 + 16)(r + 4)(r - 4)$

31. $\left(x^2 + 1\right)^2 + 3\left(x^2 + 1\right) + 2$
Let $u = x^2 + 1$
$u^2 + 3u + 2 = \left(u + 2\right)\left(u + 1\right)$
$= \left(x^2 + 1 + 2\right)\left(x^2 + 1 + 1\right)$
$= \left(x^2 + 3\right)\left(x^2 + 2\right)$

32. $12a - 6ac + 2b - bc = 6a(2 - c) + b(2 - c)$
$= (2 - c)(6a + b)$

33. $(2x-3)(x+5)=0$

$2x-3=0 \quad \text{or} \quad x+5=0$

$x=\frac{3}{2} \qquad x=-5 \quad \left\{\frac{3}{2},-5\right\}$

34. $w^2-7w=0$

$w(w-7)=0$

$w=0 \quad \text{or} \quad w-7=0$

$w=0 \qquad w=7 \quad \{0,7\}$

35. $y^2-6y=16$

$y^2-6y-16=0$

$(y+2)(y-8)=0$

$y+2=0 \quad \text{or} \quad y-8=0$

$y=-2 \qquad y=8 \quad \{-2,8\}$

36. $x(5x+4)=1$

$5x^2+4x-1=0$

$(5x-1)(x+1)=0$

$5x-1=0 \quad \text{or} \quad x+1=0$

$x=\frac{1}{5} \qquad x=-1 \quad \left\{\frac{1}{5},-1\right\}$

37. $4p-64p^3=0$

$-4p\left(16p^2-1\right)=0$

$-4p(4p+1)(4p-1)=0$

$-4p=0 \text{ or } 4p+1=0 \text{ or } 4p-1=0$

$p=0 \text{ or } 4p=-1 \text{ or } 4p=1$

$p=0 \text{ or } p=-\frac{1}{4} \text{ or } p=\frac{1}{4}$

$\left\{0,-\frac{1}{4},\frac{1}{4}\right\}$

38. $t^2+\frac{1}{2}t+\frac{1}{16}=0$

$16t^2+8t+1=0$

$(4t+1)^2=0$

$4t+1=0$

$4t=-1$

$t=-\frac{1}{4} \quad \left\{-\frac{1}{4}\right\}$

39. $h(x)=-\frac{x^2}{256}+x=0$

$x^2-256x=0$

$x(x-256)=0$

$x=0 \text{ or } x-256=0$

$x=0 \text{ or } x=256$

The rocket hits the ground 256 ft from the launch pad.

40. a. $P(t)=-0.01t^2-0.062t+127.7$

$P(4)=-0.01(4)^2-0.062(4)+127.7$

$=-0.01(16)-0.062(4)+127.7$

$=-0.16-0.248+127.7=127.292$

The population of Japan in year 4 was about 127.292 million.

b. $P(6)=-0.01(6)^2-0.062(6)+127.7$

$=-0.01(36)-0.062(6)+127.7$

$=-0.36-0.372+127.7=126.968$

The population in year 6 was about 126.968 million.

c. $P(15) = -0.01(15)^2 - 0.062(15) + 127.7$
$= -0.01(225) - 0.062(15) + 127.7$
$= -2.25 - 0.93 + 127.7 = 124.52$

The population in year 15 will be about 124.52 million.

41. $f(x) = x^2 - 6x + 8 = 0$
$(x-4)(x-2) = 0$
$x - 4 = 0$ or $x - 2 = 0$
$x = 4$ or $x = 2$
$f(0) = (0)^2 - 6(0) + 8 = 0 - 0 + 8 = 8$

x-intercepts: (4, 0), (2,0)
y-intercept: (0, 8)
Graph c.

42. $k(x) = x^3 + 4x^2 - 9x - 36 = 0$
$x^2(x+4) - 9(x+4) = 0$
$(x+4)(x^2 - 9) = 0$
$(x+4)(x+3)(x-3) = 0$
$x + 4 = 0$ or $x + 3 = 0$ or $x - 3 = 0$
$x = -4$ or $x = -3$ or $x = 3$
$k(0) = (0)^3 + 4(0)^2 - 9(0) - 36 = -36$

x-intercepts: (–4, 0), (–3,0), (3, 0)
y-intercept: (0, –36)
Graph b.

43. $p(x) = -2x^2 - 8x - 6 = 0$
$-2(x^2 + 4x + 3) = 0$
$-2(x+3)(x+1) = 0$
$-2 \neq 0$ or $x + 3 = 0$ or $x + 1 = 0$
$x = -3$ or $x = -1$
$p(0) = -2(0)^2 - 8(0) - 6 = 0 - 0 - 6 = -6$

x-intercepts: (–3, 0), (–1,0)
y-intercept: (0, –6)
Graph d.

44. $q(x) = x^3 - x^2 - 12x = 0$
$x(x^2 - x - 12) = 0$
$x(x-4)(x+3) = 0$
$x = 0$ or $x - 4 = 0$ or $x + 3 = 0$
$x = 0$ or $x = 4$ or $x = -3$
$q(0) = (0)^3 - (0)^2 - 12(0) = 0$

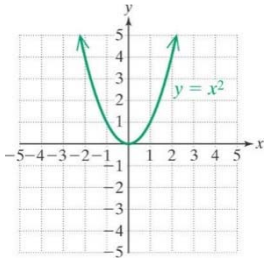
x-intercepts: (0, 0), (4,0), (–3, 0)
y-intercept: (0, 0)
Graph a.

Chapters 1 – 4 Cumulative Review Exercises

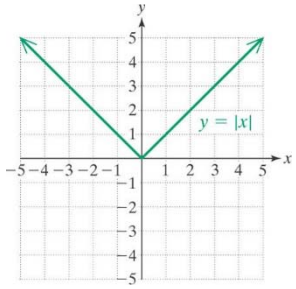
1. 

2.
$$\begin{aligned} 3x^2 - 5x + 2 - 4\left(x^2 + 3\right) &= 3x^2 - 5x + 2 - 4x^2 - 12 \\ &= -x^2 - 5x - 10 \end{aligned}$$

3. a. $y = x^2$



b. $y = |x|$



4.
$$\left(\frac{1}{3}\right)^{-2} - \left(\frac{1}{2}\right)^3 = 3^2 - \left(\frac{1}{2}\right)^3 = 9 - \frac{1}{8} = \frac{71}{8}$$

5.
$$\begin{aligned} 2\left(9.85 \times 10^7\right) &= 19.7 \times 10^7 \\ &= 1.97 \times 10^1 \times 10^7 \\ &= 1.97 \times 10^8 \text{ people} \end{aligned}$$

6. Let x = points scored by Florida State

$x + 3$ = points scored by Penn State

$$\begin{aligned} x + \left(x + 3\right) &= 49 \\ 2x + 3 &= 49 \\ 2x &= 46 \\ x &= 23 \\ x + 3 &= 26 \end{aligned}$$

Penn State scored 26 points and Florida State scored 23 points.

7.
$$\begin{aligned} x + \left(2x\right) + \left(2x - 5\right) &= 180 \\ 5x - 5 &= 180 \\ 5x &= 185 \\ x &= 37 \\ 2x &= 2\left(37\right) = 74 \\ 2x - 5 &= 2\left(37\right) - 5 = 69 \end{aligned}$$

The angles are 37°, 74°, and 69°.

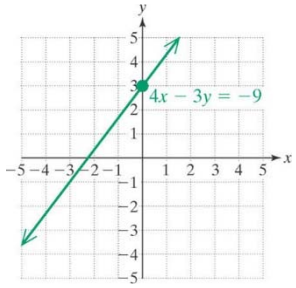
8.
$$x+4\overline{)\,x^3\qquad\quad+64}\qquad x^2-4x+16$$
$$-\left(x^3+4x^2\right)$$
$$-4x^2$$
$$-\left(-4x^2-16x\right)$$
$$16x+64$$
$$-\left(16x+64\right)$$
$$0$$

Quotient: $x^2-4x+16$

9. $4x-3y=-9$

$$-3y=-4x-9$$
$$\frac{-3y}{-3}=\frac{-4x}{-3}-\frac{9}{-3}$$
$$y=\frac{4}{3}x+3$$

Slope $=\frac{4}{3}$; y-intercept: (0, 3)



10. $y=-5$

11.
$$\left(\frac{36a^{-2}b^4}{18b^{-6}}\right)^{-3}=\left(2a^{-2}b^{4-(-6)}\right)^{-3}=\left(2a^{-2}b^{10}\right)^{-3}$$
$$=2^{-3}a^6b^{-30}=\frac{1}{2^3}a^6\frac{1}{b^{30}}=\frac{a^6}{8b^{30}}$$

12.
$$2x-y+2z=1$$
$$-3x+5y-2z=11$$
$$x+y-2z=-1$$

Add the first two equations to eliminate z:

$$2x-y+2z=1$$
$$\underline{-3x+5y-2z=11}$$
$$-x+4y=12$$

Add the first and third equations to eliminate z:

$$2x-y+2z=1$$
$$\underline{x+y-2z=-1}$$
$$3x=0$$
$$x=0$$

Substitute this result into the first sum and solve for y:

$$-0+4y=12$$
$$y=3$$

Substitute the values of x and y into the original first equation and solve for z:

$$2(0)-3+2z=1$$
$$-3+2z=1$$
$$2z=4$$
$$z=2$$

The solution is {(0, 3, 2)}.

13. **a.** Function

b. Not a function

14. $m = \frac{14}{2} = 7$

15. $P(x) = \frac{1}{6}x^2 + x - 5$

$P(6) = \frac{1}{6}(6)^2 + 6 - 5 = \frac{1}{6}(36) + 6 - 5$

$= 6 + 6 - 5 = 7$

16. $\frac{1}{3}x - \frac{1}{6} = \frac{1}{2}(x-3)$

$6\left(\frac{1}{3}x - \frac{1}{6}\right) = 6\left(\frac{1}{2}(x-3)\right)$

$2x - 1 = 3(x-3)$

$2x - 1 = 3x - 9$

$-x - 1 = -9$

$-x = -8$

$x = 8 \quad \{8\}$

17. $3x - 2y = 5$

$-2y = -3x + 5$

$\frac{-2y}{-2} = \frac{-3x}{-2} + \frac{5}{-2}$

$y = \frac{3}{2}x - \frac{5}{2}$

18. **a.** Let x = the score on the fourth test

$\frac{76 + 85 + 92 + x}{4} = 90$

$76 + 85 + 92 + x = 360$

$253 + x = 360$

$x = 107$

The fourth test score would have to be 107%; therefore, it is not possible.

$80 \le \frac{76 + 85 + 92 + x}{4} \le 89$

$320 \le 76 + 85 + 92 + x \le 356$

$320 \le 253 + x \le 356$

b. $67 \le x \le 103$

The range of values for the fourth test is between 67% and 100% inclusive.

19.

	40% Solution	15% Solution	30% Solution
Amount of Solution	x	$25-x$	25
Amount of Alcohol	$0.40x$	$0.15(25-x)$	$0.30(25)$

(amt of 40%) + (amt of 15%) = (amt of 30%)

$0.40x + 0.15(25 - x) = 0.30(25)$

$0.40x + 3.75 - 0.15x = 7.5$

$0.25x + 3.75 = 7.5$

$0.25x = 3.75$

$x = 15$

$25 - x = 25 - 15 = 10$

15 L of 40% solution and 10 L of 15% solution must be mixed.

20. $(4b-3)(2b^2+1) = 8b^3+4b-6b^2-3$
$= 8b^3-6b^2+4b-3$

21. $(5a^2+3a-1)+(3a^3-5a+6)$
$= 3a^3+5a^2-2a+5$

22. $(6w^3-5w^2-2w)\div(2w^2)$
$\frac{6w^3}{2w^2}-\frac{5w^2}{2w^2}-\frac{2w}{2w^2} = 3w-\frac{5}{2}-\frac{1}{w}$

23. $y^2-5y=14$
$y^2-5y-14=0$
$(y-7)(y+2)=0$
$y-7=0$ or $y+2=0$
$y=7$ or $y=-2$ $\{7,-2\}$

24. $a^3+9a^2+20a=0$
$a(a^2+9a+20)=0$
$a(a+5)(a+4)=0$
$a=0$ or $a+5=0$ or $a+4=0$
$a=0$ or $a=-5$ or $a=-4$ $\{0,-5,-4\}$

25. $-5<-8+|2x-5|$
$3<|2x-5|$
$2x-5>3$ or $2x-5<-3$
$2x>8$ or $2x<2$
$x>4$ or $x<1$
$(-\infty,1)\cup(4,\infty)$