Chapter 3  Systems of Linear Equations and Inequalities

Section 3.1  Practice Exercises

1.  a.  system
    b.  solution
    c.  intersect
    d.  consistent
    e.  the empty set, \{ \}
    f.  dependent
    g.  independent

3.  

\[ y = 8x - 5 \]
\[ y = 4x + 3 \]
Substitute \((-1,13)\):
\[ 13 = 8(-1) - 5 \]
\[ = -8 - 5 \]
\[ = -13 \]
Not a solution.
Substitute \((-1,1)\):
\[ 1 = 8(-1) - 5 \]
\[ = -8 - 5 \]
\[ = -13 \]
Not a solution.
Substitute \((2,11)\):
\[ 11 = 8(2) - 5 \]
\[ = 16 - 5 \]
\[ = 11 \]
\[ 11 = 4(2) + 3 \]
\[ = 8 + 3 \]
\[ = 11 \]
\((2,11)\) is a solution.

4.  

\[ y = -\frac{1}{2}x - 5 \]
\[ y = \frac{3}{4}x - 10 \]
Substitute \((4,-7)\):
\[ -7 = -\frac{1}{2}(4) - 5 \]
\[ = -2 - 5 \]
\[ = -7 \]
\[ -7 = \frac{3}{4}(4) - 10 \]
\[ = 3 - 10 \]
\[ = -7 \]
\((4,-7)\) is a solution.
Substitute \((0,-10)\):
\[ -10 = -\frac{1}{2}(0) - 5 \]
\[ = 0 - 5 \]
\[ = -5 \]
Not a solution.
Substitute \((3,-\frac{9}{2})\):
\[ -\frac{9}{2} = -\frac{1}{2}(3) - 5 \]
\[ = -\frac{3}{2} - 5 \]
\[ = -\frac{13}{2} \]
Not a solution.
Section 3.1 Solving Systems of Linear Equations by the Graphing Method

5. \[ 2x - 7y = -30 \]
\[ y = 3x + 7 \]
Substitute \((0, -30)\):
\[ 2(0) - 7(-30) = 0 + 210 = 210 \neq -30 \]
Not a solution.
Substitute \(\left(\frac{3}{2}, -5\right)\):
\[ 2\left(\frac{3}{2}\right) - 7(-5) = 3 - 35 = -32 \neq -30 \]
Not a solution.
Substitute \((-1, 4)\):
\[ 2(-1) - 7(4) = -2 - 28 = -30 = -30 \]
\[ 4 = 3(-1) + 7 \]
\[ = -3 + 7 \]
\[ = 4 \]
\((-1, 4)\) is a solution.

6. \[ x + 2y = 4 \]
\[ y = -\frac{1}{2}x + 2 \]
Substitute \((-2, 3)\):
\[ -2 + 2\left(\frac{3}{2}\right) = -2 + 6 = 4 = 4 \]
\[ 3 = -\frac{1}{2}(-2) + 2 = 1 + 2 = 3 \]
\((-2, 3)\) is a solution.
Substitute \((4, 0)\):
\[ 4 + 2(0) = 4 + 0 = 4 = 4 \]
\[ 0 = -\frac{1}{2}(4) + 2 = -2 + 2 = 0 \]
\((4, 0)\) is a solution.
Substitute \(\left(\frac{3}{2}, \frac{1}{2}\right)\):
\[ 3 + 2\left(\frac{1}{2}\right) = 3 + 1 = 4 = 4 \]
\[ \frac{1}{2} = -\frac{1}{2}(3) + 2 = -\frac{3}{2} + 2 = \frac{1}{2} \]
\(\left(\frac{3}{2}, \frac{1}{2}\right)\) is a solution.

7. \[ x - y = 6 \]
\[ 4x + 3y = -4 \]
Substitute \((4, -2)\):
\[ 4 - (-2) = 4 + 2 = 6 = 6 \]
\[ 4(4) + 3(-2) = 16 - 6 = 10 \neq -4 \]
Not a solution.
Substitute \((6, 0)\):
\[ 6 - 0 = 6 = 6 \]
\[ 4(6) + 3(0) = 24 + 0 = 24 \neq -4 \]
Not a solution.
Substitute \((2, 4)\):
\[ 2 - 4 = -2 \neq 6 \]
Not a solution.

8. \[ x - 3y = 3 \]
\[ 2x - 9y = 1 \]
Substitute \((0, 1)\):
\[ 0 - 3(1) = 0 - 3 = -3 \neq 3 \]
Not a solution.
Substitute \((4, -1)\):
\[ 4 - 3(-1) = 4 + 3 = 7 \neq 3 \]
Not a solution.
Substitute \((9, 2)\):
\[ 9 - 3(2) = 9 - 6 = 3 = 3 \]
\[ 2(9) - 9(2) = 18 - 18 = 0 \neq 1 \]
Not a solution.
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9. a. Consistent  
   b. Independent  
   c. One solution  

10. a. Consistent  
    b. Independent  
    c. One solution  

11. a. Inconsistent  
    b. Independent  
    c. Zero solutions  

12. a. Inconsistent  
    b. Independent  
    c. Zero solutions  

13. a. Consistent  
    b. Dependent  
    c. Infinitely many solutions  

14. a. Consistent  
    b. Dependent  
    c. Infinitely many solutions  

15. \[ \begin{align*} 2x + y &= -3 \\ -x + y &= 3 \end{align*} \] 
   \[ \begin{align*} y &= -2x - 3 \\ y &= x + 3 \end{align*} \] 
   The solution is \( \{(−2, 1)\} \).  

16. \[ \begin{align*} 4x - 3y &= 12 \\ 3x + 4y &= -16 \end{align*} \] 
   \[ \begin{align*} -3y &= -4x + 12 \\ 4y &= -3x - 16 \end{align*} \] 
   \[ \begin{align*} y &= \frac{4}{3}x - 4 \\ y &= -\frac{3}{4}x - 4 \end{align*} \] 
   The solution is \( \{(0, -4)\} \).  

17. \( f(x) = -2x + 3 \quad g(x) = 5x - 4 \)  
   \[ f(x) = -2x + 3 \\ g(x) = 5x - 4 \] 
   The solution is \( \{(1, 1)\} \).  

18. \( h(x) = 2x + 5 \quad g(x) = -x + 2 \)  
   \[ h(x) = 2x + 5 \\ g(x) = -x + 2 \] 
   The solution is \( \{(-1, 3)\} \).
Section 3.1 Solving Systems of Linear Equations by the Graphing Method

19. \( k(x) = \frac{1}{3}x - 5 \) \( f(x) = -\frac{2}{3}x - 2 \)

The solution is \( \{(3, -4)\} \).

20. \( f(x) = \frac{1}{2}x + 2 \) \( g(x) = \frac{5}{2}x - 2 \)

The solution is \( \{(2, 3)\} \).

21. \( x = 4 \) \( y = 2x - 3 \)

The solution is \( \{(4, 5)\} \).

22. \( 3x + 2y = 6 \) \( y = -3 \)

\( 2y = -3x + 6 \)
\( y = -\frac{3}{2}x + 3 \)

The solution is \( \{(4, -3)\} \).

23. \( y = -2x + 3 \) \( -2x = y + 1 \) \( y = -2x - 1 \)

There is no solution; \( \{ \} \).
Inconsistent system.

24. \( y = \frac{1}{3}x - 2 \) \( x = 3y - 9 \)

\( 3y = x + 9 \)
\( y = \frac{1}{3}x + 3 \)

There is no solution; \( \{ \} \).
Inconsistent system.
25. \[ y = \frac{2}{3} x - 1 \quad 2x = 3y + 3 \quad 3y = 2x - 3 \quad y = \frac{2}{3} x - 1 \]

Infinitely many solutions of the form \( \{ (x, y) \mid y = \frac{2}{3} x - 1 \} \). Dependent equations.

26. \[ 4x = 16 - 8y \quad y = -\frac{1}{2} x + 2 \]

8y = -4x + 16

Infinitely many solutions of the form \( \{ (x, y) \mid y = -\frac{1}{2} x + 2 \} \). Dependent equations.

27. \[ 2x = 4 \quad \frac{1}{2} y = -1 \]

\[ x = 2 \quad y = -2 \]

The solution is \( \{ (2, -2) \} \).

28. \[ y + 7 = 6 \quad -5 = 2x \]

\[ y = -1 \quad x = -\frac{5}{2} \]

The solution is \( \left\{ \left( -\frac{5}{2}, -1 \right) \right\} \).

29. \[ -x + 3y = 6 \quad 6y = 2x + 12 \]

\[ 3y = x + 6 \quad y = \frac{1}{3} x + 2 \]

\[ y = \frac{1}{3} x + 2 \]

30. \[ 3x = 2y - 4 \quad -4y = -6x - 8 \]

\[ 2y = 3x + 4 \quad y = \frac{3}{2} x + 2 \]

\[ y = \frac{3}{2} x + 2 \]
Section 3.1 Solving Systems of Linear Equations by the Graphing Method

Infinitely many solutions of the form \( \{ (x, y) \mid -x + 3y = 6 \} \). Dependent equations.

Infinitely many solutions of the form \( \{ (x, y) \mid 3x = 2y - 4 \} \). Dependent equations.

31. \( 2x - y = 4 \quad 4x + 2 = 2y \)
   \(-y = -2x + 4 \quad 2y = 4x + 2 \)
   \( y = 2x - 4 \quad y = 2x + 1 \)

   There is no solution; \( \{ \} \). Inconsistent system.

32. \( x = 4y + 4 \quad -2x + 8y = -16 \)
   \( 4y = x - 4 \quad 8y = 2x - 16 \)
   \( y = \frac{1}{4}x - 1 \quad y = \frac{1}{4}x - 2 \)

   There is no solution; \( \{ \} \). Inconsistent system.

33. False

35. True

37. For example: The system \( \begin{align*}
x + y &= 9 \\
2x + y &= 13
\end{align*} \)
   has solution \( \{ (4, 5) \} \).

38. For example: The system \( \begin{align*}
x + y &= 4 \\
-x + y &= 8
\end{align*} \)
   has solution \( \{ (-2, 6) \} \).
39.  \[ Cx + 2y = 11 \]
\[ C(1) + 2(3) = 11 \]
\[ C + 6 = 11 \]
\[ C = 5 \]
\[ -3x + Dy = 9 \]
\[ 3(1) + D(3) = 9 \]
\[ -3 + 3D = 9 \]
\[ 3D = 12 \]
\[ D = 4 \]

40.  \[ 3x + M y = -22 \]
\[ 3(2) + M(-4) = -22 \]
\[ 6 - 4M = -22 \]
\[ -4M = -28 \]
\[ M = 7 \]
\[ Nx + 4y = 6 \]
\[ N(2) + 4(-4) = 6 \]
\[ 2N - 16 = 6 \]
\[ 2N = 22 \]
\[ N = 11 \]

41.  \[ y = 5.62x + 15.46 \]
\[ y = -1.96x - 11.07 \]
\[ \{(3.5, -4.21)\} \]

42.  \[ y = -2.3x - 5.48 \]
\[ y = 4.62x + 26.352 \]
\[ \{(-4.6, 5.1)\} \]

43.  \[ 2.4x - 4.8y = -9.36 \]
\[ -4.8y = -2.4x - 9.36 \]
\[ y = 0.5x + 1.95 \]
\[ -1.8x + 5.4y = 12.456 \]
\[ 5.4y = 1.8x + 12.456 \]
\[ y = \frac{1}{3}x + \frac{173}{75} \]

44.  \[ 36x - 90y = -36 \]
\[ -90y = -36x - 36 \]
\[ y = 0.4x + 0.4 \]
\[ -15.5x - 5y = -80.75 \]
\[ -5y = 15.5x - 80.75 \]
\[ y = -3.1x + 16.15 \]
Section 3.2 Solving Systems of Linear Equations by the Substitution Method

Section 3.2 Practice Exercises

1. \[ y = 8x - 1 \quad 2x - 16y = 3 \]
   \[ -16y = -2x + 3 \]
   \[ y = \frac{1}{8}x - \frac{3}{16} \]
   One solution

2. \[ 4x + 6y = 1 \quad 10x + 15y = \frac{5}{2} \]
   \[ 6y = -4x + 1 \quad 15y = -10x + \frac{5}{2} \]
   \[ y = -\frac{2}{3}x + \frac{1}{6} \quad y = -\frac{2}{3}x + \frac{1}{6} \]
   Infinitely many solutions

3. \[ 2x - 4y = 0 \quad x - 2y = 9 \]
   \[ -4y = -2x \quad -2y = -x + 9 \]
   \[ y = \frac{1}{2}x \quad y = \frac{1}{2}x - \frac{9}{2} \]
   No solution

4. \[ 6x + 3y = 8 \quad 8x + 4y = -1 \]
   \[ 3y = -6x + 8 \quad 4y = -8x - 1 \]
   \[ y = -2x + \frac{8}{3} \quad y = -2x - \frac{1}{4} \]
   No solution

5. \[ -x + 2y = 10 \quad 2x - y = 11 \]
   \[ -(4) + 2(3) = 10 \quad 2(-4) - 3 = 11 \]
   \[ 4 + 6 = 10 \quad -8 - 3 = 11 \]
   \[ 10 = 10 \quad -11 = 11 \]
   \((-4, 3)\) is not a solution.

6. \[ x - y = 4 \quad 3x + 4y = 12 \]
   \[ -y = -x + 4 \quad 4y = -3x + 12 \]
   \[ y = x - 4 \quad y = -\frac{3}{4}x + 3 \]

The solution is \(\{(4, 0)\}\).
7. \( y = 2x + 3 \quad 6x + 3y = 9 \)
   \[ 3y = -6x + 9 \quad y = -2x + 3 \]
   The solution is \( \{ (0,3) \} \).

8. \( 4x + 12y = 4 \quad y = 5x + 11 \)
   \[ 4x + 12(5x + 11) = 4 \quad 4x + 60x + 132 = 4 \]
   \[ 64x = -128 \quad x = -2 \]
   \[ y = 5x + 11 = 5(-2) + 11 = -10 + 11 = 1 \]
   The solution is \( \{ (-2,1) \} \).

9. \( y = -3x - 1 \)
   \[ 2x - 3y = -8 \]
   \[ 2x - 3(-3x - 1) = -8 \]
   \[ 2x + 9x + 3 = -8 \]
   \[ 11x = -11 \]
   \[ x = -1 \]
   \[ y = -3x - 1 = -3(-1) - 1 = 3 - 1 = 2 \]
   The solution is \( \{ (-1,2) \} \).

10. \( 10y + 34 = x \)
    \[ -7x + y = -31 \]
    \[ -7(10y + 34) + y = -31 \]
    \[ -70y - 238 + y = -31 \]
    \[ -69y = 207 \]
    \[ y = -3 \]
    \[ x = 10y + 34 = 10(-3) + 34 = -30 + 34 = 4 \]
    The solution is \( \{ (4,-3) \} \).

11. \( -3x + 8y = -1 \)
    \[ 4x - 11 = y \]
    \[ -3x + 8(4x - 11) = -1 \]
    \[ -3x + 32x - 88 = -1 \]
    \[ 29x = 87 \]
    \[ x = 3 \]
    \[ y = 4x - 11 = 4(3) - 11 = 12 - 11 = 1 \]
    The solution is \( \{ (3,1) \} \).

12. \( 12x - 2y = 0 \)
    \[ -7x + y = -1 \rightarrow y = 7x - 1 \]
    \[ 12x - 2(7x - 1) = 0 \]
    \[ 12x - 14x + 2 = 0 \]
    \[ -2x = -2 \]
    \[ x = 1 \]
    \[ y = 7x - 1 = 7(1) - 1 = 7 - 1 = 6 \]
    The solution is \( \{ (1,6) \} \).

13. \( 3x + 12y = 36 \)
    \[ x - 5y = 12 \rightarrow x = 5y + 12 \]

14. \( x - 3y = -3 \rightarrow x = 3y - 3 \)
    \[ 2x + 3y = -6 \]
Section 3.2 Solving Systems of Linear Equations by the Substitution Method

\[3(5y+12)+12y=36\]
\[15y+36+12y=36\]
\[27y=0\]
\[y=0\]
\[x=5y+12=5(0)+12=0+12=12\]
The solution is \(\{(12,0)\}\).

15. \(x - y = 8 \rightarrow x = y + 8\)
\[3x + 2y = 9\]
\[3(y+8)+2y=9\]
\[3y+24+2y=9\]
\[5y=-15\]
\[y=-3\]
\[x = y + 8\]
\[=-3+8\]
\[=5\]
The solution is \(\{(5,-3)\}\).

16. \(5x-2y=10\)
\[y=x-1\]
\[5x-2(x-1)=10\]
\[5x-2x+2=10\]
\[3x=8\]
\[x=\frac{8}{3}\]
\[y=x-1=\frac{8}{3}-1=\frac{5}{3}\]
The solution is \(\left\{\left(\frac{8}{3},\frac{5}{3}\right)\right\}\).

17. \(2x-y=-1\)
\[y=-2x\]
\[2x-(-2x)=-1\]
\[4x=-1\]
\[x=-\frac{1}{4}\]
\[y=-2x=-2\left(-\frac{1}{4}\right)=\frac{1}{2}\]
The solution is \(\left\{\left(-\frac{1}{4},\frac{1}{2}\right)\right\}\).

18. \(1+3y=10\rightarrow 3y=9\rightarrow y=3\)
\[5x+2y=6\]
\[5x+2(3)=6\]
\[5x+6=6\]
\[5x=0\]
\[x=0\]
The solution is \(\{(0,3)\}\).

19. \(2x+3=7\rightarrow 2x=4\rightarrow x=2\)
\[3x-4y=6\]
\[\rightarrow -5x=2y-12\rightarrow x=\frac{2y-12}{-5}\]
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3(2) = 4\ y = 6
6 – 4\ y = 6
4\ y = 0
\ y = 0
The solution is \( \{(2,0)\} \).

21.  4\ x – 5\ y = 14
3\ y = x – 7 \Rightarrow y = \frac{x – 7}{3}
4\ x – 5\ \left(\frac{x – 7}{3}\right) = 14
3(4\ x) – 5(x – 7) = 3(14)
12\ x – 5\ x + 35 = 42
7\ x + 35 = 42
7\ x = 7
\ x = 1
y = \frac{1 – 7}{3} = \frac{-6}{3} = -2
The solution is \( \{(1,-2)\} \).

22.  x + 2\ y = 0 \rightarrow x = -2\ y
2\ x – 6\ y = -15
2(-2\ y) – 6\ y = -15
-4\ y – 6\ y = -15
-10\ y = -15
y = \frac{-15}{-10} \Rightarrow y = \frac{3}{2}
x = -2\ \left(\frac{3}{2}\right)
= -3
The solution is \( \left\{\left(-3, \frac{3}{2}\right)\right\} \).

23.  2\ x – 6\ y = -2
x = 3\ y – 1
2(3\ y – 1) – 6\ y = -2
6\ y – 2 – 6\ y = -2
-2 = -2
Infinitely many solutions of the form \( \{(x,y)| x = 3\ y – 1\} \); dependent equations.

24.  -2\ x + 4\ y = 22
x = 2\ y – 11
-2(2\ y – 11) + 4\ y = 22
-4\ y + 22 + 4\ y = 22
22 = 22
Infinitely many solutions of the form \( \{(x,y)| x = 2\ y – 11\} \); dependent equations.
Section 3.2 Solving Systems of Linear Equations by the Substitution Method

25. \[
y = \frac{1}{7}x + 3 \\
x - 7y = -4
\]
\[
x - 7\left(\frac{1}{7}x + 3\right) = -4 \\
x - x - 21 = -4 \\
-21 \neq -4
\]
There is no solution; \(\{\}\). This is an inconsistent system.

26. \[
x = -\frac{3}{2}y + \frac{1}{2} \\
4x + 6y = 7
\]
\[
4\left(-\frac{3}{2}y + \frac{1}{2}\right) + 6y = 7 \\
-6y + 2 + 6y = 7 \\
2 \neq 7
\]
There is no solution; \(\{\}\). This is an inconsistent system.

27. \[
5x - y = 10 \rightarrow y = 5x - 10 \\
2y = 10x - 5
\]
\[
2(5x - 10) = 10x - 5 \\
10x - 20 = 10x - 5 \\
-20 \neq -5
\]
There is no solution; \(\{\}\). This is an inconsistent system.

28. \[
x + 4y = 8 \rightarrow x = -4y + 8 \\
3x = 3 - 12y
\]
\[
3(-4y + 8) = 3 - 12y \\
-12y + 24 = 3 - 12y \\
24 \neq 3
\]
There is no solution; \(\{\}\). This is an inconsistent system.

29. \[
3x - y = 7 \rightarrow y = 3x - 7 \\
-14 + 6x = 2y
\]
\[
-14 + 6x = 2(3x - 7) \\
-14 + 6x = 6x - 14 \\
-14 = -14
\]
Infinitely many solutions of the form \(\{(x, y) \mid 3x - y = 7\}\); dependent equations.

30. \[
x = 4y + 1 \\
-12y = -3x + 3
\]
\[
-12y = -3(4y + 1) + 3 \\
-12y = -12y - 3 + 3 \\
0 = 0
\]
Infinitely many solutions of the form \(\{(x, y) \mid x = 4y + 1\}\); dependent equations.

31. If you get an identity, such as \(0 = 0\) or \(5 = 5\) when solving a system of equations, then the equations are dependent.

32. If you get a contradiction, such as \(0 = 6\) or \(1 = 7\) when solving a system of equations, then the system is inconsistent.

33. \[
x = 1.3y + 1.5 \\
y = 1.2x - 4.6
\]

34. \[
y = 0.8x - 1.8 \\
1.1x = -y + 9.6
\]
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\[ x = 1.3(1.2x - 4.6) + 1.5 \]
\[ x = 1.56x - 5.98 + 1.5 \]
\[-0.56x = -4.48 \]
\[ x = 8 \]
\[ y = 1.2x - 4.6 \]
\[ = 1.2(8) - 4.6 \]
\[ = 9.6 - 4.6 \]
\[ = 5 \]
The solution is \( \{(8,5)\} \).

\[ 1.1x = -(0.8x - 1.8) + 9.6 \]
\[ 1.1x = -0.8x + 1.8 + 9.6 \]
\[ 1.9x = 11.4 \]
\[ x = 6 \]
\[ y = 0.8x - 1.8 \]
\[ = 0.8(6) - 1.8 \]
\[ = 4.8 - 1.8 \]
\[ = 3 \]
The solution is \( \{(6,3)\} \).

35. \[ y = \frac{2}{3}x - \frac{1}{3} \]
\[ x = \frac{1}{4}y + \frac{17}{4} \]
\[ x = \frac{1}{4}(2 \cdot \frac{1}{3}x - \frac{1}{3}) + \frac{17}{4} \]
\[ x = \frac{1}{6}x - \frac{1}{12} + \frac{17}{4} \]
\[ \frac{5}{6}x = -\frac{1}{12} + \frac{51}{12} \]
\[ 5x = 50 \]
\[ \frac{x}{6} = \frac{50}{12} \]
\[ x = \frac{50}{6} = \frac{300}{60} = 5 \]
\[ \frac{2}{3}x - \frac{1}{3} = \frac{2}{3} \cdot \frac{1}{5} = \frac{1}{3} = \frac{10}{3} = \frac{9}{3} \]
\[ = 3 \]
The solution is \( \{(5,3)\} \).

36. \[ x = \frac{1}{6}y - \frac{5}{3} \]
\[ y = \frac{1}{5}x + \frac{21}{5} \]
\[ x = \frac{1}{6}(\frac{1}{5}x + \frac{21}{5}) - \frac{5}{3} \]
\[ x = \frac{1}{30}x + \frac{21}{30} - \frac{5}{3} \]
\[ \frac{29}{30}x = \frac{21}{30} - \frac{50}{30} \]
\[ \frac{29}{30}x = \frac{21}{30} - \frac{50}{30} \]
\[ \frac{29}{30}x = \frac{21}{30} - \frac{50}{30} \]
\[ x = -1 \]
\[ y = \frac{1}{5}x + \frac{21}{5} = \frac{1}{5}(-1) + \frac{21}{5} \]
\[ = -\frac{1}{5} + \frac{21}{5} = \frac{20}{5} = 4 \]
The solution is \( \{(-1,4)\} \).

37. \[ -2x + y = 4 \rightarrow y = 2x + 4 \]
\[ -\frac{1}{4}x + \frac{1}{8}y = \frac{1}{4} \]
\[ -\frac{1}{4}x + \frac{1}{8}(2x + 4) = \frac{1}{4} \]
\[ -\frac{1}{4}x + \frac{1}{8}x + \frac{1}{2} = \frac{1}{4} \]
\[ -\frac{1}{4}x + \frac{1}{2} = \frac{1}{4} \]
\[ \frac{1}{2} = \frac{1}{4} \]
\[ 2 \neq \frac{1}{2} \]

38. \[ 8x - y = 8 \rightarrow y = 8x - 8 \]
\[ \frac{1}{3}x - \frac{1}{24}y = \frac{1}{2} \]
\[ \frac{1}{3}x - \frac{1}{24}(8x - 8) = \frac{1}{2} \]
\[ \frac{1}{3}x - \frac{1}{3}x + \frac{1}{3} = \frac{1}{2} \]
\[ \frac{1}{3} \neq \frac{1}{2} \]
There is no solution; \( \{ \} \). This is an inconsistent system.

39. \( 3x + 2y = 6 \)
\[\begin{align*}
y &= x + 3 \\
3x + 2(x + 3) &= 6 \\
3x + 2x + 6 &= 6 \\
5x + 6 &= 6 \\
5x &= 0 \\
x &= 0 \\
y &= 0 + 3 = 3
\end{align*}\]
The solution is \( \{(0,3)\} \).

40. \(-x + 4y = -4 \)
\[\begin{align*}
y &= x - 1 \\
-x + 4(x - 1) &= -4 \\
-x + 4x - 4 &= -4 \\
3x - 4 &= -4 \\
3x &= 0 \\
x &= 0 \\
y &= 0 - 1 = -1
\end{align*}\]
The solution is \( \{(0,-1)\} \).

41. \(-300x - 125y = 1350 \)
\[\begin{align*}
y + 2 &= 8 \rightarrow y = 6 \\
-300x - 125(6) &= 1350 \\
-300x - 750 &= 1350 \\
-300x &= 2100 \\
x &= -7
\end{align*}\]
The solution is \( \{(-7,6)\} \).

42. \( 200y = 150x \)
\[\begin{align*}
y - 4 &= 1 \rightarrow y = 5 \\
200(5) &= 150x \\
1000 &= 150x \\
x &= \frac{1000}{150} = \frac{20}{3}
\end{align*}\]
The solution is \( \left\{ \left( \frac{20}{3}, 5 \right) \right\} \).

43. \( 2x - y = 6 \rightarrow y = 2x - 6 \)
\[\begin{align*}
\frac{1}{6}x - \frac{1}{12}y &= \frac{1}{2} \\
\frac{1}{6}x - \frac{1}{12}(2x-6) &= \frac{1}{2} \\
\frac{1}{6}x - \frac{1}{6}x + \frac{1}{2} &= \frac{1}{2} \\
\frac{1}{2} &= \frac{1}{2}
\end{align*}\]
Infinitely many solutions of the form \( \{(x,y)\mid 2x - y = 6\} \); dependent equations.

44. \( x - 4y = 8 \rightarrow x = 4y + 8 \)
\[\begin{align*}
\frac{1}{16}x - \frac{1}{4}y &= \frac{1}{2} \\
\frac{1}{16}(4y + 8) - \frac{1}{4}y &= \frac{1}{2} \\
\frac{1}{4}y + \frac{1}{2} - \frac{1}{4}y &= \frac{1}{2} \\
\frac{1}{2} &= \frac{1}{2}
\end{align*}\]
Infinitely many solutions of the form \( \{(x,y)\mid x - 4y = 8\} \); dependent equations.
45. \[ y = -2.7x - 5.1 \]
\[ y = 3.1x - 63.1 \]
\[ 3.1x - 63.1 = -2.7x - 5.1 \]
\[ 5.8x = 58 \]
\[ x = 10 \]
\[ y = 3.1x - 63.1 = 3.1(10) - 63.1 = 31 - 63.1 = -32.1 \]
The solution is \( \{(10, -32.1)\} \).

46. \[ y = 6.8x + 2.3 \]
\[ y = -4.1x + 56.8 \]
\[ 6.8x + 2.3 = -4.1x + 56.8 \]
\[ 10.9x = 54.5 \]
\[ x = 5 \]
\[ y = 6.8x + 2.3 = 6.8(5) + 2.3 = 34 + 2.3 = 36.3 \]
The solution is \( \{(5, 36.3)\} \).

47. \[ 4x + 4y = 5 \]
\[ x - 4y = -\frac{5}{2} \rightarrow x = 4y - \frac{5}{2} \]
\[ 4 \left( 4y - \frac{5}{2} \right) + 4y = 5 \]
\[ 16y - 10 + 4y = 5 \]
\[ 20y = 15 \]
\[ y = \frac{15}{20} = \frac{3}{4} \]
\[ x = 4y - \frac{5}{2} = 4 \left( \frac{3}{4} \right) - \frac{5}{2} = 3 - \frac{5}{2} = \frac{1}{2} \]
The solution is \( \left\{ \left( \frac{1}{2}, \frac{3}{4} \right) \right\} \).

48. \[ -2x + y = -6 \rightarrow y = 2x - 6 \]
\[ 6x - 13y = -12 \]
\[ 6x - 13(2x - 6) = -12 \]
\[ 6x - 26x + 78 = -12 \]
\[ -20x = -90 \]
\[ x = \frac{9}{2} \]
\[ y = 2x - 6 = 2 \left( \frac{9}{2} \right) - 6 = 9 - 6 = 3 \]
The solution is \( \left\{ \left( \frac{9}{2}, 3 \right) \right\} \).

49. \[ 2(x + 2y) = 12 \rightarrow x + 2y = 6 \rightarrow x = -2y + 6 \]
\[ -6x = 5y - 8 \]
\[ -6(-2y + 6) = 5y - 8 \]
\[ 12y - 36 = 5y - 8 \]
\[ 7y = 28 \]
\[ y = \frac{28}{7} = 4 \]
\[ x = -2(4) + 6 = -8 + 6 = -2 \]
The solution is \( \left\{ (-2, 4) \right\} \).

50. \[ 5x - 2y = -25 \]
\[ 10x = 3(y - 10) \rightarrow x = \frac{3}{10}(y - 10) \]
\[ 5 \left( \frac{3}{10} (y - 10) \right) - 2y = -25 \]
\[ \frac{3}{2} (y - 10) - 2y = -25 \]
\[ \frac{3}{2} y - 15 - 2y = -25 \]
\[ -\frac{1}{2} y = -10 \]
\[ y = 20 \]
\[ x = \frac{3}{10} (20 - 10) = \frac{3}{10} (10) = 3 \]
The solution is \( \left\{ (3, 20) \right\} \).
Section 3.2 Solving Systems of Linear Equations by the Substitution Method

51. \[ 5(3y - 2) = x + 4 \rightarrow 15y - 10 = x + 4 \]
\[ \rightarrow 15y - 14 = x \]
\[ 4y = 7x - 3 \]
\[ 4y = 7(15y - 14) - 3 \]
\[ 4y = 105y - 98 - 3 \]
\[ 4y = 105y - 101 \]
\[-101y = -101 \]
\[ y = 1 \]
\[ x = 15(1) - 14 \]
\[ = 15 - 14 \]
\[ = 1 \]
The solution is \( \{(1,1)\} \).

52. \[ 2x = -3(y + 3) \rightarrow x = -\frac{3}{2}(y + 3) \]
\[ 3x - 4y = -22 \]
\[ 3 \left( -\frac{3}{2}(y + 3) \right) - 4y = -22 \]
\[ -9 \left( \frac{y + 3}{2} \right) - 4y = -22 \]
\[ -9(y + 3) - 8y = -44 \]
\[ -9y - 27 - 8y = -44 \]
\[ -17y = -17 \]
\[ y = 1 \]
\[ x = -\frac{3}{2}(1 + 3) \]
\[ = -\frac{3}{2}(4) \]
\[ = -6 \]
The solution is \( \{(-6,1)\} \).

53. \[ 2x - 5 = 7 \rightarrow 2x = 12 \rightarrow x = 6 \]
\[ 4 = 3y + 1 \rightarrow 3 = 3y \rightarrow y = 1 \]
The solution is \( \{(6,1)\} \).

54. \[ -2 = 4 - 2y \rightarrow -6 = -2y \rightarrow y = 3 \]
\[ 7x - 5 = -5 \rightarrow 7x = 0 \rightarrow x = 0 \]
The solution is \( \{(0,3)\} \).

55. \[ 0.01y = 0.02x - 0.11 \rightarrow y = 2x - 11 \]
\[ 0.3x - 0.5y = 2 \rightarrow 3x - 5y = 20 \]
\[ 3x - 5(2x - 11) = 20 \]
\[ 3x - 10x + 55 = 20 \]
\[ -7x + 55 = 20 \]
\[ -7x = -35 \]
\[ x = 5 \]
\[ y = 2(5) - 11 \]
\[ = 10 - 11 \]
\[ = -1 \]
The solution is \( \{(5,-1)\} \).

56. \[ 0.3x - 0.4y = 1.3 \rightarrow 3x - 4y = 13 \]
\[ 0.01x = 0.03y + 0.01 \rightarrow x = 3y + 1 \]
\[ 3(3y + 1) - 4y = 13 \]
\[ 9y + 3 - 4y = 13 \]
\[ 5y + 3 = 13 \]
\[ 5y = 10 \]
\[ y = 2 \]
\[ x = 3(2) + 1 \]
\[ = 6 + 1 \]
\[ = 7 \]
The solution is \( \{(7,2)\} \).
57. a. points \((-4, 1)\) and \((5, 5)\)

\[
m = \frac{5 - (-4)}{5 - (-4)} = \frac{4}{9}
\]

\[
y - y_1 = m(x - x_1) \quad (x_1, y_1) = (-4, 1)
\]

\[
y - 1 = \frac{4}{9}[x - (-4)]
\]

\[
y - 1 = \frac{4}{9}x + \frac{16}{9}
\]

\[
y = \frac{4}{9}x + \frac{25}{9}
\]

b. points \((-3, 5)\) and \((4, 1)\)

\[
m = \frac{1 - 5}{4 - (-3)} = \frac{-4}{7} = \frac{-4}{7}
\]

\[
y - y_1 = m(x - x_1) \quad (x_1, y_1) = (-3, 5)
\]

\[
y - 5 = \frac{-4}{7}[x - (-3)]
\]

\[
y - 5 = \frac{-4}{7}x + \frac{12}{7}
\]

\[
y = \frac{-4}{7}x + \frac{23}{7}
\]

c. \[
\frac{4}{9}x + \frac{25}{9} = -\frac{4}{9}x + \frac{23}{7}
\]

\[
63\left[\frac{4}{9}x + \frac{25}{9}\right] = 63\left[-\frac{4}{7}x + \frac{23}{7}\right]
\]

\[
28x + 175 = -36x + 207
\]

\[
64x = 32
\]

\[
x = \frac{1}{2}
\]

\[
y = \frac{4}{9}\left(\frac{1}{2}\right) + \frac{25}{9} = \frac{2}{9} + \frac{25}{9} = \frac{27}{9} = 3
\]

The centroid is \(\left(\frac{1}{2}, 3\right)\).

58. a. points \((0, -3)\) and \((-2, 5)\)

\[
m = \frac{5 - (-3)}{-2 - 0} = \frac{8}{-2} = -4
\]

\[
y - y_1 = m(x - x_1) \quad (x_1, y_1) = (0, -3)
\]

\[
y - (-3) = -4(x - 0)
\]

\[
y + 3 = -4x
\]

\[
y = -4x - 3
\]

b. points \((-4, 4)\) and \((2, -2)\)

\[
m = \frac{-2 - 4}{2 - (-4)} = \frac{-6}{6} = -1
\]

\[
y - y_1 = m(x - x_1) \quad (x_1, y_1) = (-4, 4)
\]

\[
y - 4 = -1[x - (-4)]
\]

\[
y - 4 = -x - 4
\]

\[
y = -x
\]

c. \[-4x - 3 = -x\]

\[-3x = 3\]

\[x = -1\]

\[y = -4x - 3\]

\[y = -4(-1) - 3\]

\[= 4 - 3\]

\[= 1\]

The centroid is \((-1, 1)\).
Section 3.3 Solving Systems of Linear Equations by the Addition Method

59. a. At Glendale Lakes:
\[ y = 800x + 250 \]
At the Breakers: \[ y = 750x + 500 \]
b. \[ 800x + 250 = 750x + 500 \]
\[ 50x = 250 \]
\[ x = 5 \]
The amount spent is the same for 5 months.

60. a. Surfside: \[ y = 159.50x + 24 \]
Tropical Winds:
\[ y = 165.50x \]
\[ y = 750x + 500 \]
b. \[ 159.50x + 24 = 165.50x \]
\[ 24 = 6x \]
\[ 4 = x \]
The cost to stay is the same for 4 nights.

Section 3.3 Practice Exercises

1. a. \(-3\)
b. \(5\)

2. No solutions – parallel lines

3. One solution – different slopes

4. \[ 4x = y + 7 \rightarrow y = 4x - 7 \]
\[ -2y = -8x + 14 \rightarrow y = 4x - 7 \]
Infinitely many solutions – same line

5. Add the two equations and solve for \(y:\)
\[ 3x - y = -1 \]
\[ -3x + 4y = -14 \]
\[ 3y = -15 \]
\[ y = -5 \]
Substitute into the first equation and solve for \(x:\)
\[ 3x - (-5) = -1 \]
\[ 3x + 5 = -1 \]
\[ 3x = -6 \]
\[ x = -2 \]
The solution is \(\{( -2, -5)\}\).

6. Add the two equations and solve for \(x:\)
\[ 5x - 2y = 15 \]
\[ 3x + 2y = -7 \]
\[ 8x = 8 \]
\[ x = 1 \]
Substitute into the first equation and solve for \(y:\)
\[ 5(1) - 2y = 15 \]
\[ 5 - 2y = 15 \]
\[ -2y = 10 \]
\[ y = -5 \]
The solution is \(\{(1, -5)\}\).

7. \[ 2x + 3y = 3 \]
\[ -10x + 2y = -32 \]
Multiply the first equation by 5, add to the second equation and solve for \(y:\)

8. \[ 2x - 5y = 7 \]
\[ 3x - 10y = 13 \]
Multiply the first equation by \(-2\), add to the second equation and solve for \(x:\)
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2x + 3y = 3 → 10x + 15y = 15
-10x + 2y = -32 → -10x + 2y = -32

17y = -17
y = -1

Substitute into the first equation and solve for x:
2x + 3(-1) = 3
2x = 6
x = 3

The solution is \( \{(3, -1)\} \).

9. \( 3x + 7y = -20 \)
-5x + 3y = -84

Multiply the first equation by 5 and the second equation by 3, add the results and solve for y:
3x + 7y = -20 → 15x + 35y = -100
-5x + 3y = -84 → -15x + 9y = -252
44y = -352
y = -8

Substitute into the first equation and solve for x:
3x + 7(-8) = -20
3x - 56 = -20
3x = 36
x = 12

The solution is \( \{(12, -8)\} \).

10. \( 6x - 9y = -15 \)
5x - 2y = -40

Multiply the first equation by 2 and the second equation by -9, add the results and solve for x:
6x - 9y = -15 → 12x - 18y = -30
5x - 2y = -40 → -45x + 18y = 360
-33x = 330
x = -10

Substitute into the first equation and solve for y:
6(-10) - 9y = -15
-60 - 9y = -15
-9y = 45
y = -5

The solution is \( \{(-10, -5)\} \).

11. Write in standard form:
3x = 10y + 13 → 3x - 10y = 13
7y = 4x - 11 → -4x + 7y = -11

Multiply the first equation by 4 and the second equation by 3, add the results and solve for y:

12. Write in standard form:
-5x = 6y - 4 → -5x - 6y = -4
5y = 1 - 3x → 3x + 5y = 1

Multiply the first equation by 3 and the second equation by 5, add the results and solve for y:
Section 3.3 Solving Systems of Linear Equations by the Addition Method

\[3x - 10y = 13 \quad \rightarrow \quad 12x - 40y = 52\]
\[-4x + 7y = -11 \quad \rightarrow \quad -12x + 21y = -33\]

Substitute into the first equation and solve for \(x\):
\[3x = 10(-1) + 13\]
\[3x = -10 + 13\]
\[3x = 3\]
\[x = 1\]
The solution is \(\{(1, -1)\}\).

\[-5x - 6y = -4 \quad \rightarrow \quad -15x - 18y = -12\]
\[3x + 5y = 1 \quad \rightarrow \quad 15x + 25y = 5\]
\[-19y = 19\]  \[\Rightarrow\]  \[y = -1\]

Substitute into the first equation and solve for \(x\):
\[-5x = 6(-1) - 4\]
\[-5x = -6 - 4\]
\[-5x = -10\]
\[x = 2\]
The solution is \(\{(2, -1)\}\).

13. Multiply each equation by 10:
\[1.2x - 0.6y = 3 \quad \rightarrow \quad 12x - 6y = 30\]
\[0.8x - 1.4y = 3 \quad \rightarrow \quad 8x - 14y = 30\]
Multiply the first equation by 2 and the second equation by –3, add the results and solve for \(y\):
\[12x - 6y = 30 \quad \rightarrow \quad 24x - 12y = 60\]
\[8x - 14y = 30 \quad \rightarrow \quad -24x + 42y = -90\]
\[30y = -30\]  \[\Rightarrow\]  \[y = -1\]

Substitute into the first equation and solve for \(x\):
\[12x - 6(-1) = 30\]
\[12x + 6 = 30\]
\[12x = 24\]
\[x = 2\]
The solution is \(\{(2, -1)\}\).

14. Multiply each equation by 10:
\[1.8x + 0.8y = 1.4 \quad \rightarrow \quad 18x + 8y = 14\]
\[1.2x + 0.6y = 1.2 \quad \rightarrow \quad 12x + 6y = 12\]
Multiply the first equation by 2 and the second equation by –3, add the results and solve for \(y\):
\[18x + 8y = 14 \quad \rightarrow \quad 36x + 16y = 28\]
\[12x + 6y = 12 \quad \rightarrow \quad -36x - 18y = -36\]
\[2y = -8\]  \[\Rightarrow\]  \[y = 4\]

Substitute into the first equation and solve for \(x\):
\[18x + 8(4) = 14\]
\[18x + 32 = 14\]
\[18x = -18\]
\[x = -1\]
The solution is \(\{(-1, 4)\}\).

15. Write in standard form:
\[3x + 2 = 4y + 2 \quad \rightarrow \quad 3x - 4y = 0\]
\[7x = 3y \quad \rightarrow \quad 7x - 3y = 0\]

16. Write in standard form:
\[-4y - 3 = 2x - 3 \quad \rightarrow \quad 2x + 4y = 0\]
\[5y = 3x \quad \rightarrow \quad 3x - 5y = 0\]
Multiply the first equation by 3 and the second equation by \(-4\), add the results and solve for \(x\):

\[
\begin{align*}
3x - 4y &= 0 \quad \rightarrow \quad 9x - 12y = 0 \\
7x - 3y &= 0 \quad \rightarrow \quad -28x + 12y = 0 \\
-19x &= 0 \\
x &= 0
\end{align*}
\]

Substitute into the first equation and solve for \(y\):

\[
\begin{align*}
3(0) - 4y &= 0 \\
0 - 4y &= 0 \\
-4y &= 0 \\
y &= 0
\end{align*}
\]

The solution is \(\{(0, 0)\}\).

17. \(3x - 2y = 1\)

\(-6x + 4y = -2\)

Multiply the first equation by 2, add to the second equation and solve for \(y\):

\[
\begin{align*}
3x - 2y &= 1 \quad \rightarrow \quad 6x - 4y = 2 \\
-6x + 4y &= -2 \quad \rightarrow \quad -6x + 4y = -2 \\
0 &= 0
\end{align*}
\]

Infinitely many solutions of the form \(\{(x, y) | 3x - 2y = 1\}\); dependent equations.

18. \(3x - y = 4\)

\(6x - 2y = 8\)

Multiply the first equation by \(-2\), add to the second equation and solve for \(y\):

\[
\begin{align*}
3x - y &= 4 \quad \rightarrow \quad -6x + 2y = -8 \\
6x - 2y &= 8 \quad \rightarrow \quad 6x - 2y = 8 \\
0 &= 0
\end{align*}
\]

Infinitely many solutions of the form \(\{(x, y) | 3x - y = 4\}\); dependent equations.

19. Write in standard form:

\[
\begin{align*}
6y &= 14 - 4x \\
2x &= -3y - 7
\end{align*}
\]

Multiply the second equation by \(-2\), add to the first equation and solve for \(y\):

\[
\begin{align*}
4x + 6y &= 14 \\
2x + 3y &= -7 \quad \rightarrow \quad 4x + 6y = 14 \\
2x + 3y &= -7 \quad \rightarrow \quad -4x - 6y = 14 \\
0 &= 28
\end{align*}
\]

20. Write in standard form:

\[
\begin{align*}
2x &= 4 - y \\
y &= 2x - 2
\end{align*}
\]

Add the second equation to the first equation and solve for \(y\):

\[
\begin{align*}
2x + y &= 4 \\
-2x - y &= -2
\end{align*}
\]

\[
0 = 2
\]

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Section 3.3 Solving Systems of Linear Equations by the Addition Method

There is no solution; \{ \}. This is an inconsistent system.

21. Write in standard form:
\[ 12x - 4y = 2 \quad \rightarrow \quad 12x - 4y = 2 \]
\[ 6x = 1 + 2y \quad \rightarrow \quad 6x - 2y = 1 \]
Multiply the second equation by \(-2\), add to the first equation and solve for \(y\):
\[ 12x - 4y = 2 \quad \rightarrow \quad 12x - 4y = 2 \]
\[ 6x - 2y = 1 \quad \rightarrow \quad -12x + 4y = -2 \]
\[ 0 = 0 \]
Infinitely many solutions of the form \(\{ (x, y) \mid 12x - 4y = 2 \}\); dependent equations.

22. Write in standard form:
\[ 10x - 15y = 5 \quad \rightarrow \quad 10x - 15y = 5 \]
\[ 3y = 2x - 1 \quad \rightarrow \quad -2x + 3y = -1 \]
Multiply the second equation by \(5\), add to the first equation and solve for \(y\):
\[ 10x - 15y = 5 \quad \rightarrow \quad 10x - 15y = 5 \]
\[ -2x + 3y = -1 \quad \rightarrow \quad -10x + 15y = -5 \]
\[ 0 = 0 \]
Infinitely many solutions of the form \(\{ (x, y) \mid 10x - 15y = 5 \}\); dependent equations.

23. \[ \frac{1}{2}x + y = \frac{7}{6} \]
\[ x + 2y = 4.5 \]
Multiply the first equation by \(-2\), add to the second equation and solve for \(y\):
\[ \frac{1}{2}x + y = \frac{7}{6} \quad \rightarrow \quad -x - 2y = -\frac{7}{3} \]
\[ x + 2y = 4.5 \quad \rightarrow \quad x + 2y = 4.5 \]
\[ 0 \neq \frac{13}{6} \]
There is no solution; \{ \}. This is an inconsistent system.

24. \[ 0.2x - 0.1y = -1.2 \]
\[ x - \frac{1}{2}y = 3 \]
Multiply the first equation by \(10\) and the second equation by \(-2\), add the results and solve for \(y\):
\[ 0.2x - 0.1y = -1.2 \quad \rightarrow \quad 2x - y = -12 \]
\[ x - \frac{1}{2}y = 3 \quad \rightarrow \quad -2x + y = -6 \]
\[ 0 \neq -18 \]
There is no solution; \{ \}. This is an inconsistent system.

25. Use the substitution method if one equation has \(x\) or \(y\) already isolated.

26. It would be easier to eliminate the \(y\)-variable by multiplying the first equation by \(2\). To eliminate the \(x\)-variable, we would have to multiply the first equation by \(-7\) and the second equation by \(3\).

27. False

28. False
29. True

30. True

31. True

32. True

33. \[2x - 4y = 8\]
   \[y = 2x + 1\]
   \[2x - 4(2x + 1) = 8\]
   \[2x - 8x - 4 = 8\]
   \[-6x = 12\]
   \[x = -2\]
   \[y = 2x + 1 = 2(-2) + 1 = -4 + 1 = -3\]
   The solution is \((-2, -3)\).

34. \[8x + 6y = -8\]
   \[x = 6y - 10\]
   \[8(6y - 10) + 6y = -8\]
   \[48y - 80 + 6y = -8\]
   \[54y = 72\]
   \[y = \frac{72}{54} = \frac{4}{3}\]
   \[x = 6y - 10 = 6\left(\frac{4}{3}\right) - 10 = 8 - 10 = -2\]
   The solution is \((-2, \frac{4}{3})\).

35. \[2x + 5y = 9\]
   \[4x - 7y = -16\]
   Multiply the first equation by \(-2\), add to the second equation and solve for \(y\):
   \[2x + 5y = 9 \quad \rightarrow \quad -4x - 10y = -18\]
   \[4x - 7y = -16 \quad \rightarrow \quad 4x - 7y = -16\]
   \[-17y = -34\]
   \[y = 2\]
   Substitute into the first equation and solve for \(x\):
   \[2x + 5(-2) = 9\]
   \[2x + 10 = 9\]
   \[2x = -1\]
   \[x = -\frac{1}{2}\]
   The solution is \((-\frac{1}{2}, 2)\).

36. \[0.1x + 0.5y = 0.7\]
   \[0.2x + 0.7y = 0.8\]
   Multiply the first equation by \(-2\), add to the second equation and solve for \(y\):
   \[0.1x + 0.5y = 0.7 \quad \rightarrow \quad -0.2x - y = -1.4\]
   \[0.2x + 0.7y = 0.8 \quad \rightarrow \quad 0.2x + 0.7y = 0.8\]
   \[-0.3y = -0.6\]
   \[y = 2\]
   Substitute into the first equation and solve for \(x\):
   \[0.1x + 0.5(2) = 0.7\]
   \[0.1x + 1.0 = 0.7\]
   \[0.1x = -0.3\]
   \[x = -3\]
   The solution is \((-3, 2)\).
37. \(0.2x - 0.1y = 0.8\)
\(0.1x - 0.1y = 0.4\) → \(0.1x = 0.1y + 0.4\)
\[\rightarrow x = y + 4\]
\(0.2(y + 4) - 0.1y = 0.8\)
\(0.2y + 0.8 - 0.1y = 0.8\)
\[0.1y + 0.8 = 0.8\]
\[0.1y = 0\]
\[y = 0\]
\[x = 0 + 4\]
\[= 4\]
The solution is \(\{(4, 0)\}\).

38. \(y = \frac{1}{2}x - 3\)
\(4x + y = -3\)
\[\rightarrow 4x + \left(\frac{1}{2}x - 3\right) = -3\]
\[\frac{9}{2}x = 0\]
\[\frac{9}{2}x = 0\]
\[x = 0\]
\[y = \frac{1}{2}(0) - 3\]
\[= 0 - 3\]
\[= -3\]
The solution is \(\{(0, -3)\}\).

39. \(4x - 6y = 5\)
\(2x - 3y = 7\)
Multiply the second equation by \(-2\), add to the first equation and solve for \(y\):
\[4x - 6y = 5\quad \rightarrow\quad 4x - 6y = 5\]
\[2x - 3y = 7\quad \rightarrow\quad -4x + 6y = -14\]
\[\frac{-3}{0} \neq -9\]
There is no solution; \(\{}\). This is an inconsistent system.

40. \(3x + 6y = 7\)
\(2x + 4y = 5\)
Multiply the first equation by \(-2\) and the second equation by 3, add the results and solve for \(y\):
\[3x + 6y = 7\quad \rightarrow\quad -6x - 12y = -14\]
\[2x + 4y = 5\quad \rightarrow\quad 6x + 12y = 15\]
\[0 \neq 1\]
There is no solution; \(\{}\). This is an inconsistent system.

41. Multiply each equation by the LCD:
\[\frac{1}{4}x - \frac{1}{6}y = -2\quad \rightarrow\quad 3x - 2y = -24\]
\[-\frac{1}{6}x + \frac{1}{5}y = 4\quad \rightarrow\quad -5x + 6y = 120\]
Multiply the first equation by 3, add to the second equation and solve for \(x\):
\[3x - 2y = -24\quad \rightarrow\quad 9x - 6y = -72\]
\[-5x + 6y = 120\quad \rightarrow\quad -5x + 6y = 120\]
\[4x = 48\]
\[x = 12\]

42. Multiply each equation by the LCD:
\[\frac{1}{3}x + \frac{1}{5}y = 7\quad \rightarrow\quad 5x + 3y = 105\]
\[-\frac{1}{6}x - \frac{2}{5}y = -4\quad \rightarrow\quad 5x - 12y = -120\]
Multiply the first equation by \(-1\), add to the second equation and solve for \(y\):
\[5x + 3y = 105\quad \rightarrow\quad -5x - 3y = -105\]
\[5x - 12y = -120\quad \rightarrow\quad 5x - 12y = -120\]
\[-15y = -225\]
\[y = 15\]
Substitute into the first equation and solve for $y$:

\[3(12) - 2y = -24\]
\[36 - 2y = -24\]
\[-2y = -60\]
\[y = 30\]

The solution is \{(12,30)\}.

43. \[
\frac{1}{3} x - \frac{1}{2} y = 0
\]
\[
x = \frac{3}{2} y
\]
\[
\frac{1}{3} \left( \frac{3}{2} y \right) - \frac{1}{2} y = 0
\]
\[
\frac{1}{2} y - \frac{1}{2} y = 0
\]

Infinitely many solutions of the form \{(x,y) | x = \frac{3}{2} y\}. The equations are dependent.

44. \[
\frac{2}{5} x - \frac{2}{3} y = 0
\]
\[
y = \frac{3}{5} x
\]
\[
\frac{2}{5} x - \frac{2}{3} \left( \frac{3}{5} x \right) = 0
\]
\[
\frac{2}{5} x - \frac{2}{5} x = 0
\]

Infinitely many solutions of the form \{(x,y) | y = \frac{3}{5} x\}. The equations are dependent.

45. Write in standard form:

\[2(x + 2y) = 20 - y \rightarrow 2x + 4y = 20 - y \rightarrow 2x + 5y = 20\]
\[-7(x - y) = 16 + 3y \rightarrow -7x + 7y = 16 + 3y \rightarrow -7x + 4y = 16\]

Multiply the first equation by 4 and the second equation by -5, add the results and solve for $x$:

\[2x + 5y = 20 \rightarrow \frac{43x}{4} = 0\]
\[-7x + 4y = 16 \rightarrow \frac{43x}{4} = 0\]
\[x = 0\]

Substitute into the first equation and solve for $y$:

46. Write in standard form:

\[-3(x + y) = 10 - 4y \rightarrow -3x - 3y = 10 - 4y \rightarrow -3x + y = 10\]
\[4(x + 2y) = 50 + 3y \rightarrow 4x + 8y = 50 + 3y \rightarrow 4x + 5y = 50\]

Multiply the first equation by -5, add to the second equation and solve for $x$:

\[-3x + y = 10 \rightarrow 15x - 5y = -50\]
\[4x + 5y = 50 \rightarrow 4x + 5y = 50\]
\[19x = 0\]
\[x = 0\]

Substitute into the first equation and solve for $y$: 

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Section 3.3 Solving Systems of Linear Equations by the Addition Method

2(0) + 5y = 20
0 + 5y = 20
5y = 20
y = 4
The solution is \( \{(0, 4)\} \).

30 + x = 10
0 + y = 10
y = 10
The solution is \( \{(0, 10)\} \).

47. Solve each equation:

\[-4y = 10 \quad 4x + 3 = 1\]

\[y = -\frac{10}{4} = -\frac{5}{2} \quad 4x = -2\]

\[x = -\frac{2}{4} = -\frac{1}{2}\]

The solution is \( \left\{ \left( -\frac{1}{2}, -\frac{5}{2} \right) \right\} \).

48. Solve each equation:

\[-9x = 15 \quad 3y + 2 = 1\]

\[x = -\frac{15}{9} = -\frac{5}{3} \quad 3y = -1\]

\[y = -\frac{1}{3}\]

The solution is \( \left\{ \left( -\frac{5}{3}, -\frac{1}{3} \right) \right\} \).

49. 0.04x = -0.05y + 1.7 \quad \rightarrow x = -5y + 170

\[4x + 5y = 170 \quad \rightarrow 4x + 5y = 170\]

\[-0.03y = -2.4 + 0.07x \quad \rightarrow -3y = -240 + 7x\]

\[7x + 3y = 240\]

Multiply the first equation by \( 3 \) and the second equation by \( -5 \), add the results and solve for \( x \):

\[4x + 5y = 170 \quad \rightarrow x = \frac{150}{3} \rightarrow 12x + 15y = 510\]

\[7x + 3y = 240 \quad \rightarrow x = \frac{234}{5} \rightarrow 35x - 15y = -1200\]

\[-23x = -690\]

\[x = 30\]

Substitute into the first equation and solve for \( y \):

\[x = 30 \quad \rightarrow 4(30) + 5y = 170\]

\[120 + 5y = 170\]

\[5y = 50\]

\[y = 10\]

The solution is \( \{(30, 10)\} \).

50. \[-0.01x = -0.06y + 3.2 \quad \rightarrow x = -6y + 320\]

\[0.08y = 0.03x + 4.6 \quad \rightarrow 8y = 3x + 460\]

Multiply the first equation by \(-3\), add to the second equation, and solve for \( y \):

\[x = 30 \quad \rightarrow 3x - 18y = -960\]

\[3x + 8y = 460 \quad \rightarrow 3x + 8y = 460\]

\[-10y = -500\]

\[y = 50\]

Substitute into the first equation and solve for \( x \):

\[-x + 6(50) = 320\]

\[-x + 300 = 320\]

\[-x = 20\]

\[x = -20\]

The solution is \( \{(-20, 50)\} \).

51. Write in standard form:

52. Write in standard form:
\[
3x - 2 = \frac{1}{3}(11 + 5y) \rightarrow 3x - 2 = \frac{11}{3} + \frac{5}{3}y
\]
\[\rightarrow 3x - \frac{5}{3}y = \frac{17}{3} \rightarrow 9x - 5y = 17\]
\[x + \frac{2}{3}(2y - 3) = -2 \rightarrow x + \frac{4}{3}y - 2 = -2\]
\[\rightarrow x + \frac{4}{3}y = 0 \rightarrow 3x + 4y = 0\]

Multiply the second equation by -3, add to the first equation and solve for \(y\):
\[9x - 5y = 17 \rightarrow 9x - 5y = 17\]
\[3x + 4y = 0 \rightarrow -9x - 12y = 0\]
\[\rightarrow -17y = 17\]
\[y = -1\]

Substitute into the first equation and solve:
\[9x - 5(-1) = 17\]
\[9x + 5 = 17\]
\[9x = 12\]
\[x = \frac{12}{9} = \frac{4}{3}\]

The solution is \(\left\{ \left( \frac{4}{3}, -1 \right) \right\}\).

53. \[
\frac{1}{4}x + \frac{1}{2}y = \frac{11}{4}
\]
\[
2x + \frac{1}{3}y = \frac{7}{3}
\]

Multiply the first equation by 4 and the second equation by -6, add the results and solve for \(x\):
\[\frac{1}{4}x + \frac{1}{2}y = \frac{11}{4} \rightarrow x + 2y = 11\]
\[2x + \frac{1}{3}y = \frac{7}{3} \rightarrow -4x - 2y = -14\]
\[\rightarrow -3x = -3\]
\[x = 1\]

Substitute into the first equation above and solve for \(y\):
\[2(2y + 3) - 2x = 1 - x \rightarrow 4y + 6 - 2x = 1 - x\]
\[\rightarrow -x + 4y = -5\]
\[x + y = \frac{1}{5}(7 + y) \rightarrow 5x + 5y = 7 + y\]
\[\rightarrow 5x + 4y = 7\]

Multiply the first equation by -1, add to the second equation and solve for \(x\):
\[-x + 4y = -5 \rightarrow x - 4y = 5\]
\[5x + 4y = 7 \rightarrow 6x = 12\]
\[x = 2\]

Substitute into the first equation and solve for \(y\):
\[-2 + 4y = -5\]
\[4y = -3\]
\[y = -\frac{3}{4}\]

The solution is \(\left\{ \left( 2, -\frac{3}{4} \right) \right\}\).

54. \[
\frac{1}{10}x - \frac{1}{2}y = \frac{-8}{5}
\]
\[
\frac{1}{4}x - \frac{1}{2}y = \frac{-11}{2}
\]

Multiply the first equation by 10 and the second equation by 20, add the results and solve for \(x\):
\[\frac{1}{10}x - \frac{1}{2}y = \frac{-8}{5} \rightarrow x - 5y = -16\]
\[\frac{1}{10}x - \frac{1}{2}y = \frac{-11}{2} \rightarrow 20x + 5y = -110\]
\[\rightarrow 21x = -126\]
\[x = -6\]

Substitute into the first equation above and solve for \(y\):
Section 3.3 Solving Systems of Linear Equations by the Addition Method

1 + 2y = 11
2y = 10
y = 5
The solution is \{(1, 5)\}.

-6 - 5y = -16
-5y = -10
y = 2
The solution is \{(-6, 2)\}.

55. $4x = 3y \rightarrow x = \frac{3}{4}y$
y = \frac{4}{3}x + 2
Substitute for x and solve for y:
y = \frac{4}{3}\left(\frac{3}{4}y\right) + 2
y = y + 2
0 = 2
There is no solution; \{\}. This is an inconsistent system.

56. 4x - 2y = 6
x = \frac{1}{2}y + \frac{3}{2}
4\left(\frac{1}{2}y + \frac{3}{2}\right) - 2y = 6
2y + 6 - 2y = 6
6 = 6
Infinitely many solutions of the form \{(x, y)| 4x - 2y = 6\}; dependent equations.

57. Multiply each equation by the LCD:
\[
\frac{1}{16}c + \frac{1}{24}h = 12 \quad \rightarrow \quad 3c + 2h = 576
\]
\[
\frac{1}{14}c + \frac{1}{20}h = 14 \quad \rightarrow \quad 10c + 7h = 1960
\]
Multiply the first equation by \(-\frac{10}{3}\),
add to the second equation and solve for h:
\[
3c + 2h = 576 \quad \rightarrow \quad -10c - \frac{20}{3}h = -1920
\]
\[
10c + 7h = 1960 \quad \rightarrow \quad 10c + 7h = 1960
\]
\[
\frac{1}{3}h = 40
\]
\[
h = 120
\]
Substitute into the first equation and solve for c:
\[
9c + 7h = 2268
\]
\[
9c + 7(189) = 2268
\]
\[
9c + 1323 = 2268
\]
\[
9c = 945
\]
\[
c = 105
\]

58. Multiply each equation by the LCD:
\[
\frac{1}{21}c + \frac{1}{27}h = 12 \quad \rightarrow \quad 9c + 7h = 2268
\]
\[
\frac{1}{15}c + \frac{1}{21}h = 16 \quad \rightarrow \quad 7c + 5h = 1680
\]
Multiply the first equation by \(-\frac{7}{9}\), add to the second equation and solve for h:
\[
9c + 7h = 2268 \quad \rightarrow \quad -7c - \frac{49}{9}h = -1764
\]
\[
7c + 5h = 1680 \quad \rightarrow \quad \frac{4}{9}h = -84
\]
\[
h = 105
\]
Substitute into the first equation and solve for c:
\[
9c + 7h = 2268
\]
\[
9c + 7(189) = 2268
\]
\[
9c + 1323 = 2268
\]
\[
9c = 945
\]
\[
c = 105
\]

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59. \[9x + 11y = 47\]
\[-5x + 3y = 23\]
Multiply the first equation by \(-3\) and the second equation by \(11\), add the results and solve for \(x\):
\[
\begin{align*}
9x + 11y &= 47 \\
-27x - 33y &= -141
\end{align*}
\]
\[
\begin{align*}
-5x + 3y &= 23 \\
-55x + 33y &= 253
\end{align*}
\]
\[
\begin{align*}
-82x &= 112 \\
x &= \frac{-56}{41}
\end{align*}
\]
Multiply the first equation by \(5\) and the second equation by \(9\), add the results and solve for \(y\):
\[
\begin{align*}
9x + 11y &= 47 \\
45x + 55y &= 235
\end{align*}
\]
\[
\begin{align*}
-5x + 3y &= 23 \\
-45x + 27y &= 207
\end{align*}
\]
\[
\begin{align*}
82y &= 442 \\
y &= \frac{221}{41}
\end{align*}
\]
The solution is \(\left\{\left(-\frac{56}{41}, \frac{221}{41}\right)\right\}\).

60. \[-6x + 7y = -4\]
\[4x - 9y = 31\]
Multiply the first equation by \(9\) and the second equation by \(7\), add the results and solve for \(x\):
\[
\begin{align*}
-6x + 7y &= -4 \\
-54x + 63y &= -36
\end{align*}
\]
\[
\begin{align*}
4x - 9y &= 31 \\
28x - 63y &= 217
\end{align*}
\]
\[
-26x = 181 \\
x = -\frac{181}{26}
\]
Multiply the first equation by \(2\) and the second equation by \(3\), add the results and solve for \(y\):
\[
\begin{align*}
-6x + 7y &= -4 \\
-12x + 14y &= -8
\end{align*}
\]
\[
\begin{align*}
4x - 9y &= 31 \\
12x - 27y &= 93
\end{align*}
\]
\[
-13y = 85 \\
y = -\frac{85}{13}
\]
The solution is \(\left\{\left(-\frac{181}{26}, -\frac{85}{13}\right)\right\}\).

61. \[4x - 10y = 19\]
\[5x + 12y = -41\]
Multiply the first equation by \(6\) and the second equation by \(5\), add the results and solve for \(x\):
Problem Recognition Exercises: Solving Systems of Linear Equations

\[ 4x - 10y = 19 \rightarrow 24x - 60y = 114 \]
\[ 5x + 12y = -41 \rightarrow 25x + 60y = -205 \]

\[ 49x = -91 \]
\[ x = \frac{-91}{49} = -\frac{13}{7} \]

\[ 5x + 12y = -41 \rightarrow 20x + 48y = -164 \]
\[ 98y = -259 \]
\[ y = -\frac{259}{98} = -\frac{37}{14} \]

The solution is \( \left\{ -\frac{13}{7}, -\frac{37}{14} \right\} \).

Problem Recognition Exercises: Solving Systems of Linear Equations

1. a. \[-3x + y = -2 \quad 4x - y = 4 \]
\[ y = 3x - 2 \quad -y = -4x + 4 \]
\[ y = 4x - 4 \]

The solution is \( \{ (2,4) \} \).

2. a. \[ 3x - 2y = 4 \]
\[ x = \frac{2}{3} y + \frac{4}{3} \]
\[ -2y = -3x + 4 \]
\[ 3x = 2y + 4 \]
\[ y = \frac{3}{2} x - 2 \]
\[ 2y = 3x - 4 \]
\[ y = \frac{3}{2} x - 2 \]

Infinitely many solutions; dependent equations.

b. \[-3x + y = -2 \rightarrow y = 3x - 2 \]
\[ 4x - y = 4 \]
\[ 4x - (3x - 2) = 4 \]
\[ 4x - 3x + 2 = 4 \]
\[ x + 2 = 4 \]
\[ x = 2 \]

b. \[ 3x - 2y = 4 \]
\[ x = \frac{2}{3} y + \frac{4}{3} \]
\[ 3 \left( \frac{2}{3} y + \frac{4}{3} \right) - 2y = 4 \]
\[ 2y + 4 - 2y = 4 \]
\[ 4 = 4 \]

Infinitely many solutions;
Chapter 3  Systems of Linear Equations and Inequalities

\[ y = 3(2) - 2 \]
\[ = 6 - 2 \]
\[ = 4 \]
The solution is \( \{(2,4)\} \).

c. \[ -3x + y = -2 \]
\[ 4x - y = 4 \]
Add the equations and solve for \( x \):
\[ -3x + y = -2 \]
\[ 4x - y = 4 \]
\[ x = 2 \]
Substitute into the first equation and solve for \( y \):
\[ -3(2) + y = -2 \]
\[ -6 + y = -2 \]
\[ y = 4 \]
The solution is \( \{(2,4)\} \).

e. Write in standard form:
\[ 3x - 2y = 4 \]
\[ x = \frac{2}{3}y + \frac{4}{3} \]
\[ 3x = 2y + 4 \]
\[ 3x - 2y = 4 \]
Subtract the second equation from the first:
\[ 3x - 2y = 4 \]
\[ -3x + 2y = -4 \]
\[ 0 = 0 \]
Infinitely many solutions;
\( \{(x,y)\}\{3x - 2y = 4\} \); dependent equations.

3. a. \[ 5x = 2y \rightarrow y = \frac{5}{2}x \]
\[ y = \frac{5}{2}x + 1 \]
No solution; \( \{ \} \); inconsistent system

4. a. \[ 2y = 3x + 1 \]
\[ 4x = 4 \]
\[ y = \frac{3}{2}x + \frac{1}{2} \]
\[ x = 1 \]
The solution is \( \{(1,2)\} \).
Problem Recognition Exercises: Solving Systems of Linear Equations

b. \[
\begin{align*}
\frac{5}{2}x &= \frac{5}{2}x + 1 \\
0 &= 1
\end{align*}
\]

No solution; \(\{\}\); inconsistent system

c. Write in standard form:
\[
\begin{align*}
5x &= 2y \\
5x - 2y &= 0 \\
-5x + 2y &= 2
\end{align*}
\]

Add the equations to solve for \(x\):
\[
\begin{align*}
5x - 2y &= 0 \\
-5x + 2y &= 2 \\
0 &= 2
\end{align*}
\]

No solution; \(\{\}\); inconsistent system

5.
\[
y = -4x - 9 \\
8x + 3y = -29
\]

Substitute the first equation into the second and solve for \(x\):
\[
\begin{align*}
8x + 3(-4x - 9) &= -29 \\
8x - 12x - 27 &= -29 \\
-4x - 27 &= -29 \\
-4x &= 2 \\
x &= \frac{1}{2}
\end{align*}
\]

\[
y = -4x - 9 = -4 \left(\frac{1}{2}\right) - 9 = -2 - 9 \\
= -11
\]

The solution is \(\left\{\left(\frac{1}{2}, -11\right)\right\}\).

d. \[
\begin{align*}
2y &= 3x + 1 \\
4x &= 4 \\
x &= 1
\end{align*}
\]

\[
\begin{align*}
2y &= 3(1) + 1 = 3 + 1 = 4 \\
y &= 2
\end{align*}
\]

The solution is \(\{(1, 2)\}\).

c. Write in standard form:
\[
\begin{align*}
2y &= 3x + 1 \rightarrow -3x + 2y = 1 \\
4x &= 4 \rightarrow x = 1
\end{align*}
\]

Multiply the second equation by 3, add to the first equation and solve for \(y\):
\[
\begin{align*}
-3x + 2y &= 1 \rightarrow -3x + 2y = 1 \\
x &= 1 \rightarrow x \times 3 \\
3x &= 3 \\
2y &= 4 \\
y &= 2
\end{align*}
\]

The solution is \(\{(1, 2)\}\).

6.
\[
x + 5y = 2 \rightarrow x = -5y + 2
\]

Substitute the second equation into the first and solve for \(y\):
\[
\begin{align*}
5(-5y + 2) - 2y &= -17 \\
-25y + 10 - 2y &= -17 \\
-27y &= -27 \\
y &= 1
\end{align*}
\]

\[
\begin{align*}
x &= -5(1) + 2 \\
&= -5 + 2 \\
&= -3
\end{align*}
\]

The solution is \(\{(-3, 1)\}\).
7. \[ 5x - 3y = 2 \]
\[ 7x + 4y = -30 \]
Multiply the first equation by 4 and the second equation by 3, add the results and solve for \( x \):
\[
\begin{align*}
5x - 3y &= 2 \quad \text{by} \ 4 \\
20x - 12y &= 8
\end{align*}
\]
\[
\begin{align*}
7x + 4y &= -30 \quad \text{by} \ 3 \\
21x + 12y &= -90
\end{align*}
\]
\[
\begin{align*}
41x &= -82 \\
x &= -2
\end{align*}
\]
Substitute into the first equation and solve for \( y \):
\[
\begin{align*}
5(-2) - 3y &= 2 \\
-10 - 3y &= 2 \\
-3y &= 12 \\
y &= -4
\end{align*}
\]
The solution is \( \{(−2,−4)\} \).

8. Multiply each equation by the LCD:
\[
\begin{align*}
\frac{1}{10}x - \frac{2}{5}y &= \frac{-3}{5} \quad \text{by} \ 10 \\
x - 4y &= -6
\end{align*}
\]
\[
\begin{align*}
\frac{3}{4}x + \frac{1}{3}y &= \frac{13}{6} \quad \text{by} \ 12 \\
9x + 4y &= 26
\end{align*}
\]
Add the first equation to the second equation and solve for \( x \):
\[
\begin{align*}
x - 4y &= -6 \\
9x + 4y &= 26
\end{align*}
\]
\[
\begin{align*}
x &= 2
\end{align*}
\]
Substitute into the first equation and solve for \( y \):
\[
\begin{align*}
2 - 4y &= -6 \\
-4y &= -8 \\
y &= 2
\end{align*}
\]
The solution is \( \{(2,2)\} \).

Section 3.4 Practice Exercises

1. a. \[ 5 \cdot $12 = $60 \]
\[ x \cdot 12 = 12x \]

b. \[ 0.10 \cdot 20 = 2L \]
\[ 0.10 \cdot x = 0.10x \]

c. \[ 0.04 \cdot $5000 = $200 \]
\[ 0.04 \cdot y = 0.04y \]

d. \( b - c \); \( b + c \)

2. Systems of linear equations can be solved by the graphing method, the substitution method, and the addition method.

3. Substitution:
\[ y = 9 - 2x \]
\[ 3x - y = 16 \]

4. Addition:
\[ 7x - y = -25 \]
\[ 2x + 5y = 14 \]
Section 3.4 Applications of Systems of Linear Equations in Two Variables

\[3x - (9 - 2x) = 16\]
\[3x - 9 + 2x = 16\]
\[5x = 25\]
\[x = 5\]

\[y = 9 - 2x\]
\[= 9 - 2(5)\]
\[= 9 - 10\]
\[= -1\]

The solution is \(\{(5, -1)\}\).

Multiply the first equation by 5, add to the second equation and solve for \(x\):
\[7x - y = -25\]
\[
\begin{align*}
7x - y & = -25 \\
\times 5 & \rightarrow 35x - 5y = -125
\end{align*}
\]
\[2x + 5y = 14\]
\[
\begin{align*}
2x + 5y & = 14 \\
\rightarrow & \rightarrow \ 2x + 5y = 14
\end{align*}
\]
\[37x = -111\]
\[x = -3\]

Substitute into the first equation and solve for \(y\):
\[7(-3) - y = -25\]
\[-21 - y = -25\]
\[-y = -4\]
\[y = 4\]

The solution is \(\{(-3, 4)\}\).

5. Let \(x\) = the number of premium tickets sold
   \(y\) = the number of regular tickets sold

\[30x = \text{receipts from premium tickets}\]
\[20y = \text{receipts from regular tickets}\]
\[x + y = 1190 \rightarrow y = 1190 - x\]
\[30x + 20y = 30,180\]
\[30x + 20(1190 - x) = 30,180\]
\[30x + 23,800 - 20x = 30,180\]
\[10x = 6380\]
\[x = 638\]
\[y = 1190 - 638 = 552\]

There were 638 tickets sold at $30 each and 552 tickets sold at $20 each.

6. Let \(x\) = the cost of 1 notebook
   \(y\) = the cost of 1 pen

\[4x + 5y = 10.65\]
\[3x + 3y = 7.50 \rightarrow x + y = 2.50 \rightarrow y = 2.50 - x\]
\[4x + 5\left(2.50 - x\right) = 10.65\]
\[4x + 12.50 - 5x = 10.65\]
\[-x = -1.85\]
\[x = 1.85\]
\[y = 2.50 - 1.85\]
\[= 0.65\]

Notebooks cost $1.85 and pens cost $0.65.

7. Let \(x\) = the cost of 1 hamburger
   \(y\) = the cost of 1 fish sandwich

\[3x + 2y = 24.20\]
\[4x + y = 23.60 \rightarrow y = 23.60 - 4x\]

8. Let \(x\) = the cost of a member
   \(y\) = the cost of a nonmember

\[x + 3y = 150\]
\[2x + y = 75 \rightarrow y = 75 - 2x\]
Chapter 3  Systems of Linear Equations and Inequalities

\[
3x + 2\left(23.60 - 4x\right) = 24.20 \\
3x + 47.20 - 8x = 24.20 \\
-5x = -23 \\
x = 4.60 \\
y = 23.60 - 4\left(4.60\right) \\
= 23.60 - 18.40 \\
= 5.20 \\
\]

Hamburgers cost $4.60 and fish sandwiches cost $5.20.

9. Let \(x\) = fat in 1 scoop of vanilla  
   \(y\) = fat in 1 scoop of mud pie  
   \[2x + y = 40 \rightarrow y = 40 - 2x\]  
   \[x + 2y = 44\]  
   \[x + 2\left(40 - 2x\right) = 44\]  
   \[x + 80 - 4x = 44\]  
   \[-3x = -36\]  
   \[x = 12\]  
   \[y = 40 - 2\left(12\right)\]  
   \[= 40 - 24\]  
   \[= 16\]  

Vanilla has 12 g of fat per scoop and mud pie has 16 g of fat per scoop.

10. Let \(x\) = calories in 1 cup of popcorn  
      \(y\) = calories in 1 ounce of soda  
      \[2x + 8y = 216\]  
      \[x + 12y = 204 \rightarrow x = 204 - 12y\]  
      \[2\left(204 - 12y\right) + 8y = 216\]  
      \[408 - 24y + 8y = 216\]  
      \[-16y = -192\]  
      \[y = 12\]  
      \[x = 204 - 12\left(12\right)\]  
      \[= 204 - 144\]  
      \[= 60\]  

One cup of popcorn has 60 calories and 1 oz of soda has 12 calories.

11. Let \(x\) = the amount of 18% moisturizer cream  
      \(y\) = the amount of 24% moisturizer cream  
      \[
      \begin{array}{c|c|c|c}
      \text{18% Cr} & \text{24% Cr} & \text{22% Cr} \\
      \hline
      \text{oz cream} & x & y & 12 \\
      \text{oz moist} & 0.18x & 0.24y & 0.22\left(12\right) \\
      \end{array}
      \]
      \[
      \begin{aligned}
      x + y & = 12 \\
      0.18x + 0.24y & = 0.22\left(12\right) \\
      \end{aligned}
      \]

12. Let \(x\) = the amount of 18% acid solution  
      \(y\) = the amount of 45% acid solution  
      \[
      \begin{array}{c|c|c|c|c}
      \text{} & \text{18% acid} & \text{45% acid} & \text{36% acid} \\
      \hline
      \text{L solution} & x & y & 16 \\
      \text{L acid} & 0.18x & 0.45y & 0.36\left(16\right) \\
      \end{array}
      \]
      \[
      \begin{aligned}
      x + y & = 16 \\
      0.18x + 0.45y & = 0.36\left(16\right) \\
      \end{aligned}
      \]

Multiply the first equation by \(-0.18\), add to the second equation and solve for \(y\):
Multiply the first equation by $-0.18$, add to the second equation and solve for $y$:

\[
\begin{align*}
x + y &= 12 \\
0.18x + 0.24y &= 2.64
\end{align*}
\]

\[
\begin{align*}
x + y &= 12 \\
0.18x + 0.24y &= 2.64 \quad \Rightarrow \quad x + 0.18y = -2.16 \\
0.18x + 0.24y &= 2.64 \quad \Rightarrow \quad 0.06y = 0.48 \\
\end{align*}
\]

\[
y = 8
\]

Substitute into the first equation and solve for $x$:

\[
x + 8 = 12 \\
x = 4
\]

The mixture contains 4 oz of 18% moisturizer and 8 oz of 24% moisturizer.

13. Let $x$ = the amount of 8% nitrogen fertilizer

$y$ = the amount of 12% nitrogen fertilizer

\[
\begin{array}{c|c|c|c}
\text{oz cream} & \text{8% nit} & \text{12% nit} & \text{11% nit} \\
\hline
x & y & 8
\end{array}
\]

\[
\begin{align*}
x + y &= 8 \\
0.08x + 0.12y &= 0.11(8)
\end{align*}
\]

Multiply the first equation by $-0.08$, add to the second equation and solve for $y$:

\[
\begin{align*}
x + y &= 8 \\
0.08x + 0.12y &= 0.11(8) \quad \Rightarrow \quad -0.08x - 0.08y = -0.64 \\
0.08x + 0.12y &= 0.88 \quad \Rightarrow \quad 0.08x + 0.12y = 0.88 \\
\end{align*}
\]

\[
0.04y = 0.24 \\
y = 6
\]

Substitute into the first equation and solve for $x$:

\[
x + 6 = 8 \\
x = 2
\]

The mixture contains 2 L of 8% nitrogen fertilizer and 6 L of 12% nitrogen fertilizer.

14. Let $x$ = the amount of 30% acid solution

$y$ = the amount of 10% acid solution

\[
\begin{array}{c|c|c|c}
\text{30% acid} & \text{10% acid} & \text{12% acid} \\
\hline
\text{L solution} & x & y & 100 \\
\hline
\text{L acid} & 0.30x & 0.10y & 0.12(100)
\end{array}
\]

Multiply the first equation by $-0.10$, add to the second equation and solve for $x$:

\[
\begin{align*}
x + y &= 100 \\
0.3x + 0.1y &= 12 \quad \Rightarrow \quad 0.3x + 0.1y = 12 \\
\end{align*}
\]

\[
0.2x = 2 \\
x = 10
\]

Substitute into the first equation and solve for $y$:

\[
10 + y = 100 \\
y = 90
\]

The mixture contains 10 mL of 30% acid solution and 90 mL of 10% acid solution.
15. Let \( x \) = amount of pure (100%) bleach sol \\
y = the amount of 4% bleach solution \\

<table>
<thead>
<tr>
<th>100% bl</th>
<th>4% bl</th>
<th>12% bl</th>
</tr>
</thead>
<tbody>
<tr>
<td>oz solution</td>
<td>x</td>
<td>y</td>
</tr>
</tbody>
</table>

\[
1.00x + 0.04y = 0.12(12)
\]

Multiply the first equation by \(-0.04\), add to the second equation and solve for \( x \):

\[
x + y = 12 \\
1.00x + 0.04y = 0.12(12)
\]

Multiply the first equation by \(-0.04\), add to the second equation and solve for \( x \):

\[
x + y = 12 \\
1.00x + 0.04y = 0.12(12)
\]

Substitute into the first equation and solve for \( y \):

\[
x + y = 12 \\
1 + y = 11
\]

The mixture contains 1 oz of pure bleach and 11 oz of 4% bleach solution.

16. Let \( x \) = the amount of 25% fruit juice \\
y = the amount of 100% fruit juice \\

\[\begin{array}{c|c|c|c}
25\% \text{ fr j} & 100\% \text{ fr j} & 75\% \text{ fr j} \\
\hline
\text{oz punch} & x & y & 48 \\
\hline
\text{oz fr juice} & 0.25x & 1.00y & 0.75(48) \\
\hline
\end{array}\]

\[
x + y = 48 \\
0.25x + 1.00y = 0.75(48)
\]

Multiply the first equation by \(-0.25\), add to the second equation and solve for \( y \):

\[
x + y = 48 \\
0.25x + 1.00y = 36 \\
0.25x + 1.00y = 36
\]

Substitute into the first equation and solve for \( y \):

\[
x + 32 = 48 \\
x = 16
\]

The punch contains 16 oz of 25% fruit juice and 32 oz of 100% fruit juice.

17. Let \( x \) = the amount invested in 5% bonds \\
3\( x \) = the amount invested in 8% stocks \\

\[\begin{array}{c|c|c|c}
5\% \text{ Acct} & 8\% \text{ Acct} & \text{ Total} \\
\hline
\text{Principal} & x & 3x & \\
\hline
\text{Interest} & 0.05x & 0.08(3x) & 435 \\
\hline
\end{array}\]

\[
0.05x + 0.08(3x) = 435 \\
0.05x + 0.24x = 435 \\
0.29x = 435 \\
x = 1500 \\
3x = 3(1500) = 4500
\]

18. Let \( x \) = the amount invested in 3.5% account \\
\( \frac{1}{2}x \) = the amount invested in 2.5% account \\

\[\begin{array}{c|c|c|c|c|c|c|c|c}
3.5\% \text{ Acct} & 2.5\% \text{ Acct} & \text{ Total} \\
\hline
\text{Principal} & x & 0.5x & \\
\hline
\text{Interest} & 0.035x & 0.025(0.5x) & 247 \\
\hline
\end{array}\]

\[
0.035x + 0.025(0.5x) = 247 \\
0.035x + 0.0125x = 247 \\
0.0475x = 247 \\
x = 5200 \\
\frac{1}{2}x = \frac{1}{2}(5200) = 2600
\]

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Section 3.4 Applications of Systems of Linear Equations in Two Variables

He invested $1500 in the bond fund and $4500 in the stock fund.

Aliya invested $2600 in the savings account and $5200 in the money market account.

19. Let \( x = \) the amount borrowed at 5.5%
\( y = \) the amount borrowed at 3.5%

<table>
<thead>
<tr>
<th>5.5% Acct</th>
<th>3.5% Acct</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Principal</td>
<td>( x )</td>
<td>( y )</td>
</tr>
<tr>
<td>Interest</td>
<td>0.055( x )</td>
<td>0.035( y )</td>
</tr>
</tbody>
</table>
\[
x = y + 200
\]

0.055\( x \) + 0.035\( y \) = 245

Substitute and solve for \( y \):

0.055(\( y + 200 \)) + 0.035\( y \) = 245

0.055\( y \) + 11 + 0.035\( y \) = 245

0.09\( y \) = 234

\( y \) = 2600

Substitute into the first equation and solve for \( x \):

\( x = 2600 + 200 \)

\( x = 2800 \)

He borrowed $2800 at 5.5% and $2600 at 3.5%.

20. Let \( x = \) the amount invested at 4%
\( y = \) the amount invested at 3%

<table>
<thead>
<tr>
<th>3% Acct</th>
<th>4% Acct</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Principal</td>
<td>( x )</td>
<td>( y )</td>
</tr>
<tr>
<td>Interest</td>
<td>0.03( x )</td>
<td>0.04( y )</td>
</tr>
</tbody>
</table>
\[
x = y - 5000
\]

0.03\( x \) + 0.04\( y \) = 725

Substitute and solve for \( y \):

0.03(\( y - 5000 \)) + 0.04\( y \) = 725

0.03\( y \) - 150 + 0.04\( y \) = 725

0.07\( y \) = 875

\( y \) = 12500

\( x = 12500 - 5000 \)

= 7500

$7500 was invested at 3% and $12,500 was invested at 4%.

21. Let \( x = \) the amount borrowed at 6%
\( y = \) the amount borrowed at 7%

<table>
<thead>
<tr>
<th>6% Acct</th>
<th>7% Acct</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Principal</td>
<td>( x )</td>
<td>( y )</td>
</tr>
<tr>
<td>Interest</td>
<td>0.06( x )</td>
<td>0.07( y )</td>
</tr>
</tbody>
</table>
\[
x + y = 15,000
\]

\[
0.06\( x \) + 0.07\( y \) = \frac{4750}{5} = 950
\]

Multiply the first equation by \(-0.06\), add to the second equation and solve for \( y \):

22. Let \( x = \) the amount invested at 5%
\( y = \) the amount invested at 6%

<table>
<thead>
<tr>
<th>5% Acct</th>
<th>6% Acct</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Principal</td>
<td>( x )</td>
<td>( y )</td>
</tr>
<tr>
<td>Interest</td>
<td>0.05( x )</td>
<td>0.06( y )</td>
</tr>
</tbody>
</table>
\[
x + y = 15,500
\]

\[
0.05\( x \) + 0.06\( y \) = \frac{3500}{4} = 875
\]

Multiply the first equation by \(-0.06\), add to the second equation and solve for \( x \):
Chapter 3  Systems of Linear Equations and Inequalities

\[ x + y = 15,000 \rightarrow -0.06x - 0.06y = -900 \\
0.06x + 0.07y = 950 \rightarrow 0.06x + 0.07y = 950 \\
\]  
\[ 0.01y = 50 \\
y = 5000 \]

Substitute into the first equation and solve for \( x \):
\[ x + 5000 = 15,000 \]
\[ x = 10,000 \]
Alina borrowed $10,000 from the bank charging 6% interest and $5000 from the bank charging 7% interest.

\[ x + y = 15,500 \rightarrow -0.06x - 0.06y = -930 \\
0.05x + 0.06y = 875 \rightarrow 0.05x + 0.06y = 875 \\
-0.01x = -55 \\
x = 5500 \]

Substitute into the first equation and solve for \( y \):
\[ 5500 + y = 15,500 \]
\[ y = 10,000 \]
Didi should invest $5500 in the 5% fund and $10,000 in the 6% fund.

23. Let \( b \) = the speed of the boat in still water  
\( c \) = the speed of the current  
\( b + c \) = speed of boat with the current  
\( b - c \) = speed of boat against the current

\begin{tabular}{|c|c|c|}
\hline
Distance & Rate & Time \\
\hline
With current & 16 & \( b + c \) & 2 \\
Against current & 16 & \( b - c \) & 4 \\
\hline
\end{tabular}

(rate)(time) = (distance)
\[ (b+c)(2) = 16 \]
\[ (b-c)(4) = 16 \]
Divide the first equation by 2, the second equation by 4, add the results, and solve:
\[ (b+c)(2) = 16 \rightarrow 2b + 2c = 8 \]
\[ (b-c)(4) = 16 \rightarrow 4b - 4c = 48 \]
\[ 2b = 12 \]
\[ b = 6 \]
Substitute and solve for \( c \):
\[ 6 + c = 8 \]
\[ c = 2 \]

The speed of the boat is 6 mph and the speed of the current is 2 mph.

24. Let \( p \) = the speed of the plane in still air  
Let \( w \) = the speed of the wind  
\( p + w \) = speed of the plane with the wind  
\( p - w \) = speed of plane against the wind

\begin{tabular}{|c|c|c|}
\hline
Distance & Rate & Time \\
\hline
Tailwind & 720 & \( p + w \) & 3 \\
Headwind & 720 & \( p - w \) & 4 \\
\hline
\end{tabular}

(rate)(time) = (distance)
\[ (p+w)(3) = 720 \]
\[ (p-w)(4) = 720 \]
Divide the first equation by 3, the second equation by 4, add the results, and solve:
\[ (p+w)(3) = 720 \rightarrow p + w = 240 \]
\[ (p-w)(4) = 720 \rightarrow p - w = 180 \]
\[ 2p = 420 \]
\[ p = 210 \]
Substitute and solve for \( w \):
\[ 210 + w = 240 \]
\[ w = 30 \]

The speed of the plane is 210 mph in still air and the speed of the wind is 30 mph.
25. Let \( p \) = the speed of the plane in still air
   Let \( w \) = the speed of the wind
   \( p + w \) = speed of the plane with the wind
   \( p - w \) = speed of plane against the wind

   **Distance** | **Rate** | **Time** 
   --- | --- | --- 
   Tailwind | 3200 | \( p + w \) | 4 
   Headwind | 3200 | \( p - w \) | 5 

\[(r \cdot t) = d\]

\[(p + w)(4) = 3200\]
\[(p - w)(5) = 3200\]

Divide the first equation by 4, the second equation by 5, add the results, and solve:

\[(p + w)\frac{4}{4} = 800\]
\[(p - w)\frac{5}{5} = 640\]

\[2p = 1440\]
\[p = 720\]

Substitute and solve for \( w \):

\[720 + w = 800\]
\[w = 80\]

The speed of the plane is 720 km/hr in still air and the speed of the wind is 80 km/hr.

26. Let \( b \) = the speed of the boat in still water
   Let \( c \) = the speed of the current
   \( b + c \) = speed of boat with the current
   \( b - c \) = speed of boat against the current

   **Distance** | **Rate** | **Time** 
   --- | --- | --- 
   With current | 100 | \( b + c \) | 2.5 
   Against current | 100 | \( b - c \) | \(\frac{10}{3}\) 

\[(r \cdot t) = d\]

\[(b + c)(2.5) = 100\]
\[(b - c)\left(\frac{10}{3}\right) = 100\]

Divide the first equation by 2.5, the second equation by \( \frac{10}{3} \), add the results, and solve:

\[(b + c)(2.5) = 100\]
\[(b - c)\frac{10}{3} = 100\]

\[2b = 70\]
\[b = 35\]

Substitute and solve for \( c \):

\[35 + c = 40\]
\[c = 5\]

The speed of the boat is 35 mph in still water and the speed of the current is 5 mph.

27. Let \( x \) = the walking speed
   Let \( y \) = the speed of the moving sidewalk
   \( x + y \) = speed of walking with sidewalk
   \( x - y \) = speed of walking against sidewalk

   **Distance** | **Rate** | **Time** 
   --- | --- | --- 
   With walk | 100 | \( x + y \) | 20 
   Against walk | 60 | \( x - y \) | 30 

28. Let \( b \) = the speed of the bike in still air
   Let \( w \) = the speed of the wind
   \( b + w \) = speed of the bike with the wind
   \( b - w \) = speed of bike against the wind

   **Distance** | **Rate** | **Time** 
   --- | --- | --- 
   Tailwind | 24 | \( b + w \) | 2 
   Headwind | 24 | \( b - w \) | 3 

\[(r \cdot t) = d\]
(rate)(time) = (distance)

\((x + y)(20) = 100\)
\((x - y)(30) = 60\)

Divide the first equation by 20, the second equation by 30, add the results, and solve:

\[
\frac{(x + y)(20)}{20} \rightarrow x + y = 5
\]
\[
\frac{(x - y)(30)}{30} \rightarrow x - y = 2
\]
\[
\frac{2x}{2} = 7
\]
\[
x = 3.5
\]

Substitute and solve for \(y\):

\[
3.5 + y = 5
\]
\[
y = 1.5
\]

Stephen’s speed on nonmoving ground is 3.5 ft/sec. The sidewalk moves at 1.5 ft/sec.

29. Let \(x\) = one acute angle
   Let \(y\) = the other acute angle
   \[x = 3y + 6\]
   \[x + y = 90\]
   Substitute and solve:
   \[3y + 6 + y = 90\]
   \[4y = 84\]
   \[y = 21\]
   \[x = 3(21) + 6 = 63 + 6 = 69\]
The two acute angles measure 69º and 21º.

30. Let \(x\) = one of two equal angles
   Let \(y\) = the other angle
   \[y = x - 3\]
   \[x + x + y = 180\]
   Substitute and solve:
   \[x + x + x - 3 = 180\]
   \[3x = 183\]
   \[x = 61\]
   \[y = 61 - 3 = 58\]
The angles measure 61º, 61º, and 58º.

31. Let \(x\) = one angle
   Let \(y\) = the other angle
   \[y = 3x - 2\]
   \[x + y = 180\]
   Substitute and solve:

32. Let \(x\) = one angle
   Let \(y\) = the other angle
   \[y = 5x\]
   \[x + y = 180\]
   Substitute and solve:
Section 3.4 Applications of Systems of Linear Equations in Two Variables

\[ x + 3x - 2 = 180 \]
\[ 4x = 182 \]
\[ x = 45.5 \]
\[ y = 3(45.5) - 2 \]
\[ = 136.5 - 2 \]
\[ = 134.5 \]

The two angles measure 45.5º and 134.5º.

33. Let \( x \) = one angle
Let \( y \) = the other angle
\[ y = 2x + 6 \]
\[ x + y = 90 \]
Substitute and solve:
\[ x + 2x + 6 = 90 \]
\[ 3x = 84 \]
\[ x = 28 \]
\[ y = 2(28) + 6 \]
\[ = 56 + 6 \]
\[ = 62 \]
The two angles measure 28º and 62º.

34. Let \( x \) = one angle
Let \( y \) = the other angle
\[ y = 2x + 15 \]
\[ x + y = 90 \]
Substitute and solve:
\[ x + 2x + 15 = 90 \]
\[ 3x = 75 \]
\[ x = 25 \]
\[ y = 2(25) + 15 \]
\[ = 50 + 15 \]
\[ = 65 \]
The angles measure 25º and 65º.

35. Let \( x \) = the amount of pure (100%) gold
\( y \) = the amount of 60% gold

<table>
<thead>
<tr>
<th>g mix</th>
<th>x</th>
<th>y</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>g gold</td>
<td>1.00x</td>
<td>0.60y</td>
<td>0.75(20)</td>
</tr>
</tbody>
</table>
\[ x + y = 20 \]
\[ 1.00x + 0.60y = 0.75(20) \]
Multiply the first equation by –0.60, add to the second equation and solve for \( x \):

36. Let \( x \) = the amount of 15% disinfectant
\( y \) = the amount of 55% disinfectant

<table>
<thead>
<tr>
<th>oz mix</th>
<th>x</th>
<th>y</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>oz gold</td>
<td>0.15x</td>
<td>0.55y</td>
<td>0.17(50)</td>
</tr>
</tbody>
</table>
\[ x + y = 50 \]
\[ 0.15x + 0.55y = 0.17(50) \]
Multiply the first equation by –0.15, add to the second equation and solve for \( y \):
Chapter 3  Systems of Linear Equations and Inequalities

\[ x + y = 15,500 \rightarrow -0.06x - 0.06y = -930 \]
\[ 0.05x + 0.06y = 875 \rightarrow -0.01x = -55 \]
\[ x = 5,500 \]

7.5 g of pure gold must be used.

37. Let \( b \) = the speed of the boat in still water
Let \( c \) = the speed of the current
\( b + c \) = speed of boat with the current
\( b - c \) = speed of boat against the current

<table>
<thead>
<tr>
<th>Distance</th>
<th>Rate</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>With current</td>
<td>16</td>
<td>( b + c )</td>
</tr>
<tr>
<td>Against current</td>
<td>10</td>
<td>( b - c )</td>
</tr>
</tbody>
</table>

\((\text{rate})(\text{time}) = (\text{distance})\)
\[(b + c)(2.5) = 16 \rightarrow 2.5b + 2.5c = 16\]
\[(b - c)(2.5) = 10 \rightarrow 2.5b - 2.5c = 10\]

Add the two equations, and solve:
\[2.5b + 2.5c = 16\]
\[2.5b - 2.5c = 10\]
\[5b = 26\]
\[b = 5.2\]

Substitute and solve for \( c \):
\[2.5(5.2) + 2.5c = 16\]
\[13 + 2.5c = 16\]
\[2.5c = 3\]
\[c = 1.2\]

The speed of the boat in still water is 5.2 mph and the speed of the current is 1.2 mph.

38. Let \( b \) = the speed of the kayak in still water
Let \( c \) = the speed of the current
\( b + c \) = speed of boat with the current
\( b - c \) = speed of boat against the current

<table>
<thead>
<tr>
<th>Distance</th>
<th>Rate</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>With current</td>
<td>31.5</td>
<td>( b + c )</td>
</tr>
<tr>
<td>Against current</td>
<td>31.5</td>
<td>( b - c )</td>
</tr>
</tbody>
</table>

\((\text{rate})(\text{time}) = (\text{distance})\)
\[(b + c)(7) = 31.5\]
\[(b - c)(9) = 31.5\]

Divide the first equation by 7, the second equation by 9, add the results, and solve:
\[(b + c)(7) = 31.5 \quad \frac{\text{div} \ 7}{b + c = 4.5}\]
\[(b - c)(9) = 31.5 \quad \frac{\text{div} \ 9}{b - c = 3.5}\]

\[2b = 8\]
\[b = 4\]

Substitute and solve for \( c \):
\[4 + c = 4.5\]
\[c = 0.5\]

The speed of the kayak in still water is 4 mph and the speed of the current is 0.5 mph.
39. Let \( x \) = the cost of a grandstand ticket  
y = the cost of a general admission ticket  
\[ 6x + 2y = 2330 \]  
\[ 4x + 4y = 2020 \]  
Multiply the first equation by \(-2\), add to the second equation and solve for \( x \):  
\[ 12x - 4y = -4660 \]  
\[ 4x + 4y = 2020 \]  
\[ -8x = -2640 \]  
\[ x = 330 \]  
Substitute and solve for \( y \):  
\[ 6(330) + 2y = 2330 \]  
\[ 1980 + 2y = 2330 \]  
\[ 2y = 350 \]  
\[ y = 175 \]  
Grandstand tickets cost $330 and general admission tickets cost $175.

40. Let \( x \) = the number of two-point baskets  
Let \( y \) = the number of three-point baskets  
\[ x + y = 8 \rightarrow x = 8 - y \]  
\[ 2x + 3y = 19 \]  
Substitute and solve:  
\[ 2(8 - y) + 3y = 19 \]  
\[ 16 - 2y + 3y = 19 \]  
\[ y = 3 \]  
\[ x = 8 - 3 \]  
\[ = 5 \]  
The player made 5 two-point baskets and 3 three-point baskets.

41. Let \( x \) = the amount invested at 2%  
y = the amount invested at 1.3%  
\begin{array}{ccc}
\text{2\% Acct} & \text{1.3\% Acct} & \text{Total} \\
\hline
\text{Principal} & x & y & 3,000 \\
\text{Interest} & 0.02x & 0.013y & 51.25 \\
\hline
\end{array}  
\[ x + y = 3,000 \]  
\[ 0.02x + 0.013y = 51.25 \]  
Multiply the first equation by \(-0.02\), add to the second equation and solve for \( y \):  
\[ x + y = 3,000 \rightarrow -.02x -.02y = -60 \]  
\[ .02x + .013y = 51.25 \]  
\[ -0.007y = -8.75 \]  
\[ y = 1250 \]  
Substitute into the first equation and solve for \( x \):  
\[ x + 1500 = 8000 \]  
\[ x = 6500 \]  

42. Let \( x \) = the amount invested at 3%  
y = the amount invested at 1.8%  
\begin{array}{ccc}
\text{3\% Acct} & \text{1.8\% Acct} & \text{Total} \\
\hline
\text{Principal} & x & y & 8000 \\
\text{Interest} & 0.03x & 0.018y & 222 \\
\hline
\end{array}  
\[ x + y = 8000 \]  
\[ 0.03x + 0.018y = 222 \]  
Multiply the first equation by \(-0.03\), add to the second equation and solve for \( y \):  
\[ x + y = 8000 \rightarrow -.03x -.03y = -240 \]  
\[ .03x + .018y = 222 \]  
\[ .03x + .018y = 222 \]  
\[ -0.012y = -18 \]  
\[ y = 1500 \]  
Substitute into the first equation and solve for \( x \):  
\[ x + 1500 = 8000 \]  
\[ x = 6500 \]
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\[ x + 1250 = 3000 \]
\[ y = 1750 \]

Svetlana invested $1750 at 2% and $1250 at 1.3%.

43. Let \( w \) = the width of the rectangle
Let \( l \) = the length of the rectangle
\[ l = w + 1 \]
\[ 2l + 2w = 42 \]
Substitute and solve:
\[ 2(w+1) + 2w = 42 \]
\[ 2w + 2 + 2w = 42 \]
\[ 4w + 2 = 42 \]
\[ 4w = 40 \]
\[ w = 10 \]
\[ l = 10 + 1 = 11 \]
The width is 10 m and the length is 11 m.

44. Let \( x \) = one angle
Let \( y \) = the other angle
\[ y = \frac{1}{4}x \]
\[ x + y = 90 \]
Substitute and solve:
\[ x + \frac{1}{4}x = 90 \]
\[ \frac{5}{4}x = 90 \]
\[ x = 72 \]
\[ y = \frac{1}{4}(72) = 18 \]
The two angles measure 72º and 18º.

45. Let \( d \) = the number of $1 coins
\[ f = \text{the number of 50 cent pieces} \]
\[ d + f = 21 \rightarrow f = 21 - d \]
\[ 1d + 0.50f = 15.50 \]
\[ d + 0.50(21 - d) = 15.50 \]
\[ d + 10.50 - 0.50d = 15.50 \]
\[ 0.50d = 5.00 \]
\[ d = 10 \]
\[ f = 21 - 10 = 11 \]
The collection contains 10 - $1 coins and 11 – 50 cent pieces.

46. Let \( d \) = the number of dimes
\[ n = \text{the number of nickels} \]
\[ d + n = 30 \rightarrow n = 30 - d \]
\[ 0.10d + 0.05n = 1.90 \]
\[ 0.10d + 0.05(30 - d) = 1.90 \]
\[ 0.10d + 1.50 - 0.05d = 1.90 \]
\[ 0.05d = 0.40 \]
\[ d = 8 \]
\[ n = 30 - 8 = 22 \]
Jacob has 8 dimes and 22 nickels.

47. a. \( f(x) = 60x \)

b. \( g(x) = 50x + 100 \)
c. Substitute and solve:

48. a. \( c(x) = 20 + 0.25x \)

b. \( m(x) = 30 + 0.20x \)
c. Substitute and solve:
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\[ 60x = 50x + 100 \]
\[ 10x = 100 \]
\[ x = 10 \]
10 months

\[ 20 + 0.25x = 30 + 0.20x \]
\[ 0.05x = 10 \]
\[ x = 200 \]

The rental fees are the same when the cars are driven 200 mi.

Section 3.5 Practice Exercises

1. a. linear
b. is not; is
c. dashed; is not
d. solid; is

2. \[ 5 < x + 1 \text{ and } -2x + 6 \geq -6 \]
\[ 4 < x \text{ and } -2x \geq -12 \]
\[ 4 < x \text{ and } x \leq 6 \quad (4,6] \]

3. \[ 5 - x \leq 4 \text{ and } 6 > 3x - 3 \]
\[ -x \leq -1 \text{ and } 9 > 3x \]
\[ x \geq 1 \text{ and } x < 3 \quad [1,3) \]

4. \[ 4 - y < 3y + 12 \text{ or } -2(y + 3) \geq 12 \]
\[ -8 < 4y \text{ or } -2y - 6 \geq 12 \]
\[ 4y > -8 \text{ or } -2y \geq 18 \]
\[ y > -2 \text{ or } y \leq -9 \]
\[ (-\infty, -9] \cup (-2, \infty) \]

5. \[ -2x < 4 \text{ or } 3x - 1 \leq -13 \]
\[ x > -2 \text{ or } 3x \leq -12 \]
\[ x > -2 \text{ or } x \leq -4 \]
\[ (-\infty, -4] \cup (-2, \infty) \]

6. \[ 2x - y > 8 \]

a. \[ 2(3) - (-5) = 6 + 5 = 11 > 8 \quad \text{Yes} \]
b. \[ 2(-1) - (-10) = -2 + 10 = 8 \not\geq 8 \quad \text{No} \]
c. \[ 2(4) - (-2) = 8 + 2 = 10 > 8 \quad \text{Yes} \]
d. \[ 2(0) - (0) = 0 - 0 = 0 \not\geq 8 \quad \text{No} \]

7. \[ 3y + x < 5 \]

a. \[ 3(7) + (-1) = 21 - 1 = 20 \not< 5 \quad \text{No} \]
b. \[ 3(0) + (5) = 0 + 5 = 5 \not< 5 \quad \text{No} \]
c. \[ 3(0) + (0) = 0 + 0 = 0 < 5 \quad \text{Yes} \]
d. \[ 3(-3) + (2) = -9 + 2 = -7 < 5 \quad \text{Yes} \]

8. \[ y \leq -2 \]

a. \[ -3 \leq -2 \quad \text{Yes} \]
b. \[ -2 \leq -2 \quad \text{Yes} \]
c. \[ 0 \not\leq -2 \quad \text{No} \]
d. \[ 2 \not\leq -2 \quad \text{No} \]

9. \[ x \geq 5 \]

a. \[ 4 \not< 5 \quad \text{No} \]
b. \[ 5 \geq 5 \quad \text{Yes} \]
c. \[ 8 \geq 5 \quad \text{Yes} \]
d. \[ 0 \not\geq 5 \quad \text{No} \]
10. Use a dashed line when the inequality is strict ( < or > ).

11. To choose the correct inequality symbol, three observations must be made. First, notice the shading occurs below the line. Second, since the coefficient of \( y \) is negative in the given statement, the direction of the inequality will change. Third, the boundary line is dashed indicating no equality. Thus use the symbol \( > \) for the inequality: \( x - y > 2 \).

12. To choose the correct inequality symbol, three observations must be made. First, notice the shading occurs below the line. Second, since the coefficient of \( y \) is positive in the given statement, the direction of the inequality will not change. Third, the boundary line is solid indicating equality. Thus use the symbol \( \leq \) for the inequality: \( y \leq -2x + 3 \).

13. To choose the correct inequality symbol, three observations must be made. First, notice the shading occurs above the line. Second, since the coefficient of \( y \) is positive in the given statement, the direction of the inequality will not change. Third, the boundary line is solid indicating equality. Thus use the symbol \( \geq \) for the inequality: \( y \geq -4 \).

14. Since the boundary is a vertical line, to choose the correct inequality symbol, two observations must be made. First, notice the shading occurs to the left of the line. Second, the boundary line is dashed indicating no equality. Thus use the symbol \( < \) for the inequality: \( x < 3 \).

15. The graph of \( x \geq 0 \) includes Quadrant I and Quadrant IV. The graph of \( y \leq 0 \) includes Quadrant III and Quadrant IV. The intersection of the graphs occurs in Quadrant IV. Thus, the statements are \( x \geq 0 \) and \( y \leq 0 \).

16. The graph of \( x \geq 0 \) includes Quadrant I and Quadrant IV. The graph of \( y \geq 0 \) includes Quadrant I and Quadrant II. The intersection of the graphs occurs in Quadrant I. Thus, the statements are \( x \geq 0 \) and \( y \geq 0 \).
17. \( x - 2y > 4 \)
Graph the related equation \( x - 2y = 4 \) by using a dashed line.
Test point above \((0,0)\): \(0 - 2(0) > 4\)
\(0 > 4\)
Test point below \((0,-3)\): \(0 - 2(-3) > 4\)
\(6 > 4\)
\((0,0)\) is not a solution. \((0,-3)\) is a solution.
Shade the region below the boundary line.

18. \( x - 3y \geq 6 \)
Graph the related equation \( x - 3y = 6 \) by using a solid line.
Test point above \((0,0)\): \(0 - 3(0) > 6\)
\(0 > 6\)
Test point below \((0,-3)\): \(0 - 3(-3) > 6\)
\(9 > 6\)
\((0,0)\) is not a solution. \((0,-3)\) is a solution.
Shade the region below the boundary line.

19. \( 5x - 2y < 10 \)
Graph the related equation \( 5x - 2y = 10 \) by using a dashed line.
Test point above \((0,0)\): \(5(0) - 2(0) < 10\)
\(0 < 10\)
Test point below \((2,-3)\): \(5(2) - 2(-3) < 10\)
\(16 < 10\)
\((0,0)\) is a solution. \((2,-3)\) is not a solution.
Shade the region above the boundary line.
20. \( x - 3y < 8 \)
Graph the related equation \( x - 3y = 8 \) by using a dashed line.
Test point above \((0,0)\): \(0 - 3(0) < 8\)
Test point below \((0,-4)\): \(0 - 3(-4) < 8\)
\((0,0)\) is a solution. \((0,-4)\) is not a solution.
Shade the region above the boundary line.

21. \( 2x \leq -6y + 12 \)
Graph the related equation \( 2x = -6y + 12 \) by using a solid line.
Test point above \((0,3)\): \(2(0) \leq -6(3) + 12\)
Test point below \((0,0)\): \(2(0) \leq -6(0) + 12\)
\((0,3)\) is not a solution. \((0,0)\) is a solution.
Shade the region below the boundary line.
22. \(4x < 3y + 12\)
Graph the related equation \(4x = 3y + 12\) by using a dashed line.
Test point above \((0,0)\): Test point below \((0,-5)\):
\[4(0) < 3(0) + 12 \quad 4(0) < 3(-5) + 12\]
\[0 < 12 \quad 0 < -3\]
\((0,0)\) is a solution. \((0,-5)\) is not a solution.
Shade the region above the boundary line.

23. \(2y \leq 4x\)
Graph the related equation \(2y = 4x\) by using a solid line.
Test point above \((0,1)\): Test point below \((0,-1)\):
\[2(1) \leq 4(0) \quad 2(-1) \leq 4(0)\]
\[2 \leq 0 \quad -2 \leq 0\]
\((0,1)\) is not a solution. \((0,-1)\) is a solution.
Shade the region below the boundary line.

24. \(-6x < 2y\)
Graph the related equation \(-6x = 2y\) by using a dashed line.
Test point above \((0,1)\): Test point below \((0,-1)\):
\[-6(0) < 2(1) \quad -6(0) < 2(-1)\]
\[0 < 2 \quad 0 < -2\]
\((0,1)\) is a solution. \((0,-1)\) is not a solution.
Shade the region above the boundary line.
25. \( y \geq -2 \)
Graph the related equation \( y = -2 \) by using a solid line.
Test point above \((0,0)\): \(0 \geq -2\)
Test point below \((0,-3)\): \(-3 \geq -2\)
\((0,0)\) is a solution. \((0,-3)\) is not a solution.
Shade the region above the boundary line.

26. \( y \geq 5 \)
Graph the related equation \( y = 5 \) by using a solid line.
Test point above \((0,6)\): \(6 \geq 5\)
Test point below \((0,0)\): \(0 \geq 5\)
\((0,6)\) is a solution. \((0,0)\) is not a solution.
Shade the region above the boundary line.
27. \(4x < 5 \) or \( x < \frac{5}{4}\) represents all the points to the left of the vertical line \( x = \frac{5}{4}\). The boundary is a dashed line. Shade the region to the left of the boundary line.

![Graph with shaded region]

28. \(x + 6 < 7 \) or \( x < 1\) represents all the points to the left of the vertical line \( x = 1\). The boundary is a dashed line. Shade the region to the left of the boundary line.

![Graph with shaded region]

29. \(y \geq \frac{2}{5}x - 4\)

Graph the related equation \(y = \frac{2}{5}x - 4\) by using a solid line.

Test point above \((0,0)\): \(0 \geq \frac{2}{5}(0) - 4\)

Test point below \((0, -5)\): \(-5 \geq \frac{2}{5}(0) - 4\)

\((0,0)\) is a solution. \((0, -5)\) is not a solution. Shade the region above the boundary line.

![Graph with shaded region]
30. \( y \geq -\frac{5}{2}x - 4 \)

Graph the related equation \( y = -\frac{5}{2}x - 4 \) by using a solid line.

Test point above \((0,0)\):

\[
0 \geq -\frac{5}{2}(0) - 4
\]

Test point below \((0,-5)\):

\[
-5 \geq -\frac{5}{2}(0) - 4
\]

\((0,0)\) is a solution. \((0,-5)\) is not a solution.

Shade the region above the boundary line.

31. \( y \leq \frac{1}{3}x + 6 \)

Graph the related equation \( y = \frac{1}{3}x + 6 \) by using a solid line.

Test point above \((0,7)\):

\[
7 \leq \frac{1}{3}(0) + 6
\]

Test point below \((0,0)\):

\[
0 \leq \frac{1}{3}(0) + 6
\]

\((0,7)\) is not a solution. \((0,0)\) is a solution.

Shade the region below the boundary line.
32. \( y \leq -\frac{1}{4}x + 2 \)

Graph the related equation \( y = -\frac{1}{4}x + 2 \) by using a solid line.

Test point above \((0,3)\): Test point below \((0,0)\):
\[
3 \leq -\frac{1}{4}(0) + 2 \quad \quad 0 \leq -\frac{1}{4}(0) + 2 \\
3 \leq 2 \quad \quad 0 \leq 2
\]

\((0,3)\) is not a solution. \((0,0)\) is a solution.

Shade the region below the boundary line.

33. \( y - 5x > 0 \)

Graph the related equation \( y - 5x = 0 \) by using a dashed line.

Test point above \((0,3)\): Test point below \((0,-3)\):
\[
3 - 5(0) > 0 \quad \quad -3 - 5(0) > 0 \\
3 > 0 \quad \quad -3 > 0
\]

\((0,3)\) is a solution. \((0,-3)\) is not a solution.

Shade the region above the boundary line.
34. \( y - \frac{1}{2} x > 0 \)

Graph the related equation \( y - \frac{1}{2} x = 0 \) by using a dashed line.

Test point above \((0,3)\): Test point below \((0,-3)\):

\[
\begin{align*}
3 - \frac{1}{2}(0) & > 0 \\
3 & > 0
\end{align*}
\]

\[
\begin{align*}
-3 - \frac{1}{2}(0) & > 0 \\
-3 & > 0
\end{align*}
\]

\((0,3)\) is a solution. \((0,-3)\) is not a solution.

Shade the region above the boundary line.

35. \( \frac{x}{5} + \frac{y}{4} < 1 \)

Graph the related equation \( \frac{x}{5} + \frac{y}{4} = 1 \) by using a dashed line.

Test point above \((0,5)\): Test point below \((0,0)\):

\[
\begin{align*}
\frac{0}{5} + \frac{5}{4} & < 1 \\
\frac{5}{4} & < 1
\end{align*}
\]

\[
\begin{align*}
\frac{0}{5} + \frac{0}{4} & < 1 \\
0 & < 1
\end{align*}
\]

\((0,5)\) is not a solution. \((0,0)\) is a solution.

Shade the region below the boundary line.
36. \[ x + \frac{y}{2} \geq 2 \]

Graph the related equation \( x + \frac{y}{2} = 2 \) by using a solid line.

Test point above \((0,5)\): \[ 0 + \frac{5}{2} \geq 2 \]
\[ \frac{5}{2} \geq 2 \]

\((0,5)\) is a solution.

Test point below \((0,0)\): \[ 0 + \frac{0}{2} \geq 2 \]
\[ 0 \geq 2 \]

\((0,0)\) is not a solution.

Shade the region above the boundary line.

37. \[ 0.1x + 0.2y \leq 0.6 \]

Graph the related equation \( 0.1x + 0.2y = 0.6 \) by using a solid line.

Test point above \((0,5)\): \[ 0.1(0) + 0.2(5) \leq 0.6 \]
\[ 1 \leq 0.6 \]

\((0,5)\) is not a solution.

Test point below \((0,0)\): \[ 0.1(0) + 0.2(0) \leq 0.6 \]
\[ 0 \leq 0.6 \]

\((0,0)\) is a solution.

Shade the region below the boundary line.

38. \[ 0.3x - 0.2y < 0.6 \]

Graph the related equation \( 0.3x - 0.2y = 0.6 \) by using a dashed line.

Test point above \((0,0)\): \[ 0.3(0) - 0.2(0) < 0.6 \]
\[ 0 < 0.6 \]

Test point below \((0,-5)\): \[ 0.3(0) - 0.2(-5) < 0.6 \]
\[ 1 < 0.6 \]
(0,0) is a solution. (0,−5) is not a solution.
Shade the region above the boundary line.

39. \[ x \leq -\frac{2}{3}y \]
Graph the related equation \( x = -\frac{2}{3}y \) by using a solid line.
Test point above (0,3): Test point below (0,−3):
\[
\begin{align*}
0 & \leq -\frac{2}{3}(3) \\
0 & \leq -2 \\
0 & \leq -\frac{2}{3}(-3) \\
0 & \leq 2
\end{align*}
\]
(0,3) is not a solution. (0,−3) is a solution.
Shade the region below the boundary line.

40. \[ x \geq -\frac{5}{4}y \]
Graph the related equation \( x = -\frac{5}{4}y \) by using a solid line.
Test point above (0,3): Test point below (0,−3):
\[
\begin{align*}
0 & \geq -\frac{5}{4}(3) \\
0 & \geq -\frac{15}{4} \\
0 & \geq -\frac{5}{4}(-3) \\
0 & \geq \frac{15}{4}
\end{align*}
\]
(0,3) is a solution. (0,−3) is not a solution.
Shade the region above the boundary line.
41. $y < 4$ and $y > -x + 2$

$y < 4$ represents the points below the horizontal line $y = 4$.
Shade the region below the boundary line using a dashed line border.
Graph the related equation $y = -x + 2$ by using a dashed line.
Test point above $(0, 3)$: Test point below $(0, 0)$:
\[ 3 > -(0) + 2 \quad \quad \quad 0 > -(0) + 2 \]
\[ 3 > 2 \quad \quad \quad 0 > 2 \]
$(0, 3)$ is a solution. $(0, 0)$ is not a solution.
Shade the region above the boundary line.
The solution is the intersection of the graphs.

42. $y < 3$ and $x + 2y < 6$

$y < 3$ represents the points below the horizontal line $y = 3$.
Shade the region below the boundary line using a dashed line border.
Graph the related equation $x + 2y = 6$ by using a dashed line.
Test point above $(0, 4)$: Test point below $(0, 0)$:
\[ 0 + 2(4) < 6 \quad \quad \quad 0 + 2(0) < 6 \]
\[ 8 < 6 \quad \quad \quad 0 < 6 \]
$(0, 4)$ is not a solution. $(0, 0)$ is a solution.
Shade the region below the boundary line.
The solution is the intersection of the graphs.
43. \(2x + y \leq 5 \) or \( x \geq 3 \)

Graph the related equation \(2x + y = 5\) by using a solid line.

Test point above \((0, 6)\):
\[2(0) + 6 \leq 5\]
\[6 \leq 5\]

Test point below \((0, 0)\):
\[2(0) + 0 \leq 5\]
\[0 \leq 5\]

\((0, 6)\) is not a solution. \((0, 0)\) is a solution.

Shade the region below the boundary line.

\(x \geq 3\) represents the points to the right of the vertical line \(x = 3\).

Shade the region to the right of the boundary line using a solid line border. The solution is the union of the graphs.

44. \(x + 3y \geq 3 \) or \( x \leq -2 \)

Graph the related equation \(x + 3y = 3\) by using a solid line.

Test point above \((0, 4)\):
\[0 + 3(4) \geq 3\]
\[12 \geq 3\]

Test point below \((0, 0)\):
\[0 + 3(0) \geq 3\]
\[0 \geq 3\]

\((0, 4)\) is a solution. \((0, 0)\) is not a solution.

Shade the region above the boundary line.

\(x \leq -2\) represents the points to the left of the vertical line \(x = -2\).

Shade the region to the left of the boundary line using a solid line border. The solution is the union of the graphs.
45. \( x + y < 3 \) and \( 4x + y < 6 \)

Graph the related equation \( x + y = 3 \) by using a dashed line.

Test point above \((0,4)\):

\[ 0 + 4 < 3 \]
\[ 4 < 3 \]

\((0,4)\) is not a solution.

Test point below \((0,0)\):

\[ 0 + 0 < 3 \]
\[ 0 < 3 \]

\((0,0)\) is a solution.

Shade the region below the boundary line.

Graph the related equation \( 4x + y = 6 \) by using a dashed line.

Test point above \((0,7)\):

\[ 4(0) + 7 < 6 \]
\[ 7 < 6 \]

\((0,7)\) is not a solution.

Test point below \((0,0)\):

\[ 4(0) + 0 < 6 \]
\[ 0 < 6 \]

\((0,0)\) is a solution.

Shade the region below the boundary line.

The solution is the intersection of the graphs.

46. Graph the related equation \( 3x + y = 9 \) by using a dashed line.

Test point above \((3,3)\):

\[ 3(3) + 3 < 9 \]
\[ 12 < 9 \]

\((3,3)\) is not a solution.

Test point below \((0,0)\):

\[ 3(0) + 0 < 9 \]
\[ 0 < 9 \]

\((0,0)\) is a solution.

Shade the region below the boundary line.

The solution is the intersection of the graphs.
$x + y < 4$ and $3x + y < 9$

Graph the related equation $x + y = 4$ by using a dashed line.

Test point above $(0,5)$: $0 + 5 < 4$

Test point below $(0,0)$: $0 + 0 < 4$

$5 < 4$ and $0 < 4$

$(0,5)$ is not a solution. $(0,0)$ is a solution.

Shade the region below the boundary line.

47. $2x - y \leq 2$ or $2x + 3y \geq 6$

Graph the related equation $2x - y = 2$ by using a solid line.

Test point above $(0,0)$: $2(0) - 0 \leq 2$

Test point below $(0,-3)$: $2(0) - (-3) \leq 2$

$0 \leq 2$ and $3 \leq 2$

$(0,0)$ is a solution. $(0,-3)$ is not a solution.

Shade the region above the boundary line.

Graph the related equation $2x + 3y = 6$ by using a solid line.

Test point above $(0,3)$: $2(0) + 3(3) \geq 6$

Test point below $(0,0)$: $2(0) + 3(0) \geq 6$

$9 \geq 6$ and $0 \geq 6$

$(0,3)$ is a solution. $(0,0)$ is not a solution.

Shade the region above the boundary line.

The solution is the union of the graphs.
48. \(3x + 2y \geq 4\) or \(x - y \leq 3\)

Graph the related equation \(3x + 2y = 4\) by using a solid line.

Test point above \((0,3)\): \(3(0) + 2(3) \geq 4\)\
\(6 \geq 4\)

\((0,3)\) is a solution.

Test point below \((0,0)\): \(3(0) + 2(0) \geq 4\)\
\(0 \geq 4\)

\((0,0)\) is not a solution.

Shade the region above the boundary line.

Graph the related equation \(x - y = 3\) by using a solid line.

Test point above \((0,0)\): \(0 - 0 \leq 3\)\
\(0 \leq 3\)

\((0,0)\) is a solution.

Test point below \((0,-4)\): \(0 - (-4) \leq 3\)\
\(4 \leq 3\)

\((0,-4)\) is not a solution.

Shade the region above the boundary line.

The solution is the union of the graphs.

49. \(x > 4\) and \(y < 2\)

\(x > 4\) represents the points to the right of the vertical line \(x = 4\).

Shade the region to the right of the boundary line using a dashed line border.

\(y < 2\) represents the points below the horizontal line \(y = 2\).

Shade the region below the boundary line using a dashed line border. The solution is the intersection of the graphs.
50. \( x < 3 \) and \( y > 4 \)
    \( x < 3 \) represents the points to the left of the vertical line \( x = 3 \).
    Shade the region to the left of the boundary line using a dashed line border.
    \( y > 4 \) represents the points above the horizontal line \( y = 4 \).
    Shade the region above the boundary line using a dashed line border. The solution is the intersection of the graphs.

51. \( x \leq -2 \) or \( y \leq 0 \)
    \( x \leq -2 \) represents the points to the left of the vertical line \( x = -2 \).
    Shade the region to the left of the boundary line using a solid line border.
    \( y \leq 0 \) represents the points below the horizontal line \( y = 0 \).
    Shade the region below the boundary line using a solid line border. The solution is the union of the graphs.

52. \( x \geq 0 \) or \( y \geq -3 \)
    \( x \geq 0 \) represents the points to the right of the vertical line \( x = 0 \).
    Shade the region to the right of the boundary line using a solid line border.
    \( y \geq -3 \) represents the points above the horizontal line \( y = -3 \).
    Shade the region above the boundary line using a solid line border. The solution is the union of the graphs.
53. \( x > 0 \) and \( x + y < 6 \)
   \( x > 0 \) represents the points to the right of the vertical line \( x = 0 \).
   Shade the region to the right of the boundary line using a dashed line border.
   Graph the related equation \( x + y = 6 \) by using a dashed line.
   Test point above \((0,7)\): \( 0 + 7 < 6 \)
   \( 0 + 7 < 6 \)
   \( 7 < 6 \)
   \((0,7)\) is not a solution.
   Test point below \((0,0)\): \( 0 + 0 < 6 \)
   \( 0 + 0 < 6 \)
   \( 0 < 6 \)
   \((0,0)\) is a solution.
   Shade the region below the boundary line.
   The solution is the intersection of the graphs.

54. \( x < 0 \) and \( x + y < 2 \)
   \( x < 0 \) represents the points to the left of the vertical line \( x = 0 \).
   Shade the region to the left of the boundary line using a dashed line border.
   Graph the related equation \( x + y = 2 \) by using a dashed line.
   Test point above \((0,3)\): \( 0 + 3 < 2 \)
   \( 0 + 3 < 2 \)
   \( 3 < 2 \)
   \((0,3)\) is not a solution.
   Test point below \((0,0)\): \( 0 + 0 < 2 \)
   \( 0 + 0 < 2 \)
   \( 0 < 2 \)
   \((0,0)\) is a solution.
   Shade the region below the boundary line.
Chapter 3 Systems of Linear Equations and Inequalities

The solution is the intersection of the graphs.

\[ y \leq 0 \text{ or } x - y \leq -4 \]

\( y \leq 0 \) represents the points below the horizontal line \( y = 0 \).

Shade the region below the boundary line using a solid line border.

Graph the related equation \( x - y = -4 \) by using a solid line.

Test point above \((0,5)\): \[ 0 - 5 \leq -4 \]
\[ -5 \leq -4 \]

\((0,5)\) is a solution.

Test point below \((0,0)\): \[ 0 - 0 \leq -4 \]
\[ 0 \leq -4 \]

\((0,0)\) is not a solution.

Shade the region above the boundary line.

The solution is the union of the graphs.

\[ x + y \leq 3 \text{ and } x \geq 0 \text{ and } y \geq 0 \]

Graph the related equation \( x + y = 3 \) by using a solid line.

Test point above \((0,4)\): \[ 0 + 4 \leq 3 \]
\[ 4 \leq 3 \]

\((0,4)\) is not a solution.

Test point below \((0,0)\): \[ 0 + 0 \leq 3 \]
\[ 0 \leq 3 \]

\((0,0)\) is a solution.

Shade the region below the boundary line.

\( x \geq 0 \) represents the points to the right of the vertical line \( x = 0 \).

Shade the region to the right of the boundary line using a solid line border.
Section 3.5 Linear Inequalities and Systems of Linear Inequalities in Two Variables

\( y \geq 0 \) represents the points above the horizontal line \( y = 0 \).
Shade the region above the boundary line using a solid line border.
The solution is the intersection of the graphs.

57. \( x - y \leq 2 \) and \( x \geq 0 \) and \( y \geq 0 \)
Graph the related equation \( x - y = 2 \) by using a solid line.
Test point above \((0,0)\): \( 0 - 0 \leq 2 \)
\( 0 \leq 2 \)
Test point below \((0,-3)\): \( 0 - (-3) \leq 2 \)
\( 3 \leq 2 \)
\((0,0)\) is a solution. \((0,-3)\) is not a solution.
Shade the region above the boundary line.
\( x \geq 0 \) represents the points to the right of the vertical line \( x = 0 \).
Shade the region to the right of the boundary line using a solid line border.
\( y \geq 0 \) represents the points above the horizontal line \( y = 0 \).
Shade the region above the boundary line using a solid line border.
The solution is the intersection of the graphs.

58. \( x \geq 0 \) and \( y \geq 0 \) and \( x + y \leq 8 \) and \( 3x + 5y \leq 30 \)
\( x \geq 0 \) represents the points to the right of the vertical line \( x = 0 \).
Shade the region to the right of the boundary line using a solid line border.
\( y \geq 0 \) represents the points above the horizontal line \( y = 0 \).
Shade the region above the boundary line using a solid line border.
Graph the related equation \( x + y = 8 \) by using a solid line.
Test point above \((0,9)\): Test point below \((0,0)\):
\[
\begin{align*}
0 + 9 &\leq 8 \\
9 &\leq 8
\end{align*}
\]
\((0,9)\) is not a solution. \((0,0)\) is a solution.
Shade the region below the boundary line.

Graph the related equation \( 3x + 5y = 30 \) by using a solid line.
Test point above \((0,7)\): Test point below \((0,0)\):
\[
\begin{align*}
3(0) + 5(7) &\leq 30 \\
35 &\leq 30
\end{align*}
\]
\((0,7)\) is not a solution. \((0,0)\) is a solution.
Shade the region below the boundary line.
The solution is the intersection of the graphs.

59. \( x \geq 0 \) and \( y \geq 0 \) and \( x + y \leq 5 \) and \( x + 2y \leq 6 \)
\( x \geq 0 \) represents the points to the right of the vertical line \( x = 0 \).
Shade the region to the right of the boundary line using a solid line border.
\( y \geq 0 \) represents the points above the horizontal line \( y = 0 \).
Shade the region above the boundary line using a solid line border.
Graph the related equation \( x + y = 5 \) by using a solid line.
Test point above \((0,6)\): Test point below \((0,0)\):
\[
\begin{align*}
0 + 6 &\leq 5 \\
6 &\leq 5
\end{align*}
\]
\((0,6)\) is not a solution. \((0,0)\) is a solution.
Shade the region below the boundary line.
Section 3.5 Linear Inequalities and Systems of Linear Inequalities in Two Variables

Graph the related equation \( x + 2y = 6 \) by using a solid line.
Test point above \((0,4)\): \(0 + 2(4) \leq 6\)
Test point below \((0,0)\):
\[0 + 2(0) \leq 6\]
\[8 \leq 6\]
\((0,4)\) is not a solution. \((0,0)\) is a solution.
Shade the region below the boundary line.
The solution is the intersection of the graphs.

60. a. \[2x + 2y \leq 50\]
b. \[x \geq 0 \text{ and } y \geq 0 \text{ and } 2x + 2y \leq 50\]
x \geq 0 represents the points to the right of the vertical line \(x = 0\). Shade the region to the right of the boundary line using a solid line border.
y \geq 0 represents the points above the horizontal line \(y = 0\). Shade the region above the boundary line using a solid line border.
Graph the related equation \(2x + 2y = 50\) by using a solid line.
Test point above \((0,27)\):
\[2(0) + 2(27) \leq 50\]
\[54 \leq 50\]
\((0,27)\) is not a solution.
Test point below \((0,0)\):
\[2(0) + 2(0) \leq 50\]
\[0 \leq 50\]
\((0,0)\) is a solution.
Shade the region below the boundary line.
Chapter 3   Systems of Linear Equations and Inequalities

61.  a.  \[ 2x + 2y \leq 40 \]

\[ x \geq 0 \text{ and } y \geq 0 \text{ and } 2x + 2y \leq 40 \]

\[ x \geq 0 \] represents the points to the right of the vertical line \( x = 0 \). Shade the region to the right of the boundary line using a solid line border.

\[ y \geq 0 \] represents the points above the horizontal line \( y = 0 \). Shade the region above the boundary line using a solid line border.

Graph the related equation \( 2x + 2y = 40 \) by using a solid line.

Test point above \((0,21)\):  
\[ 2(0) + 2(21) \leq 40 \]
\[ 42 \leq 40 \]

\((0,21)\) is not a solution.  

Test point below \((0,0)\):  
\[ 2(0) + 2(0) \leq 40 \]
\[ 0 \leq 40 \]

\((0,0)\) is a solution.

Shade the region below the boundary line.

62.  a.  \[ x \geq 0, \ y \geq 0 \]

b.  \[ 4x + 3y \leq 24 \]

c.  \[ 3x + y \leq 12 \]

d.  \[ x \geq 0 \text{ and } y \geq 0 \text{ and } 4x + 3y \leq 24 \text{ and } 3x + y \leq 12 \]

\[ x \geq 0 \] represents the points to the right of the vertical line \( x = 0 \). Shade the region to the right of the boundary line using a solid line border.

\[ y \geq 0 \] represents the points above the horizontal line \( y = 0 \). Shade the region above the boundary line using a solid line border.

Graph the related equation \( 4x + 3y = 24 \) by using a solid line.

Test point above \((0,9)\):  
\[ 4(0) + 3(9) \leq 24 \]
\[ 27 \leq 24 \]

\((0,9)\) is not a solution.  

Test point below \((0,0)\):  
\[ 4(0) + 3(0) \leq 24 \]
\[ 0 \leq 24 \]

\((0,0)\) is a solution.

Shade the region below the boundary line.
Graph the related equation $3x + y = 12$ by using a solid line.

Test point above $(0,13)$: $3(0) + (13) \leq 12$

Test point below $(0,0)$: $3(0) + (0) \leq 12$

$(0,13)$ is not a solution. $(0,0)$ is a solution.

Shade the region below the boundary line.

The solution is the intersection of the graphs.

e. Yes. The point $(3, 1)$ represents 3 Model A desks and 1 Model B desk being produced.

f. No. The point $(5, 4)$ represents 5 Model A desks and 4 Model B desk being produced. Producing this combination of desks would exceed the number of available hours for staining and finishing and for assembly.

63. a. $x \geq 0$, $y \geq 0$

b. $x \leq 40$, $y \leq 40$

c. $x + y \geq 65$

d. $x \geq 0$ and $y \geq 0$ and $x \leq 40$ and $y \leq 40$ and $x + y \geq 65$

$x \geq 0$ represents the points to the right of the vertical line $x = 0$. Shade the region to the right of the boundary line using a solid line border.

$y \geq 0$ represents the points above the horizontal line $y = 0$. Shade the region above the boundary line using a solid line border.

$x \leq 40$ represents the points to the left of the vertical line $x = 40$. Shade the region to the left of the boundary line using a solid line border.

$y \leq 40$ represents the points below the horizontal line $y = 40$. Shade the region below the boundary line using a solid line border.

Graph the related equation $x + y = 65$ by using a solid line.
Chapter 3  Systems of Linear Equations and Inequalities

Test point above \((0,66)\):  
\[0 + 66 \geq 65\]  
\[66 \geq 65\]  
\((0,66)\) is a solution.

Test point below \((0,0)\):  
\[0 + 0 \geq 65\]  
\[0 \geq 65\]  
\((0,0)\) is not a solution.

Shade the region above the boundary line.

The solution is the intersection of the graphs.

\[\begin{array}{c}
\text{Yes. The point} (35, 40) \text{ means that Karen works 35 hours and Todd works 40 hours.} \\
\text{f. No. The point} (20, 40) \text{ means that Karen works 20 hours and Todd works 40 hours. This does not satisfy the constraint that there must be at least 65 hours total.}
\end{array}\]

Section 3.6  Practice Exercises

1.  
   a.  linear
   b.  ordered triples

2.  
   \[-5x + 3y = -1 \quad 4x - 2y = -2\]
   \[-5(-4) + 3(-7) = -1 \quad 4(-4) - 2(-7) = -2\]
   \[20 - 21 = -1 \quad -16 + 14 = -2\]
   \[-1 = -1 \quad -2 = -2\]
   Yes, \((-4,-7)\) is a solution.

3.  
   a. \[3x + y = 4 \rightarrow y = -3x + 4\]
       \[4x + y = 5\]
       \[4x + (-3x + 4) = 5\]
       \[4x - 3x + 4 = 5\]
       \[x = 1\]
       \[y = -3x + 4\]
       \[= -3(1) + 4\]
       \[= -3 + 4 = 1\]

4.  
   a. \[2x - 5y = 3 \rightarrow 2x = 5y + 3 \rightarrow x = \frac{5}{2}y + \frac{3}{2}\]
       \[-4x + 10y = 3\]
       \[-4\left(\frac{5}{2}y + \frac{3}{2}\right) + 10y = 3\]
       \[-10y - 6 + 10y = 3\]
       \[-6 \neq 3\]
Section 3.6 Systems of Linear Equations in Three Variables and Applications

The solution is \( \{(1,1)\} \).

b. \[
\begin{align*}
3x + y &= 4 \\
4x + y &= 5
\end{align*}
\]
Multiply the first equation by \(-1\), add to the second equation and solve for \(x\):
\[
\begin{align*}
3x + y &= 4 \quad \rightarrow \quad -3x - y = -4 \\
4x + y &= 5 \\
\hline
x &= 1
\end{align*}
\]
Substitute into the first equation and solve for \(y\):
\[
\begin{align*}
3(1) + y &= 4 \\
y &= 1
\end{align*}
\]
The solution is \( \{(1,1)\} \).

b. \[
\begin{align*}
2x - 5y &= 3 \\
-4x + 10y &= 3
\end{align*}
\]
Multiply the first equation by 2, add to the second equation and solve for \(x\):
\[
\begin{align*}
2x - 5y &= 3 \quad \rightarrow \quad 4x - 10y = 6 \\
-4x + 10y &= 3 \\
\hline
0 &= 9
\end{align*}
\]
There is no solution; \( \{ \} \). This is an inconsistent system.

5. Let \(b\) = the speed of the bike in still air
Let \(w\) = the speed of the wind
\[
\begin{align*}
b + w &= \text{speed of the bike with the wind} \\
b - w &= \text{speed of bike against the wind}
\end{align*}
\]

<table>
<thead>
<tr>
<th>Distance</th>
<th>Rate</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tailwind</td>
<td>(b + w)</td>
<td>(4/3)</td>
</tr>
<tr>
<td>Headwind</td>
<td>(b - w)</td>
<td>(2)</td>
</tr>
</tbody>
</table>

(rate)(time) = (distance)
\[
\begin{align*}
(b + w) \left( \frac{4}{3} \right) &= 24 \\
(b - w) (2) &= 24
\end{align*}
\]

Divide the first equation by 4/3, the second equation by 2, add the results, and solve:
\[
\begin{align*}
(b + w) \left( \frac{4}{3} \right) &= 24 \quad \text{div} \quad 4/3 \quad b + w = 18 \\
(b - w) (2) &= 24 \quad \text{div} \quad 2 \quad b - w = 12 \\
2b &= 30 \\
b &= 15
\end{align*}
\]
Substitute and solve for \(w\):
\[
\begin{align*}
15 + w &= 18 \\
w &= 3
\end{align*}
\]
Marge’s speed is 15 mph in still air. The wind speed is 3 mph.

6. The number of solutions to a system of three equations in three variables is one solution, no solution, or infinitely many solutions.
7. \(2x + y + z = 10\)
   \(4x + 2y - 3z = 10\)
   \(x - 3y + 2z = 8\)
Substitute \((2,1,7)\):
\[
2(2) - 1 + 7 = 4 - 1 + 7 = 10 = 10
\]
\[
4(2) + 2(1) - 3(7) = 8 + 2 - 21 = -11 \neq 10
\]
Not a solution.
Substitute \((3,-10,-6)\):
\[
2(3) - (-10) + (-6) = 6 + 10 - 6 = 10 = 10
\]
\[
4(3) + 2(-10) - 3(-6) = 12 - 20 + 18 = 10
\]
\[
3 - 3(-10) + 2(-6) = 3 + 30 - 12 = 21 \neq 8
\]
Not a solution.
Substitute \((4,0,2)\):
\[
2(4) - (0) + (2) = 8 - 0 + 2 = 10 = 10
\]
\[
4(4) + 2(0) - 3(2) = 16 + 0 - 6 = 10 = 10
\]
\[
4 - 3(0) + 2(2) = 4 - 0 + 4 = 8 = 8
\]
\[
4(0,2) \text{ is a solution.}
\]
8. \(-3x - 3y - 6z = -24\)
   \(-9x - 6y + 3z = -45\)
   \(9x + 3y - 9z = 33\)
Substitute \((1,1,3)\):
\[
-3(1) - 3(1) - 6(3) = -3 - 3 - 18 = -24 = -24
\]
\[
-9(1) - 6(1) + 3(3) = -9 - 6 + 9 = -6 \neq -45
\]
Not a solution.
Substitute \((0,0,4)\):
\[
-3(0) - 3(0) - 6(4) = 0 - 0 - 24 = -24 = -24
\]
\[
-9(0) - 6(0) + 3(4) = 0 - 0 + 12 = 12 \neq -45
\]
Not a solution.
Substitute \((4,2,1)\):
\[
-3(4) - 3(2) - 6(1) = -12 - 6 - 6 = -24 = -24
\]
\[
-9(4) - 6(2) + 3(1) = -36 - 12 + 3 = -45 = -45
\]
\[
9(4) + 3(2) - 9(1) = 36 + 6 - 9 = 33 = 33
\]
\[
(4,2,1) \text{ is a solution.}
\]
9. \(-x - y - 4z = -6\)
   \(x - 3y + z = -1\)
   \(4x + y - z = 4\)
Substitute \((12,2,-2)\):
\[
-(12) - (2) - 4(-2) = -12 - 2 + 8 = -6 = -6
\]
\[
12 - 3(2) + (-2) = 12 - 6 - 2 = 4 \neq -1
\]
Not a solution.
Substitute \((4,2,1)\):
\[
-(4) - (2) - 4(1) = -4 - 2 - 4 = -10 \neq -6
\]
Not a solution.
Substitute \((1,1,1)\):
\[
-(1) - (1) - 4(1) = -1 - 1 - 4 = -6 = -6
\]
\[
1 - 3(1) + (1) = 1 - 3 + 1 = -1 = -1
\]
\[
4(1) + (1) - (1) = 4 + 1 - 1 = 4 = 4
\]
\[
(1,1,1) \text{ is a solution.}
\]
10. \(x + 2y - z = 5\)
   \(x - 3y + z = -5\)
   \(-2x + y - z = -4\)
Substitute \((0,4,3)\):
\[
(0) + 2(4) - (3) = 0 + 8 - 3 = 5 = 5
\]
\[
(0) - 3(4) + (3) = 0 - 12 + 3 = -9 \neq -5
\]
Not a solution.
Substitute \((3,6,10)\):
\[
(3) + 2(6) - (10) = 3 + 12 - 10 = 5 = 5
\]
\[
(3) - 3(6) + (10) = 3 - 18 + 10 = -5 = -5
\]
\[
-2(3) + (6) - (10) = -6 + 6 - 10 = -10 \neq -4
\]
Not a solution.
Substitute \((3,3,1)\):
\[
(3) + 2(3) - (1) = 3 + 6 - 1 = 8 \neq 5
\]
Not a solution.
11. \[ \begin{align*}
2x + y - 3z &= -12 \\
3x - 2y - z &= 3 \\
x + 5y + 2z &= -3
\end{align*} \]

Multiply the first equation by 2 and add to the second equation to eliminate \(y\):

\[
\begin{align*}
2x + y - 3z &= -12 \quad \rightarrow \quad x^2 \quad 4x + 2y - 6z &= -24 \\
3x - 2y - z &= 3 \quad \rightarrow \quad 3x - 2y - z &= 3 \\
\hline
7x \\
-7z &= -21 \\
x - z &= -3
\end{align*}
\]

Multiply the first equation by \(-5\) and add to the third equation to eliminate \(y\):

\[
\begin{align*}
2x + y - 3z &= -12 \quad \rightarrow \quad -10x - 5y + 15z &= 60 \\
x + 5y + 2z &= -3 \quad \rightarrow \quad -x + 5y + 2z &= -3 \\
\hline
-11x \\
+ 17z &= 57
\end{align*}
\]

Multiply the first result by 11 and add to the second result to eliminate \(x\):

\[
\begin{align*}
x - z &= -3 \quad \rightarrow \quad 11x - 11z &= -33 \\
-11x + 17z &= 57 \quad \rightarrow \quad -11x + 17z &= 57 \\
\hline
6z &= 24 \\
z &= 4
\end{align*}
\]

Substitute and solve for \(x\) and \(y\):

\[
\begin{align*}
x - z &= -3 \\
x - 4 &= -3 \\
x &= 1
\end{align*}
\]

\[
\begin{align*}
x &= 1 \\
y &= -2
\end{align*}
\]

The solution is \(\{(1, -2, 4)\}\).

12. \[ \begin{align*}
-3x - 2y + 4z &= -15 \\
2x + 5y - 3z &= 3 \\
4x - y + 7z &= 15
\end{align*} \]

Multiply the third equation by \(-2\) and add to the first equation to eliminate \(y\):

\[
\begin{align*}
-3x - 2y + 4z &= -15 \quad \rightarrow \quad -3x - 2y + 4z &= -15 \\
4x - y + 7z &= 15 \quad \rightarrow \quad -8x + 2y - 14z &= -30 \\
\hline
-11x \\
-10z &= -45
\end{align*}
\]

Multiply the third equation by 5 and add to the second equation to eliminate \(y\):

\[
\begin{align*}
2x + 5y - 3z &= 3 \quad \rightarrow \quad 2x + 5y - 3z &= 3 \\
x + 4 - y + 7z &= 15 \quad \rightarrow \quad 20x - 5y + 35z &= 75 \\
\hline
22x \\
+ 32z &= 78
\end{align*}
\]

Multiply the first result by 2 and add to the second result to eliminate \(x\):

\[
\begin{align*}
-11x - 10z &= -45 \quad \rightarrow \quad -22x - 20z &= -90 \\
22x + 32z &= 78 \quad \rightarrow \quad 22x + 32z &= 78 \\
\hline
12z &= -12 \\
z &= -1
\end{align*}
\]

Substitute and solve for \(x\) and \(y\):

\[
\begin{align*}
-11x - 10z &= -45 \\
-11x - 10(-1) &= -45 \\
-11x + 10 &= -45 \\
-11x &= -55 \\
x &= 5 \\
4x - y + 7z &= 15 \\
4(5) - y + 7(-1) &= 15 \\
20 - y - 7 &= 15 \\
-y &= 2 \\
y &= -2
\end{align*}
\]

The solution is \(\{(5, -2, -1)\}\).
13. \(x - 3y - 4z = -7\)
\(5x + 2y + 2z = -1\)
\(4x - y - 5z = -6\)

Multiply the third equation by \(-3\) and add to the first equation to eliminate \(y\):
\(x - 3y - 4z = -7\) \(\rightarrow\) \(x - 3y - 4z = -7\)
\(4x - y - 5z = -6\) \(\rightarrow\) \(-12x + 3y + 15z = 18\)

\(-11x + 11z = 11\)
\(-x + z = 1\)

Multiply the third equation by 2 and add to the second equation to eliminate \(y\):
\(5x + 2y + 2z = -1\) \(\rightarrow\) \(5x + 2y + 2z = -1\)
\(4x - y - 5z = -6\) \(\rightarrow\) \(8x - 2y - 10z = -12\)

\(13x - 8z = -13\)

Multiply the first result by 8 and add to the second result to eliminate \(z\):
\(-x + z = 1\) \(\rightarrow\) \(-x + z = 1\)
\(4(-1) - y - 5(0) = -6\)
\(-(-1) + z = 1\) \(\rightarrow\) \(-4 - y - 0 = -6\)
\(z = 0\) \(\rightarrow\) \(-y = -2\)
\(y = 2\)

The solution is \((1,2,0)\).

14. \(6x - 5y + z = 7\)
\(5x + 3y + 2z = 0\)
\(-2x + y - 3z = 11\)

Multiply the third equation by 5 and add to the first equation to eliminate \(y\):
\(6x - 5y + z = 7\) \(\rightarrow\) \(6x - 5y + z = 7\)
\(-2x + y - 3z = 11\) \(\rightarrow\) \(-10x + 5y - 15z = 55\)

\(-4x - 14z = 62\)

Multiply the third equation by \(-3\) and add to the second equation to eliminate \(y\):
\(5x + 3y + 2z = 0\) \(\rightarrow\) \(5x + 3y + 2z = 0\)
\(-2x + y - 3z = 11\) \(\rightarrow\) \(6x - 3y + 9z = -33\)

\(11x + 11z = -33\)
\(x + z = -3\)

Multiply the second result by 4 and add to the first result to eliminate \(x\):
Section 3.6 Systems of Linear Equations in Three Variables and Applications

-4x - 14z = 62 \rightarrow -4x - 14 = 62
x + z = -3 \rightarrow 4x + 4z = -12
10z = 50
z = -5

Substitute and solve for x and y:
x + z = -3
2x + y - 3z = 11
x + (-5) = -3
2(2) + y - 3(-5) = 11
x = 2
-4 + y + 15 = 11
y = 0

The solution is \{(2, 0, -5)\}.

16. \quad y = 2x + z + 1 \rightarrow 2x - y + z = -1
-3x - 1 = -2y + 2z \rightarrow -3x + 2y - 2z = 1
5x + 3z = 16 - 3y \rightarrow 5x + 3y + 3z = 16

Multiply the first equation by -3 and add to the third equation to eliminate z:
2x - y + z = -1 \rightarrow -6x + 3y - 3z = 3
5x + 3y + 3z = 16 \rightarrow 5x + 3y + 3z = 16
-6x + 3y = -19

Multiply the first equation by 2 and add to the second equation to eliminate z:
2x - y + z = -1 \rightarrow 4x - 2y + 2z = -2
-3x + 2y - 2z = 1 \rightarrow -3x + 2y - 2z = -1

Substitute and solve for y and z:

The solution is \{(1, -4, -2)\}.

17. \quad x + y + z = 6
-x + y - z = -2
2x + 3y + z = 11

Add the first and second equations to eliminate z:
x + y + z = 6
-x + y - z = -2
2y = 4
y = 2

Add the second and third equations to eliminate z:

18. \quad x - y - z = -11
x + y - z = 15
2x - y + z = -9

Add the first and third equations to eliminate z:
x - y - z = -11
2x - y + z = -9
3x - 2y = -20

Add the second and third equations to eliminate z:
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\[-x + y - z = -2\]
\[2x + 3y + z = 11\]
\[x + 4y = 9\]

Substitute \( y = 2 \) and solve for \( x \):
\[x + 4(2) = 9\]
\[x + 8 = 9\]
\[x = 1\]

Substitute into the first equation and solve for \( z \):
\[1 + 2 + z = 6\]
\[z = 3\]
The solution is \( \{(1, 2, 3)\} \).

19. \[2x - 3y + 2z = -1\]
\[x + 2y = -4\]
\[x + z = 1\]

Multiply the third equation by \(-2\) and add to the first equation to eliminate \( z \):
\[2x - 3y + 2z = -1 \rightarrow 2x - 3y + 2z = -1\]
\[x + z = 1 \xrightarrow{\times 2} -2x - 2z = -2\]
\[-3y = -3\]
\[y = 1\]

20. \[x + y + z = 2\]
\[2x - z = 5\]
\[3y + z = 2\]

Add the first and second equations to eliminate \( z \):
\[x + y + z = 2\]
\[2x - z = 5\]
\[3x + y = 7\]

21. \[4x + 9y = 8\]
\[8x + 6z = -1\]
\[6y + 6z = -1\]

Multiply the third equation by \(-1\) and add to the second equation to eliminate \( z \):
\[8x + 6z = -1 \rightarrow 8x + 6z = -1\]
\[6y + 6z = -1 \xrightarrow{\times 1} -6y - 6z = 1\]
\[8x - 6y = 0\]
Add the second and third equations to eliminate \( z \):
\[
2x - z = 5 \\
3y + z = 2 \\
2x + 3y = 7
\]
Multiply the first result by \(-3\) and add to the second result to eliminate \( y \):
\[
3x + y = 7 \quad \rightarrow \quad -9x - 3y = -21 \\
2x + 3y = 7 \\
\Rightarrow 7x = -14 \\
x = 2
\]
Substitute and solve for \( y \) and \( z \):
\[
2x - z = 5 \\
2(2) - z = 5 \\
4 - z = 5 \\
-2 = z \\
z = 1
\]
\[
\frac{3y + z}{2} = 2 \\
\frac{3y + (-1)}{2} = 2 \\
3y = 3 \\
y = 1
\]
The solution is \( \{(2, 1, -1)\} \).

Multiply the first equation by \(-2\) and add to this result to eliminate \( x \):
\[
4x + 9y = 8 \quad \rightarrow \quad -8x - 18y = -16 \\
8x - 6y = 0 \\
\Rightarrow 24y = -16 \\
y = \frac{2}{3}
\]
Substitute and solve for \( x \) and \( z \):
\[
4x + 9y = 8 \\
4x + 6z = -1 \\
4x = 2 \\
6z = -5 \\
x = \frac{1}{2} \\
z = \frac{5}{6}
\]
The solution is \( \left\{ \left( \frac{1}{2}, \frac{2}{3}, -\frac{5}{6} \right) \right\} \).

\[
22. \quad 3x + 2z = 11 \\
y - 7z = 4 \\
x - 6y = 1
\]
Multiply the third equation by \(-3\) and add to the first equation to eliminate \( x \):
\[
3x + 2z = 11 \\
3x + 2z = 11 \\
x - 6y = 1 \quad \rightarrow \quad -3x + 18y = -3 \\
18y + 2z = 8
\]
Multiply the second equation by \(-18\) and add to this result to eliminate \( y \):
\[
y - 7z = 4 \quad \rightarrow \quad -18y + 126z = -72 \\
18y + 2z = 8 \\
\Rightarrow 128z = -64 \\
z = -\frac{1}{2}
\]
Substitute and solve for \( x \) and \( y \):
\[
3x + 2z = 11 \\
3x + 2 \left( -\frac{1}{2} \right) = 11 \\
3x - 1 = 11 \\
3x = 12 \\
x = 4
\]
\[
y - 7 \left( -\frac{1}{2} \right) = 4 \\
y + \frac{7}{2} = 4 \\
y = \frac{1}{2}
\]
The solution is \( \left\{ \left( \frac{1}{2}, \frac{2}{3}, -\frac{5}{6} \right) \right\} \).
23. Let \( x \) = the first angle
Let \( y \) = the second angle
Let \( z \) = the third angle
\[
x + y + z = 180
\]
\[
y = 2x + 5
\]
\[
z = 3x - 11
\]
Substitute the second and third equations into the first and solve for \( x \):
\[
x + (2x + 5) + (3x - 11) = 180
\]
\[
6x - 6 = 180
\]
\[
6x = 186
\]
\[
x = 31
\]
Substitute and solve for \( y \) and \( z \):
\[
y = 2x + 5
\]
\[
z = 3x - 11
\]
\[
y = 2(31) + 5 = 67
\]
\[
z = 3(31) - 11 = 82
\]
The angles are 31º, 67º, and 82º.

24. Let \( x \) = the first angle
Let \( y \) = the second angle
Let \( z \) = the third angle
\[
x + y + z = 180
\]
\[
y = 2x
\]
\[
z = 5x - 4
\]
Substitute the second and third equations into the first and solve for \( x \):
\[
x + (2x) + (5x - 4) = 180
\]
\[
8x - 4 = 180
\]
\[
8x = 184
\]
\[
x = 23
\]
Substitute and solve for \( y \) and \( z \):
\[
y = 2x
\]
\[
z = 5x - 4
\]
\[
y = 2(23) = 46
\]
\[
z = 5(23) - 4 = 111
\]
The angles are 23º, 46º, and 111º.

25. Let \( x \) = the shortest side
Let \( y \) = the middle side
Let \( z \) = the longest side
\[
x + y + z = 55 \quad \rightarrow \quad x + y + z = 55
\]
\[
x = y - 8 \quad \rightarrow \quad x - y = -8
\]
\[
z = x + y - 1 \rightarrow -x - y + z = -1
\]
Add the first and third equations to eliminate \( x \):
\[
x + y + z = 55
\]
\[
-x - y + z = -1
\]
\[
2z = 54
\]
\[
z = 27
\]
Add the second and third equations to eliminate \( x \):
\[
x - y = -8
\]
\[
-x - y + z = -1
\]
\[
-2y + z = -9
\]
Substitute and solve for \( x \) and \( y \):
\[
2x + y = 40
\]
\[
x = 8
\]
Substitute and solve for \( y \) and \( z \):
The lengths of the sides are 10 cm, 18 cm, and 27 cm.

27. Let $x =$ the fiber in the supplement
Let $y =$ the fiber in the oatmeal
Let $z =$ the fiber in the cereal

\[ 3x + y + 4z = 19 \]
\[ 2x + 4y + 2z = 25 \]
\[ 5x + 3y + 2z = 30 \]

Multiply the first equation by $-4$ and add to the second equation to eliminate $y$:

\[ 3x + y + 4z = 19 \rightarrow \underbrace{-12x - 4y - 16z = -76}_{\text{Multiply by } -4} \]
\[ 2x + 4y + 2z = 25 \rightarrow \underbrace{2x + 4y + 2z = 25}_{\text{Multiply by } 1} \]
\[ -10x - 14z = -51 \]

Multiply the first equation by $-3$ and add to the third equation to eliminate $y$:

\[ 3x + y + 4z = 19 \rightarrow \underbrace{-9x - 3y - 12z = -57}_{\text{Multiply by } -3} \]
\[ 5x + 3y + 2z = 30 \rightarrow \underbrace{5x + 3y + 2z = 30}_{\text{Multiply by } 1} \]
\[ -4x - 10z = -27 \]

Multiply the second new equation by $-2.5$ and add to the first new equation to eliminate $x$:

\[ -10x - 14z = -51 \rightarrow \underbrace{-10x - 14z = -51}_{\text{Multiply by } -10} \]
\[ -4x - 10z = -27 \rightarrow \underbrace{10x + 25z = 67.5}_{\text{Multiply by } -2.5} \]
\[ 11z = 16.5 \]
\[ z = 1.5 \]

Substitute and solve for $x$ and $y$:

\[ -4x - 10z = -27 \]
\[ -4x - 10(1.5) = -27 \]
\[ -4x - 15 = -27 \]
\[ -4x = -12 \]
\[ x = 3 \]

\[ 3x + y + 4z = 19 \]
\[ 3(3) + y + 4(1.5) = 19 \]
\[ 9 + y + 6 = 19 \]
\[ 15 + y = 19 \]
\[ y = 4 \]

The fiber supplement has 3 g; the oatmeal has 4 g; and the cereal has 1.5 g.
28. Let \( x = \) the calcium in milk
Let \( y = \) the calcium in ice cream
Let \( z = \) the calcium in the supplement
\[
\begin{align*}
x + y + z &= 1180 \\
2x + y + z &= 1680 \\
x + 2y + z &= 1260
\end{align*}
\]
Multiply the first equation by \(-1\) and add to the second equation to eliminate \( y \) and \( z \):
\[
\begin{align*}
x + y + z &= 1180 \\
2x + y + z &= 1680
\rightarrow (1) \\
x &= 500
\end{align*}
\]
Substitute into the second and third equations:
\[
\begin{align*}
2x + y + z &= 1680 \\
x + 2y + z &= 1260 \\
500 + 2y + z &= 1260 \\
1000 + y + z &= 1680 \\
y + z &= 680
\end{align*}
\]
Multiply the first new equation by \(-2\) and add to the second new equation to eliminate \( y \):
\[
\begin{align*}
y + z &= 680 \\
2y + z &= 760
\rightarrow (2) \\
-2x - 2z &= -1360 \\
2y + z &= 760
\rightarrow (3)
\end{align*}
\]
Substitute and solve for \( y \):
\[
\begin{align*}
x + y + z &= 1180 \\
500 + y + 600 &= 1180 \\
y + 1100 &= 1180 \\
y &= 80
\end{align*}
\]
The milk has 500 mg; the ice cream has 80 mg; and the calcium supplement has 600 mg.

29. Let \( x = \) the number of par 3 holes
Let \( y = \) the number of par 4 holes
Let \( z = \) the number of par 5 holes
\[
\begin{align*}
x + y + z &= 18 \\
y &= 3x \\
z &= x + 3
\end{align*}
\]
Substitute the second and third equations into the first and solve for \( x \):

30. Let \( x = \) the number of ounces of peanuts
Let \( y = \) the number of ounces of pecans
Let \( z = \) the number of ounces of cashews
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\[ x + (3x) + (x + 3) = 18 \]
\[ 5x + 3 = 18 \]
\[ 5x = 15 \]
\[ x = 3 \]

Substitute and solve for \( y \) and \( z \):

\[ y = 3x \]
\[ z = x + 3 \]
\[ y = 9 \]
\[ z = 6 \]

There are three par 3 holes, nine par 4 holes, and six par 5 holes.

\[ x + y + z = 48 \]
\[ \rightarrow x + y + z = 48 \]
\[ x = y + z \rightarrow x - y - z = 0 \]
\[ z = 2y \rightarrow -2y + z = 0 \]

Add the first and second equations to eliminate \( z \):

\[ x + y + z = 48 \]
\[ x - y - z = 0 \]
\[ \frac{2x}{2} = 48 \]
\[ x = 24 \]

Add the second and third equations to eliminate \( z \):

\[ x - y - z = 0 \]
\[ -2y + z = 0 \]
\[ x - 3y = 0 \]

Substitute and solve for \( y \) and \( z \):

\[ x - 3y = 0 \]
\[ z = 2y \]
\[ 24 - 3y = 0 \]
\[ z = 2(8) \]
\[ -3y = -24 \]
\[ z = 16 \]
\[ y = 8 \]

There are 24 oz of peanuts, 8 oz of pecans, and 16 oz of cashews in the mixture.

31. Let \( x \) = the price of a hat
Let \( y \) = the price of a T-shirt
Let \( z \) = the price of a jacket

\[ 3x + 2y + z = 140 \]
\[ 2x + 2y + 2z = 170 \]
\[ x + 3y + 2z = 180 \]

Multiply the first equation by \(-2\) and add to the second equation to eliminate \( z \):

\[ 3x + 2y + z = 140 \rightarrow -6x - 4y - 2z = -280 \]
\[ 2x + 2y + 2z = 170 \rightarrow 2x + 2y + 2z = 170 \]
\[ -4x - 2y = -110 \]

Multiply the third equation by \(-1\) and add to the second equation to eliminate \( z \):
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\[
2x + 2y + 2z = 170 \quad \Rightarrow \quad 2x + 2y + 2z = 170
\]
\[
x + 3y + 2z = 180 \quad \Rightarrow \quad x - 3y - 2z = -180
\]
\[
\phantom{x} \quad \Rightarrow \quad x - y = -10
\]
Multiply the second result by \(-2\) and add to the first result to eliminate \(y\):
\[
-4x - 2y = -110 \quad \Rightarrow \quad -4x - 2y = -110
\]
\[
x - y = -10 \quad \Rightarrow \quad -2x + 2y = 20
\]
\[
\phantom{x} \quad \Rightarrow \quad -6x = -90
\]
\[
x = 15
\]
Multiply the third equation by \(2\) and add to the first equation to eliminate \(z\):
\[
x + 4y + 2z = 1040
\]
\[
0.08x + 0.11(4y) + 0.10(2z) = 106
\]
\[
\phantom{0.08x} \quad \Rightarrow \quad x = z + 80
\]
\[
x + 4y + 2z = 1040
\]
\[
8x + 44y + 20z = 10,600
\]
\[
x = -z = 80
\]
Multiply the third equation by \(2\) and add to the first equation to eliminate \(z\):
\[
x + 4y + 2z = 1040 \quad \Rightarrow \quad x + 4y + 2z = 1040
\]
\[
x = -z = 80 \quad \Rightarrow \quad 2x - 2z = 160
\]
\[
\phantom{2x} \quad \Rightarrow \quad 3x + 4y = 1200
\]
Multiply the third equation by \(20\) and add to the second equation to eliminate \(z\):
\[
8x + 44y + 20z = 10600
\]
\[
\quad \Rightarrow \quad 8x + 44y + 20z = 10600
\]
\[
x = -z = 80
\]
\[
\quad \Rightarrow \quad 20x - 20z = 1600
\]
\[
\phantom{20x} \quad \Rightarrow \quad 28x + 44y = 12200
\]
Multiply the first result by \(-11\) and add to the second result to eliminate \(y\):

32.  Let \(x\) = the cost per night Paris
Let \(y\) = the cost per night Stockholm
Let \(z\) = the cost per night Oslo
\[
x + 4y + 2z = 1040
\]
\[
0.08x + 0.11(4y) + 0.10(2z) = 106
\]
\[
\phantom{0.08x} \quad \Rightarrow \quad x = z + 80
\]
\[
x + 4y + 2z = 1040
\]
\[
8x + 44y + 20z = 10,600
\]
\[
x = -z = 80
\]
Multiply the third equation by \(2\) and add to the first equation to eliminate \(z\):
\[
x + 4y + 2z = 1040 \quad \Rightarrow \quad x + 4y + 2z = 1040
\]
\[
x = -z = 80 \quad \Rightarrow \quad 2x - 2z = 160
\]
\[
\phantom{2x} \quad \Rightarrow \quad 3x + 4y = 1200
\]
Multiply the third equation by \(20\) and add to the second equation to eliminate \(z\):
\[
8x + 44y + 20z = 10600
\]
\[
\quad \Rightarrow \quad 8x + 44y + 20z = 10600
\]
\[
x = -z = 80
\]
\[
\quad \Rightarrow \quad 20x - 20z = 1600
\]
\[
\phantom{20x} \quad \Rightarrow \quad 28x + 44y = 12200
\]
Multiply the first result by \(-11\) and add to the second result to eliminate \(y\):
3x + 4y = 1200 \rightarrow -33x - 44y = -13200 \\
28x + 44y = 12200 \rightarrow 28x + 44y = 12200 \\
\hspace{2cm} -5x = -1000 \\
\hspace{2cm} x = 200

Substitute and solve for \(y\) and \(z\):

\[\begin{align*}
3x + 4y &= 1200 \\
3(200) + 4y &= 1200 \\
600 + 4y &= 1200 \\
4y &= 600 \\
y &= 150
\end{align*}\]

Hotel costs per night are: Paris $200, Stockholm $150, and Oslo $120.

33. Let \(x\) = the amount invested in small caps
Let \(y\) = the amount invested in global markets
Let \(z\) = the amount invested in the balanced fund

\[\begin{align*}
x + y + z &= 25,000 \rightarrow x + 4y + 2z &= 1040 \\
0.06x + 0.10y + 0.09z &= 2106 \rightarrow 6x + 10y + 9z &= 216,000
\end{align*}\]

Substitute into the first two equations to eliminate \(y\):

\[\hspace{2cm} x + z = 25,000 \quad 6x + 10y + 9z = 216,000\]
\[\hspace{2cm} x + 2z + z = 25,000 \quad 6x + 10(2z) + 9z = 216,000\]
\[\hspace{2cm} x + 3z = 25,000 \quad 6x + 20z + 9z = 216,000\]
\[\hspace{3cm} 6x + 29z = 216,000\]

Multiply the first new equation by \(-6\) and add to the second new equation to eliminate \(x\):

\[\hspace{2cm} x + 3z = 25,000 \quad -6x - 18z = -150,000\]
\[\hspace{2cm} 6x + 29z = 216,000 \quad 6x + 29z = 216,000\]
\[\hspace{5cm} 11z = 66,000\]
\[\hspace{5cm} z = 6000\]

Substitute and solve for \(x\):

\[\begin{align*}
x + 3z &= 25,000 \\
x + 3(6000) &= 25,000 \\
x + 18,000 &= 25,000 \\
x &= 7000
\end{align*}\]

Substitute and solve for \(y\):
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\[ x + y + z = 25,000 \]
\[ 7000 + y + 6000 = 25,000 \]
\[ y + 13,000 = 25,000 \]
\[ y = 12,000 \]

Walter invested $7000 in small caps, $12,000 in global markets, and $6000 in the balanced fund.

34. Let \( x \) = the amount deposited in checking
Let \( y \) = the amount deposited in savings
Let \( z \) = the amount deposited in money market

\[ \begin{align*}
  x + y + z &= 8000 \\
  0.012x + 0.025y + 0.03z &= 202 \\
  3x &= z
\end{align*} \]

Substitute into the first two equations to eliminate \( z \):
\[ \begin{align*}
  x + y + z &= 8000 \\
  x + y + 3x &= 8000 \\
  4x + y &= 8000
\end{align*} \]
\[ \begin{align*}
  12x + 25y + 30z &= 202,000 \\
  12x + 25y + 90x &= 202,000 \\
  102x + 25y &= 202,000
\end{align*} \]

Multiply the first new equation by \(-25\) and add to the second new equation to eliminate \( y \):
\[ \begin{align*}
  4x + y &= 8000 \\
  100x - 25y &= -200,000 \\
  102x + 25y &= 202,000 \\
  2x &= 2000 \\
  x &= 1000
\end{align*} \]

Substitute and solve for \( y \):
\[ \begin{align*}
  4x + y &= 8000 \\
  4(1000) + y &= 8000 \\
  4000 + y &= 8000 \\
  y &= 4000
\end{align*} \]

Substitute and solve for \( z \):
\[ \begin{align*}
  x + y + z &= 8000 \\
  1000 + 4000 + z &= 8000 \\
  5000 + z &= 8000 \\
  z &= 3000
\end{align*} \]

Raeann deposited $1000 in checking (at 1.2%), $4000 in savings (at 2.5%), and $3000 in the money market (at 3%).
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35. \[\begin{align*}
2x + y + 3z &= 2 \\
x - y + 2z &= -4 \\
-2x + 2y - 4z &= 8
\end{align*}\]
Add the first and second equations to eliminate \(y\):
\[\begin{align*}
2x + y + 3z &= 2 \\
x - y + 2z &= -4 \\
\underline{3x + 5z} &= -2
\end{align*}\]
Multiply the second equation by 2 and add to the third equation to eliminate \(y\):
\[\begin{align*}
x - y + 2z &= -4 \\
\underline{-2x + 2y - 4z} &= 8
\end{align*}\]
\[0 = 0\]
The equations are dependent.

36. \[\begin{align*}
x + y = z &\rightarrow x + y - z = 0 \\
2x + 4y - 2z &= 6 \\
3x + 6y - 3z &= 9
\end{align*}\]
Multiply the second equation by \(-3\), multiply the third equation by \(2\), and add to eliminate \(x\):
\[\begin{align*}
2x + 4y - 2z &= 6 \\
\underline{3x + 6y - 3z} &= 9
\end{align*}\]
\[6x + 12y - 6z = 18\]
\[0 = 0\]
The equations are dependent.

37. \[\begin{align*}
6x - 2y + 2z &= 2 \\
4x + 8y - 2z &= 5 \\
-2x - 4y + z &= -2
\end{align*}\]
Multiply the third equation by 2 and add to the second equation to eliminate \(z\):
\[\begin{align*}
4x + 8y - 2z &= 5 \\
-2x - 4y + z &= -2
\end{align*}\]
\[0 = 1\]
The system is inconsistent. There is no solution.

38. \[\begin{align*}
3x + 2y + z &= 3 \\
x - 3y + z &= 4 \\
-6x - 4y - 2z &= 1
\end{align*}\]
Multiply the first equation by \(2\) and add to the third equation to eliminate \(z\):
\[\begin{align*}
3x + 2y + z &= 3 \\
\underline{-6x - 4y - 2z} &= 1
\end{align*}\]
\[0 \neq 7\]
The system is inconsistent. There is no solution.

39. Multiply by the LCD of each equation:
\[\begin{align*}
\frac{1}{2}x + \frac{2}{3}y &= \frac{5}{2} \rightarrow 3x + 4y = 15 \\
\frac{1}{3}x - \frac{1}{2}z &= -\frac{3}{10} \rightarrow 2x - 5z = -3 \\
\frac{1}{3}y - \frac{1}{3}z &= \frac{3}{4} \rightarrow 4y - 3z = 9
\end{align*}\]
Multiply the first equation by \(-1\) and add to the third equation to eliminate \(y\):
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3x + 4y = 15  \rightarrow 3x - 4y = -15
4y - 3z = 9 \rightarrow 4y - 3z = 9
\begin{align*}
-3x & = -3z - 6 \\
2x - 5z & = -3
\end{align*}
\begin{align*}
\text{Multiply this result by } \frac{2}{3} \text{ and add to the second equation to eliminate } x: \\
-3x - 3z = -6 & \rightarrow -2x - 2z = -4 \\
2x - 5z = -3 & \rightarrow 2x - 5z = -3
\end{align*}
\begin{align*}
-7z = -7 \\
z = 1
\end{align*}
Substitute and solve for x and y:
\begin{align*}
2x - 5z & = -3 \\
4y - 3z & = 9
\end{align*}
\begin{align*}
2x - 5 & = -3 \\
4y - 3 & = 9
\end{align*}
\begin{align*}
x & = 2 \\
y & = 12
\end{align*}
\begin{align*}
x & = 1 \\
y & = 3
\end{align*}
The solution is \( \{(1,3,1)\} \).

40. Multiply by the LCD of each equation:
\begin{align*}
\frac{1}{2}x + \frac{1}{4}y + z = 3 & \rightarrow 2x + y + 4z = 12 \\
\frac{1}{8}x + \frac{1}{4}y + \frac{1}{6}z = \frac{3}{8} & \rightarrow x + 2y + 3z = 9 \\
x - y - \frac{2}{3}z = \frac{1}{3} & \rightarrow 3x - 3y - 2z = 1
\end{align*}
Add the second and third equations to eliminate z:
\begin{align*}
x + 2y + 2z & = 9 \\
3x - 3y - 2z & = 1
\end{align*}
\begin{align*}
4x - y & = 10
\end{align*}
Multiply the third equation by 2 and add to the first equation to eliminate z:
\begin{align*}
2x + y + 4z = 12 \\
3x - 3y - 2z = 1 \rightarrow 6x - 6y - 4z = 2
\end{align*}
\begin{align*}
8x - 5y & = 14
\end{align*}
Multiply the first result by \(-2\) and add to the second result to eliminate x:
\begin{align*}
-3x + y - z & = 8 \\
-4x + 2y + 3z & = -3 \\
2x + 3y - 2z & = -1
\end{align*}
Multiply the first equation by 3 and add to the second equation to eliminate z:
\begin{align*}
-3x + y - z & = 8 \rightarrow -9x + 3y - 3z = 24 \\
-4x + 2y + 3z & = -3 \\
\frac{-13x + 5y}{-20} & = 21
\end{align*}
Multiply the first equation by \(-2\) and add to the third equation to eliminate z:
\begin{align*}
-3x + y - z & = 8 \rightarrow 6x - 2y + 2z = -16 \\
2x + 3y - 2z & = -1 \rightarrow 2x + 3y - 2z = -1 \\
8x + y & = \frac{-17}{8}
\end{align*}
Multiply the second result by \(-5\) and add to the first result to eliminate y:
\begin{align*}
-13x + 5y & = 21 \\
8x + y & = \frac{-17}{8} \rightarrow -40x - 5y = 85
\end{align*}
\begin{align*}
-53x & = 106 \\
x & = -2
Section 3.6 Systems of Linear Equations in Three Variables and Applications

42. \(4x - y = 10 \quad \rightarrow \quad 8x - 2y = -20\)
\(8x - 5y = 14 \quad \rightarrow \quad 8x - 5y = 14\)

Substitute and solve for \(y\) and \(z\):
\[8x + y = -17 \quad \rightarrow \quad -3x + y - z = 8\]
\[8(-2) + y = -17 \quad \rightarrow \quad -3(-2) + (-1) - z = 8\]
\[-16 + y = -17 \quad \rightarrow \quad 6 - 1 - z = 8\]
\[y = -1 \quad \rightarrow \quad -z = 3\]

The solution is \(\{(2, -1, -3)\}\).

43. \(2x + y = 3(z - 1) \quad \rightarrow \quad 2x + y - 3z = -3\)
\(3x - 2(y - 2z) = 1 \quad \rightarrow \quad 3x - 2y + 4z = 1\)
\(2(2x - 3z) = -6 - 2y \quad \rightarrow \quad 4x + 2y - 6z = -6\)

Multiply the first equation by \(-2\) and add to the third equation to eliminate \(z\):
\[2x + y = -3 \quad \rightarrow \quad 2x + 16z = -10\]
\[-7x - 3y + 4z = 8\]

Multiply the third equation by \(-4\) and add to the second equation to eliminate \(z\):
Chapter 3  Systems of Linear Equations and Inequalities

2x + y - 3z = -3 → 2x - 2y + 6z = 6
4x + 2y - 6z = -6 → 4x + 2y - 6z = -6

The equations are dependent.

45. Multiply each equation by 10:

\[
\begin{align*}
-0.1y + 0.2z &= 0.2 \quad \rightarrow \quad -y + 2z = 2 \\
0.1x + 0.1y + 0.1z &= 0.2 \quad \rightarrow \quad x + y + z = 2 \\
-0.1x -0.3z &= 0.2 \quad \rightarrow \quad -x -3z = 2
\end{align*}
\]

Add the first and second equations to eliminate y:

\[
\begin{align*}
-y + 2z &= 2 \\
x + y + z &= 2 \\
x &= -3z = 2
\end{align*}
\]

Add this result to the third equation to eliminate x:

\[
\begin{align*}
x + 3z &= 4 \\
x - 3z &= 2 \\
0 &= 6
\end{align*}
\]

The system is inconsistent. There is no solution.

46. Multiply each equation by 10:

\[
\begin{align*}
-0.4x -0.3y &= 0.2 \quad \rightarrow \quad -4x -3y = 0 \\
0.3y + 0.1z &= -0.1 \quad \rightarrow \quad 3y + z = -1 \\
0.4x -0.1z &= 1.2 \quad \rightarrow \quad 4x - z = 12
\end{align*}
\]

Add the second and third equations to eliminate z:

\[
\begin{align*}
3y + z &= -1 \\
4x - z &= 12 \\
4x + 3y &= 11
\end{align*}
\]

Add the first equation to this result to eliminate x:

\[
\begin{align*}
-4x - 3y &= 0 \\
4x + 3y &= 11 \\
0 &= 11
\end{align*}
\]

The system is inconsistent. There is no solution.

47. 2x - 4y + 8z = 0

\[
\begin{align*}
x - 3y + z &= 0 \\
x - 2y + 5z &= 0
\end{align*}
\]

Add the second and third equations to eliminate x:

\[
\begin{align*}
x - 3y + z &= 0 \\
x - 2y + 5z &= 0 \\
-5y + 6z &= 0
\end{align*}
\]

48. 2x - 4y + z = 0

\[
\begin{align*}
x - 3y - z &= 0 \\
3x - y + 2z &= 0
\end{align*}
\]

Add the first and second equations to eliminate z:

\[
\begin{align*}
2x - 4y + z &= 0 \\
x - 3y - z &= 0 \\
3x - 7y &= 0
\end{align*}
\]
Multiply the second equation by 2 and add to the first equation to eliminate $x$:

$$2x - 4y + 8z = 0 \quad \rightarrow \quad 2x - 4y + 8z = 0$$
$$-x - 3y + z = 0 \quad \rightarrow \quad -2x - 6y + 2z = 0$$

$$\begin{align*}
-10y + 10z &= 0 \\
\end{align*}$$

Multiply the first result by $-2$ and add to the second result to eliminate $y$:

$$\begin{align*}
-5y + 6z &= 0 \\
-10y + 10z &= 0
\end{align*} \quad \rightarrow \quad
\begin{align*}
10y - 12z &= 0 \\
10y + 10z &= 0
\end{align*}$$

$$\begin{align*}
\begin{align*}
-2z &= 0 \\
z &= 0
\end{align*}
\end{align*}$$

Substitute and solve for $x$ and $y$:

$$\begin{align*}
-5y + 6z &= 0 \\
-5y + 6(0) &= 0 \\
-5y &= 0
\end{align*}$$

$$\begin{align*}
\begin{align*}
x - 2y + 5z &= 0 \\
x - 2(0) + 5(0) &= 0 \\
x &= 0
\end{align*}
\end{align*}$$

$$\begin{align*}
\begin{align*}
y &= 0 \\
x &= 0
\end{align*}
\end{align*}$$

The solution is $\{(0,0,0)\}$.

Multiply the second equation by 2 and add to the third equation to eliminate $z$:

$$\begin{align*}
x - 3y - z &= 0 \quad \rightarrow \quad x^2 - 3y - z = 0 \\
3x - y + 2z &= 0 \quad \rightarrow \quad 3x - y + 2z = 0
\end{align*}$$

$$\begin{align*}
5x - 7y &= 0
\end{align*}$$

Multiply the first result by $-2$ and add to the second result to eliminate $y$:

$$\begin{align*}
3x - 7y &= 0 \\
5x - 7y &= 0
\end{align*} \quad \rightarrow \quad
\begin{align*}
-3x + 7y &= 0 \\
5x - 7y &= 0
\end{align*}$$

$$\begin{align*}
2x &= 0 \\
x &= 0
\end{align*}$$

Multiply the first result by $-1$ and add to the second result to eliminate $y$:

$$\begin{align*}
3x - 7y &= 0 \\
5x - 7y &= 0
\end{align*} \quad \rightarrow \quad
\begin{align*}
-2x + 7y &= 0 \\
5x - 7y &= 0
\end{align*}$$

$$\begin{align*}
2x &= 0 \\
2x &= 0
\end{align*}$$

Substitute and solve for $y$ and $z$:

$$\begin{align*}
3(0) - 7y &= 0 \\
2(0) - 4(0) + z &= 0
\end{align*}$$

$$\begin{align*}
-7y &= 0 \\
0 - 0 + z &= 0
\end{align*}$$

$$\begin{align*}
y &= 0 \\
z &= 0
\end{align*}$$

The solution is $\{(0,0,0)\}$.

49. $4x - 2y - 3z = 0$

$$\begin{align*}
-8x - y + z &= 0 \\
2x - y - \frac{3}{2}z &= 0
\end{align*}$$

Multiply the third equation by $-2$ and add to the first equation to eliminate $y$:

$$\begin{align*}
4x - 2y - 3z &= 0 \quad \rightarrow \quad 4x - 2y - 3z = 0 \\
2x - y - \frac{3}{2}z &= 0 \quad \rightarrow \quad -4x + 2y + 3z = 0
\end{align*}$$

$$\begin{align*}
\begin{align*}
0 &= 0
\end{align*}
\end{align*}$$

The equations are dependent.

50. $5x + y = 0$

$$\begin{align*}
4y - z &= 0 \\
5x + 5y - z &= 0
\end{align*}$$

Multiply the second equation by $-1$ and add to the third equation to eliminate $z$:

$$\begin{align*}
4y - z &= 0 \\
5x + 5y - z &= 0
\end{align*} \quad \rightarrow \quad
\begin{align*}
-4y + z &= 0 \\
5x + 5y - z &= 0
\end{align*}$$

$$\begin{align*}
\begin{align*}
5x + y &= 0
\end{align*}
\end{align*}$$

Multiply the first equation by $-1$ and add to the this result to eliminate $y$:

$$\begin{align*}
5x + y &= 0 \quad \rightarrow \quad -5x - y = 0 \\
5x + y &= 0
\end{align*}$$

$$\begin{align*}
0 &= 0
\end{align*}$$

The equations are dependent.
Section 3.7 Practice Exercises

1. a. matrix; rows; columns
   b. column; one; square
   c. coefficient; augmented
   d. row echelon

2. Let \( x \) = amount of pure (100\%) acid sol
   \( y \) = the amount of 50\% acid solution
   \[
   \begin{array}{cccc}
   \text{oz solution} & x & y & \text{20} \\
   \text{oz bleach} & 1.00x & 0.50y & 0.70(20) \\
   \end{array}
   \]
   Multiply the first equation by \(-0.50\), add to the second equation and solve for \( x \):
   \[
   x + y = 20 \rightarrow -0.50x - 0.50y = -10 \\
   1x + 0.50y = 14 \rightarrow 1.00x + 0.50y = 14 \\
   0.50x = 4 \\
   x = 8 \\
   \]
   Substitute into the first equation and solve for \( y \):
   \[
   8 + y = 20 \rightarrow y = 12 \\
   \]
   Mix 12 L of the 50\% mixture with 8L of pure acid.

3. \( x - 6y = 9 \)
   \( x + 2y = 13 \)
   Multiply the first equation by \(-1\), add to the second equation and solve for \( y \):
   \[
   \begin{align*}
   x - 6y &= 9 \rightarrow x = 9 + 6y \\\n   x + 2y &= 13 \rightarrow x = 13 - 2y \\
   \end{align*}
   \]
   \[
   8y = 4 \rightarrow y = \frac{1}{2} \\
   \]
   Substitute into the first equation and solve for \( x \):
   \[
   x - 6\left(\frac{1}{2}\right) = 9 \rightarrow x = 3 \\
   \]
   The solution is \( \left\{ \left( 12, \frac{1}{2} \right) \right\} \).

4. \( x + y - z = 8 \)
   \( x - 2y + z = 3 \)
   \( x + 3y + 2z = 7 \)
   Add the first and second equations to eliminate \( z \):
   \[
   \begin{align*}
   x + y - z &= 8 \\
   x - 2y + z &= 3 \\
   \end{align*}
   \]
   \[
   2x - y = 11 \\
   \]
   Multiply the second equation by \(-2\) and add to the third equation to eliminate \( z \):

5. \( 2x - y + z = -4 \)
   \( -x + y + 3z = -7 \)
   \( x + 3y - 4z = 22 \)
   Add the first and second equations to eliminate \( y \):
   \[
   \begin{align*}
   2x - y + z &= -4 \\
   -x + y + 3z &= -7 \\
   \end{align*}
   \]
   \[
   x + 4z = -11 \\
   \]
   Multiply the first equation by 3 and add to the third equation to eliminate \( y \):
Section 3.7 Solving Systems of Linear Equations by Using Matrices

\[ x - 2y + z = 3 \quad \rightarrow \quad -2x + 4y - 2z = -6 \]
\[ x + 3y + 2z = 7 \quad \rightarrow \quad x + 3y + 2z = 7 \]
\[ \underline{-x + 7y} = 1 \]

Multiply the second result by 2 and add to the first result to eliminate \( x \):
\[ 2x - y = 11 \quad \rightarrow \quad 2x - y = 11 \]
\[ -x + 7y = 1 \quad \rightarrow \quad -2x + 14y = 2 \]
\[ 13y = 13 \]
\[ y = 1 \]

Substitute and solve for \( x \) and \( z \):
\[ 2x - y = 11 \]
\[ x - 2y + z = 3 \]
\[ 2x - 1 = 11 \quad \quad \rightarrow \quad \quad 6 - 2(1) + z = 3 \]
\[ 2x = 12 \quad \quad \rightarrow \quad \quad 6 - 2 + z = 3 \]
\[ x = 6 \quad \quad \rightarrow \quad \quad z = -1 \]

The solution is \( \{(6,1,-1)\} \).

\[ 2x - y + z = -4 \quad \rightarrow \quad 6x - 3y + 3z = -12 \]
\[ x + 3y - 4z = 22 \quad \rightarrow \quad x + 3y - 4z = 22 \]
\[ 7x - z = 10 \]

Multiply the second result by 2 and add to the first result to eliminate \( z \):
\[ x + 4z = -11 \quad \rightarrow \quad x + 4z = -11 \]
\[ 7x - z = 10 \quad \rightarrow \quad 28x - 4z = 40 \]
\[ 29x = 29 \]
\[ x = 1 \]

Substitute and solve for \( y \) and \( z \):
\[ x + 4z = -11 \quad \quad \rightarrow \quad \quad -x + y + 3z = -7 \]
\[ x = 1 \quad \quad \rightarrow \quad \quad 1 + 4z = -11 \quad \quad \rightarrow \quad \quad 1 + y + 3(-3) = -7 \]
\[ 4z = -12 \quad \quad \rightarrow \quad \quad z = -3 \]
\[ y = 3 \]

The solution is \( \{(1,3,-3)\} \).

6. 4×1, column matrix
7. 3×1, column matrix
8. 3×3, square matrix
9. 2×2, square matrix
10. 1×2, row matrix
11. 1×4, row matrix
12. 2×4, none of these
13. 2×3, none of these
14. 3×2, none of these
15. \[
\begin{bmatrix}
1 & -2 & -1 \\
2 & 1 & -7
\end{bmatrix}
\]
16. \[
\begin{bmatrix}
1 & -3 & 3 \\
2 & -5 & 4
\end{bmatrix}
\]
17. \[
\begin{bmatrix}
1 & -2 & 1 & 5 \\
2 & 6 & 3 & -2 \\
3 & -1 & -2 & 1
\end{bmatrix}
\]
18. \[
\begin{bmatrix}
5 & 0 & 2 & 17 \\
8 & -1 & 6 & 26 \\
8 & 3 & -12 & 24
\end{bmatrix}
\]
19. \[4x + 3y = 6 \]
\[12x + 5y = -6 \]
20. \[-2x + 5y = -15 \]
\[-7x + 15y = -45 \]

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21. \[ x = 4 \]
    \[ y = -1 \]
    \[ z = 7 \]

22. \[ x = 0.5 \]
    \[ y = 6.1 \]
    \[ z = 3.9 \]

23. a. \( 7 \)
    b. \( -2 \)

24. a. \(-13\)
    b. \(0\)

25. \[
Z = \begin{bmatrix}
2 & 1 & 11 \\
2 & -1 & 1 \\
\end{bmatrix} \xrightarrow{R_5 \rightarrow R_3} \begin{bmatrix}
1 & \frac{1}{2} & \frac{11}{2} \\
2 & -1 & 1 \\
\end{bmatrix}
\]

26. \[
J = \begin{bmatrix}
1 & 1 & 7 \\
0 & 3 & -6 \\
\end{bmatrix} \xrightarrow{R_1 \rightarrow R_3} \begin{bmatrix}
1 & 1 & 7 \\
0 & 1 & -2 \\
\end{bmatrix}
\]

27. \[
K = \begin{bmatrix}
5 & 2 & 1 \\
1 & -4 & 3 \\
\end{bmatrix} \xrightarrow{R_3 \Rightarrow R_3} \begin{bmatrix}
1 & -4 & 3 \\
5 & 2 & 1 \\
\end{bmatrix}
\]

28. \[
L = \begin{bmatrix}
9 & 6 & 13 \\
-7 & 2 & 19 \\
\end{bmatrix} \xrightarrow{R_2 \Rightarrow R_3} \begin{bmatrix}
-7 & 2 & 19 \\
9 & 6 & 13 \\
\end{bmatrix}
\]

29. \[
M = \begin{bmatrix}
1 & 5 & 2 \\
-3 & -4 & -1 \\
\end{bmatrix} \xrightarrow{3R_1 + R_2 \Rightarrow R_2} \begin{bmatrix}
1 & 5 & 2 \\
0 & 11 & 5 \\
\end{bmatrix}
\]

30. \[
N = \begin{bmatrix}
1 & 3 & -5 \\
-2 & 2 & 12 \\
\end{bmatrix} \xrightarrow{2R_1 + R_2 \Rightarrow R_1} \begin{bmatrix}
1 & 3 & -5 \\
0 & 8 & 2 \\
\end{bmatrix}
\]

31. a. \[
\begin{bmatrix}
1 & 3 & 0 & -1 \\
4 & 1 & -5 & 6 \\
-2 & 0 & -3 & 10 \\
\end{bmatrix} \xrightarrow{-4R_1 + R_4 \Rightarrow R_4} \begin{bmatrix}
1 & 3 & 0 & -1 \\
0 & -11 & -5 & 10 \\
-2 & 0 & -3 & 10 \\
\end{bmatrix}
\]

32. a. \[
\begin{bmatrix}
1 & 2 & 0 & 10 \\
5 & 1 & -4 & 3 \\
-3 & 4 & 5 & 2 \\
\end{bmatrix} \xrightarrow{-5R_1 + R_2 \Rightarrow R_3} \begin{bmatrix}
1 & 2 & 0 & 10 \\
0 & -9 & -4 & -47 \\
-3 & 4 & 5 & 2 \\
\end{bmatrix}
\]

b. \[
\begin{bmatrix}
1 & 3 & 0 & -1 \\
0 & -11 & -5 & 10 \\
0 & 6 & -3 & 8 \\
\end{bmatrix} \xrightarrow{3R_1 + R_3 \Rightarrow R_3} \begin{bmatrix}
1 & 2 & 0 & 10 \\
0 & -9 & -4 & -47 \\
0 & 10 & 5 & 32 \\
\end{bmatrix}
\]

33. True

34. True

35. True

36. True

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37. Interchange rows 1 and 2.

38. Multiply row 3 by 2. Replace row 3 with the result.

39. Multiply row 1 by \(-3\) and add to row 2. Replace row 2 with the result.

40. Multiply row 2 by 4 and add to row 3. Replace row 3 with the result.

41. \(x - 2y = -1\)
\[
\begin{bmatrix}
1 & -2 & -1 \\
2 & 1 & -7
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -2 & -1 \\
0 & 5 & -5
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -2 & -1 \\
0 & 1 & -1
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & -3 \\
0 & 1 & -1
\end{bmatrix}
\]
The solution is \(\{(−3,−1)\}\).

42. \(x - 3y = 3\)
\[
\begin{bmatrix}
1 & -3 & 3 \\
2 & -5 & 4
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -3 & 3 \\
0 & 1 & -2
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & -3 \\
0 & 1 & -2
\end{bmatrix}
\]
The solution is \(\{(−3,−2)\}\).

43. \(x + 3y = 6\)
\(-4x - 9y = 3\)
\[
\begin{bmatrix}
1 & 3 & 6 \\
-4 & -9 & 3
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 3 & 6 \\
0 & 3 & 27
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 3 & 6 \\
0 & 1 & 9
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & -21 \\
0 & 1 & 9
\end{bmatrix}
\]
The solution is \(\{(−21,9)\}\).

44. \(2x - 3y = -2\)
\(x + 2y = 13\)
\[
\begin{bmatrix}
2 & -3 & -2 \\
1 & 2 & 13
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 2 & 13 \\
2 & -3 & -2
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 2 & 13 \\
0 & -7 & -28
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 2 & 13 \\
0 & 1 & 4
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & 5 \\
0 & 1 & 4
\end{bmatrix}
\]
The solution is \(\{(5,4)\}\).
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45. \( x + 3y = 3 \)
\( 4x + 12y = 12 \)
\[
\begin{bmatrix}
1 & 3 & 3 \\
4 & 12 & 12
\end{bmatrix}
\to
\begin{bmatrix}
1 & 3 & 3 \\
0 & 0 & 0
\end{bmatrix}
\]
Infinitely many solutions of the form \( \{(x, y) \mid x + 3y = 3\} \). The equations are dependent.

46. \( 2x + 5y = 1 \)
\( -4x - 10y = -2 \)
\[
\begin{bmatrix}
2 & 5 & 1 \\
-4 & -10 & -2
\end{bmatrix}
\to
\begin{bmatrix}
2 & 5 & 1 \\
0 & 0 & 0
\end{bmatrix}
\]
Infinitely many solutions of the form \( \{(x, y) \mid 2x + 5y = 1\} \). The equations are dependent.

47. \( x - y = 4 \)
\( 2x + y = 5 \)
\[
\begin{bmatrix}
1 & -1 & 4 \\
2 & 1 & 5
\end{bmatrix}
\to
\begin{bmatrix}
1 & -1 & 4 \\
0 & 3 & -3
\end{bmatrix}
\to
\begin{bmatrix}
1 & -1 & 4 \\
0 & 1 & -1
\end{bmatrix}
\to
\begin{bmatrix}
1 & 0 & 3 \\
0 & 1 & -1
\end{bmatrix}
\]
The solution is \( \{(3, -1)\} \).

48. \( 2x - y = 0 \)
\( x + y = 3 \)
\[
\begin{bmatrix}
2 & -1 & 0 \\
1 & 1 & 3
\end{bmatrix}
\to
\begin{bmatrix}
2 & -1 & 0 \\
1 & -1 & 0
\end{bmatrix}
\to
\begin{bmatrix}
1 & 1 & 3 \\
0 & -3 & -6
\end{bmatrix}
\to
\begin{bmatrix}
1 & 1 & 3 \\
0 & 1 & 2
\end{bmatrix}
\]
The solution is \( \{(1, 2)\} \).

49. \( x + 3y = -1 \)
\( -3x - 6y = 12 \)
\[
\begin{bmatrix}
1 & 3 & -1 \\
-3 & -6 & 12
\end{bmatrix}
\to
\begin{bmatrix}
1 & 3 & -1 \\
0 & 3 & 9
\end{bmatrix}
\to
\begin{bmatrix}
1 & 3 & -1 \\
0 & 1 & 3
\end{bmatrix}
\to
\begin{bmatrix}
1 & 0 & -10 \\
0 & 1 & 3
\end{bmatrix}
\]
The solution is \( \{(-10, 3)\} \).
Section 3.7 Solving Systems of Linear Equations by Using Matrices

50. \[ \begin{align*}
   x + y &= 4 \\
   2x - 4y &= -4
\end{align*} \]

\[
\begin{bmatrix}
   1 & 1 & 4 \\
   2 & -4 & -4
\end{bmatrix}
\xrightarrow{-2R_1 + R_2 \Rightarrow R_2}
\begin{bmatrix}
   1 & 1 & 4 \\
   0 & -6 & -12
\end{bmatrix}
\xrightarrow{-\frac{1}{6}R_2 \Rightarrow R_2}
\begin{bmatrix}
   1 & 1 & 4 \\
   0 & 1 & 2
\end{bmatrix}
\]

The solution is \( \{(2, 2)\} \).

51. \[ \begin{align*}
   3x + y &= -4 \\
   -6x - 2y &= 3
\end{align*} \]

\[
\begin{bmatrix}
   3 & 1 & -4 \\
   -6 & -2 & 3
\end{bmatrix}
\xrightarrow{2R_1 + R_2 \Rightarrow R_1}
\begin{bmatrix}
   3 & 1 & -4 \\
   0 & 0 & -5
\end{bmatrix}
\]

There is no solution; \( \{\} \). The system is inconsistent.

52. \[ \begin{align*}
   2x + y &= 4 \\
   6x + 3y &= -1
\end{align*} \]

\[
\begin{bmatrix}
   2 & 1 & 4 \\
   6 & 3 & -1
\end{bmatrix}
\xrightarrow{-3R_1 + R_2 \Rightarrow R_2}
\begin{bmatrix}
   2 & 1 & 4 \\
   0 & 0 & -13
\end{bmatrix}
\]

There is no solution; \( \{\} \). The system is inconsistent.

53. \[ \begin{align*}
   x + y + z &= 6 \\
   x - y + z &= 2 \\
   x + y - z &= 0
\end{align*} \]

\[
\begin{bmatrix}
   1 & 1 & 1 & 6 \\
   1 & -1 & 1 & 2 \\
   1 & 1 & -1 & 0
\end{bmatrix}
\xrightarrow{-R_1 + R_2 \Rightarrow R_2}
\begin{bmatrix}
   1 & 1 & 1 & 6 \\
   0 & -2 & 0 & -4 \\
   0 & 0 & 2 & 0
\end{bmatrix}
\xrightarrow{\frac{1}{2}R_2 \Rightarrow R_2}
\begin{bmatrix}
   1 & 1 & 1 & 6 \\
   0 & 1 & 0 & 2 \\
   0 & 0 & -2 & -6
\end{bmatrix}
\]

The solution is \( \{(1, 2, 3)\} \).

54. \[ \begin{align*}
   2x - 3y - 2z &= 11 \\
   x + 3y + 8z &= -1 \\
   3x - y + 14z &= -2
\end{align*} \]
The solution is \[ \{ (7, 2, -\frac{3}{2}) \} \].

55.  
\[ x - 2y = 5 - z \rightarrow x - 2y + z = 5 \]
\[ 2x + 6y + 3z = -10 \rightarrow 2x + 6y + 3z = -10 \]
\[ 3x - y - 2z = 5 \rightarrow 3x - y - 2z = 5 \]

The solution is \[ \{ (1, -2, 0) \} \].

56.  
\[ 5x = 10z + 15 \rightarrow 5x - 10z = 15 \]
\[ x - y + 6z = 23 \rightarrow x - y + 6z = 23 \]
\[ x + 3y - 12z = 13 \rightarrow x + 3y - 12z = 13 \]
Section 3.7 Solving Systems of Linear Equations by Using Matrices

\[
\begin{bmatrix}
5 & 0 & -10 & 15 \\
1 & -1 & 6 & 23 \\
1 & 3 & -12 & 13
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -1 & 6 & 23 \\
5 & 0 & -10 & 15 \\
1 & 3 & -12 & 13
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -1 & 6 & 23 \\
0 & 1 & -8 & -20 \\
0 & 4 & -18 & -10
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & -2 & 3 \\
0 & 1 & -8 & -20 \\
0 & 0 & 1 & 5
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & 0 & 13 \\
0 & 1 & 0 & 20 \\
0 & 0 & 1 & 5
\end{bmatrix}
\]

The solution is \( \{(13, 20, 5)\} \).

57. \texttt{rref([A])} \\
\[
\begin{bmatrix}
1 & 0 & -3 \\
0 & 1 & -1
\end{bmatrix}
\]

58. \texttt{rref([A])} \\
\[
\begin{bmatrix}
1 & 0 & -3 \\
0 & 1 & -2
\end{bmatrix}
\]

59. \texttt{rref([A])} \\
\[
\begin{bmatrix}
1 & 0 & -21 \\
0 & 1 & 9
\end{bmatrix}
\]

60. \texttt{rref([A])} \\
\[
\begin{bmatrix}
1 & 0 & 5 \\
0 & 1 & 4
\end{bmatrix}
\]

61. \texttt{rref([A])} \\
\[
\begin{bmatrix}
1 & 0 & 0 & 1 \\
0 & 1 & 0 & 2 \\
0 & 0 & 1 & 3
\end{bmatrix}
\]

62. \texttt{rref([A])} \\
\[
\begin{bmatrix}
1 & 0 & 0 & 7 \\
0 & 1 & 0 & 2 \\
0 & 0 & 1 & -1.5
\end{bmatrix}
\]

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Chapter 3  Systems of Linear Equations and Inequalities

Chapter 3  Group Activity

1. [Image of a graph showing height of a softball versus time]

2. Answers will vary.

3. Answers will vary.

4. Answers will vary based on the choices for the points \( (t_1, y_1) \), \( (t_2, y_2) \), and \( (t_3, y_3) \). However, \( a \), \( b \), and \( c \) should be close to \( a = -16 \), \( b = 32.5 \), and \( c = 5 \).

5. Answers will vary, but should be close to \( y = -16t^2 + 32.5t + 5 \). The equation represents the height of the ball \( t \) seconds after being thrown.

6. 
\[
y = -16t^2 + 32.5t + 5 \\
y = -16 \left( \frac{1}{2} \right)^2 + 32.5 \left( \frac{1}{2} \right) + 5 \\
= -4 + 16.25 + 5 \\
= 17.25
\]

Answers should be close to 17.25 ft.

7. 
\[
y = -16(1.4)^2 + 32.5(1.4) + 5 \\
= -31.36 + 45.5 + 5 \\
= 19.14 \\
= 19 \text{ ft}
\]

Answers will vary but should be close to the observed value of 19 ft.
Chapter 3  Review Exercises

Section 3.1

1. a. \(-5x - 7y = 4\)
   \[ y = -\frac{1}{2} x - 1 \]
   Substitute \((2,2)\):
   \[-5(2) - 7(2) = -10 - 14 = -24 \neq 4 \]
   \((2,2)\) is not a solution.

   b. Substitute \((2,-2)\):
   \[-5(2) - 7(-2) = -10 + 14 = 4 = 4 \]
   \[-2 = -\frac{1}{2} (2) - 1 = -1 - 1 = -2 \]
   \((2,-2)\) is a solution.

2. False

3. True

4. True

5. \(f(x) = x - 1\)
   \[ g(x) = 2x - 4 \]
   The solution is \(\{(3,2)\}\).

6. \(y = 2x + 7\)
   \[ y = -x - 5 \]
   The solution is \(\{(-4, -1)\}\).

7. \(6x + 2y = 4\)
   \[ 2y = -6x + 4 \]
   \[ y = -3x + 2 \]

8. \(y = \frac{1}{2} x - 2\)
   \[ -4x + 8y = -8 \]
   \[ 8y = 4x - 8 \]
   \[ y = \frac{1}{2} x - 1 \]
Infinitely many solutions of the form \( \{(x, y) | 6x + 2y = 4\} \); dependent equations.

**Section 3.2**

9. \[ y = \frac{3}{4}x - 4 \]
\[-x + 2y = -6 \]
\[-x + 2\left(\frac{3}{4}x - 4\right) = -6 \]
\[-x + \frac{3}{2}x - 8 = -6 \]
\[\frac{1}{2}x = 2 \]
\[x = 4 \]
\[y = \frac{3}{4}x - 4 = \frac{3}{4}(4) - 4 = 3 - 4 = -1 \]
The solution is \( \{(4, -1)\} \).

10. \[ 6x + y = 5 \]
\[5x + y = 3 \rightarrow y = -5x + 3 \]
\[6x + y = 5 \]
\[6x + (-5x + 3) = 5 \]
\[-x + 3 = 5 \]
\[x = 2 \]
\[y = -5x + 3 = -5(2) + 3 = -10 + 3 = -7 \]
The solution is \( \{(2, -7)\} \).

11. \[ 2(x + y) = 10 - 3y \rightarrow 2x + 2y = 10 - 3y \]
\[0.4x + y = 1.2 \rightarrow y = 1.2 - 0.4x \]
\[2x + 5(1.2 - 0.4x) = 10 \]
\[2x + 6 - 2x = 10 \]
\[6 \neq 10 \]
There is no solution; \( \{ \} \). The system is inconsistent.

12. \[ 3x = 11y - 9 \]
\[y = \frac{3}{11}x + \frac{6}{11} \]
\[3x = 11\left(\frac{3}{11}x + \frac{6}{11}\right) - 9 \]
\[3x = 3x + 6 - 9 \]
\[0 \neq -3 \]
There is no solution; \( \{ \} \). The system is inconsistent.

13. \[ 6(5x - y) = 90 \rightarrow 300x - 60y = 90 \]
\[10x = 2y + 3 \rightarrow 2y = 10x - 3 \]
\[\rightarrow y = 5x - \frac{3}{2} \]
\[300x - 60\left(5x - \frac{3}{2}\right) = 90 \]
\[300x - 300x + 90 = 90 \]
\[90 = 90 \]

14. \[ 4x + y = 7 \rightarrow y = -4x + 7 \]
\[x + \frac{1}{4}y = \frac{7}{4} \]
\[x + \frac{1}{4}(-4x + 7) = \frac{7}{4} \]
\[x - x + \frac{7}{4} = \frac{7}{4} \]
\[\frac{7}{4} = \frac{7}{4} \]
Infinitely many solutions of the form \( \{(x, y) \mid 10x - 2y = 3\} \); dependent equations.

15. \( y = 105 + 45x \)
\( y = 48.50x \)
\( 105 + 45x = 48.50x \)
\( 105 = 3.5x \)
\( 30 = x \)
The cost would be the same for 30 months.

16. \( y = 44 + 81.50x \)
\( y = 87x \)
\( 44 + 81.50x = 87x \)
\( 44 = 5.50x \)
\( 8 = x \)
The cost would be the same for 8 days.

Section 3.3

17. Multiply each equation by its LCD:
\( \frac{2}{5}x + \frac{3}{5}y = 1 \rightarrow x \cdot \frac{5}{2} \Rightarrow 2x + 3y = 5 \)
\( x - \frac{2}{3}y = \frac{1}{3} \rightarrow x \cdot 3 \Rightarrow 3x - 2y = 1 \)
Multiply the first equation by 2 and the second equation by 3, add the results and solve for \( x \):
\( 2x + 3y = 5 \rightarrow x^2 \Rightarrow 4x + 6y = 10 \)
\( 3x - 2y = 1 \rightarrow x^3 \Rightarrow 9x - 6y = 3 \)
\( \frac{13x}{13} = 13 \)
\( x = 1 \)
Substitute into the first equation and solve for \( y \):
\( 2(1) + 3y = 5 \)
\( 2 + 3y = 5 \)
\( 3y = 3 \)
\( y = 1 \)
The solution is \( \{(1, 1)\} \).

18. \( 4x + 3y = 5 \)
\( 3x - 4y = 10 \)
Multiply the first equation by 4 and the second equation by 3, add the results and solve for \( x \):
\( 4x + 3y = 5 \rightarrow x^4 \Rightarrow 16x + 12y = 20 \)
\( 3x - 4y = 10 \rightarrow x^3 \Rightarrow 9x - 12y = 30 \)
\( \frac{25x}{25} = 50 \)
\( x = 2 \)
Substitute into the first equation and solve for \( y \):
\( 4(2) + 3y = 5 \)
\( 8 + 3y = 5 \)
\( 3y = -3 \)
\( y = -1 \)
The solution is \( \{(2, -1)\} \).

19. \( 3x + 4y = 2 \)
\( 2x + 5y = -1 \)

20. Multiply each equation by its LCD:
Multiply the first equation by 5 and the second equation by –4, add the results and solve for \( x \):
\[
\begin{align*}
5x + 4y &= 2 \\
-4x + 5y &= -1
\end{align*}
\]
\[
\begin{align*}
15x + 20y &= 10 \\
-8x - 20y &= 4
\end{align*}
\]
\[
7x = 14 \quad \Rightarrow \quad x = 2
\]
Substitute into the first equation and solve for \( y \):
\[
3(2) + 4y = 2
\]
\[
6 + 4y = 2
\]
\[
4y = -4
\]
\[
y = -1
\]
The solution is \( \{(2, -1)\} \).

21. Write in standard form:
\[
2y = 3x - 8 \quad \Rightarrow \quad -3x + 2y = -8
\]
\[
6x = -4y + 4 \quad \Rightarrow \quad -6x + 4y = 4
\]
Multiply the first equation by \(-2\), add to the second equation and solve for \( x \):
\[
\begin{align*}
-3x + 2y &= -8 \\
-6x + 4y &= 4
\end{align*}
\]
\[
6x - 4y = 16
\]
\[
-6x + 4y = 4
\]
\[
0 \neq 20
\]
The system is inconsistent.

22. Write in standard form:
\[
3x + y = 16 \quad \Rightarrow \quad 3x + y = 16
\]
\[
3(x + y) = y + 2x + 2 \quad \Rightarrow \quad x + 2y = 2
\]
Multiply the first equation by \(-2\), add to the second equation and solve for \( x \):
\[
\begin{align*}
3x + y &= 16 \\
x + 2y &= 2
\end{align*}
\]
\[
-x = -30
\]
\[
x = 6
\]
Substitute into the first equation and solve for \( y \):
\[
3(6) + y = 16
\]
\[
18 + y = 16
\]
\[
y = -2
\]
The solution is \( \{(6, -2)\} \).

23. Write in standard form:
\[
-(y + 4x) = 2x - 9 \quad \Rightarrow \quad -6x - y = -9
\]
\[
-2x + 2y = -10 \quad \Rightarrow \quad -2x + 2y = -10
\]
Multiply the first equation by 2, add to the second equation and solve for \( x \):
\[
\begin{align*}
-6x - y &= -9 \\
-4x &= -18
\end{align*}
\]
\[
x = 4.5
\]
Substitute into the first equation and solve for \( y \):
\[
-6(4) - y = -9
\]
\[
-24 - y = -9
\]
\[
y = -15
\]
$-6x - y = -9 \quad \rightarrow \quad x^2 \quad -12x - 2y = -18$

$-2x + 2y = -10 \quad \rightarrow \quad -2x + 2y = -10$

$\frac{-14x}{-14} = -28$

$x = 2$

The solution is $\{(2, -3)\}$.

24. Write in standard form:

$-(4x - 35) = 3y \quad \rightarrow \quad -4x - 3y = -35$

$-(x - 15) = y \quad \rightarrow \quad -x - y = -15$

Multiply the second equation by $-3$, add to the first equation and solve for $x$:

$-4x - 3y = -35 \quad \rightarrow \quad -4x - 3y = -35$

$-x - y = -15 \quad \rightarrow \quad 3x + 3y = 45$

$-x = 10$

$x = -10$

Substitute into the second equation and solve for $y$:

$-(-10) - y = -15$

$10 - y = -15$

$-y = -25$

$y = 25$

The solution is $\{(-10, 25)\}$.

25. Multiply each equation by 10:

$-0.4x + 0.3y = 1.8 \rightarrow -4x + 3y = 18$

$0.6x - 0.2y = -1.2 \rightarrow 6x - 2y = -12$

Multiply the first equation by 2 and the second equation by 3, add the results and solve for $x$:

$-4x + 3y = 18 \quad \rightarrow \quad x^2 \quad -8x + 6y = 36$

$6x - 2y = -12 \quad \rightarrow \quad 3x \quad 18x - 6y = -36$

$\frac{10x}{10} = 0$

$x = 0$

Substitute into the first equation and solve for $y$:

$-4(0) + 3y = 18$

$0 + 3y = 18$

$y = 6$

The solution is $\{(0, 6)\}$.

26. Multiply each equation by 100:

$0.02x - 0.01y = -0.11 \rightarrow 2x - y = -11$

$0.01x + 0.04y = 0.26 \rightarrow x + 4y = 26$

Multiply the first equation by 4, add to the second equation and solve for $x$:

$2x - y = -11 \quad \rightarrow \quad x^4 \quad 8x - 4y = -44$

$x + 4y = 26 \quad \rightarrow \quad x^3 \quad x + 4y = 26$

$9x = -18$

$x = -2$

Substitute into the first equation and solve for $y$:

$2(-2) - y = -11$

$-4 - y = -11$

$-y = -7$

$y = 7$

The solution is $\{(-2, 7)\}$.

Section 3.4

27. Let $x =$ the amount invested at 5%

$y =$ the amount invested at 3.5%

Let $x =$ the number of student tickets

Let $y =$ the number of adult tickets

Let $lx =$ receipts from student tickets
Chapter 3  Systems of Linear Equations and Inequalities

<table>
<thead>
<tr>
<th>$5%$ Acct</th>
<th>$3.5%$ Acct</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Principal</td>
<td>$x$</td>
<td>$y$</td>
</tr>
<tr>
<td>Interest</td>
<td>$0.05x$</td>
<td>$0.035y$</td>
</tr>
</tbody>
</table>

$0.05x + 0.035y = 303.75$

Substitute and solve for $y$:

$0.05(2y) + 0.035y = 303.75$

$0.10y + 0.035y = 303.75$

$0.135y = 303.75$

$y = 2250$

Substitute into the first equation and solve for $x$:

$x = 2y = 2(2250) = 4500$

$4500$ was invested at $5\%$.

29. Let $x =$ the amount of $20\%$ saline solution

$y =$ the amount of $50\%$ saline solution

<table>
<thead>
<tr>
<th>20$%$ sal</th>
<th>50$%$ sal</th>
<th>31.25$%$ sal</th>
</tr>
</thead>
<tbody>
<tr>
<td>L solution</td>
<td>$x$</td>
<td>$y$</td>
</tr>
<tr>
<td>L saline</td>
<td>0.20$x$</td>
<td>0.50$y$</td>
</tr>
</tbody>
</table>

$x + y = 16$

$0.20x + 0.50y = 0.3125(16)$

Multiply the first equation by $-0.20$, add to the second equation and solve for $y$:

$x + y = 16 \rightarrow -0.20x - 0.20y = -3.2$

$0.20x + 0.50y = 5 \rightarrow 0.20x + 0.50y = 5.0$

$0.30y = 1.8$

$y = 6$

Substitute into the first equation and solve for $x$:

$x + 6 = 16$

$x = 10$

The mixture contains $10$ L of $20\%$ saline solution and $6$ L of $50\%$ saline solution.

30. Let $p =$ the speed of the plane in still air

Let $w =$ the speed of the wind

$p + w =$ speed of the plane with the wind

$p - w =$ speed of plane against the wind

<table>
<thead>
<tr>
<th>Distance</th>
<th>Rate</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tailwind</td>
<td>280</td>
<td>$p + w$</td>
</tr>
<tr>
<td>Headwind</td>
<td>280</td>
<td>$p - w$</td>
</tr>
</tbody>
</table>

$(rate)(time) = (distance)$

$(p + w)(1.75) = 280$

$(p - w)(2) = 280$

Divide the first equation by $1.75$, the second equation by $2$, add the results, and solve:

$(p + w)(1.75) = 280 \div 1.75 \rightarrow p + w = 160$

$(p - w)(2) = 280 \div 2 \rightarrow p - w = 140$

$2p = 300$

$p = 150$

Substitute and solve for $w$:

$150 + w = 160 \rightarrow w = 10$

The speed of the plane is $150$ mph in still air and the speed of the wind is $10$ mph.
31. \( f(x) = 915x + 275 \)

b. \( g(x) = 12.95 + 0.08x \)

c. Substitute and solve:
\[
\begin{align*}
915x + 275 &= 965x \\
50x &= 275 \\
x &= 5.5
\end{align*}
\]
5.5 months.

32. Let \( x = \) one angle
Let \( y = \) the other angle
\[
\begin{align*}
y &= 5x + 6 \\
x + y &= 90
\end{align*}
\]
Substitute and solve:
\[
\begin{align*}
x + 5x + 6 &= 90 \\
6x &= 84 \\
x &= 14
\end{align*}
\]
\[
y = 5(14) + 6 = 70 + 6 = 76
\]
The two angles measure 14º and 76º.

Section 3.5

33. \( 2x > -y + 5 \)
Graph the related equation \( 2x = -y + 5 \) by using a dashed line.
Test point above \((0,6)\): Test point below \((0,0)\):
\[
\begin{align*}
2(0) &> -(6) + 5 \\
0 &> -1
\end{align*}
\]
\( (0,6) \) is a solution. \( (0,0) \) is not a solution.
Shade the region above the boundary line.

34. \( 2x \leq -8 - 3y \)
Graph the related equation \( 2x = -8 - 3y \) by using a solid line.
Test point above \((0,0)\): Test point below \((0,-3)\):
\[
\begin{align*}
2(0) &\leq -8 - 3(0) \\
0 &\leq -8
\end{align*}
\]
\( (0,0) \) is not a solution. \( (0,-3) \) is a solution.
Shade the region below the boundary line.
35. \( x > -3 \) represents all the points to the right of the vertical line \( x = -3 \). The boundary is a dashed line.
Shade the region to the right of the boundary line.

![Graph of x > -3](image)

36. \( x \leq 2 \) represents all the points to the left of the vertical line \( x = 2 \). The boundary is a solid line.
Shade the region to the left of the boundary line.

![Graph of x <= 2](image)

37. \( x \geq \frac{1}{2} y \)

Graph the related equation \( x = \frac{1}{2} y \) by using a solid line.
Test point above \((0,2)\):
\[
0 \geq \frac{1}{2} (2)
\]
\[
0 \geq 1\]
\((0,2)\) is not a solution.

Test point below \((0,-2)\):
\[
0 \geq \frac{1}{2} (-2)
\]
\[
0 \geq -1\]
\((0,-2)\) is a solution.
Shade the region below the boundary line.

38. \( x < \frac{2}{5} y \)

Graph the related equation \( x = \frac{2}{5} y \) by using a dashed line.

Test point above \((0,5)\): \(0 < \frac{2}{5}(5)\)

Test point below \(0,-5)\): \(0 < \frac{2}{5}(-5)\)

\((0,5)\) is a solution. \((0,-5)\) is not a solution.

Shade the region above the boundary line.

39. \(2x - y > -2 \) and \(2x - y \leq 2\)

Graph the related equation \(2x - y = -2\) by using a dashed line.

Test point above \((0,3)\): \(2(0)-3 > -2\)

Test point below \((0,0)\): \(2(0)-0 > -2\)

\((0,3)\) is not a solution. \((0,0)\) is a solution.

Shade the region below the boundary line.

Graph the related equation \(2x - y = 2\) by using a solid line.
Chapter 3  Systems of Linear Equations and Inequalities

Test point above \((0,0)\):  Test point below \((0,-3)\):
\[
2(0) - 0 \leq 2 \quad 2(0) - (-3) \leq 2 \\
0 \leq 2 \quad 3 \leq -2
\]
\((0,0)\) is a solution.  \((0,-3)\) is not a solution.

Shade the region above the boundary line.
The solution is the intersection of the graphs.

40.  \(3x + y \geq 6\) or \(3x + y < -6\)
Graph the related equation \(3x + y = 6\) by using a solid line.
Test point above \((0,7)\):  Test point below \((0,0)\):
\[
3(0) + 7 \geq 6 \quad 3(0) + 0 \geq 6 \\
7 \geq 6 \quad 0 \geq 6
\]
\((0,7)\) is a solution.  \((0,0)\) is not a solution.
Shade the region above the boundary line.
Graph the related equation \(3x + y = -6\) by using a dashed line.
Test point above \((0,0)\):  Test point below \((0,-7)\):
\[
3(0) + 0 < -6 \quad 3(0) + (-7) < -6 \\
0 < -6 \quad -7 < -6
\]
\((0,0)\) is not a solution.  \((0,-7)\) is a solution.
Shade the region below the boundary line.
The solution is the union of the graphs.
41. \( y \geq x \) or \( y \leq -x \)

Graph the related equation \( y = x \) by using a solid line.

Test point above \((0,5)\): \( 5 \geq 0 \)

\((0,5)\) is a solution.

Test point below \((0,-5)\): \( -5 \geq 0 \)

\((0,-5)\) is not a solution.

Shade the region above the boundary line.

Graph the related equation \( y = -x \) by using a solid line.

Test point above \((1,5)\): \( 5 \leq -1 \)

\((1,5)\) is not a solution.

Test point below \((1,-5)\): \( -5 \leq -1 \)

\((1,-5)\) is a solution.

Shade the region below the boundary line.

The solution is the union of the graphs.

![Graph](image)

42. \( x \geq 0 \) and \( y \geq 0 \) and \( y \leq -\frac{2}{3}x + 4 \)

\( x \geq 0 \) represents the points to the right of the vertical line \( x = 0 \).

Shade the region to the right of the boundary line using a solid line border.

\( y \geq 0 \) represents the points above the horizontal line \( y = 0 \).

Shade the region above the boundary line using a solid line border.

Graph the related equation \( y = -\frac{2}{3}x + 4 \) by using a solid line.

Test point above \((0,5)\): \( 5 \leq -\frac{2}{3}(0) + 4 \)

\((0,5)\) is not a solution.

Test point below \((0,0)\): \( 0 \leq -\frac{2}{3}(0) + 4 \)

\((0,0)\) is a solution.

Shade the region below the boundary line.

The solution is the intersection of the graphs.
43. a. \( x \geq 0, \ y \geq 0 \)

b. \( x + y \leq 100 \)

c. \( x \geq 4y \)

d. \( x \geq 0 \) and \( y \geq 0 \) and \( x + y \leq 100 \) and \( x \geq 4y \)

\( x \geq 0 \) represents the points to the right of the vertical line \( x = 0 \). Shade the region to the right of the boundary line using a solid line border.

\( y \geq 0 \) represents the points above the horizontal line \( y = 0 \). Shade the region above the boundary line using a solid line border.

Graph the related equation \( x + y = 100 \) by using a solid line.

Test point above \((0,101)\):

\[
0 + 101 \leq 100
\]

\[
101 \leq 100
\]

\((0,101)\) is not a solution.

Test point below \((0,0)\):

\[
0 + 0 \leq 100
\]

\[
0 \leq 100
\]

\((0,0)\) is a solution.

Shade the region below the boundary line.

Graph the related equation \( x = 4y \) by using a solid line.

Test point above \((0,1)\):

\[
0 \geq 4(1)
\]

\[
0 \geq 4
\]

\((0,1)\) is not a solution.

Test point below \((0,-1)\):

\[
0 \geq 4(-1)
\]

\[
0 \geq -4
\]

\((0,-1)\) is a solution.

Shade the region below the boundary line.

The solution is the intersection of the graphs.
Section 3.6

44. \[5x + 5y + 5z = 30\]
    \[-x + y + z = 2\]
    \[10x + 6y - 2z = 4\]

Multiply the second equation by \(-5\) and add to the first equation to eliminate \(z\):

\[
5x + 5y + 5z = 30 \quad \rightarrow \quad 5x + 5y + 5z = 30
\]
\[
-x + y + z = 2 \quad \rightarrow \quad 5x - 5y - 5z = -10
\]
\[
10x = 20
\]
\[
x = 2
\]

Multiply the second equation by 2 and add to the third equation to eliminate \(z\):

\[
-2x + 2y + 2z = 4
\]
\[
10x + 6y - 2z = 4 \quad \rightarrow \quad 10x + 6y - 2z = 4
\]
\[
8x + 8y = 8
\]

Substitute and solve for \(y\) and \(z\):

\[
8x + 8y = 8
\]
\[
8(2) + 8y = 8
\]
\[
16 + 8y = 8
\]
\[
-2 + (-1) + z = 2
\]
\[
y = -8
\]
\[
-2 + (-1) + z = 2
\]
\[
y = -8
\]
\[
-3 + z = 2
\]
\[
y = -8
\]
\[
z = 5
\]

The solution is \(\{(2, -1, 5)\}\).

45. \[5x + 3y - z = 5\]
    \[x + 2y + z = 6\]
    \[-x - 2y - z = 8\]

Add the second and third equations to eliminate \(z\):

\[
x + 2y + z = 6
\]
\[
-x - 2y - z = 8
\]
\[
0 \neq 14
\]

The system is inconsistent. There is no solution.

46. \[x + y + z = 4\]
    \[-x - 2y - 3z = -6\]
    \[2x + 4y + 6z = 12\]

Multiply the second equation by 2 and add to the third equations to eliminate \(z\):

\[
-2x - 4y - 6z = -12
\]
\[
2x + 4y + 6z = 12 \quad \rightarrow \quad 2x + 4y + 6z = 12
\]
\[
0 = 0
\]

The equations are dependent.
47.  \[
\begin{align*}
3x + 4z &= 5 \\
2y + 3z &= 2 \\
2x - 5y &= 8
\end{align*}
\]
Multiply the first equation by 3 and the second equation by –4, and add the results to eliminate \(z\):
\[
\begin{align*}
3x + 4z &= 5 \\ 2y + 3z &= 2 \\
\rightarrow \quad 9x + 12z &= 15 \\
-8y - 12z &= -8 \\ 9x - 8y &= 7
\end{align*}
\]
Multiply the third equation by –9 and this result by 2, and add to eliminate \(x\):
\[
\begin{align*}
2x - 5y &= 8 \\ 9x - 8y &= 7 \\
\rightarrow \quad 18x + 45y &= -72 \\
18x - 16y &= 14 \\
29y &= -58 \\
y &= -2
\end{align*}
\]
Substitute and solve for \(x\) and \(z\):
\[
\begin{align*}
2x - 5y &= 8 \\
2x - 5(-2) &= 8 \\
2x + 10 &= 8 \\
2x &= -2 \\
x &= -1
\end{align*}
\]
\[
\begin{align*}
2y + 3z &= 2 \\
2(-2) + 3z &= 2 \\
-4 + 3z &= 2 \\
3z &= 6 \\
z &= 2
\end{align*}
\]
The solution is \([-1, -2, 2]\).

48.  Let \(x\) = the shortest leg
Let \(y\) = the longer leg
Let \(z\) = the hypotenuse
\[
\begin{align*}
x + y + z &= 30 \\
y &= 2x + 2 \\
z &= 3x - 2
\end{align*}
\]
Substitute the second and third equations into the first equation and solve:
\[
\begin{align*}
x + (2x + 2) + (3x - 2) &= 30 \\
6x &= 30 \\
x &= 5
\end{align*}
\]
\[
\begin{align*}
y &= 2x + 2 \\
y &= 2(5) + 2 \\
y &= 10 + 2 \\
y &= 12 \\
z &= 3x - 2 \\
z &= 3(5) - 2 \\
z &= 15 - 2 \\
z &= 13
\end{align*}
\]
The lengths of the sides are 5 ft, 12 ft, and 13 ft.

49.  Let \(x\) = the rate of the slowest pump
Let \(y\) = the rate of the middle rate pump
Let \(z\) = the rate of the fastest pump
\[
\begin{align*}
x + y + z &= 950 \\
x &= z - 150 \\
z &= x + y - 150
\end{align*}
\]
Add the first and third equations to eliminate \(x\):
\[
\begin{align*}
x + y + z &= 950 \\
-x - y + z &= -150 \\
2z &= 800 \\
z &= 400
\end{align*}
\]
50.  Let \(x\) = the first angle
Let \(y\) = the second angle
Let \(z\) = the third angle
\[
\begin{align*}
x + y + z &= 180 \\
x &= y - 9 \\
z &= 3x + 26
\end{align*}
\]
Substitute the second and third equations into the first and solve for \(x\):
\[
\begin{align*}
x + (y - 9) + (3x + 26) &= 180 \\
5x + 17 &= 180 \\
5x &= 163 \\
x &= 32.6
\end{align*}
\]
Substitute and solve for \(y\) and \(z\):
Substitute and solve for $x$ and $y$:

\[
\begin{align*}
  x &= z - 150 & x + y + z &= 950 \\
  x &= 400 - 150 & 250 + y + 400 &= 950 \\
  x &= 250 & y + 650 &= 950 \\
  \quad & & y &= 300
\end{align*}
\]

The pumps can drain 250, 300, and 400 gal/hr.

Section 3.7

51. $3 \times 3$

52. $3 \times 2$

53. $1 \times 4$

54. $3 \times 1$

55. \[
\begin{bmatrix}
  1 & 1 & 3 \\
  1 & -1 & -1
\end{bmatrix}
\]

56. \[
\begin{bmatrix}
  1 & -1 & 1 & 4 \\
  2 & -1 & 3 & 8 \\
 -2 & 2 & -1 & -9
\end{bmatrix}
\]

57. $x = 9$

58. $x = -5$

59. a. 4

60. a. \[
\begin{bmatrix}
  1 & 2 & 0 & -3 \\
  4 & -1 & 1 & 0 \\
 -3 & 2 & 2 & 5
\end{bmatrix}
\]

\[
\frac{4R_1 + R_2 \Rightarrow R_1}{R_1 \rightarrow R_1} \Rightarrow \begin{bmatrix}
  1 & 2 & 0 & -3 \\
  0 & -9 & 1 & 12 \\
 -3 & 2 & 2 & 5
\end{bmatrix}
\]

b. \[
\begin{bmatrix}
  1 & 3 & 1 \\
  4 & -1 & 6
\end{bmatrix}
\]

\[
\frac{4R_1 + R_2 \Rightarrow R_1}{R_1 \rightarrow R_1} \Rightarrow \begin{bmatrix}
  1 & 3 & 1 \\
  0 & -13 & 2
\end{bmatrix}
\]

b. \[
\begin{bmatrix}
  1 & 2 & 0 & -3 \\
  0 & -9 & 1 & 12 \\
  0 & 8 & 2 & -4
\end{bmatrix}
\]

The angles are $29^\circ$, $38^\circ$, and $113^\circ$. 
Chapter 3  Systems of Linear Equations and Inequalities

61. \[ x + y = 3 \]
\[ x - y = -1 \]

\[
\begin{bmatrix}
1 & 1 & 3 \\
1 & -1 & -1
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 1 & 3 \\
0 & -2 & -4
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 1 & 3 \\
0 & 1 & 2
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & 1 \\
0 & 1 & 2
\end{bmatrix}
\]

The solution is \( \{(1,2)\} \).

62. \[ 4x + 3y = 6 \]
\[ 12x + 5y = -6 \]

\[
\begin{bmatrix}
4 & 3 & 6 \\
12 & 5 & 6
\end{bmatrix}
\rightarrow
\begin{bmatrix}
0 & -4 & -24 \\
0 & 1 & 6
\end{bmatrix}
\rightarrow
\begin{bmatrix}
0 & 1 & 6 \\
0 & 1 & 6
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & -3 \\
0 & 1 & 6
\end{bmatrix}
\]

The solution is \( \{(-3,6)\} \).

63. \[ x - y + z = -4 \]
\[ 2x + y - 2z = 9 \]
\[ x + 2y + z = 5 \]

\[
\begin{bmatrix}
1 & -1 & 1 & -4 \\
2 & 1 & -2 & 9 \\
1 & 2 & 1 & 5
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -1 & 1 & -4 \\
0 & 3 & 0 & 17 \\
0 & 3 & 0 & 19
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -1 & 1 & -4 \\
0 & 3 & 0 & 9 \\
0 & 3 & 0 & 17
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -1 & 1 & -4 \\
0 & 1 & 0 & -3 \\
0 & 3 & -4 & 17
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & 1 & -1 \\
0 & 1 & 0 & -3 \\
0 & 0 & 1 & -2
\end{bmatrix}
\]

The solution is \( \{(1,3,-2)\} \).

64. \[ x - y + z = 4 \]
\[ 2x - y + 3z = 8 \]
\[ -2x + 2y - z = -9 \]
Chapter 3 Test

1. \(4x - 3y = -5 \quad \) 
   \(12x + 2y = 7\)

Substitute \((\frac{1}{4}, 2)\):

\[4\left(\frac{1}{4}\right) - 3(2) = 1 - 6 = -5 = -5\]

\[12\left(\frac{1}{4}\right) + 2(2) = 3 + 4 = 7 = 7\]

\((\frac{1}{4}, 2)\) is a solution.

2. b. The system is consistent and independent. There is one solution.

3. c. The system is inconsistent and independent. There are no solutions.

4. a. The system is consistent and dependent. There are infinitely many solutions.

5. \(4x - 2y = -4\)
   \(3x + y = 7\)

The solution is \(\{(1,4)\}\).

6. \(f(x) = x + 3\)
   \(g(x) = -\frac{3}{2}x - 2\)

The solution is \(\{(-2,1)\}\).

7. \(3x + 5y = 13\)
   \(y = x + 9\)

8. \(6x + 8y = 5\)
   \(3x - 2y = 1\)

Multiply the second equation by 4, add to the first equation and solve for \(x\):
Chapter 3  Systems of Linear Equations and Inequalities

9. Write in standard form:
   \[7y = 5x - 21 \rightarrow -5x + 7y = -21\]
   \[9y + 2x = -27 \rightarrow 2x + 9y = -27\]
   Multiply the first equation by 2 and the second equation by 5, add the results and solve for \(y\):
   \[-5x + 7y = -21 \rightarrow 10x + 14y = -42\]
   \[2x + 9y = -27 \rightarrow 10x + 45y = -135\]
   \[59y = -177\]
   \[y = -3\]
   Substitute into the first equation and solve for \(x\):
   \[2x + 9(-3) = -27\]
   \[2x - 27 = -27\]
   \[2x = 0\]
   \[x = 0\]
   The solution is \(\{(0, -3)\}\).

10. Multiply each equation by the LCD:

11. Multiply each equation by the LCD:

12. Substitute and solve for \(x\):
\[
\frac{1}{5}x = \frac{1}{2}y + \frac{17}{5} \rightarrow 2x = 5y + 34
\]
\[
\rightarrow 2x - 5y = 34
\]
\[
\frac{1}{4}(x + 2) = -\frac{1}{6}y \rightarrow 3x + 6 = -2y
\]
\[
\rightarrow 3x + 2y = -6
\]
Multiply the first equation by 2 and the second equation by 5, add the results and solve for \(x\):
\[
\begin{align*}
2x - 5y &= 34 \quad \rightarrow x^2 \quad 4x - 10y = 68 \\
3x + 2y &= -6 \quad \rightarrow x^5 \quad 15x + 10y = -30
\end{align*}
\]
\[
\begin{align*}
19x &= 38 \\
x &= 2
\end{align*}
\]
Substitute into the first equation and solve for \(y\):
\[
2(2) - 5y = 34
\]
\[
4 - 5y = 34
\]
\[
-5y = 30
\]
\[
y = -6
\]
The solution is \(\{(2, -6)\}\).

13. Multiply each equation by the LCD:
\[
-0.03y + 0.06x = 0.3 \rightarrow -3y + 6x = 30
\]
\[
\rightarrow 6x - 3y = 30
\]
\[
0.4x - 2 = -0.5y \rightarrow 4x - 20 = -5y
\]
\[
\rightarrow 4x + 5y = 20
\]
Multiply the first equation by 5 and the second equation by 3, add the results and solve for \(x\):
\[
\begin{align*}
6x - 3y &= 30 \quad \rightarrow x^5 \quad 30x - 15y = 150 \\
4x + 5y &= 20 \quad \rightarrow x^3 \quad 12x + 15y = 60
\end{align*}
\]
\[
\begin{align*}
42x &= 210 \\
x &= 5
\end{align*}
\]
Substitute into the first equation and solve for \(y\):
\[
6(5) - 3y = 30
\]
\[
30 - 3y = 30
\]
\[
-3y = 0
\]
\[
y = 0
\]
The solution is \(\{(5, 0)\}\).
Chapter 3  Systems of Linear Equations and Inequalities

14.  \(2x - 5y \geq 10\)

Graph the related equation \(2x - 5y = 10\) by using a solid line.

Test point above \((0, 0)\):  
\[2(0) - 5(0) \geq 10\]
\[0 \geq 10\]

\((0, 0)\) is not a solution.

Test point below \((0, -3)\):
\[2(0) - 5(-3) \geq 10\]
\[15 \geq 10\]

\((0, -3)\) is a solution.

Shade the region below the boundary line.

15.  \(x + y < 3\) and \(3x - 2y > -6\)

Graph the related equation \(x + y = 3\) by using a dashed line.

Test point above \((0, 4)\):
\[0 + 4 < 3\]
\[4 < 3\]

\((0, 3)\) is not a solution.

Test point below \((0, 0)\):
\[0 + 0 < 3\]
\[0 < 3\]

\((0, 0)\) is a solution.

Shade the region below the boundary line.

Graph the related equation \(3x - 2y = -6\) by using a dashed line.

Test point above \((0, 4)\):
\[3(0) - 2(4) > -6\]
\[-8 > -6\]

\((0, 4)\) is not a solution.

Test point below \((0, 0)\):
\[3(0) - 2(0) > -6\]
\[0 > -6\]

\((0, 0)\) is a solution.

Shade the region below the boundary line.

The solution is the intersection of the graphs.
16. \( 5x \leq 5 \) or \( x + y \leq 0 \)

Graph the related equation \( 5x = 5 \) or \( x = 1 \) by using a solid line.
Shade the region to the left of the boundary line.

Graph the related equation \( x + y = 0 \) by using a solid line.

Test point above \((0,1)\): \( 0 + 1 \leq 0 \)
\( 1 \leq 0 \)
\((0,1)\) is not a solution.

Test point below \((0,-1)\): \( 0 + (-1) \leq 0 \)
\( -1 \leq 0 \)
\((0,-1)\) is a solution.

Shade the region below the boundary line.
The solution is the union of the graphs.

17. a. \( x \geq 0, \ y \geq 0 \)

b. \( 300x + 400y \geq 1200 \)
\( x \geq 0 \) and \( y \geq 0 \) and \( 300x + 400y \geq 1200 \)
\( x \geq 0 \) represents the points to the right of the vertical line \( x = 0 \). Shade the region to the right of the boundary line using a solid line border.

c. \( y \geq 0 \) represents the points above the horizontal line \( y = 0 \). Shade the region above the boundary line using a solid line border.

Graph the related equation \( 300x + 400y = 1200 \) by using a solid line.

Test point above \((0,4)\): \( 300(0) + 400(4) \geq 1200 \)
\( 1600 \geq 1200 \)
\((0,4)\) is a solution.

Test point below \((0,0)\): \( 300(0) + 400(0) \geq 1200 \)
\( 0 \geq 1200 \)
\((0,0)\) is not a solution.

Shade the region above the boundary line.
The solution is the intersection of the graphs.
Chapter 3  Systems of Linear Equations and Inequalities

18.  \[2x + 2y + 4z = -6\]  
    \[3x + y + 2z = 29\]  
    \[x - y - z = 44\]  
Multiply the second equation by \(-2\) and add to the first equation to eliminate \(z\):  
\[2x + 2y + 4z = -6\]  
\[3x + y + 2z = 29\]  
\[\rightarrow -6x - 2y - 4z = -58\]  
\[\rightarrow -4x = -64\]  
\[x = 16\]  
Add the second and third equations to eliminate \(y\):  
\[2x + 2y + 4z = -6\]  
\[3x + y + 2z = 29\]  
\[\rightarrow -x + 2y + 2z = 8\]  
Substitute and solve for \(x\) and \(z\):  
\[\begin{align*} 3x + y + 2z &= 29 \\ x - y - z &= 44 \\ 4x + z &= 73 \end{align*}\]  
\[\begin{align*} 4(16) + z &= 73 \\ 64 + z &= 73 \\ z &= 9 \end{align*}\]  
\[3(16) + y + 2(9) = 29\]  
\[48 + y + 18 = 29\]  
\[y = -37\]  
The solution is \(\{(16, -37, 9)\}\).

19.  Write each equation in standard form:  
\[2(x + z) = 6 + x - 3y \rightarrow x + 3y + 2z = 6\]  
\[2x = 11 + y - z \rightarrow 2x - y + z = 11\]  
\[x + 2(y + z) = 8 \rightarrow x + 2y + 2z = 8\]  
Multiply the third equation by \(-1\) and add to the first equation to eliminate \(x\):  
\[x + 3y + 2z = 6 \rightarrow x + 3y + 2z = 6\]  
\[x + 2y + 2z = 8 \rightarrow -x - 2y - 2z = -8\]  
\[y = -2\]  
Multiply the second equation by \(-2\) and add to the first equation to eliminate \(z\):  
\[2x - y + z = 11\]  
\[2(2) - (-2) + z = 11\]  
\[4 + 2 + z = 11\]  
\[z = 5\]  
The solution is \(\{(2, -2, 5)\}\).
20. Let $x =$ the amount borrowed at 6.5% 
$y =$ the amount borrowed at 5% 
$x + y = 5000 \rightarrow y = 5000 - x$ 
$0.065x + 0.05y = 268$ 
Substitute and solve for $x$. 
$0.065x + 0.05(5000 - x) = 268$ 
$0.065x + 250 - 0.05x = 268$ 
$0.015x = 18$ 
$x = 1200$ 
Substitute into the first equation and solve for $y$. 
$1200 + y = 5000$ 
$y = 3800$ 
She borrowed $1200 at 6.5\% and $3800 at 5\%.

21. Let $x =$ the amount of 20\% acid solution 
Let $y =$ the amount of 60\% acid solution 

<table>
<thead>
<tr>
<th>20% acid</th>
<th>60% acid</th>
<th>44% acid</th>
</tr>
</thead>
<tbody>
<tr>
<td>L solution</td>
<td>$x$</td>
<td>$y$</td>
</tr>
<tr>
<td>L acid</td>
<td>0.20$x$</td>
<td>0.60$y$</td>
</tr>
</tbody>
</table>

$x + y = 200$ 
$0.20x + 0.60y = 0.44(200)$ 
Multiply the first equation by $-0.20$, add to the second equation and solve for $y$: 
$x + y = 200 \rightarrow -0.20x - 0.20y = -40$ 
$0.20x + 0.60y = 88 \rightarrow 0.20x + 0.60y = 88$ 
$0.40y = 48$ 
$y = 120$ 
Substitute into the first equation and solve for $x$: 
$x + 120 = 200$ 
$x = 80$ 
The mixture contains 80 L of 20\% acid solution and 120 L of 60\% acid solution.

22. Let $x =$ one angle 
Let $y =$ the other angle 

$2y = x - 60$ 
$x + y = 90 \rightarrow y = 90 - x$ 
Substitute and solve: 
$2(90 - x) = x - 60$ 
$180 - 2x = x - 60$ 
$-3x = -240$ 
$x = 80$ 
$y = 90 - 80 = 10$ 
The two angles measure 80\° and 10\°.
23. Let \( x \) = number of orders Joanne can process
   Let \( y \) = number of orders Kent can process
   Let \( z \) = number of orders Geoff can process

\[
\begin{align*}
x + y + z &= 504 \\
y &= x + 20 \\
z &= x + y - 104 \to -x - y + z = -104
\end{align*}
\]

Add the first and third equations to eliminate \( x \):

\[
\begin{align*}
x + y + z &= 504 \\
-2y &= 524
\end{align*}
\]

Substitute and solve for \( x \) and \( y \):

\[
\begin{align*}
2y + z &= 524 \\
-x + y &= 20 \\
x &= 162 \\
y &= -142 \\
z &= 200
\end{align*}
\]

Joanne processes 142 orders, Kent processes 162 orders, and Geoff processes 200 orders.

24. For example:

\[
\begin{bmatrix}
2 & 1 \\
0 & -4 \\
2.6 & 7
\end{bmatrix}
\]

25. a. \[
\begin{bmatrix}
1 & 2 & 1 \\
4 & 0 & 1 \\
-5 & -6 & 3
\end{bmatrix}
\]

\[
\begin{align*}
-4R_1 + R_2 & \Rightarrow R_2 \\
1 & 2 & 1 & -3 \\
0 & -8 & -3 & 10 \\
-5 & -6 & 3 & 0
\end{align*}
\]

b. \[
\begin{align*}
5R_1 + R_1 & \Rightarrow R_1 \\
1 & 2 & 1 & -3 \\
0 & -4 & 8 & -15
\end{align*}
\]

26. \[
\begin{align*}
5x - 4y &= 34 \\
x - 2y &= 8
\end{align*}
\]

\[
\begin{bmatrix}
5 & -4 & 34 \\
1 & -2 & 8
\end{bmatrix}
\]

\[
\begin{align*}
R_1 & \Leftrightarrow R_2 \\
\Rightarrow R_2 & \Leftrightarrow R_1 \\
\Rightarrow R_2 & \Leftrightarrow R_1 \\
\Rightarrow R_2 & \Leftrightarrow R_1
\end{align*}
\]

\[
\begin{bmatrix}
1 & -2 & 8 \\
0 & 6 & -6
\end{bmatrix}
\]

\[
\begin{align*}
\frac{1}{6}R_2 & \Rightarrow R_2 \\
1 & -2 & 8 \\
0 & 1 & -1
\end{align*}
\]
The solution is $\{(6,-1)\}$.

27. $x + y + z = 1$
$2x + y = 0$
$-2y - z = 5$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & 0 & 0 \\ 0 & -2 & -1 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & -2 & -2 \\ 0 & -2 & -1 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & -2 & -2 \\ 0 & -2 & -1 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -1 & -1 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 3 & 9 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

The solution is $\{(2,-4,3)\}$.

Chapters 1 – 3 Cumulative Review Exercises

1. $\begin{bmatrix} 3^2 - 8(7-5) \\ 3^2 - 8(2) \\ 9 - 8(2) \end{bmatrix} = \begin{bmatrix} 9 - 16 \\ -14 \end{bmatrix}$

2. $7 \left[ 4 - 2(w - 5) - 3(2w + 1) \right] + 20$
$= 7 \left[ 4 - 2w + 10 - 6w - 3 \right] + 20$
$= 7 \left[ -8w + 11 \right] + 20$
$= -56w + 77 + 20 = -56w + 97$

3. $-5(2x - 1) - 2(3x + 1) = 7 - 2(8x + 1)$
$-10x + 5 - 6x - 2 = 7 - 16x - 2$
$-16x + 3 = -16x + 5$
$3 \neq 5$

There is no solution: $\{\}$. 

4. $\frac{1}{2}(a - 2) - \frac{3}{4}(2a + 1) = \frac{-1}{6}$
$12 \left[ \frac{1}{2}(a - 2) - \frac{3}{4}(2a + 1) \right] = 12 \left[ \frac{-1}{6} \right]$
Chapter 3  Systems of Linear Equations and Inequalities

5. \(-4 = \left| 2x - 3 \right| - 9\)
   \[5 = \left| 2x - 3 \right|\]
   \(2x - 3 = 5\) or \(2x - 3 = -5\)
   \(2x = 8\) or \(2x = -2\)
   \(x = 4\) or \(x = -1\) \(\{4,-1\}\)

6. \(-3y - 2(y+1) < 8\)
   \(-3y - 2y - 2 < 8\)
   \(-5y - 2 < 8\)
   \(-5y < 10\)
   \(y > -2\) \((-2,\infty)\)

7. \(4x > 16\) or \(-6x - 3 \geq 9\)
   \(x > 4\) or \(-6x \geq 12\)
   \(x > 4\) or \(x \leq -2\) \((-\infty,-2]\cup(4,\infty)\)

8. \(4x > 16\) and \(-6x - 3 \geq 9\)
   \(x > 4\) and \(-6x \geq 12\)
   \(x > 4\) and \(x \leq -2\) \(\{\}\)

9. \(0 \leq \frac{3x - 9}{6} \leq 5\)
   \(0 \leq 3x - 9 \leq 30\)
   \(9 \leq 3x \leq 39\)
   \(3 \leq x \leq 13\) \([3,13]\)

10. \(\left| x - 4 \right| + 1 < 11\)
    \(\left| x - 4 \right| < 10\)
    \(-10 < x - 4 < 10\)
    \(-6 < x < 14\) \((-6,14)\)

11. \(4 < \left| 2x + 4 \right|\)
    \(2x + 4 > 4\) or \(2x + 4 < -4\)
    \(2x > 0\) or \(2x < -8\)
    \(x > 0\) or \(x < -4\)
    \((-\infty,-4)\cup(0,\infty)\)

12. \(x - 5y \leq 5\)
    Graph the related equation \(x - 5y = 5\) by using a solid line.
    Test point above \((0,0)\): \(0 - 5(0) \leq 5\)
    Test point below \((0, -3)\): \(0 - 5(-3) \leq 5\)
    \(0 \leq 5\)
    \(15 \leq 5\)
    \((0,0)\) is a solution. \((0,-3)\) is not a solution.
    Shade the region above the boundary line.
13. \(5x - 2y = 15\)
   \[-2y = -5x + 15\]
   \[y = \frac{-5}{2}x - \frac{15}{2}\]

   \[\text{Slope: } \frac{5}{2} \quad \text{y-intercept: } \left(0, -\frac{15}{2}\right)\]

   \[5x - 2(0) = 15\]
   \[5x = 15\]
   \[x = 3 \quad x\text{-intercept: } (3, 0)\]

14. \(y = \frac{-1}{3}x - 4\)

15. \(x = -2\)

16. \[m = \frac{-10 - (-10)}{6 - 4}\]
   \[= \frac{-10 + 10}{2}\]
   \[= \frac{0}{2} = 0\]

17. \[m = \frac{-4 - (-8)}{2 - 3}\]
   \[= \frac{4}{-1}\]
   \[= -4\]

   \[y - (-8) = -4(x - 3)\]
   \[y + 8 = -4x + 12\]
   \[y = -4x + 4\]

18. Multiply the second equation by the LCD: \(2x - 3y = 6\)
   \[\frac{1}{2}x - \frac{3}{4}y = 1 \rightarrow 2x - 3y = 4\]

   Multiply the second equation by \(-1\), add to the first equation and solve for \(x\):
   \[2x - 3y = 6 \quad \rightarrow \quad 2x - 3y = 6\]
   \[2x - 3y = 4 \quad \rightarrow \quad -2x + 3y = -4\]
   \[0 \neq 2\]

   There is no solution; \{\}. The system is inconsistent.
19. \[ 2x + y = 4 \]
    \[ y = 3x - 1 \]
    \[ 2x + (3x - 1) = 4 \]
    \[ 5x - 1 = 4 \]
    \[ 5x = 5 \]
    \[ x = 1 \]
    \[ y = 3x - 1 = 3(1) - 1 = 3 - 1 = 2 \]

The solution is \((1, 2)\).

20. Let \(x\) = the length of the longest side
    \(y\) = the length of the middle side
    \(z\) = the length of the shortest side

\[ x + y + z = 7 \Rightarrow x + y + z = 7 \]
\[ x = 2z \Rightarrow x = 2z \]
\[ y + z = x + 1 \Rightarrow x + y + z = 1 \]

Substitute the second equation into the other equations to eliminate \(x\):
\[ x + y + z = 7 \Rightarrow -x + y + z = 1 \]
\[ 2z + y + z = 7 \Rightarrow -2z + y + z = 1 \]
\[ 3z + y = 7 \Rightarrow -2z + y + z = 1 \]
\[ y - z = 1 \]
\[ y = z + 1 \]

Substitute the second new equation into the first new equation to eliminate \(y\):
\[ 3z + y = 7 \]
\[ 3z + z + 1 = 7 \]
\[ 4z = 6 \]
\[ z = \frac{3}{2} = 1 \frac{1}{2} \]

Substitute and solve for \(y\) in the second new equation:
\[ y = z + 1 \]
\[ y = \frac{3}{2} + 1 \]
\[ y = \frac{5}{2} = 2 \frac{1}{2} \]

Substitute and solve for \(x\) in the second original equation:
\[ x = 2z \]
\[ x = 2 \left(1 \frac{1}{2}\right) \]
\[ x = 3 \]

The lengths of the sides are \(1 \frac{1}{2}\) m, \(2 \frac{1}{2}\) m, and 3 m.

21. a. \[ f(x) = 12x + 225 \]
    b. \[ g(x) = 10x + 300 \]
    c. \[ 10x + 300 = 12x + 225 \]
    \[ 2x = 75 \]
    \[ x = 37.5 \]
    \[ 30 \text{ months} \]
22. \[3x = 3 - 2y - 3z \rightarrow 3x + 2y + 3z = 3\]
\[4x - 5y + 7z = 1 \rightarrow 4x - 5y + 7z = 1\]
\[2x + 3y - 2z = 6 \rightarrow 2x + 3y - 2z = 6\]
Multiply the first equation by \(-2\) and the third equation by \(3\), and add the results to eliminate \(x\):
\[3x + 2y + 3z = 3 \rightarrow -6x - 4y - 6z = -6\]
\[2x + 3y - 2z = 6 \rightarrow 6x + 9y - 6z = 18\]
\[\frac{5y - 12z = 12}{-11y + 11z = -11}\]
Multiply the third equation by \(-2\) and add to the second equation to eliminate \(x\):
\[4x - 5y + 7z = 1 \rightarrow 4x - 5y + 7z = 1\]
\[2x + 3y - 2z = 6 \rightarrow -4x - 6y + 4z = -12\]
\[y + z = -1\]
Multiply the second result by \(5\) and add to the first result to eliminate \(y\):
\[5y - 12z = 12\]
\[-y + z = -1 \rightarrow -5y + 5z = -5\]
\[-7z = 7\]
\[z = -1\]
Substitute and solve for \(y\) and \(z\):
\[-y + (1) = -1\]
\[2x + 3(0) - 2(-1) = 6\]
\[-y = 0\]
\[2x + 0 + 2 = 6\]
\[y = 0\]
\[2x = 4\]
\[x = 2\]
The solution is \(\{(2, 0, -1)\}\).

23. \(2 \times 3\)

24. For example:
\[
\begin{bmatrix}
2 & 3 & 1 \\
-1 & 4 & 2 \\
6 & & \\
\end{bmatrix}
\]

25. \(2x - 4y = -2\)
\(4x + y = 5\)
\[
\begin{bmatrix}
2 & -4 & -2 \\
4 & 1 & 5 \\
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -2 & -1 \\
4 & 1 & 5 \\
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -2 & -1 \\
0 & 9 & 9 \\
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -2 & -1 \\
0 & 1 & 1 \\
\end{bmatrix}
\]
The solution is \(\{(1,1)\}\).